On fuzzy soft connected topological spaces

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Abstract. In this work, we introduce notion of connectedness on fuzzy soft topological spaces and present fundamentals properties. We also investigate effect to fuzzy soft connectedness. Moreover, C_i-connectedness which plays an important role in fuzzy topological space extend to fuzzy soft topological spaces.

1. Introduction

There are several set theories dealing with uncertainties such as fuzzy set theory [24], intuitionistic fuzzy set theory, vague set theory, and rough set theory. But they have their own difficulties. For example, constructing membership function is a very important problem in fuzzy set theory. Due to these and similar reasons, soft set theory was defined by Molodtsov [17]. He introduced soft set theory as a new mathematical tool to model uncertainties. He presented the paper [17] about fundamental properties of the soft set theory and application areas such as Riemann-integration, Perron integration, game theory, operations research, probability theory, etc.

Recently, many researchers study on soft set theory. Not only applied mathematicians but also theoretic mathematicians are interested in this topic. For example, in algebra Aktaş [3] defined the concept of soft group. Then, Shabir [20], Aygünoğlu [4], Zorlutuna [26], Roy [19] and Peyghan [18] wrote several articles about soft topological structures and Hussain [14] wrote a paper entitled a note on soft connectedness.

After defining the soft set, fuzzy soft set theory was proposed by Maji [16] and its fundamental properties were investigated by Çağman [8]. Also, topological researches were done on this topic by Tanay [22], Varol [23], Gain [11], Şimşekler [21] and Gündüz [12].

2. Preliminary

In this section, we will give some fundamental definitions and theorems about fuzzy soft sets, fuzzy soft topology and fuzzy soft continuous mappings. Throughout this paper, $\mathcal{P}(X)$, E and I^X denote power set of X, set of parameter and set of all fuzzy sets over X, respectively, where $I = [0, 1]$.

Definition 2.1 [24] A fuzzy set μ on X is a mapping $\mu : X \to I$. The value $\mu(x)$ represents the degree of membership of x in the fuzzy set μ for $x \in X$.

Definition 2.2 [24] Let $\mu, \nu \in I^X$. Then, some basic set operations for fuzzy sets are defined as follows:

   i. (Inclusion) $\mu \subseteq \nu$ iff $\mu(x) \leqslant \nu(x)$ for all $x \in X$
  ii. (Equality) $\mu = \nu$ iff $\mu(x) = \nu(x)$ for all $x \in X$
 iii. (Union) $\eta = \mu \cup \nu$ iff $\eta(x) = \mu(x) \vee \nu(x)$ for all $x \in X$
 iv. (Intersection) $\xi = \mu \cap \nu$ iff $\xi(x) = \mu(x) \wedge \nu(x)$ for all $x \in X$
  v. (Complement) $\lambda = \mu^c$ iff $\lambda(x) = 1 - \mu(x)$ for all $x \in X$.
 vi. (Empty set) If $\mu(x) = 0$ for all $x \in X$, then μ is called empty fuzzy set and denoted by $\tilde{\emptyset}$
vii. (Universal set) If $\mu(x) = 1$ for all $x \in X$, then μ is called universal fuzzy set and denoted by $\tilde{X}$.

Definition 2.3 [17] A pair (F, A) is called a soft set over X, if F is a mapping defined by $F : A \to \mathcal{P}(X)$, where $A \subseteq E$.

Following definition was given by Çağman [9] and it is a new comment about soft sets.

Definition 2.4 [9] A soft set F over X is a set valued function from E to $\mathcal{P}(X)$. It can be written a set of ordered pairs

$$F = \big\{(e, F(e)) : e \in E\big\}.$$

Note that if $F(e) = \emptyset$, then the element $(e, F(e))$ is not appeared in F. Set of all soft sets over X is denoted by $\mathbb{S}$.

According to above definition we will redefine fuzzy soft set and its set operations.

Definition 2.5 A fuzzy soft set $\tilde{f}$ over X is a set valued function from E to I^X. It can be written a set of ordered pairs

$$\tilde{f} = \Big\{\big(e, \{(x, \mu_{\tilde{f}(e)}(x)) : x \in X\}\big) : e \in E\Big\}$$

where $\mu_{\tilde{f}(e)} \in I^X$ for all $e \in E$. Note that if $\tilde{f}(e) = \tilde{\emptyset}$, then the element $(e, \tilde{f}(e))$ is not appeared in $\tilde{f}$. Set of all fuzzy soft sets over X is denoted by $\mathbb{FS}_X^E$.

Definition 2.6 Let $\tilde{f}, \tilde{g} \in \mathbb{FS}_X^E$. Then some basic set operations of fuzzy soft sets are defined as follows:

- *i.* (Inclusion) $\tilde{f} \sqsubseteq \tilde{g}$ iff $\tilde{f}(e) \subseteq \tilde{g}(e)$ for all $e \in E$.
- *ii.* (Equality) $\tilde{f} = \tilde{g}$ iff $\tilde{f} \sqsubseteq \tilde{g}$ and $\tilde{g} \sqsubseteq \tilde{f}$
- *iii.* (Union) $\tilde{h} = \tilde{f} \sqcup \tilde{g}$ iff $\tilde{h}(e) = \tilde{f}(e) \cup \tilde{g}(e)$ for all $e \in E$.
- *iv.* (Intersection) $\tilde{h} = \tilde{f} \sqcap \tilde{g}$ iff $\tilde{h}(e) = \tilde{f}(e) \cap \tilde{g}(e)$ for all $e \in E$.
- *v.* (Complement) $\tilde{h} = \tilde{f}^c$ iff $\tilde{h}(e) = \left(\tilde{f}(e)\right)^c$ for all $e \in E$
- *vi.* (Null Fuzzy Soft Set) $\tilde{f}$ is called the null fuzzy soft set and denoted by $\tilde{0}_X$, if $\tilde{f}(e) = \tilde{\emptyset}$ for all $e \in E$.
- *vii.* (Universal Fuzzy Soft Set) $\tilde{f}$ is called the universal fuzzy soft set and denoted by $\tilde{1}_X$, if $\tilde{f}(e) = \tilde{X}$ for all $e \in E$.

