



# Per Unit Coding for Combined Economic Emission Load Dispatch using Smart Algorithms

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## Abstract

This paper proposes per unit coding for combined economic emission load dispatch problem. In the proposed coding, it is possible to apply the percent effects of elements in any number and with high accuracy in objective function. In the proposed per unit coding, each function is transformed into per unit form based on its own maximum value and has a value from 0 to 1. In this paper, particle swarm optimization is used for solving economic emission load dispatch problem. In order to show the advantages of the proposed method, 25 independent case studies are conducted on systems holding three and six power units with different influence percentages of each function are investigated. The obtained results are compared with those of other methods such as Biogeography Based Optimization, Tabu Search, NSGA-II and etc. The obtained results properly show the superiority of the proposed method to combine economic emission dispatch problem over the penalty factor technique and other conventional combined approaches.

*Keywords:* Smart grid Economic emission dispatch; multi-objective; optimization; particle swarm optimization; per unit coding.

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## 1. Introduction

The aim of economic load dispatch (ELD) in thermal power plants is to minimize the operating costs, major of which is the plants' fuel cost [1]. Thermal power plants operation is accompanied by the considerable amounts of emissions such as sulfur oxide ( $SO_x$ ), nitrogen oxide ( $NO_x$ ), and carbon oxide ( $CO_x$ ). Extensive investigations have been reported to decrease the influences of emissions on the economic dispatch problems. The investigations involve the use of linear programming techniques [2], fuzzy methods [3-4], and heuristic algorithms [5-9]. The problem of economic load dispatch along with the emission problem is solved separately in [10-11]. The optimization based on the linear programming considering one objective in each instance is presented in [12].

The multi-objective emission and ELD function converted to a single-object problem by linear function is presented in [13-14]. In this conversion, since the ELD is in \$/hr and emission amount is kg/hr and because the cost of each

function might separately be some times greater than the other ones, a penalty factor or a balance factor is applied on the objective function [15]. The conducted investigations show that this penalty factor leads to equalization in both functions allowing the algorithm to consider the influence of both elements with a similar weighting factor. In addition, the objective function will then be in \$/hr applying such penalty factor. Selecting the penalty factor of emission based on maximum generators fuel cost results in manifold value increase of a separately function during the algorithm run process and the search process fails to operate properly. Consequently, the algorithm ends in non-similar solutions in each program run and faces with errors and difficulties in optimum solutions selection. Another problem of applying such methods falls in the fact that the emission influence rate in objective function should be considered in economic dispatching process. These methods are not able to respond to this circumstance and just find out the mediocrity of the functions, which is

linearly converted to one function. In order to overcome such problem, a method, is proposed through which several functions – in any number – are changed to per unit regarding their own base values and consequently each function has a similar value in [0-1] during algorithm implementation. In this case, it can determine that the optimization should be accomplished based on percentage of each single-object function. The power plants fuel cost optimization along with the emission influences reduction based on the desired percentages are considered in applying the proposed coding in the economic emission load dispatch.

PSO is a modern heuristic algorithm proper for solving non-convex optimization problems. The search approach of this algorithm depends on the population and particle swarm. J. Kennedy and R. C. Eberhart initially presented PSO in 1995 based on the analysis of group behavior of birds and fish categories [16]. In PSO, each particle's decision is adopted according to its own previous experiments and the experiences of its neighbors. The simple concept, easy implementation, relative ability in continuing program implementation and parameters control even under error existence, and the calculative efficiency are some of the distinctive advantageous of this technique [17].

In order to evaluate the performance of the proposed per unit coding (PUC) applied to combine the economic emission load dispatch (EELD) problem's objective function, and to observe advantageous of the proposed coding, 25 independent case studies are conducted on systems holding three and six power units. In these cases, the emission level of  $NO_x$ ,  $CO_x$  and  $SO_x$  have been decreased in desired percents in addition to simultaneously reduction of plants fuel cost. In this paper, all cases are executed for 30 times making it possible to evaluate the convergence pattern of the algorithm to similar solutions. Since the objective function in the proposed technique, unlike other methods, is expressed in per unit form, the results are compared with those of other methods such as Biogeography Based Optimization (BBO) [20], Oppositional BBO (OBBO) [20], Tabu Search [21], and NSGA-II [19], for which the objective functions are in \$/hr. The results show the superiority of the proposed per unit coding approach over other mentioned methods.

## 2. Economic Emission Dispatch Formulation

Total system fuel costs reduction along with the emission decrease of each pollutant is aimed in solving EELD problem. Authors of [18-21] have mentioned the formulation and EELD problem constraints in details. The problem variables are the

generated powers of the generators defined as follows:

$$[P_G] = [P_{G1}, P_{G2}, \dots, P_{GNg}]^T$$

$$\text{by minimizing: } F = [F_{FC}, F_{NX}, F_{CX}, F_{SX}] \quad (1)$$

$$\text{Subjected to: } h(P_{Gi})=0 \text{ and } g(P_{Gi}) \leq 0$$

where  $Ng$  is the number of the last generator and  $P_{Gi}$  is the generated real power of  $i^{th}$  generator.  $h(P_{Gi})$  is the equality constraint and  $g(P_{Gi})$  is the inequality constraint of the problem.  $F$  is the multi-objective function minimizing of which is aimed. The objective functions of this problem are separately as follows:

### A) Minimizing Fuel Cost

It is aimed to minimize the thermal power plants' fuel costs, the objective function of which is a 2<sup>nd</sup> order function defined as follows:

$$F_{FC} = \min \sum_{i=1}^{Ng} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \quad \$/hr \quad (2)$$

where  $a_i$ ,  $b_i$  and  $c_i$  are the constants related to the thermal power plants' fuel costs.

