# A Novel Model for Bus Stop Location Appropriate for Public Transit Network Design: The Case of Central Business Districts (CBD) of Tehran 

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#### Abstract

In this paper, a novel multi-objective bus stop location model is proposed, which considers not only the coverage of demand and minimization of access time but also the necessities of suitable stops for transit network design phase. Three objective functions are considered including minimizing (I) sum of the total access distance (time), (II) the weighted combination of stops, and (III) the number of stops. A sum-weighted method is used to solve the proposed multi-objective model considering the different scenarios of weights. A detailed analysis is carried out Tehran CBD to generate sensible stops results.


Keywords: Bus stop Location, Multi-objective, Network design, Public transportation.
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## 1. Introduction

Public transportation has been recognized as a viable option for urban sustainable transportation, and includes some advantages such as air pollution reduction, energy saving, mobility improvement, and congestion minimization through the urban network.

For a public transportation system to become an attractive alternative travel mode, it must be minimized the access time, delay time and travel time to increase usage convenience, although other environmental service characteristics such as the pedestrian environment can be extended simultaneously.

In order to ensure a high level of service for public transportation users, walking distances to stops should be as short as possible. Farewell and Marx [8] state that people consider longer walk time to be much less convenient than in-vehicle travel time and proposed a maximum walking distance of

400 meters to a transit stop. This means that by reducing access (walking) time, we can attract more people to use public transportation system.

The problem of finding the location of bus stops is known as bus stop location problem (BSLP), and it is one of the most important problems in the domain of transit network design problem (TNDP). BSLP has been studied by many researchers. Minimizing the number of stops and minimizing the access distance (time) are two main objectives in BSLP.

Schöbel [1] considered the location of stops along the edges of an already existing public transportation network to achieve a maximal covering on demand points with a minimal number of stops. A demand point is serviced by a stop if the distance between them is less than a pre-specified value. In the study of Murray [2] the optimal number of bus stops is investigated. The model uses a strategic approach for measuring the degree of redundancy and inefficiency in bus stop coverage for
an existing public transportation network, using the location set covering problem (LSCP). The object of the LSCP is to minimize the number of bus stops to provide complete access coverage on the whole service area. Ceder [3] studied BSLP in order to find the minimum number of stops and their location in a general network, so that no passenger will be further away than a pre-selected distance.

Retnani et al [5] propose two objectives for BSLP to determine the location of new stops along the edges of public transportation network. In the first one, they minimize the number of stops to simplify accessing of the demand facilities to the closest stop within a pre-defined distance. Besides there are a lot of research to minimize the number of stops, see e.g., Gleason [11], Murray et al. [9], Laporte et al. [20], Wu and Murray [21].

In the second objective that was studied by Retnani et al., they fixed the number of new stops and minimized the sum of distances between demand facilities and stops. They implemented their general ideas in two different environments, the plane, where demand facilities are represented by coordinates, and in networks, where they are nodes of a graph. In Murray and Wu [14], the accessibility distance is also used as objective function, and a discrete version of this problem has been formulated as an integer program.

Schöbel [1] pointed out that the best distribution of bus stops is not obvious, since even from the commuter's point of view, the following two conflicting effects occur:

1) Many stops are advantageous, since they increase accessibility
2) Each additional stop increases the transportation(in-vehicle) time, since it decreases the average speed of a bus

Fletterman [7] offered a two-objective optimization model for BSLP in which both of the total distances between commuters and stops, and the number of stops are minimized. Fletterman wanted to find a trade-off between the access distance and the number of stops. The selected stops were used as nodes (input) for the TNDP by Fletterman. However, in most studies that is usual to design routes and after that find the location of stops along the routes, but because of the importance of access time and its critical role to attract people, Fletterman [7] and Fan and Machemehl [25] used the selected stops as nodes for the TNDP, and these nodes are zone delegations.

Fletterman implemented this model on a planar graph, and based on the selected stops it is not possible to guarantee that in the results there are appropriate routes in the transit network design phase.