Theorem 2.7 [23] Let $\{\tilde{f}_i\}_{i \in \Lambda} \subseteq \mathbb{FS}_X^E$ and $\tilde{g} \in \mathbb{FS}_X^E$. Then

- *i.* $\tilde{g} \sqcap \left(\bigsqcup_{i \in \Lambda} \tilde{f}_i\right) = \bigsqcup_{i \in \Lambda}(\tilde{g} \sqcap \tilde{f}_i)$
- *ii.* $\tilde{g} \sqcup \left(\bigsqcap_{i \in \Lambda} \tilde{f}_i\right) = \bigsqcap_{i \in \Lambda}(\tilde{g} \sqcup \tilde{f}_i)$
- *iii.* $\left(\bigsqcup_{i \in \Lambda} \tilde{f}_i\right)^c = \bigsqcap_{i \in \Lambda} \tilde{f}_i^c$
- *iv.* $\left(\bigsqcap_{i \in \Lambda} \tilde{f}_i\right)^c = \bigsqcup_{i \in \Lambda} \tilde{f}_i^c$
- *v.* $\tilde{0}_X \sqsubseteq \tilde{f} \sqsubseteq \tilde{1}_X$, $\tilde{1}_X^c = \tilde{0}_X$ and $\tilde{0}_X^c = \tilde{1}_X$,
- *vi.* $\tilde{g} \sqcup \tilde{g}^c = \tilde{1}_X$ and $\left(\tilde{g}^c\right)^c = \tilde{g}$.

Definition 2.8 [15] Let $\mathbb{FS}_X^E$ and $\mathbb{FS}_Y^K$ be the families of all fuzzy soft sets over X and Y, respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two functions. Then the φ_ψ is called a fuzzy soft mapping from X to Y, and denoted by $\varphi_\psi : \mathbb{FS}_X^E \to \mathbb{FS}_Y^K$.

- *i.* Let $\tilde{f} \in \mathbb{FS}_X^E$. Then the image of $\tilde{f}$ under the fuzzy soft mapping φ_ψ is the fuzzy soft set over Y defined by $\varphi_\psi(\tilde{f})$, where

$$\varphi_\psi(\tilde{f})(k)(y) = \begin{cases} \bigvee_{x \in \varphi^{-1}(y)} \left(\bigvee_{e \in \psi^{-1}(k)} f(e)\right)(x), & \text{if } \varphi^{-1}(y) \neq \emptyset, \psi^{-1}(k) \neq \emptyset \\ 0_Y, & \text{otherwise.} \end{cases}$$

- *ii.* Let $\tilde{g} \in \mathbb{FS}_Y^K$. Then the pre-image of $\tilde{g}$ under the fuzzy soft mapping φ_ψ is a fuzzy soft set over X defined by $\varphi_\psi^{-1}(\tilde{g})$, where

$$\varphi_\psi^{-1}(\tilde{g})(e)(x) = g(\psi(e))(\varphi(x)).$$

If φ and ψ are injective (surjective) then the fuzzy soft mapping φ_ψ is said to be injective (surjective).

Theorem 2.9 [15] Let $\varphi_\psi : \mathbb{FS}_X^E \to \mathbb{FS}_Y^K$ be a fuzzy soft mapping, $\tilde{f} \in \mathbb{FS}_X^E$ and $\{\tilde{f}_i : i \in I\} \subseteq \mathbb{FS}_X^E$. Then

- *i.* If $\tilde{f}_1 \sqsubseteq \tilde{f}_2$, then $\varphi_\psi(\tilde{f}_1) \sqsubseteq \varphi_\psi(\tilde{f}_2)$
- *ii.* $\varphi_\psi\left(\bigsqcup_{i \in \Lambda} \tilde{f}_i\right) = \bigsqcup_{i \in \Lambda} \varphi_\psi(\tilde{f}_i)$
- *iii.* $\varphi_\psi\left(\bigsqcap_{i \in \Lambda} \tilde{f}_i\right) \sqsubseteq \bigsqcap_{i \in \Lambda} \varphi_\psi(\tilde{f}_i)$
- *iv.* $\left(\varphi_\psi(\tilde{f})\right)^c \sqsubseteq \varphi_\psi\left(\tilde{f}^c\right)$
- *v.* If φ_ψ surjective, then $\varphi_\psi(\tilde{1}_X) = \tilde{1}_Y$
- *vi.* $\varphi_\psi(\tilde{0}_X) = \tilde{0}_Y$
- *vii.* $\tilde{f} \sqsubseteq \varphi_\psi^{-1}\left(\varphi_\psi(\tilde{f})\right)$, the equality holds if φ_ψ is injective.

Theorem 2.10 [15] Let $\varphi_\psi : \mathbb{FS}_X^E \to \mathbb{FS}_Y^K$ be a fuzzy soft mapping, $\tilde{g} \in \mathbb{FS}_Y^K$ and $\{\tilde{g}_j : j \in J\} \subseteq \mathbb{FS}_Y^K$. Then

   i. If $\tilde{g}_1 \sqsubseteq \tilde{g}_2$, then $\varphi_\psi^{-1}(\tilde{g}_1) \sqsubseteq \varphi_\psi^{-1}(\tilde{g}_2)$

  ii. $\varphi_\psi^{-1}\left(\bigsqcup_{j \in J} \tilde{g}_j\right) = \bigsqcup_{j \in J} \varphi_\psi^{-1}(\tilde{g}_j)$

 iii. $\varphi_\psi^{-1}\left(\bigsqcap_{j \in J} \tilde{g}_j\right) = \bigsqcap_{j \in J} \varphi_\psi^{-1}(\tilde{g}_j)$

 iv. $\left(\varphi_\psi^{-1}(\tilde{g})\right)^c = \varphi_\psi^{-1}(\tilde{g}^c)$

  v. $\varphi_\psi^{-1}(\tilde{1}_Y) = \tilde{1}_X$ and $\varphi_\psi^{-1}(\tilde{0}_Y) = \tilde{0}_X$

 vi. $\varphi_\psi\left(\varphi_\psi^{-1}(\tilde{g})\right) \sqsubseteq \tilde{g}$, the equality holds if φ_ψ is surjective.

Definition 2.11 [22] A fuzzy soft topological space is a triplet (X, τ, E) where X is a nonempty set and τ a family of fuzzy soft sets over X satisfying the following properties:

   i. $\tilde{0}_X, \tilde{1}_X \in \tau$,

  ii. If $\tilde{f}, \tilde{g} \in \tau$, then $\tilde{f} \sqcap \tilde{g} \in \tau$,

 iii. If $\{\tilde{f}_i : i \in \Lambda\} \subseteq \tau$, then $\bigsqcup_{i \in \Lambda} \tilde{f}_i \in \tau$.