### B) Minimizing $NO_x$ Emission

Here, it is aimed to minimize the  $NO_x$  emission of plants, the objective function of which is a 2<sup>nd</sup> order function defined as follows:

$$F_{NX} = \min \sum_{i=1}^{Ng} (a_{Ni} + b_{Ni} P_{Gi} + c_{Ni} P_{Gi}^2) \quad \text{ton/hr} \quad (3)$$

where  $a_{Ni}$ ,  $b_{Ni}$  and  $c_{Ni}$  are the constants related to the  $NO_x$  emission amount of power plants.

### C) Minimizing $CO_x$ Emission

Here, it is aimed to minimize the  $CO_x$  emission of plants.

$$F_{CX} = \min \sum_{i=1}^{Ng} (a_{Ci} + b_{Ci} P_{Gi} + c_{Ci} P_{Gi}^2) \quad \text{ton/hr} \quad (4)$$

where  $a_{Ci}$ ,  $b_{Ci}$  and  $c_{Ci}$  are the constants related to the  $CO_x$  emission amount of power plants.

### D) Minimizing $SO_x$ Emission

It is aimed here to minimize the  $SO_x$  emission of plants, the objective function of which is a 2<sup>nd</sup> order function defined as follows:

$$F_{SX} = \min \sum_{i=1}^{Ng} (a_{Si} + b_{Si} P_{Gi} + c_{Si} P_{Gi}^2) \quad \text{ton/hr} \quad (5)$$

where  $a_{Si}$ ,  $b_{Si}$  and  $c_{Si}$  are the constants related to the  $SO_x$  emission amount of power plants.

The generated power of plants is the sum of load and the power losses of transmission system. In other words, the equality constraint is as follows:

$$\sum_{i=1}^{Ng} (P_{Gi}) - P_{load} - P_{loss} = 0 \quad (6)$$

where  $P_{load}$  is the load demand power and  $P_{loss}$  is the power loss in the transmission system and is obtained as follows:

$$P_{loss} = \sum_{i=1}^{Ng} \sum_{j=1}^{Ng} P_{Gi} B_{ij} P_{Gj} \quad (7)$$

The inequality constraint is the generated power of the power plants, which falls between a maximum and a minimum values as follows:

$$P_{Gi \min} \leq P_{Gi} \leq P_{Gi \max} \quad (8)$$

### 3. Economic Emission Dispatch Combination

#### A) A Review on Major Methods of Economic Emission Dispatch Combination

Several methods are presented for EELD problem combination. However, it seems that these methods do not satisfy some of the expectations and do not accurately combine the emission and economic load dispatch functions. Using the penalty factor in emission section is one of the most commonly used methods in EELD problem combination [22] and is applied on majority of the investigations such as [23-26]. The objective function of this method is as follows:

$$\min TC = (F_{FC} + h \times F_{EC}) \quad (\$/hr) \quad (9)$$

where  $h$  is the price penalty factor and is as follows:

$$h_i = \frac{F_{FC, \max}}{F_{EC, \max}} = \frac{a_i + b_i P_{Gi, \max} + c_i P_{Gi, \max}^2}{\alpha_i + \beta_i P_{Gi, \max} + \lambda_i P_{Gi, \max}^2} \quad (\$/ton) \quad (10)$$

where  $TC$  is the total cost,  $F_{FC}$  is the plant's fuel cost,  $F_{EC}$  is the emission amount,  $F_{FC, \max}$  and  $F_{EC, \max}$  is the maximum plant's fuel cost and maximum emission and  $\alpha_i$ ,  $\beta_i$  and  $\lambda_i$  are the emission constants.

Steps used to find the price penalty factor for a particular load demand:

(1) Find the ratio between maximum fuel cost and maximum emission of each generator.

(2) Arrange the values of price penalty factor in ascending order.

(3) Add the maximum capacity of each unit one at a time, starting from the smallest unit until

$$\sum P_{Gi, \max} \geq P_d$$

(4) At this stage,  $h_i$  associated with the last unit in the process is the price penalty factor for given load.

(5) The above procedure gives the approximate value of penalty factor; so exact value is computed using interpolation method

The main reason of applying the penalty factor,  $h$ , is to harmonize  $TC$  measuring unit ( $\$/hr$  as it is obvious) and to equalize the weights of  $F_{FC}$  and  $F_{EC}$  functions in the fitness function allowing the algorithm to consider the influences of both functions similarly in the objective function. Several techniques focusing on correction of this method to be applied on specific problems are presented in different papers. However, the performance basis of the majority of presented methods stands on utilizing the penalty factor and this structure.

The investigations conducted in this paper imply that the fuel cost of plants in objective function sometimes 100 times more than the emission amount. Therefore, applying penalty factor can just to some extent equalize the weight of several functions in objective function and just to some extent consider the similar influence algorithm of two functions in optimization process. Despite, convergence to non-similar solutions in runs depicts the non-accuracy of applying penalty factor. For example, in Table 4 of [24], the optimum solution of fuel cost problem is  $8364.3019 (\$/hr)$ , the  $SO_x$  emission amount is  $8.97419 (ton/hr)$ , and the penalty factor applied on  $SO_x$  is  $970.03157 (\$/ton)$ . In order to

simultaneously optimize the functions with similar weight, it is necessary to have  $F_{FC} = h \times F_{EC}$ , while even under the optimized condition  $F_{FC} < (h \times F_{EC})$  is valid. This is a limitation of the above method and the algorithm cannot consider the influences of fuel cost and emission functions with similar weight. Investigations are conducted on simultaneously applying this penalty factor on objective function possessing more than three functions show no appropriate performances because the penalty factor value automatically varies as the nature of the functions varies. For example, if it is aimed to decrease the  $SO_x$  and  $NO_x$  emission simultaneously with fuel cost reduction, two penalty factors are required according to the mentioned method, which are obtained based on the maximum fuel cost. Here, the simultaneous reduction with similar fuel and emission functions weight is under consideration. Therefore, the method is reliable when

$F_{FC} = h_1 \times F_{NX} = h_2 \times F_{SX}$  is valid in objective function. The investigations show that the optimization of several functions with similar weight in objective function is not possible.