The above literature review outlined minimizing the total access distance (time) and the number of stops as the two main objectives for BSLP. Before Fletterman's model, these objectives were utilized separately, but in Fletterman's model both of them were used for BSLP and after that the selected stops were as input for TNDP.

The purpose of this paper is to find stops as input nodes for TNDP, not only do the stops maintain the two main BSLP objectives as mentioned before, they are also appropriate for the transit network design phase. To this aim, we considered candidate stops on potential roads, suitable for route planning, by proposing a three-objective model. The first objective is to minimize the total access distance (time). The second one is used to minimize the weighted combination of stops with the goal of suitable stops for TNDP by considering proper weights for candidate nodes. The third objective function minimizes the number of stops. The detailed description of this model will be discussed in the third section.

The remainder of the paper is organized as follows: First, in Section 2 the problem is defined in detail and the model assumptions will be reviewed then the mathematical model of the problem is given in Section 3. In Section 4 we review some concepts regarding to multi-objective optimization problem, then the sum-weighted method is discussed. In section 5 we give the numerical analysis on a real case study in Central Business Districts of Tehran. Section 6 concludes the study and lay out the future work for continuing research.

## 2. Problem definition

In BSLP candidate stops are supposed as facilities and users are considered as demand nodes. The goal is to determine the minimum number of bus stops, which are suitable for TNDP and users can access them with minimum access distance (time).

In order to formulate the problem and to achieve the goal, the following assumptions are made:

1) Access speed is the same for all passengers.
2) Suitability of stops for TNDP is defined as nearness to features of ideal stops in the current bus network, which will be discussed in section 3.2.
3) The population is distributed uniformly in the study area (this assumption is not far from reality)


Fig.1. suitability of stops for TNDP
Minimizing the summation of aggregate access distance (time), minimizing the weighted combination of stops (the goal of this objective function is to find suitable stops for TNDP) and minimizing the number of stops are considered the objective functions of the model.

To further clarify the issue, according to Fig. 1, two candidate stops A and B have the similar situation about accessibility (specified demands can access stops with equal distance), and one of the candidate stops should be selected due to minimizing the number of stops. When we talk about suitability of a stop for TNDP, some features are important to be considered like: being a junction stop, traffic flow in the 400 -meter radius buffer around each stop and land use and etc (further discussion will be presented in section 3.2 ). With respect to mentioned features, candidate stop A is more suitable for TNDP than candidate stop B. Previous models for BSLP, like Fletterman's model[7], don't guarantee to select stop A, in this paper, the new defined objective, minimizing the weighted combination of stops, selects stop A which is more appropriate for TNDP.

In the next section, the problem is formulated as an integer programming (IP) model.

## 3. The model

In this section, at first, the indices, the parameters, and the decision variables of the proposed model are expressed as the following:

## Set and Indices :

$C$ : the set of candidate stops
$q:$ an index for demand nodes, $\mathrm{q}=1,2, \ldots, \mathrm{Q}$
$n:$ an index for a candidate bus stop, $n \epsilon C$
Parameters:
$d_{q n}$ : distance between $\mathrm{q}^{\text {th }}$ demand node and $\mathrm{n}^{\text {th }}$ candidate stop
$d e_{q}:$ number of commuters in $\mathrm{q}^{\text {th }}$ demand node
$d_{\max }$ : maximum walking distance to access the nearest stop
$P$ : additional penalty factor for unmet demand
$w_{n}$ : the weight of $\mathrm{n}^{\text {th }}$ candidate stop
$Q$ : number of demand node
Decision variables:
$a_{q n}=\left\{\begin{array}{lc}1 & \text { if } q^{\text {th }} \text { demand node is assigned to } n^{\text {th }} \text { stop } \\ 0 & \text { o. } w .\end{array}\right.$
$h_{n}=\left\{\begin{array}{lc}1 & \text { if } n^{\text {th }} \text { candidate stop is selected } \\ 0 & \text { o.w. }\end{array}\right.$
Then, the proposed BSLP model is formulated as follows:
$\min f_{1}=$
$\sum_{q=1}^{Q}\left[\sum_{n \epsilon C}\left(d e_{q} d_{q n} a_{q n}\right)+\left(1-\sum_{n \epsilon C} a_{q n}\right) d e_{q} \quad P\right]$
$\min f_{2}=\sum_{n \in c} \frac{1}{w_{n}} h_{n}$
$\min f_{3}=\sum_{n \in C}$
s.t.
$d_{q n} a_{q n} \leq d_{\max } \quad \forall q=\{1 \ldots Q\}, n \in C$
$\sum_{n \in C} a_{q n} \leq 1 \quad \forall q=\{1 \ldots Q\}$
$a_{q n} \leq h_{n} \quad \forall q=\{1 \ldots Q\} \quad, n \in C$
$h_{n} \in\{0,1\} \quad \forall n \in C$
$a_{q n} \in\{0,1\} \quad \forall q=\{1 \ldots Q\} \quad, n \in C$
The objective function (1) minimizes the total access distance (time) of the users to the bus stops. Second term considers pre-determined additional penalty $P$ for each unmet demand node.