Then the family τ is called a fuzzy soft topology on X. Every member of τ is called fuzzy soft open. $\tilde{g}$ is called fuzzy soft closed in (X, τ, E) if $\tilde{g}^c \in \tau$.

Definition 2.12 Let (X, τ, E) be a fuzzy soft topological space and $\tilde{f} \in \mathbb{FS}_X^E$. If $\tilde{f}$ is fuzzy soft open and fuzzy soft closed, then it is called fuzzy soft clopen set. In case $\tilde{f} \neq \tilde{1}_X$ and $\tilde{f} \neq \tilde{0}_X$, $\tilde{f}$ is called fuzzy soft proper set.

***Example* 2.13** $\tau = \{\tilde{1}_X, \tilde{0}_X\}$ and $\sigma = \mathbb{FS}_X^E$ are fuzzy soft topologies on X.

Definition 2.14 [22] Let (X, τ, E) be a fuzzy soft topological space and $\tilde{f} \in \mathbb{FS}_X^E$. Fuzzy soft interior of $\tilde{f}$ denoted by $int(\tilde{f})$ is the union of all fuzzy soft open subsets of $\tilde{f}$.

Definition 2.15 [22] Let (X, τ, E) be a fuzzy soft topological space and $\tilde{f} \in \mathbb{FS}_X^E$. Fuzzy soft closure of $\tilde{f}$ denoted by $cl(\tilde{f})$ is the intersection of all fuzzy soft closed supersets of $\tilde{f}$.

It can be seen that $int(\tilde{f})$ and $cl(\tilde{f})$ are the largest fuzzy soft open set which contained in $\tilde{f}$ and the smallest fuzzy soft closed set which contains $\tilde{f}$ over X, respectively.

Theorem 2.16 [23] Let (X, τ, E) be a fuzzy soft topological space and $\tilde{f}, \tilde{g} \in \mathbb{FS}_X^E$. Then,

   i. If $\tilde{f} \sqsubseteq \tilde{g}$, then $int(\tilde{f}) \sqsubseteq int(\tilde{g})$ and $cl(\tilde{f}) \sqsubseteq cl(\tilde{g})$

  ii. $\tilde{f}$ is a soft open set iff $int(\tilde{f}) = \tilde{f}$

 iii. $\tilde{f}$ is a soft closed set iff $cl(\tilde{f}) = \tilde{f}$

 iv. $\left(cl(\tilde{f})\right)^c = int(\tilde{f}^c)$ and $cl(\tilde{f}^c) = \left(int(\tilde{f})\right)^c$

Definition 2.17 [23] Let (X, τ, E) and (Y, σ, K) be two fuzzy soft topological spaces. A fuzzy soft mapping $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ is called fuzzy soft continuous mapping if $\varphi_\psi^{-1}(\tilde{g}) \in \tau$ for all $\tilde{g} \in \sigma$.

***Example* 2.18** Let consider the fuzzy soft topology τ in Example 2.13 and σ be a fuzzy soft topology on Y. If $\varphi_\psi : (X, \sigma, E) \to (Y, \sigma, K)$ is a fuzzy soft mapping, then the fuzzy soft mapping φ_ψ is a fuzzy soft continuous mapping.

3. Fuzzy soft connectedness

In this section, we give definition of fuzzy soft connected spaces and their fundamental properties. Moreover, we will introduce SC_i-connectedness ($i = 1, 2, 3, 4$).

Definition 3.1 Let (X, τ, E) be a fuzzy soft topological space and $\tilde{f} \in \mathbb{FS}_X^E$. If there are two fuzzy soft proper open sets $\tilde{g}_1$ and $\tilde{g}_2$ such that $\tilde{f} \sqsubseteq \tilde{g}_1 \sqcup \tilde{g}_2$ and $\tilde{g}_1 \sqcap \tilde{g}_2 = \tilde{0}_X$, then the fuzzy soft set $\tilde{f}$ is called fuzzy soft disconnected set. If there does not exist such two fuzzy soft proper open sets, then the fuzzy soft set $\tilde{f}$ is called fuzzy soft connected set.

In above definition, if we take $\tilde{1}_X$ instead of $\tilde{f}$, then the (X, τ, E) is called fuzzy soft disconnected (connected) space.

***Example* 3.2** If we consider the fuzzy soft topological spaces (X, τ, E) and (X, σ, E) in Example 2.13, (X, τ, E) is a fuzzy soft connected topological space, but (X, σ, E) is a fuzzy soft disconnected topological space.

Theorem 3.3 Let (X, τ, E) be a fuzzy soft topological space. (X, τ, E) fuzzy soft connected if and only if there does not exist a fuzzy soft proper clopen set $\tilde{f}$ in (X, τ, E).

Proof. $(\Rightarrow)$: Let (X, τ, E) be a fuzzy soft connected space. Suppose that there exist a fuzzy soft proper clopen set $\tilde{f}$ in (X, τ, E) such that $\tilde{f} \sqcup \tilde{f}^c = \tilde{1}_X$ and $\tilde{f} \sqcap \tilde{f}^c = \tilde{0}_X$. It is a contradiction.
    $(\Leftarrow)$: It is clear. ∎

Theorem 3.4 Let (X, τ, E) be a fuzzy soft connected topological space and $\sigma \subseteq \tau$. Then, (X, σ, E) is a fuzzy soft connected topological space.

Proof. It is clear. ∎

Theorem 3.5 Let (X, τ, E) and (Y, σ, K) be two fuzzy soft topological spaces, $\tilde{f} \in \mathbb{FS}_X^E$ and $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ be a fuzzy soft continuous function. If $\tilde{f}$ is a fuzzy soft connected set, then $\varphi_\psi(\tilde{f})$ is a fuzzy soft connected set.