As it is obvious, this technique is not able to optimize more than two separate functions simultaneously applying similar percentage. In this paper, in order to overcome the mentioned problems, a new per unit coding is proposed to convert multi-objective functions to a single-objective model in which each function with any number is considered in per unit form and consequently, each function has a similar weight in the objective function. Therefore, the ability of controlling the influence percent of each independent function on objective function is provided.

#### B) Proposed Per Unit Coding for Combined Multi-objective Function to a Single-objective Model

This paper proposes per unit coding for combined multi-objective functions to a single-objective model in solving multi-objective optimization problems. Here, each function based on the own maximum amount are expressed in per unit form separately. In this state, each function's weight falls similarly in [0 1] and the algorithm would be able to consider the influence of each function similarly. Therefore, there would be no need to apply the penalty factor. The final combined objective function investigated by the proposed technique is as follows:

$$F = \sum_{a=1}^l f_a^{pu} \quad (pu) \quad (11)$$

$$f_a^{pu} = \frac{f_a \left( \sum_{y=1}^h kx_{a,y} \right)}{f_a \left( \sum_{y=1}^h kx_{a,y}^{\max} \right)} \quad (pu) \quad (12)$$

where,  $F$  was the linearized single-objective function, and  $f_a^{pu}$  is function  $a$  in p.u form. in Eq. (12),  $x_{a,y}$  represents the  $y^{th}$  variable of function  $a$ , and  $x_{a,y}^{\max}$  represents the maximum value of  $x$ , and  $k$  is the constant coefficient of each function. In Eq. (11), the multi-objective function is transformed linearly into a normalized single-objective function in which each of the independent functions have a same weight between 0 and 1.

If one wants, during linearizing process, to increase or decrease the impact of some of the functions in the objective function, weighting coefficients can be used for each function. A necessary and sufficient condition for the use of weighting coefficients is that the sum of the coefficients should be equal 1. That is,

$$F = \sum_{a=1}^l f_a^{pu} \times \lambda_a \quad (13)$$

$$\sum_{a=1}^l \lambda_a = 1 \quad (14)$$

where,  $\lambda_a$  is weighting factor for the function  $a$ .

#### C) The Proposed Technique for Economic Emission Dispatch Combination

In order to assess the capabilities of the proposed technique, it is aimed to transform four independent  $F_{FC}$ ,  $F_{NX}$ ,  $F_{CX}$ , and  $F_{SX}$  functions linearly into a single-objective function and separately specify the influence of each function on the objective function. The final combined objective function investigated by the proposed technique for EELD problem is the following:

$$\text{minimize } (F = n_f F_{FC}^{pu} + n_n F_{NX}^{pu} + n_c F_{CX}^{pu} + n_s F_{SX}^{pu}) \quad (pu) \quad (15)$$

where  $F_{FC}^{pu}$  is the plants fuel cost, which is the per unit form of  $F_{FC}$  in its own maximum amount base and equals to the following:

$$F_{FC}^{pu} = \frac{F_{FC}}{F_{FC, \max}} \quad (pu) \quad (16)$$

$$F_{FC, \max} = \sum_{i=1}^{Ng} (a_i + b_i P_{Gi, \max} + C_i P_{Gi, \max}^2) \quad (\$/hr) \quad (17)$$

In (15),  $F_{NX}^{pu}$  is the  $NO_X$  emission amount and is the per unit form of  $F_{NX}$  in its own maximum amount and equals to the following:

$$F_{NX}^{pu} = \frac{F_{NX}}{F_{NX, \max}} \quad (pu) \quad (18)$$

$$F_{NX, \max} = \sum_{i=1}^{Ng} (a_{Ni} + b_{Ni} P_{Gi, \max} + C_{Ni} P_{Gi, \max}^2) \quad (ton/hr) \quad (19)$$

In (15),  $F_{CX}^{pu}$  is the  $CO_X$  emission amount and is the per unit form of  $F_{CX}$  in its own maximum amount and equals to the following:

$$F_{CX}^{pu} = \frac{F_{CX}}{F_{CX, \max}} \quad (pu) \quad (20)$$

$$F_{CX, \max} = \sum_{i=1}^{Ng} (a_{Ci} + b_{Ci} P_{Gi, \max} + C_{Ci} P_{Gi, \max}^2) \quad (ton/hr) \quad (21)$$

In (15),  $F_{SX}^{pu}$  is the  $SO_X$  emission amount and equals to the following:

$$F_{SX}^{pu} = \frac{F_{SX}}{F_{SX, \max}} \quad (pu) \quad (22)$$

$$F_{SX, \max} = \sum_{i=1}^{N_g} (a_{Si} + b_{Si} P_{Gi, \max} + C_{Si} P_{Gi, \max}^2) \quad (ton/hr) \quad (23)$$

In (15),  $n_f$ ,  $n_n$ ,  $n_c$ , and  $n_s$  are the influence constants of each function in the objective function and are related to plants fuel cost,  $NO_X$ ,  $CO_X$ , and  $SO_X$  emission amounts, respectively. These constants should be selected in a way that the sum of them equals to unit (1). In other words, the following should be valid:

$$n_f + n_n + n_c + n_s = 1 \quad (24)$$

These constants, which are positive and less than 1, are the determining factor of the influence of each independent function in the objective function.