The objective function (2) minimizes the weighted combination of stops; the goal of this objective function is to find suitable stops for TNDP.

The objective function (3) minimizes the number of bus stops in the network.

The constraint (4) ensures that no user is connected to a bus stop that is further away than the maximum walking distance, $d_{\max }$. So, a user can only be connected to a certain bus stop if the distance between them does not exceed a certain value.

The constraints (5) restricts demand node from being assigned to multiple bus stops. If a user is surrounded by multiple bus stop within the maximum walking distance, the commuter is assigned to the closest, resulting in $\sum_{n \epsilon C} a_{q n}=1$. If no bus stop lays within the maximum distance of the user, the user is recognized as an unmet demand,
resulting in $\sum_{n \epsilon C} a_{q n}=0$ in second term of the objective in Eq. (1).

The constraint (6) ensures to assign demand node only to open bus stops.

The last two constraints, (7) and (8), show the binary decision variables of the model.

### 3.1. Model simplification

Solving the binary integer problem (1)-(8) can be considerably computational demanding due to its many binary variables. We can reduce the size of the problem by putting zero for some unnecessary variables, although, some OR software like AIMMS do the preprocessing by default. With respect to the constraint in (4) some stops are not accessible for $q^{\text {th }}$ demand node, and we can preset zero or eliminate the variable $a_{q n}$ if the related distance between the center of $\mathrm{q}^{\text {th }}$ demand node to nth candidate stop, $d_{q n}$, is more than the pre-selected distance $d_{\max }$. By doing this work for every demand node, we can reduce the size of the integer program before solving it. The rule for eliminating the these variables is given as

$$
a_{q n}=0 \text { for }(q, n) \text { such that } d_{q n}>d_{\max }
$$

The next subsection deals with defining the model's parameters, and how they have been obtained.

### 3.2. Parameters in the proposed model

The presented model in the last section includes parameters such as the set of candidate stops ( $C$ ), the weight of nth candidate stop $\left(w_{n}\right)$, population of $q$ th area $\left(d e_{q}\right)$, the distance between the center of qth area to nth candidate stop $\left(d_{q n}\right)$, maximum walking distance ( $d_{\max }$ ), additional penalty factor for unmet demand $(P)$. The way of obtaining some of these parameters are presented in the following:

## A. Set of candidate stops( $C$ )

At first, we consider potential roads for line planning phase (roads with width greater than 10 meters), every junction in these roads are considered as candidate stops. If the distance between two consequent junctions is greater than 200 meters, we add some mid road candidate stops.