Proof. Suppose that $\varphi_\psi(\tilde{f})$ is a fuzzy soft disconnected set. Then, there exist two fuzzy soft proper open sets $\tilde{g}$ and $\tilde{h}$ such that $\varphi_\psi(\tilde{f}) \sqsubseteq \tilde{g} \sqcup \tilde{h}$ and $\tilde{g} \sqcap \tilde{h} = \tilde{0}_Y$. Therefore, from Theorem 2.10

$$\tilde{f} \sqsubseteq \varphi^{-1}\big(\varphi_\psi(\tilde{f})\big) \sqsubseteq \varphi_\psi^{-1}(\tilde{g} \sqcup \tilde{h}) = \varphi_\psi^{-1}(\tilde{g}) \sqcup \varphi_\psi^{-1}(\tilde{h})$$

and

$$\varphi_\psi^{-1}(\tilde{g} \sqcap \tilde{h}) = \varphi_\psi^{-1}(\tilde{g}) \sqcap \varphi_\psi^{-1}(\tilde{h}) = \varphi_\psi^{-1}(\tilde{0}_Y) = \tilde{0}_X.$$

So, we have a contradiction and this complete the proof. ∎

Theorem 3.6 Let (X, τ, E) and (Y, σ, K) be two fuzzy soft topological spaces and $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ be a fuzzy soft continuous and surjective function. If (X, τ, E) is a fuzzy soft connected space, then (Y, σ, K) is also a fuzzy soft connected space.

Proof. Suppose that (Y, σ, K) is a fuzzy soft disconnected space. Therefore, there exist two fuzzy soft proper open sets $\tilde{g}_1$ and $\tilde{g}_2$ such that $\tilde{g}_1 \sqcup \tilde{g}_2 = \tilde{1}_Y$, $\tilde{g}_1 \sqcap \tilde{g}_2 = \tilde{0}_Y$. By Theorem 2.10 $\varphi_\psi^{-1}(\tilde{g}_1) \sqcup \varphi_\psi^{-1}(\tilde{g}_2) = \tilde{1}_X$ and $\varphi_\psi^{-1}(\tilde{g}_1) \sqcap \varphi_\psi^{-1}(\tilde{g}_2) = \tilde{0}_X$. So, we have a contradiction and this complete the proof. ∎

Definition 3.7 Let (X, τ, E) be a fuzzy soft topological space. If there exist $\tilde{f}, \tilde{g} \in \mathbb{FS}_X^E$ which are proper, such that $cl(\tilde{f}) \sqcap \tilde{g} = \tilde{0}_X$ and $\tilde{f} \sqcap cl(\tilde{g}) = \tilde{0}_X$ then the fuzzy soft sets $\tilde{f}$ and $\tilde{g}$ are called fuzzy soft separated sets.

Theorem 3.8 Let (X, τ, E) be a fuzzy soft topological space, $\tilde{f}$ and $\tilde{g}$ be two fuzzy soft open sets. If $\tilde{f} \sqcap \tilde{g} = \tilde{0}_X$, then $\tilde{f}$ and $\tilde{g}$ are fuzzy soft separated sets.

Proof. Let $\tilde{f}, \tilde{g} \in \tau$ and $\tilde{f} \sqcap \tilde{g} = \tilde{0}_X$. Then, $\tilde{f}^c \sqcup \tilde{g}^c = \tilde{1}_X$. Therefore, $\tilde{f} \sqsubseteq \tilde{g}^c$ and $\tilde{g} \sqsubseteq \tilde{f}^c$, $\tilde{f}^c$ and $\tilde{g}^c$ are fuzzy soft closed sets. From Theorem 2.16, we have

$$cl(\tilde{f}) \sqsubseteq cl(\tilde{g}^c) = \tilde{g}^c \text{ and } cl(\tilde{g}) \sqsubseteq cl(\tilde{f}^c) = \tilde{f}^c.$$

So, $cl(\tilde{f}) \sqcap \tilde{g} = \tilde{0}_X$ and $\tilde{f} \sqcap cl(\tilde{g}) = \tilde{0}_X$. ∎

Theorem 3.9 Let (X, τ, E) be a fuzzy soft topological space, $\tilde{f}$ and $\tilde{g}$ be two fuzzy soft closed sets. If $\tilde{f} \sqcap \tilde{g} = \tilde{0}_X$, then $\tilde{f}$ and $\tilde{g}$ are fuzzy soft separated sets.

Proof. Let $\tilde{f}^c, \tilde{g}^c \in \tau$ and $\tilde{f} \sqcap \tilde{g} = \tilde{0}_X$. By Theorem 2.16, $cl(\tilde{f}) = \tilde{f}$ and $cl(\tilde{g}) = \tilde{g}$. Therefore we have

$$\tilde{f} \sqcap \tilde{g} = cl(\tilde{f}) \sqcap \tilde{g} = \tilde{0}_X$$

and

$$\tilde{f} \sqcap \tilde{g} = \tilde{f} \sqcap cl(\tilde{g}) = \tilde{0}_X$$

So, $\tilde{f}$ and $\tilde{g}$ are fuzzy soft separated sets. ∎

Theorem 3.10 Let (X, τ, E) be a fuzzy soft topological space and $\tilde{f} \in \mathbb{FS}_X^E$. $\tilde{f}$ is fuzzy soft connected if and only if it cannot be written as union of fuzzy soft separated sets.

Proof. ($\Rightarrow$:) If $\tilde{f} = \tilde{0}_X$, then it is fuzzy soft connected set. Thus, let $\tilde{f} \neq \tilde{0}_X$. Assume that $\tilde{f}$ can be written as union of fuzzy soft separated sets $\tilde{g}$ and $\tilde{h}$. But this situation contradicts by hypothesis.

  ($\Leftarrow$:) It is clear. ∎

Theorem 3.11 A fuzzy soft topological space (X, τ, E) is connected if and only if $\tilde{1}_X$ cannot be written as union of fuzzy soft separated sets.

Proof. ($\Rightarrow$) : Suppose that $\tilde{1}_X$ can be written as union of fuzzy soft separated sets $\tilde{f}$ and $\tilde{g}$. Then, $\tilde{1}_X = \tilde{f} \sqcup \tilde{g}$, $cl(\tilde{f}) \sqcap \tilde{g} = \tilde{0}_X$ and $\tilde{f} \sqcap cl(\tilde{g}) = \tilde{0}_X$. Therefore, we have $\tilde{f} \sqcap \tilde{g} = \tilde{0}_X$, $\tilde{f} = \tilde{g}^c$ and $\tilde{g} = \tilde{f}^c$. Thus,

$$\begin{aligned} cl(\tilde{f}) &= cl(\tilde{f}) \sqcap \tilde{1}_X \\ &= cl(\tilde{f}) \sqcap (\tilde{f} \sqcup \tilde{g}) \\ &= (cl(\tilde{f}) \sqcap \tilde{f}) \sqcup (cl(\tilde{f}) \sqcap \tilde{g}) \\ &= \tilde{f}. \end{aligned}$$

So, $\tilde{f}$ is a fuzzy soft closed set. Similarly, $\tilde{g}$ is also a fuzzy soft closed set. From $\tilde{f} = \tilde{g}^c$ and $\tilde{g} = \tilde{f}^c$, $\tilde{f}$ and $\tilde{g}$ are fuzzy soft open sets. This is a contradiction.