The proposed weighting coefficients in (14) and (24) equations are used in literature to create the same weight in the objective function.

In [26-29], the weighting coefficients along with penalty factors are proposed for better equalizing of various functions' weights in the objective function. In this case, the objective function is expressed in \$/h.

In [30], the varying weighting coefficients are used without penalty coefficients to equalize the weights of functions in objective function. For this case, weighting coefficient takes various values in each iteration and functions, starting from zero and increasing to 1. In this state, any of the functions in \$/hr, ton/hr constitutes an objective function. In this state, a function that have maximum value and weighting coefficient near zero is selected as the optimum solution for the problem, thus, optimal solution has error.

In the proposed method, unlike other methods, each function in the objective function has a value between 0 and 1 and all of the functions do not need penalty factor. In the proposed coding, the use of weighting coefficients in the objective function is only used to increase or decrease the effect of each function, and not to equalize the weights of the functions.

Therefore, if the weighting coefficients are not used in Eq. (13), any function in the objective functions and in turn optimization will has the same influence. The reason for not using a mean or minimum value of any function in Eq. (12) is to limit the amount of each function in per unit between 0 and 1.

## 4. Particle Swarm Optimization

### A) An Overview on PSO

J. Kennedy and R. C. Eberhart founded the PSO algorithm based on the behavior of individuals (particles or ingredients) of a group. This technique refers to the zoology and the moving model of subjects within a group. It seems that the group members share data and this leads to the group's performance increase. The PSO algorithm's search rely on the parallel utilization of a group of particles. Each particle represents a solution. In this algorithm, a particle moves towards the optimum value according to three factors of present velocity, previous experience, and neighbors' experience [27].

In n-dimensional search space, the position and the velocity of  $i^{th}$  particle is illustrated by  $X_i = (X_{i1}, X_{i2}, \dots, X_{in})$  and  $V_i = (V_{i1}, V_{i2}, \dots, V_{in})$  vectors, respectively; dimensions of which show the number of the particles. Vectors ( $P_{best} = X_{i1}^p, X_{i2}^p, \dots, X_{in}^p$ ) and ( $G_{best} = X_{i1}^G, X_{i2}^G, \dots, X_{in}^G$ ) define the best position of the  $i^{th}$  particle and the best hitherto position of  $i^{th}$  particle's neighbors, respectively. The corrected velocity and position of each particle after iteration can be depicted as follows:

$$V_i^{k+1} = \omega \times V_i^k + c_1 \times r_1 \times (P_{best}^k - X_i^k) + c_2 \times r_2 \times (G_{best}^k - X_i^k) \quad (25)$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \quad (26)$$

where  $V_i^k$  is the velocity of the  $i^{th}$  particle in the  $k^{th}$  iteration,  $\omega$  is the weight inertia factor,  $c_1$  and  $c_2$  are the acceleration factors,  $r_1$  and  $r_2$  are random numbers within  $[0..1]$ , and  $X_i^k$  is the position of the  $i^{th}$  particle in the  $k^{th}$  iteration.

During the updating process, the values of parameters such as  $\omega$  should be determined in a progressive form. Constants  $c_1$  and  $c_2$  show the random weights of the acceleration parts, which pull each particle towards the best individual and group answers. Generally, in order to increase the convergence characteristics, the inertia weight factor is designed in a linearly decreasing form. Reduction from  $\omega_{\max}$  to  $\omega_{\min}$  is accomplished as follows:

$$\omega^k = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} \times k \quad (27)$$

where  $k$  is the number of iteration and  $iter_{\max}$  is the maximum number of iterations [16].

### B) Solving EELD Problem through the Proposed Method using PSO Algorithm

The EELD problem solving process through the objective function expressed in per unit form based on the optimized PSO approach is as to the following steps:

Step 1) creating random initial population and particle initial velocity;

Step 2) calculating the cost, sorting the cost, and selecting  $P_{best}$  and  $G_{best}$  ;

Step 3) updating the position and the velocity of particles according to (25) and (26);

Step 4) correcting new particles positions to satisfy the problem constraints;

Step 5) jumping to step 2 if the program ending criterion is not achieved;

Step 6) applying the best values of the particles, which cause the objective function cost in per unit form minimization in (2), (3), (4), and (5) to obtain system optimum fuel cost in (\$/hr), the optimum amount of  $NO_x$  ,  $CO_x$  , and  $SO_x$  emissions in (ton/hr).

## 5. Numerical Experimentations

The proposed technique is applied on two different power systems as follows: A) System with three power units with network losses and objective function with three variables aiming economic load dispatch problem solving considering  $NO_x$  and  $SO_x$  emissions. B) System with six power units with network losses and objective function with four variables aiming economic load dispatch problem solving considering  $NO_x$  ,  $CO_x$  , and  $SO_x$  emissions.

For each sample of problem, 30 independent experimentations are conducted to compare the problem solving quality and convergence features. In all cases,  $c_1$  and  $c_2$  values are 2.1 and 1.9; and the weight inertia factors variation domain is [0.3 0.9]. Initial population size is 100 and the iteration number is 1000.