## B. The weight of $n^{\text {th }}$ candidate stop $\left(w_{n}\right)$

Since the selected stops will be used as nodes for TNDP, it is so important to choose the most suitable stops. The model, which is presented in this paper, utilizes proper weight to determine how a candidate node is suitable for TNDP. One way to
obtain these weights could be learned from current good stops that are selected by experts, we call them ideal stops. At first we analyzed the features of ideal stops in the current best routes, and found that some more effective features for every candidate stops like the traffic flow in the 400-meter radius buffer around each stop, being a junction stop or not, land use (nearness to universities, hospitals, shopping malls and cinemas) in the 400 -meter radius buffer around each stop, and lane width of the street where the stop exists there. After that we consider some ideal stops which were selected by some experts from current situation. The ideal values for these features are supposed the average values for the features of ideal stops. Then we compare every feature value of candidate stops with the ideal feature. After that, all feature values were normalized to a value between zero and one. Then we obtained the distance between every features of candidate stops with ideal stops by the following meter:

Where, $F_{i_{j}}^{N}$ is the $\mathrm{j}^{\text {th }}$ feature normalized (its value between zero and one) value of $\mathrm{i}^{\text {th }}$ candidate stop, $\tilde{F}_{j}$ is the $\mathrm{j}^{\text {th }}$ ideal feature normalized value.

The distance between $\mathrm{i}^{\text {th }}$ candidate stop and ideal stops can be calculated as:
$d\left(s_{i}, \tilde{s}\right)=\sqrt{\sum_{j \in F} d_{\mathrm{F}}^{2}\left(F_{i_{j}}^{N}, \tilde{F}_{j}^{N}\right)}$
Where F is the features set, that mentioned in the last paragraph.

Candidate stops were clustered with respect to their distance with ideal stops into k clusters by using $k$-mean Clustering method. We defined the weight set as follows:

$$
W=\left\{\left.\frac{1}{2^{i}} \right\rvert\, i^{\prime \cdots}=0, \ldots, k-1\right\}
$$

For 4 clusters, the weight set will be $W=$ $\{1,0.5,0.25,0.125\}$.

The weight ' 1 ', the maximum weight, is for the cluster which has the minimum distance from ideal stop (the most suitable for transit network design w.r.t. defined features), and so the weight ' 0.125 ', the minimum weight is for the cluster which has the maximum distance from ideal stop. It is obvious that by defining these weights, how much the second objective value is close to third objective value, we can have suitable stops for transit network design.

## C. Maximum walking distance $\left(d_{\max }\right)$

The value of maximum walking distance varies from one study to another one. Saka [15] suggests a maximum walking distance of 800 m , and in the work of Alterkawi [16] the walking distance is limited to
between 300 m and 400 m . Viewed in terms of physical distance, a suitable access standard in urban areas is typically stipulated as 400 meters (Demetsky and Lin [4]; Levinson [6]; Federal Transit Administration [22]; Ammons [23], Murray and Wu [14]).

### 3.3. The proposed multi-objective solving method

In this section, some required multi-objective background is defined in the following paragraph. After that, we review the sum weighted method for multi-objective optimization problems. At the end of this section, we utilize a new objective function, which is a weighted convex sum of the normalized objectives, for our problem.

Abounacer and et al [10] proposed an epsilonconstraint method for any three-objective optimization problem provided that the problem involves at least two integer and conflicting objectives, and they prove that it generates the exact Pareto front.

Xinmin [19] proposed an approach, in his thesis, which reflects the better practical meaning of objective weights in the adaptive environment. His method was used for time-cost optimization problem, and in our study we can extend his method for three objective functions as follows:

$$
\begin{aligned}
& \min f(x)=w_{1}{ }^{o} \frac{-f_{1}^{\max }+f_{1}(x)-\alpha}{f_{1}^{\max }-f_{1}^{\min }+\alpha} \\
&+w_{2}{ }^{o} \frac{-f_{2}^{\max }+f_{2}(x)-\alpha}{f_{2}^{\max }-f_{2}^{\min }+\alpha} \\
&+w_{3}{ }^{o} \frac{-f_{3}^{\max }+f_{3}(x)-\alpha}{f_{3}^{\max }-f_{3}^{\min }+\alpha}
\end{aligned}
$$

$$
\begin{equation*}
x \in S \tag{9}
\end{equation*}
$$

Where $\mathrm{f}_{\mathrm{i}}^{\text {max }}$ and $\mathrm{f}_{\mathrm{i}}^{\text {min }}, \mathrm{i}=1,2,3$, are respectively the maximum and minimum value of $i^{\text {th }}$ objective function. $\alpha$ is a small positive random number between 0 and 1 .
$\frac{-f_{i}^{\text {max }}+f_{i}(x)-\alpha}{f_{i}^{\text {max }}-f_{i}^{\text {min }}+\alpha}$ is the $i^{\text {th }}$ component in the new minimization problem, and its value is between -1 and 0 .
$\mathrm{w}_{\mathrm{i}}^{o}, \mathrm{i}=1,2,3$, is the weight corresponding to $\mathrm{i}^{\text {th }}$ component in the new minimization problem.