  ($\Leftarrow$) : Suppose that (X, τ, E) is not a fuzzy soft connected space. Then, there exist a fuzzy soft proper clopen set $\tilde{f}$. But this contradict by hypothesis. ∎

Theorem 3.12 Let (X, τ, E) be a fuzzy soft topological space and $\tilde{f} \in \mathbb{FS}_X^E$ be a fuzzy soft open connected set. If $\tilde{f} \sqsubseteq \tilde{g} \sqsubseteq cl(\tilde{f})$, then $\tilde{g}$ is a fuzzy soft connected set.

Proof. Suppose that $\tilde{g}$ is a fuzzy soft disconnected set. Then, there exist two fuzzy soft open proper sets $\tilde{h}_1$ and $\tilde{h}_2$ such that

$$\tilde{h}_1 \sqcap \tilde{h}_2 = \tilde{0}_X \ \text{ and } \ \tilde{g} \sqsubseteq \tilde{h}_1 \sqcup \tilde{h}_2.$$

Therefore

$$\tilde{f} = \left[\tilde{f} \sqcap \tilde{h}_1 \right] \sqcup \left[\tilde{f} \sqcap \tilde{h}_2 \right]$$

and

$$\left[\tilde{f} \sqcap \tilde{h}_1 \right] \sqcap \left[\tilde{f} \sqcap \tilde{h}_2 \right] = \tilde{0}_X.$$

But it is a contradiction. Thus $\tilde{g}$ is a fuzzy soft connected set. ∎

Remark 1 *Let (X, τ, E) be a fuzzy soft topological space and $\tilde{f} \in \mathbb{FS}_X^E$ be a fuzzy soft open set. If $\tilde{f}$ is a fuzzy soft connected set, then $cl(\tilde{f})$ is a fuzzy soft connected set.*

In following definition, we will introduce fuzzy soft C_i−connected (or namely SC_i) spaces $(i = 1, 2, 3, 4)$ by helping of fuzzy C_i−connectedness [2]. Definition of SC_i−connected spaces can be seen as extension of fuzzy soft connected space.

Definition 3.13 Let (X, τ, E) be a fuzzy soft topological space and $\tilde{f} \in \mathbb{FS}_X^E$. $\tilde{f}$ is called

   i. SC_1−connected iff does not exist two non null fuzzy soft open sets $\tilde{g}$ and $\tilde{h}$ such that $\tilde{f} \sqsubseteq \tilde{g} \sqcup \tilde{h}$, $\tilde{g} \sqcap \tilde{h} \sqsubseteq \tilde{f}^c$, $\tilde{f} \sqcap \tilde{g} \neq \tilde{0}_X$ and $\tilde{f} \sqcap \tilde{h} \neq \tilde{0}_X$.

   ii. SC_2−connected iff does not exist two non null fuzzy soft open sets $\tilde{g}$ and $\tilde{h}$ such that $\tilde{f} \sqsubseteq \tilde{g} \sqcup \tilde{h}$, $\tilde{f} \sqcap \tilde{g} \sqcap \tilde{h} = \tilde{0}_X$ $\tilde{f} \sqcap \tilde{g} \neq \tilde{0}_X$ and $\tilde{f} \sqcap \tilde{h} \neq \tilde{0}_X$.

   iii. SC_3−connected iff does not exist two non null fuzzy soft open sets $\tilde{g}$ and $\tilde{h}$ such that $\tilde{f} \sqsubseteq \tilde{g} \sqcup \tilde{h}$, $\tilde{g} \sqcap \tilde{h} \sqsubseteq \tilde{f}^c$, $\tilde{g} \not\sqsubseteq \tilde{f}^c$ and $\tilde{h} \not\sqsubseteq \tilde{f}^c$.

   iv. SC_4−connected iff does not exist two non null fuzzy soft open sets $\tilde{g}$ and $\tilde{h}$ such that $\tilde{f} \sqsubseteq \tilde{g} \sqcup \tilde{h}$, $\tilde{f} \sqcap \tilde{g} \sqcap \tilde{h} = \tilde{0}_X$, $\tilde{g} \not\sqsubseteq \tilde{f}^c$ and $\tilde{h} \not\sqsubseteq \tilde{f}^c$.

From Definition 3.13, relations between SC_i−connectedness $(i = 1, 2, 3, 4)$ can be described by the following diagram:

$$
\begin{array}{ccc}
SC_1 & \longrightarrow & SC_2 \\
\downarrow & & \downarrow \\
SC_3 & \longrightarrow & SC_4
\end{array}
$$
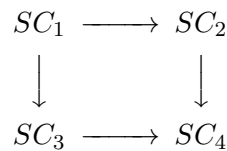
In the following examples, we illustrate all reverse implications.

Example 3.14 Let $X = [0, 1]$ and $E = \{a, b\}$. Moreover, define soft sets $\tilde{f}$, $\tilde{g}$ and $\tilde{h}$ as

following:

$$\tilde{f} = \Big\{ \big(a, \{(x, \mu_{f(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{f(b)}(x)) : x \in X\}\big) \Big\}$$

$$\tilde{g} = \Big\{ \big(a, \{(x, \mu_{g(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{g(b)}(x)) : x \in X\}\big) \Big\}$$

$$\tilde{h} = \Big\{ \big(a, \{(x, \mu_{h(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{h(b)}(x)) : x \in X\}\big) \Big\},$$

where

$$\mu_{g(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \leqslant 1 \\ 1, & \text{if } 0 \leqslant x \leqslant \frac{1}{3} \end{cases} \quad \text{and} \quad \mu_{g(b)}(x) = \begin{cases} 1, & \text{if } \frac{1}{3} < x \leqslant 1 \\ \frac{1}{3}, & \text{if } 0 \leqslant x \leqslant \frac{1}{3}, \end{cases}$$

$$\mu_{h(a)}(x) = \begin{cases} 1, & \text{if } \frac{1}{3} < x \leqslant 1 \\ \frac{1}{3}, & \text{if } 0 \leqslant x \leqslant \frac{1}{3} \end{cases} \quad \text{and} \quad \mu_{h(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \leqslant 1 \\ 1, & \text{if } 0 \leqslant x \leqslant \frac{1}{3} \end{cases}$$

$\mu_{f(a)}(x) = \mu_{f(b)}(x) = \dfrac{3}{4}$ for all $x \in [0,1]$. $\tau = \{\tilde{0}_X, \tilde{1}_X, \tilde{g}, \tilde{h}, \tilde{g} \sqcap \tilde{h}\}$ is a fuzzy soft topology on X. It can be see clearly that $\tilde{f}$ is SC_4−connected but SC_3−disconnected.