### A) System with three power units

In this section, it is aimed to optimize economic load dispatch problem considering  $NO_x$  and  $SO_x$  emissions in a system with three power units and network power losses. In order to have more accurate investigation of the optimized problem, nine independent case studies with different influence percentage of each function are conducted. Total system load is 850 MW. Input data and  $B$  coefficients of network losses are as in [19].

The case studies conducted in this section aim to decrease the plants fuel costs and the emission amount with different influence percentage on the objective function, simultaneously. The coefficients related to the influence percentage of each function are in Table 1.

The results of the above case studies are shown in Table 2. As it is obvious, in case study 1, it is aimed just to decrease the fuel costs. The system fuel cost, is  $8344.5871(\$/hr)$ , which is the least among the case studies. Decreasing the fuel costs influence from 100% in case 1 to 75% in case 2; and adding  $SO_x$  emission influence results in the fuel cost increases by  $3.2349(\$/hr)$ . As shown in Table 2, from case studies 1 to 6, and as the fuel costs influence percentage consecutive reduction, the fuel costs regularly increase in a way that it reaches from  $8344.5871(\$/hr)$  in case study 1 to  $8363.6921(\$/hr)$  in case study 6. In case study 7, as the fuel costs influence is increased by 16.6% in comparison with the case 6, proportionally, the fuel costs decreases by  $2.333(\$/hr)$ . The maximum fuel costs amount is obtained in case 9, where the influence coefficient on objective function is 0.0%.

Evaluating the results shown in Table 2 depicts that as the  $SO_x$  influence is increasing from 0.0% in case 1 to 25% in case 2, proportionally, the  $SO_x$  emission amount is decreased from  $9.01958(ton/hr)$  in case 1 to  $8.99748(ton/hr)$  in case 2.

Table.1.  
Influence percentage of each function in three units system

Case	$n_f$	$n_n$	$n_s$
1	1	0	0
2	0.75	0	0.25
3	0.75	0.25	0
4	0.5	0.25	0.25
5	0.4	0.2	0.4
6	0.33	0.33	0.33
7	0.5	0.5	0
8	0	1	0
9	0	0	1

In continuous, as the influence of  $SO_x$  emission is increased by 15% in case 5, the emission amount proportionally decreases by  $0.02215(ton/hr)$ . The least  $SO_x$  emission amount is achieved in case study 9, where it is aimed just to

decrease  $SO_x$  emission amount and is  $8.9659(\text{ton/hr})$   
 Maximum  $SO_x$  emission amount is obtained in case 1, where the influence coefficient of this emission is 0.0%. The least amount of  $NO_x$  emission is  $0.0959203(\text{ton/hr})$  achieved in case 8, where it is aimed just to decrease  $NO_x$  emission amount. As the influence percentage of  $NO_x$  emission is decreased from 100% in case 8 to 50% in case 7, the emission amount is increased to  $0.0959254(\text{ton/hr})$ . As shown in Table 2, the emission amount increases and reaches to  $0.09605(\text{ton/hr})$  as the  $NO_x$  influence percentage reach to 25% in case 3. Maximum amount of  $NO_x$  emission is reported in case study 1, where the influence percentage of this emission in objective function is 0.0%.

If it is aimed to equally consider the influence percentage of each function in the objective function, the best result is for case 6, where the influence of each three function is equal ( $n_f = n_n = n_s = 0.333$ ) or where there is no equations (13-14).

In Table 3, the results of case study 1, the minimum plants fuel costs considering no emission influence are compared with results of other intelligent algorithms such as BBO, OBBO, Tabu Search, NSGA-II. In this comparison, the objective function is just consisting of fuel costs function.

In Table 4, the results of case 6, that is the minimum cost of  $SO_x$  emission without considering other emissions and fuel cost and are compared with those of the other algorithms. As it is obvious, the proposed technique as well as OBBO approach is similarly able to find out the global optimum point better than BBO, Tabu search, and NSGA-II techniques.

Table.2.  
Results obtained in three units system with different combination percentages

Case Study	1	2	3	4	5	6	7	8	9
P <sub>1</sub>	435.1872	464.2551	490.9134	500.3264	504.1676	505.9775	501.4434	508.5703	552.1329
P <sub>2</sub>	299.9904	279.8189	265.3399	257.3302	253.9810	252.5721	256.5656	250.4495	219.5785
P <sub>3</sub>	130.6513	121.2015	108.7317	107.1916	106.6472	106.2258	106.8288	105.7226	92.8070
F <sub>FC</sub>	8344.5871	8347.822	8356.551	8360.801	8362.7380	8363.6921	8361.359	8365.108	8396.484
F <sub>NX</sub>	0.098686	0.09694	0.09605	0.09595	0.09595	0.0959268	0.09592	0.0959204	0.096823
F <sub>SX</sub>	9.01958	8.99748	8.98137	8.97692	8.97533	8.974626	8.97646	8.97367	8.96593
TP	865.8292	865.2756	864.9850	864.8483	864.7959	864.7758	864.8371	864.7459	864.5184
TC	0.725583	0.725220	0.71723	0.715986	0.717102	0.712234	0.707655	0.688114	0.72075
P <sub>loss</sub>	15.8292	15.2756	14.9850	14.8483	14.79596	14.77582	14.8371	14.74592	14.51842

\* P: Power [MW]; F<sub>FC</sub>: Fuel cost [\$/hr]; F<sub>NX</sub>: NO<sub>x</sub> emission amount [ton/hr]; F<sub>SX</sub>: SO<sub>x</sub> emission amount [kg/hr]; TP: total power [MW]; TC: total cost [PU]; P<sub>loss</sub>: power loss [MW];