We denote the latter minimization problem with $P_{N}\left(w^{o}\right)$. By this new definition of objective function in $P_{N}\left(w^{o}\right)$, in Eq.(9), objective weights are meaningful.

In the next section, we move on to the application of the proposed model to a real general
network: The Central Business Districts (CBD) of Tehran.

## 4. Numerical analysis on The CBD of Tehran

Tehran's CBD (districts 6, 7, 11 and 12) is the case study of our problem and concentrates the commercial activities, surrounded by several residential areas. The candidate stops in CBD of Tehran, according to section 3.2, are provided by ArcGIS 10, and they are shown in Fig..2. The maximum walking distance to a bus stop is set 400 m in this study, and the additional penalty factor for unmet demand is set to 10 .


Fig.2. Candidate stops on potential roads in CBD of Tehran
With respect to the new objective function in $P_{N}\left(w^{o}\right)$, Eq.(9), we vary the weights $w_{1}{ }^{o}, w_{2}{ }^{o}$ and $w_{3}{ }^{\circ}$, in 0.2 interval, which results in $21^{1}$ combinations in total. This method was introduced by Guan and et al [18] for their multi-objective problem. They dealt with a weighted combination of objectives, which the range of objectives was different from each other (e.g. the value of one of the objectives is about 40 times greater than the other one's value). In their results, they had similar solutions in most cases, w.r.t. different weights scenarios, and it's because of unbalanced range of objectives values, and consequently changing weights does not change the objectives values.

The proposed IP model was developed using AIMMS 3.12 and solved using the CPLEX 12.3 solver. Our experiments were perfomed in an ASUS

[^0]with an Intel Core i7-2630 QM. 2.0 GHz with 8 GB RAM.

In the following, we examine the results of the 21 combinatorial cases for the weighting factors, $w_{1}{ }^{o}, w_{2}{ }^{o}$ and $w_{3}{ }^{o}$, we divide the results into 5 parts with different conditions of the weights, and all the results are brought and summarized in the table 1:

## 1. The case with either $w_{1}{ }^{o}, w_{2}{ }^{o}$ or $w_{3}{ }^{o}=1$

This case implies that only one of the objectives receives the absolute concentration while the rest is disregarded. With $w_{i}^{o}=1, \quad \mathrm{i}=1,2,3, \quad$ the corresponding objective function value will be minimized. According to table 1 , when $w_{1}{ }^{o}=1$, the first objective value ( $\left|f_{1}\right|$ ) is 4014376.81316 km . This case is similar to Retnani's first model [5]. In the case with $w_{2}{ }^{o}$ or $w_{3}{ }^{o}=1$, both minimum values of the second and the third objective functions are zero. There are some studies with respect to the case $w_{3}{ }^{o}=1$, like Gleason [11], Schöbel [1], and the Retnani's second model [5].

## 2. The case with $w_{1}{ }^{0}=0$

When $w_{1}{ }^{o}=0$, the first objective, the access distance (time) of users, is disregarded completely. In this case, none of stops are selected, and the value of the first objective function is actually the penalty for unmet demand. According to table..., the values of other objective functions are zero $\left(\left|f_{2}\right|=\left|f_{3}\right|=0\right)$.

## 3. The case with $w_{2}{ }^{o}=0$

Fletterman's model is one of the corresponding studies with respect to this case. When $w_{2}{ }^{\circ}=0$, the selected stops are not necessarily suitable for transit network design problem, as we mentioned before. We define a column in table 1 which shows the ratio of second objective value to third objective value. We told before in the section 3.2, with respect to defined weights for candidate stops, if the second objective function value approaches to the third one, then the selected stops will be suitable for TNDP. We can see in table 1 when $w_{2}{ }^{\circ}=0$, the ratio in the last column is bigger than the rest cases and it can be interpreted that the model with respect to this case doesn't select appropriate stops for TNDP.