Example 3.15 Let $X = [0,1]$ and $E = \{a,b\}$. Moreover, define soft sets $\tilde{g}$, $\tilde{h}$ and $\tilde{f}$ as following:

$$\tilde{g} = \Big\{ \big(a, \{(x, \mu_{g(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{g(b)}(x)) : x \in X\}\big) \Big\}$$

$$\tilde{h} = \Big\{ \big(a, \{(x, \mu_{h(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{h(b)}(x)) : x \in X\}\big) \Big\},$$

$$\tilde{f} = \tilde{g} \sqcup \tilde{h}$$

where

$$\mu_{g(a)}(x) = \begin{cases} 0, & \text{if } \frac{1}{3} < x \leqslant 1 \\ \frac{1}{3}, & \text{if } 0 \leqslant x \leqslant \frac{1}{3} \end{cases} \quad \text{and} \quad \mu_{g(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \leqslant 1 \\ 0, & \text{if } 0 \leqslant x \leqslant \frac{1}{3}, \end{cases}$$

$$\mu_{h(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \leqslant 1 \\ 0, & \text{if } 0 \leqslant x \leqslant \frac{1}{3}, \end{cases} \quad \text{and} \quad \mu_{h(b)}(x) = \begin{cases} 0, & \text{if } \frac{1}{3} < x \leqslant 1 \\ \frac{1}{3}, & \text{if } 0 \leqslant x \leqslant \frac{1}{3} \end{cases}$$

$\tau = \{\tilde{0}_X, \tilde{1}_X, \tilde{g}, \tilde{h}, \tilde{g} \sqcup \tilde{h}\}$ is a fuzzy soft topology on X. It can be seen clearly that $\tilde{f}$ is SC_4−connected but SC_2−disconnected.

Example 3.16 Let $X = [0,1]$ and $E = \{a,b\}$. Moreover, define soft sets $\tilde{f}$, $\tilde{g}$ and $\tilde{h}$ as

following:

$$\tilde{f} = \Big\{ \big(a, \{(x, \mu_{f(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{f(b)}(x)) : x \in X\}\big) \Big\}$$

$$\tilde{g} = \Big\{ \big(a, \{(x, \mu_{g(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{g(b)}(x)) : x \in X\}\big) \Big\}$$

$$\tilde{h} = \Big\{ \big(a, \{(x, \mu_{h(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{h(b)}(x)) : x \in X\}\big) \Big\},$$

where

$$\mu_{g(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \leqslant 1 \\ \frac{2}{3}, & \text{if } 0 \leqslant x \leqslant \frac{1}{3} \end{cases} \quad \text{and} \quad \mu_{g(b)}(x) = \begin{cases} \frac{2}{3}, & \text{if } \frac{1}{3} < x \leqslant 1 \\ \frac{1}{3}, & \text{if } 0 \leqslant x \leqslant \frac{1}{3}, \end{cases}$$

$$\mu_{h(a)}(x) = \begin{cases} \frac{2}{3}, & \text{if } \frac{1}{3} < x \leqslant 1 \\ \frac{1}{3}, & \text{if } 0 \leqslant x \leqslant \frac{1}{3}, \end{cases} \quad \text{and} \quad \mu_{h(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \leqslant 1 \\ \frac{2}{3}, & \text{if } 0 \leqslant x \leqslant \frac{1}{3} \end{cases}$$

$\mu_{f(a)}(x) = \mu_{f(b)}(x) = \frac{1}{3}$ for all $x \in [0,1]$. $\tau = \{\tilde{0}_X, \tilde{1}_X, \tilde{g}, \tilde{h}, \tilde{g} \sqcap \tilde{h}, \tilde{g} \sqcup \tilde{h}\}$ is a fuzzy soft topology on X. It can be seen clearly that $\tilde{f}$ is SC_3-connected and SC_2-connected but SC_1-disconnected.

Example 3.17 Let $X = [0,1]$ and $E = \{a, b\}$. Moreover, define soft sets $\tilde{f}$, $\tilde{g}$ and $\tilde{h}$ as following:

$$\tilde{f} = \Big\{ \big(a, \{(x, \mu_{f(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{f(b)}(x)) : x \in X\}\big) \Big\}$$

$$\tilde{g} = \Big\{ \big(a, \{(x, \mu_{g(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{g(b)}(x)) : x \in X\}\big) \Big\}$$

$$\tilde{h} = \Big\{ \big(a, \{(x, \mu_{h(a)}(x)) : x \in X\}\big), \big(b, \{(x, \mu_{h(b)}(x)) : x \in X\}\big) \Big\},$$

where

$$\mu_{g(a)}(x) = \begin{cases} 0, & \text{if } \frac{2}{3} < x \leqslant 1 \\ \frac{2}{3}, & \text{if } 0 \leqslant x \leqslant \frac{2}{3} \end{cases} \quad \text{and} \quad \mu_{g(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{2}{3} < x \leqslant 1 \\ 0, & \text{if } 0 \leqslant x \leqslant \frac{2}{3}, \end{cases}$$

$$\mu_{h(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{2}{3} < x \leqslant 1 \\ 0, & \text{if } 0 \leqslant x \leqslant \frac{2}{3}, \end{cases} \quad \text{and} \quad \mu_{h(b)}(x) = \begin{cases} 0, & \text{if } \frac{2}{3} < x \leqslant 1 \\ \frac{2}{3}, & \text{if } 0 \leqslant x \leqslant \frac{2}{3} \end{cases}$$

$$\mu_{f(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{2}{3} < x \leqslant 1 \\ \frac{2}{3}, & \text{if } 0 \leqslant x \leqslant \frac{2}{3} \end{cases} \quad \text{and} \quad \mu_{f(b)}(x) = \begin{cases} \frac{2}{3}, & \text{if } \frac{2}{3} < x \leqslant 1 \\ \frac{1}{3}, & \text{if } 0 \leqslant x \leqslant \frac{2}{3} \end{cases}$$

$\tau = \{\tilde{0}_X, \tilde{1}_X, \tilde{g}, \tilde{h}, \tilde{g} \sqcup \tilde{h}\}$ is a fuzzy soft topology on X. It can be seen clearly that $\tilde{f}$ is SC_3-connected but SC_2-disconnected and SC_1-disconnected.