Table.3.  
Minimum fuel cost for three units system

Units	PM	OBBO	BBO	Tabu Search	NSGA-II
P <sub>1</sub>	435.1872	435.1981	435.1966	435.69	436.366
P <sub>2</sub>	299.9904	299.9697	299.9723	298.828	298.187
P <sub>3</sub>	130.6513	130.6604	130.6600	131.28	131.228
F <sub>FC</sub>	8344.5871	8344.5875	8344.5927	5344.598	8344.651
F <sub>NX</sub>	0.09868611	0.098686	0.098686	0.09863	0.098922
F <sub>SX</sub>	9.0195847	9.021948	9.02195	9.02146	9.02541
TP	865.8292	865.8282	865.8289	865.798	865.781
P <sub>loss</sub>	15.8292	15.8282	15.8289	15.798	15.781

\*PM: Proposed Method;

Table.4.  
Minimum SO<sub>x</sub> emission for three units System

Units	PM	OBBO	BBO	Tabu Search	NSGA-II
P <sub>1</sub>	552.1329	552.1113	552.1111	549.247	538.527
P <sub>2</sub>	219.5785	219.4441	219.4441	234.582	227.817
P <sub>3</sub>	92.8070	92.9597	92.96053	81.893	98.185
F <sub>FC</sub>	8396.484	8396.4616	8396.4665	8403.485	8385.177
F <sub>NX</sub>	0.096823	0.096817	0.096817	0.096817	0.096325
F <sub>SX</sub>	8.965931	8.965931	8.965937	8.974	8.96670
TP	864.5184	864.5151	864.5158	865.722	864.528
P <sub>loss</sub>	14.5184	14.5151	14.5158	15.722	14.528

Table.5.  
Minimum  $NO_x$  emission for three units system

Units	PM	OBBO	BBO	Tabu search	NSGAI
$P_1$	508.5703	508.5800	508.5813	502.914	508.367
$P_2$	250.4495	250.4423	250.4433	254.294	250.444
$P_3$	105.7226	105.7228	105.7212	108.592	105.934
$F_{FC}$	8365.108	8365.1088	8365.1146	8371.143	8364.993
$F_{NX}$	0.095923	0.095923	0.095923	0.0958	0.095924
$F_{SX}$	8.97367	8.973662	8.973667	8.9860	8.97374
TP	864.7459	864.7451	864.7459	865.8	864.745
$P_{loss}$	14.7459	14.7451	14.7459	15.8	14.745

In Table 5, the results of case 8, that is the minimum  $NO_x$  emission levels without considering other emissions and fuel cost influences are compared with results of the other algorithms. In Table 6, the results obtained from case 6, is compared with those of other methods. In this case, it is aimed to obtain combinations of power generation aiming to simultaneously decrease emission amount and plants fuel costs with equal influence. The convergence pattern of the algorithm during program implementation for case studies detailed in Tables 3-6 are shown in Figure1.

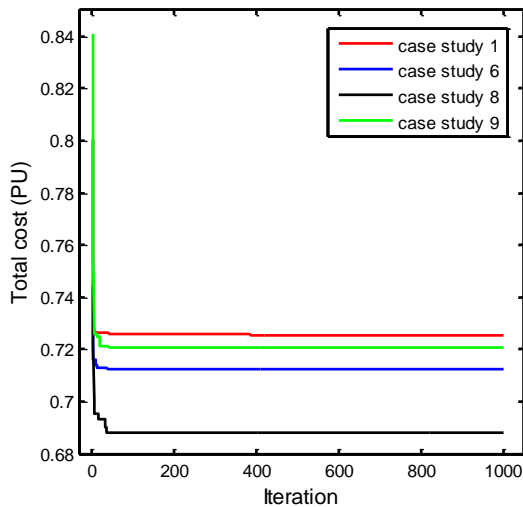


Fig. 1. Convergence pattern of algorithm with objective function in Per Unit form

### B) System with Six Power Units and Objective Function with Four Variables

In this section, it is aimed to optimize the economic load dispatch problem considering network losses,  $NO_x$ ,  $CO_x$  and  $SO_x$  emissions in a six-generator system. In order to investigate the optimization problem more accurately, sixteen case studies are conducted with different influence percentage of each function on the objective function. Total system provides 1800 MW load

amount. The input data and  $B$  factor for network losses are as in [18].

The case studies in this section are conducted to decrease fuel cost and different emission percentages simultaneously and separately according to Table 7 and the results are presented in Table 8.

Case study 10, in Table 7 aims to 100% decrease the fuel costs of the plants ( $F_{fc}$ ) considering no emission influence. The cost is 18900.94( $\$/hr$ ), which is the least among all case studies in this section. In case 11, it is tried just to decrease  $NO_x$  emission and the fuel cost is 0.0%. Comparing this with the results of case 10, shows the fuel cost is increased to 81.958( $\$/hr$ ) and the

$NO_x$  emission amount decreases to 61.932( $kg/hr$ ) and is the least among the case studies. In case study 12, it is aimed just to decrease  $CO_x$  emission while the influence of other functions is 0.0%. The fuel costs increases by 2.12( $\$/hr$ ) and the  $CO_x$  emission decreases to 80.768( $kg/hr$ ), which is the

least among case studies. The most important point in comparing the results of these three case studies is the fuel costs increase in case studies 11 and 12, in which the influence percentage of fuel costs is 0.0%. However, as it is shown, the fuel cost increase in 11 is up to 81.958( $\$/hr$ ) in comparison with case 10, while it is 2.12( $\$/hr$ ) in 12. The investigation results show that the coefficients, which cause fuel cost reduction, simultaneously cause  $CO_x$  and  $SO_x$  emission reduction since the similarity of  $CO_x$  emission with fuel cost coefficients. This is not faced in case studies conducted on three-generator system.