## 4. The case with $w_{3}{ }^{o}=0$

When $w_{3}{ }^{\circ}=0$, the third objective, minimizing the number of stops, will be discarded. Since the second and the third objective don't conflict together, the number of selected stops will be controlled by the second weight, $w_{2}$, e.g. with small value for $w_{2}$, the number of selected stops, $\left|f_{3}\right|$, is increased and reach to maximum point, $\left|f_{3}\right|=636$, when $w_{2}=0$.

## 5. The case with $w_{1}{ }^{o},{w_{2}}^{o}$ or $w_{3}{ }^{o} \neq 0$ or 1

In this case, there is a balanced trade-off among three objectives. According to table 1, the higher the $\mathrm{i}^{\text {th }}$ weight, the lower corresponding objective function value.

When $w_{1}{ }^{o}=0.6, w_{2}^{o}=0.2$ and $w_{3}{ }^{o}=0.2$, access time is the main concern (in this case $\left|f_{1}\right|=$ 4559366.46853 km , that is near to the minimum of the first objective value ). When $w_{1}{ }^{o}=0.2$ , $w_{2}{ }^{o}=0.4$ and $w_{3}{ }^{o}=0.4$, both the selecting appropriate stops for TNDP and minimum number of them are the main concern (in this case, the number of selected stops is 73) and the result for this case is shown in Fig. 3. When $w_{1}{ }^{o}=0.2, w_{2}{ }^{o}=0.2$ and $w_{3}{ }^{0}=0.6$, minimizing the number of stops is the main concern( according to table 1 , when all weights take value between 0 and 1 , the third objective function reaches to its minimum value, that is 57, by the last combination of weights)and Fig. 4 shows the results for this case.


Fig.3. Selected stops related to ${w_{1}}^{\boldsymbol{o}}=0.2,{w_{2}}^{\boldsymbol{o}}=0.4,{w_{3}}^{\boldsymbol{o}}=0.4$


Fig.4. Selected stops related to $w_{1}{ }^{o}=0.2, w_{2}{ }^{\boldsymbol{o}}=0.2, w_{3}{ }^{\boldsymbol{o}}=0.6$
As it is obvious from table 1 , according to the new definition of objective function in $P_{N}\left(w^{o}\right)$,

Eq.(9), there are different objective values, $\left|f_{1}\right|,\left|f_{2}\right|$ and $\left|f_{3}\right|$, for different scenarios of weights, and Fig. 5 shows us a good view of these values. As a result, by defining the new objective function, we have more different solutions in proportion to Guan's study [18].

Smaller points in Fig. 5 imply the projection of main points in every plane (XY, XZ, and YZplanes), and according to these points, the first objective and the second objective conflict together
and the same situation is valid for the first objective and the third objective, since by increasing one of the objectives, another one decreases. As we can see, the second and the third objective can be optimized simultaneously. Since the small points on the YZ-plane, the plane related to the second and the third objective, are not in the straight line, therefore, both objectives have different effects on the solutions and it emphasizes that the existence of the second objective is obligatory.

Table. 1
Solutions for the various combinations of the values of weighting factors in Eq.(9)