Example 3.18 In the Example 3.16, if we take $\mu_{f(a)}(x) = \mu_{f(b)}(x) = \frac{2}{3}$ for all $x \in [0,1]$, then $\tilde{f}$ is SC_2-connected but SC_3-disconnected.

Theorem 3.19 Let $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ be a fuzzy soft surjective continuous mapping and $\tilde{f} \in \mathbb{FS}_X^E$. If $\tilde{f}$ is a SC_1-connected, then $\varphi_\psi(\tilde{f})$ is SC_1-connected.

Proof. Suppose that $\varphi_\psi(\tilde{f})$ is SC_1-connected. Then, there exist two non null fuzzy soft open sets $\tilde{g}$ and $\tilde{h}$ in (Y, σ, K) such that

$$\varphi_\psi(\tilde{f}) \sqsubseteq \tilde{g} \sqcup \tilde{h},$$
$$\tilde{g} \sqcap \tilde{h} \sqsubseteq \big(\varphi_\psi(\tilde{f})\big)^c,$$
$$\varphi_\psi(\tilde{f}) \sqcap \tilde{g} \neq \tilde{0}_Y,$$
$$\varphi_\psi(\tilde{f}) \sqcap \tilde{h} \neq \tilde{0}_Y.$$

Thus, by Theorem 2.10 we have

$$\varphi_\psi(\tilde{f}) \sqsubseteq \tilde{g} \sqcup \tilde{h},$$
$$\tilde{g} \sqcap \tilde{h} \sqsubseteq \big(\varphi_\psi(\tilde{f})\big)^c,$$
$$\varphi_\psi(\tilde{f}) \sqcap \tilde{g} \neq \tilde{0}_X,$$
$$\varphi_\psi(\tilde{f}) \sqcap \tilde{h} \neq \tilde{0}_X.$$

But this contradict by hypothesis. So, $\varphi_\psi(\tilde{f})$ is SC_1-connected. ∎

Theorem 3.20 Let $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ be a fuzzy soft surjective continuous mapping and $\tilde{f} \in \mathbb{FS}_X^E$. If $\tilde{f}$ is a SC_2-connected, then $\varphi_\psi(\tilde{f})$ is SC_2-connected.

Proof. it can be proved similar way to Theorem 3.19. ∎

Theorem 3.21 Let $\varphi_\psi : (X, \tau) \to (Y, \sigma)$ be fuzzy soft continuous surjective mapping and $\tilde{f} \in \mathbb{FS}_X^E$. If $\tilde{f}$ is a SC_3-connected, then $\varphi_\psi(\tilde{f})$ is a SC_3-connected.

Proof. Assume that, $\varphi_\psi(\tilde{f})$ is not SC_3-connected. Then, there exist two non null fuzzy soft open sets $\tilde{g}$ and $\tilde{h}$ in (Y, σ, K) such that

$$\varphi_\psi(\tilde{f}) \sqsubseteq \tilde{g} \sqcup \tilde{h},$$
$$\tilde{g} \sqcap \tilde{h} \sqsubseteq \big(\varphi_\psi(\tilde{f})\big)^c,$$
$$\tilde{g} \not\sqsubseteq \big(\varphi_\psi(\tilde{f})\big)^c,$$
$$\tilde{h} \not\sqsubseteq \big(\varphi_\psi(\tilde{f})\big)^c.$$

By Theorem 2.10,

$$\tilde{f} \sqsubseteq \varphi_\psi^{-1}\big(\varphi_\psi(\tilde{f})\big) \sqsubseteq \varphi_\psi^{-1}\big(\tilde{g} \sqcup \varphi_\psi^{-1}(\tilde{h})\big) = \varphi_\psi^{-1}(\tilde{g}) \sqcup \varphi_\psi^{-1}(\tilde{h})$$

and

$$\varphi_\psi^{-1}\big(\tilde{g} \sqcap \varphi_\psi^{-1}(\tilde{h})\big) = \varphi_\psi^{-1}(\tilde{g}) \sqcap \varphi_\psi^{-1}(\tilde{h}) \sqsubseteq \tilde{f}^c.$$

Since, $\tilde{f} \sqsubseteq \varphi_\psi^{-1}\big(\varphi_\psi(\tilde{f})\big)$ implies $\big(\varphi_\psi^{-1}(\varphi_\psi(\tilde{f}))\big)^c \sqsubseteq \tilde{f}^c$ and φ_ψ is a fuzzy soft continuous function, so $\varphi_\psi^{-1}(\tilde{g}), \varphi_\psi^{-1}(\tilde{h}) \in \sigma$. Moreover, from $\tilde{g} \not\sqsubseteq \big(\varphi_\psi(\tilde{f})\big)^c$ and $\tilde{h} \not\sqsubseteq \big(\varphi_\psi(\tilde{f})\big)^c$, there exist $y_1, y_2 \in Y$ such that

$$g_e(y_1) \geqslant 1 - \varphi_\psi(\tilde{f})(k)(y_1) \tag{1}$$

$$h_e(y_2) \geqslant 1 - \varphi_\psi(\tilde{f})(k)(y_2) \tag{2}$$

We claim that $\varphi_\psi^{-1}(\tilde{g}) \not\sqsubseteq \tilde{f}^c$ and $\varphi_\psi^{-1}(\tilde{h}) \not\sqsubseteq \tilde{f}^c$. To prove the claim. we suppose $\varphi_\psi^{-1}(\tilde{g}) \sqsubseteq \tilde{f}^c$. Clearly, this claim contradicts (1). Similarly, $\varphi_\psi^{-1}(\tilde{h}) \sqsubseteq \tilde{f}^c$ contradicts (2). So, $\varphi_\psi(\tilde{f})$ is SC_3-connected. ∎

Theorem 3.22 Let $\varphi_\psi : (X, \tau) \to (Y, \sigma)$ be fuzzy soft continuous surjevtive mapping and $\tilde{f} \in \mathbb{FS}_X^E$. If $\tilde{f}$ is a SC_4-connected, then $\varphi_\psi(\tilde{f})$ is SC_4 connected.