In case study 13, it is aimed to just decrease the  $SO_x$  emission considering no other function. With comparing the results of cases 13 and 10 it is seen that, as expected, as the fuel cost influence is 100% decreased, the fuel cost is increased by



0.04(\$/hr) and in turn, as the  $SO_x$  emission influence is increased by 100%, its amount is reduced by 0.003(kg/hr). These variations are small in comparison with the other case studies due to the similarity of fuel cost and  $SO_x$  emission as mentioned before. The powers of the plants, which cause fuel cost reduction, simultaneously decrease the  $SO_x$  emission amount.

It is obvious from case studies 10, 11, and 14 that fuel cost influence percentage is decreased by 50% in 14 in comparison with 10 and the  $NO_x$  emission influence is increased by 50%. Therefore, according to the claims of this paper, the fuel costs in case study 14 should increase and  $NO_x$  emission should be decreased in comparison with 10. In case 14 and in compare with the case 11, the  $NO_x$  emission is decreased by 50% and the fuel cost percentage is increased by 50%. Therefore, the fuel cost reduction and  $NO_x$  emission increase in case 14 in comparison with 11 which are expected. A review on results of Table 8 shows that the fuel costs in case study 14 is increased by 75.013(\$/hr) and  $NO_x$  emission is decreased by 61.134(kg/hr) in comparison with case study 10. Comparing case 10 results with that of case study shows that fuel costs are decreased by 6.945(\$/hr) and  $NO_x$  emission is increased by 0.7977(kg/hr).

Comparing results obtained from case study 19 with that of 20 shows that the fuel cost increases by 46.6(\$/hr) in 20 as fuel costs influence decreases from 60% to 33%. As  $NO_x$  emission influence increases from 0% to 33% in case study 20, its amount decreases by 43.961(kg/hr) as expected. As it is obvious,  $CO_x$  emission amount proportionally increases from 1036.15(kg/hr) to 65.18882(ton/hr) as the  $CO_x$  emission influence decreases from 20% to 0% in case 20.

As shown in table 8, in case study 18 the influence of  $NO_x$  emission is 10% and as it decreases to 0% in case 19 the emission amount increases by 22.6882(kg/hr).

In case study 20, the influence percentage of  $NO_x$  emission increases up to 33%, which is 33% more than case 19 and 23% higher than 18. Therefore, as it was expected, the  $NO_x$  emission amount in case 20 decreases by 43.961(kg/hr) in

comparison with case study 19 and by 21.2728(kg/hr) with case 18. This shows the proportionality exists between the influence percentage in objective function and the value obtained for the emission. In continuous, as the influence of this emission decreases from 33% in case study 20 to 20% in case 22, the emission amount proportionally increases by 14.862(kg/hr).

Table.6.

Best Compromise Solution of Fuel cost,  $SO_x$ , and  $NO_x$  emission for Three units System

Units	PM	OBBO	BBO	NSGA-II
P <sub>1</sub>	505.9775	507.1197	507.1195	496.328
P <sub>2</sub>	252.5721	251.6420	251.6426	260.426
P <sub>3</sub>	106.2258	106.0003	106.0004	108.144
F <sub>FC</sub>	8363.692	8364.306	8364.312	8358.89
F <sub>NX</sub>	0.095926	0.095924	0.095924	0.09599
F <sub>SX</sub>	8.974626	8.974195	8.974201	8.97870
TP	864.7758	864.7625	864.7525	864.898
P <sub>loss</sub>	14.7758	14.7625	14.76258	14.898

Table.7.

Influence percentage of each function in six units system

Case	n <sub>r</sub>	n <sub>n</sub>	n <sub>c</sub>	n <sub>s</sub>
10	1	0	0	0
11	0	1	0	0
12	0	0	1	0
13	0	0	0	1
14	0.5	0.5	0	0
15	0.5	0	0.5	0
16	0.5	0	0	0.5
17	0	0.5	0	0.5
18	0.7	0.1	0	0.2
19	0.6	0	0.2	0.2
20	0.33	0.33	0	0.33
21	0	0.33	0.33	0.33
22	0.5	0.2	0.2	0.1
23	0.4	0.2	0.2	0.2
24	0.3	0.1	0.2	0.4
25	0.25	0.25	0.25	0.25

This proportionality between influence percentage in objective function and the obtained emission amount well shows high capability of the proposed method in per unit form in combining several emissions with different percentages with fuel costs.

### 6. Conclusion

This paper proposes per unit coding for converted multi-objective functions to a single-objective model for solving multi-objective optimization problems. In this paper to show the effectiveness of the proposed coding, it is applied to combined economic emission load dispatch problems.

In proposed coding, it is possible to combine any number of functions. Each function is transformed into per unit form based on its own maximum value and has a value from 0 to 1 per unit and totally forms the objective function. One of the advantageous of the proposed method is its ability in combining several functions with desired influence percentages and possibility of combining several independent functions with high accuracy. In this method, penalty factor application is not required. In order to depict the advantages of the proposed method, 25 independent case studies with different influence percentages of each function are investigated. In the first section, nine cases are conducted on a three-generator system and three independent functions results of which show the success of the proposed technique and

proportionality of the cases and the mentioned advantages. In the second section, sixteen independent cases are conducted on a six-generator system with four independent functions. Results show the possibility of combining more than three independent functions in the objective function. Comparing results with different influence percentages in objective function shows the linear relation between influence percentage and optimized values. Unlike other approaches, the objective function of the proposed method is not in (\$/hr) and is in per unit form. The results prove the effectiveness of the proposed coding and show that it could be used as a reliable tool for combined multi-objective function in optimization problems.