|  | $w_{1}{ }^{o}$ | $w_{2}{ }^{\circ}$ | $w_{3}{ }^{\circ}$ | firstObj | secondObj | thirdObj | secondObj/thirdObj |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0.2 | 0.8 | 29357382432.74 | 0 | 0 | - |
| 2 | 0 | 0.4 | 0.6 | 29357382432.74 | 0 | 0 | - |
| 3 | 0 | 0.6 | 0.4 | 29357382432.74 | 0 | 0 | - |
| 4 | 0 | 0.8 | 0.2 | 29357382432.74 | 0 | 0 | - |
| 5 | 0.2 | 0 | 0.8 | 16799692103.01 | 94 | 48 | 1.96 |
| 6 | 0.2 | 0.2 | 0.6 | 15547070004.29 | 84 | 57 | 1.47 |
| 7 | 0.2 | 0.4 | 0.4 | 13644868809.36 | 95 | 73 | 1.30 |
| 8 | 0.2 | 0.6 | 0.2 | 12551815350.93 | 106 | 84 | 1.26 |
| 9 | 0.2 | 0.8 | 0 | 11579762990.02 | 117 | 100 | 1.17 |
| 10 | 0.4 | 0 | 0.6 | 6789932466.96 | 349 | 142 | 2.46 |
| 11 | 0.4 | 0.2 | 0.4 | 7025273728.70 | 235 | 144 | 1.63 |
| 12 | 0.4 | 0.4 | 0.2 | 6531066390.21 | 241 | 158 | 1.53 |
| 13 | 0.4 | 0.6 | 0 | 6308262217.53 | 245 | 174 | 1.41 |
| 14 | 0.6 | 0 | 0.4 | 4559366468.53 | 463 | 195 | 2.37 |
| 15 | 0.6 | 0.2 | 0.2 | 4697438021.68 | 353 | 198 | 1.78 |
| 16 | 0.6 | 0.4 | 0 | 4914071386.82 | 325 | 203 | 1.60 |
| 17 | 0.8 | 0 | 0.2 | 4194990758.59 | 523 | 215 | 2.43 |
| 18 | 0.8 | 0.2 | 0 | 4244890616.60 | 418 | 228 | 1.83 |
| 19 | 1 | 0 | 0 | 4014376813.16 | 1527 | 636 | 2.40 |
| 20 | 0 | 1 | 0 | 29357382432.74 | 0 | 0 | - |
| 21 | 0 | 0 | 1 | 29357382432.74 | 0 | 0 | - |



Fig.5. The values of objective functions according to table 1
We implemented the model with different values of maximum walking distances, $d_{\max }$, and the spending time, according to different $d_{\max }$ values and number of cases, is shown in table 2.

Table. 2
The effect of $\boldsymbol{d}_{\max }$ on implementation time

| The effect of $\boldsymbol{d}_{\max }$ on implementation time |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\boldsymbol{d}_{\max }$ | The number of <br> cases (w.r.t <br> different weights) | implementation <br> time |  |
| 2 | 500 | 21 | 733 s |  |
| 3 | 600 | 21 | 1421 s |  |
| 4 | 800 | 10 | 3634 s |  |

As we can see from the table 2 when $d_{\max }$ increases, the feasible region will be extended, so it needs more time to solve the model.

We know that $d_{\text {max }}$ more than 400 meters is not attractive for the public transportation users.

## 5. Conclusion

This paper presents a novel model of simultaneous minimization of total access distance, weighted combination of bus stops (suitability for network design phase) and number of stops. The model is formulated as a constrained integer programming model. The model outputs balance the access distance, number of stops and suitability of stops for transit network design phase.

A detailed analysis is carried out with the CBD of Tehran, in which sensitivity analysis on the weighting factors for the three components of the objective function is conducted. In all the numerical cases investigated, the model does generate sensible
stops results. The results shows us that the existence of the second objective function, the weighted combination of stops, which its goal is to find suitable stops for TNDP, is necessary for BSLP which the selected stops are as inputs for TNDP.

By defining the objective function (9), the better practical meaning of objective weights is acquired and a variety of solutions are obtained. The value of the maximum walking distance, $d_{\max }$, has a strong effect on the implementation time.

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[^0]:    ${ }^{1}$ We can replace $\mathrm{W}_{\mathrm{i}}^{0}$ by $0.2 * x_{i}, \mathrm{i}=1,2,3$, so we have $\sum_{i=1}^{3} 0.2 *$ $x_{i}=1 \rightarrow \sum_{i=1}^{3} x_{i}=5 ; x_{i} \geq 0$ and integer, $\mathrm{i}=1,2,3$
    Therefore the number of cases is $\binom{5+3-1}{5}=21$