Proof. It can be proved similarly way in Theorem 3.21. ∎

Theorem 3.23 Let (X, τ, E) be a fuzzy soft topological space, $\tilde{f}_1$ and $\tilde{f}_2$ be two SC_1-connected fuzzy soft sets such that $\tilde{f}_1 \sqcap \tilde{f}_2 \neq \tilde{0}_X$. Then, $\tilde{f}_1 \sqcup \tilde{f}_2$ is SC_1-connected.

Proof. It is easy. ∎

Remark 2 *From Theorem 3.23, we can say easily that if $\tilde{f}_1$ and $\tilde{f}_2$ be two SC_2-connected fuzzy soft sets such that $\tilde{f}_1 \sqcap \tilde{f}_2 \neq \tilde{0}_X$, then $\tilde{f}_1 \sqcup \tilde{f}_2$ is SC_2-connected.*

Theorem 3.24 Let (X, τ, E) be a fuzzy soft topological space and $\{\tilde{f}_k : k \in \Lambda\} \subseteq \mathbb{FS}_X^E$ be family of SC_1-connected fuzzy soft sets such that $\tilde{f}_i \sqcap \tilde{f}_j \neq \tilde{0}_X$ for $i, j \in \Lambda$ $(i \neq j)$. Then, $\bigsqcup_{k \in \Lambda} \tilde{f}_k$ is is a SC_1-connected fuzzy soft set.

Proof. It can be proved by using Theorem 3.23. ∎

4. Conclusion

In this study we introduced fuzzy soft connectedness and presented fundamentals properties such as protecting image of connected fuzzy soft set under the fuzzy soft continuous mapping and SC_i-connectedness. For future works, we consider to study on intuitionistic fuzzy soft connected sets.

References

[1] B. Ahmad, and A. Kharal, *On fuzzy soft sets*, Advances in Fuzzy Systems, 1 (2009), 1-6.
[2] N. Ajmal and J.K. Kohli, *Connectedness in fuzzy topological spaces*, Fuzzy Sets and Systems, 31 (1989), 369-388.
[3] H. Aktaş and N. Çağman, *Soft sets and soft group*, Information Science, 177 (2007), 2726-2735.
[4] A. Aygünoğlu and H. Aygün, *Some notes on soft topological spaces*, Neural Comput and Appl., 21(1) (2011), 113-119.
[5] N. Çağman, and S. Enginoğlu, *Soft set theory and uni-int decision making*, European Journal of Operational Research, 207 (2010), 848-855.

[6] N. Çağman, F. Çıtak and S. Enginoğlu, *Fuzzy parameterized fuzzy soft set theory and its applications*, Turk. J. Fuzzy Syst., 1(1) (2010), 21-35.
[7] N. Çağman, S. Karataş and S. Enginoğlu, *Soft topology*, Computers and Mathematics with Applications, 62 (2011), 351-358.
[8] N. Çağman, S. Enginoğlu and F. Çıtak, *Fuzzy soft set theory and its applications*, Iranian Journal of Fuzzy Systems, 8(3) (2011), 137-147.
[9] N. Çağman, *Contributions to the theory of soft sets*, Journal of New Results in Science, 4 (2014), 33-41.
[10] C. L. Chang, *Fuzzy topological spaces*, J. Math. Appl., 24 (1968), 182-193.
[11] P.K. Gain, P. Mukherjee, R.P. Chakraborty and M. Pal, *On some structural properties of fuzzy soft topological spaces*, Intern. J. Fuzzy Mathematical Archive, 1 (2013), 1-15.
[12] Ç. Gündüz and S. Bayramov, *Some results on fuzzy soft topological spaces* Hindawi Publishing Corporation Mathematical Problems in Engineering Volume 2013, Article ID 835308.
[13] S. Hussain and B. Ahmad, *Some properties of soft topological spaces*, Computers and Mathematics with Applications, 62 (2011), 4058-4067.
[14] S. Hussain, *A note on soft connectedness*, Journal of Egyptian Mathematical Society, 23(1) 2015, 6-11.
[15] A. Kharal and B. Ahmad, *Mappings on fuzzy soft classes*, Hindawi Publishing Corporation, Advances in Fuzzy Systems, Article ID 407890, 2009.
[16] P. K. Maji, R. Biswas and A. R. Roy, *Fuzzy soft sets*, J. Fuzzy Math., 9(3) (2001), 589-602.
[17] D. Molodtsov, *Soft set theory-first results*, Computers Math. Appl., 37(5) (1999), 19-31.
[18] E. Peyghan, B. Samadi and A. Tayebi, *About soft topological spaces* Journal of New Results in Science, 2 (2013), 60-75.
[19] S. Roy and T.K. Samanta, *A note on fuzzy soft topological spaces*, Annals of Fuzzy Mathematics and Informatics, 3(2) (2012), 305-311.
[20] M. Shabir and M. Naz, *On soft topological spaces*, Computers and Mathematics with Applications, 61 (2011), 1786-1799.
[21] T. Şimşekler and Ş. Yüksel, *Fuzzy soft topological spaces*, Annals of Fuzzy Mathematics and Informatics, 5(1) (2013), 87-96.
[22] B. Tanay and M. B. Kandemir, *Topological structures of fuzzy soft sets*, Computers and Mathematics with Applications, 61 (2011) 412-418.
[23] B. P. Varol and H. Aygün, *Fuzzy soft topology*, Hacettepe Journal of Mathematics and Statistics Volume 41 (3)(2012), 407-419.
[24] L. A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965), 338-353.
[25] X. D. Zhao, *Connectedness on fuzzy topological spaces*, Fuzzy Sets and Systems, 20 (1986), 223-240.
[26] İ. Zorlutuna, M. Akdağ, W. K. Min, S. Atmaca, *Remarks on soft topological spaces*, Annals of Fuzzy Mathematics and Informatics, 3(2) (2012), 171-185. Rings, 1(4) (2002) 203-211.