Table.8.  
Results obtained in six units system with different combination percentages

Case Study	10	11	12	13	14	15	16	17
P <sub>1</sub>	250.000	230.5542	250.0000	250.0000	234.8706	249.9999	250.0000	250.0000
P <sub>2</sub>	230.000	230.0000	230.0000	230.0000	230.0000	229.9999	230.0000	230.0000
P <sub>3</sub>	500.000	500.0000	500.0000	500.0000	500.0000	500.0000	499.9998	500.0000
P <sub>4</sub>	265.000	265.0000	265.0000	265.0000	265.0000	265.000	265.000	265.0000
P <sub>5</sub>	419.3343	500.0000	432.4286	419.5379	497.2769	430.7080	418.8232	430.4048
P <sub>6</sub>	277.2856	227.4992	265.5453	277.1018	225.2794	267.0791	277.7468	267.3498
F <sub>FC</sub>	18900.94	18982.89	18903.06	18900.98	18975.95	18902.55	18901.00	18902.47
F <sub>NX</sub>	2.18437	2.122438	2.167115	2.184075	2.123236	2.169226	2.185106	2.169598
F <sub>CX</sub>	64.2228	65.95838	64.14203	64.22029	65.83152	64.14340	64.22926	64.14393
F <sub>SX</sub>	11.34014	11.3879	11.34130	11.34014	11.383913	11.34101	11.34014	11.34096
TP	1941.62	1953.053	1942.974	1941.639	1952.427	1942.787	1941.570	1942.754
TC	0.855078	0.655946	0.577030	0.855120	0.757332	0.716097	0.855099	0.716115
P <sub>loss</sub>	141.6200	153.0534	142.9740	141.6398	152.4270	142.7872	141.5702	142.7546
Case Study	18	19	20	21	22	23	24	25
P <sub>1</sub>	250.0000	250.0000	244.0527	249.9999	249.9999	250.0000	250.9998	250.0000
P <sub>2</sub>	230.0000	230.0000	230.0000	230.0000	230.0000	230.0000	230.0000	229.9999
P <sub>3</sub>	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	500.0000	499.9999
P <sub>4</sub>	265.0000	265.000	265.0000	265.0000	256.0000	265.000	265.000	265.000
P <sub>5</sub>	448.5481	427.6561	481.6364	456.1017	456.3433	451.7012	456.4921	451.5359
P <sub>6</sub>	251.3043	269.8063	228.9465	244.7102	244.5000	248.5456	244.3706	248.6898
F <sub>FC</sub>	18911.03	18901.82	18948.42	18916.77	18916.97	18913.27	18917.11	18913.15
F <sub>NX</sub>	2.150359	2.173048	2.129087	2.144131	2.143949	2.147634	2.143837	2.147772
F <sub>CX</sub>	64.26214	64.15267	65.18882	64.39959	64.40483	64.31330	64.40808	64.31040
F <sub>SX</sub>	11.34594	11.34060	11.36785	11.34931	11.34943	11.34725	11.34950	11.34718
TP	1944.852	1942.462	1949.635	1945.811	1945.843	1945.246	1945.862	1945.226
TC	0.836444	0.799527	0.790879	0.699426	0.789524	0.76184	0.789528	0.761849
P <sub>loss</sub>	144.8525	142.4625	149.6357	145.8119	145.8434	145.2468	145.8628	145.2260

\*F<sub>CX</sub>: CO<sub>x</sub> emission amount [ton/hr];

## References

- [1] H. Aliyari, R. Effatnejad, "Novel Optimization Based On the Genetic Algorithm for Economic Dispatch of 30 Bus IEEE Test Systems", *Majlesi Journal of Energy Management*, Vol. 3, No. 3, pp. 1-5, 2014.
- [2] A. Farag, S. Al-Baiyat and T.C. Cheng, "Economic Load Dispatch Multi objective optimization Procedures Using Linear Programming Techniques", *IEEE Trans Power Syst.*, Vol. 10, pp. 731-738, 1995.
- [3] D. Srinivasan, C. S. Chang and A. C. Liew, "Multi objective Generation Schedule using Fuzzy Optimal Search Technique", *IEE Proceedings on Generation, Transmission & Distribution*, Vol. 141, pp. 231-241, 1994.
- [4] C. M. Huang, H. T. Yang and C. Huang, "Bi-Objective Power Dispatch Using Fuzzy Satisfaction Maximizing Decision Approach", *IEEE Transaction on Power Systems*, Vol. 12, pp. 1715-1721, 1997.

- [5] S. Jiang, Y. Shen, "A novel hybrid particle swarm optimization and gravitational search algorithm for solving economic emission load dispatch problems with various practical constraints", *Electrical Power and Energy Systems*, Vol. 55, pp. 628-644, 2014.
- [6] K. Bhattacharjee, A. Bhattacharya, S. Halder nee Dey, "Solution of Economic Emission Load Dispatch problems of power systems by Real Coded Chemical Reaction algorithm", *Electrical Power and Energy Systems*, Vol. 59, pp. 176-187, 2014.
- [7] A. Dogan, S. Ozyon, C. Yasar, T. Liao, "Artificial bee colony algorithm with dynamic population size to combined economic and emission dispatch problem", *Electrical Power and Energy Systems*, Vol. pp. 144-153, 2014.
- [8] T. Niknam, H. Doagou Mojarrad, B. Bahmani Firouzi, "A new optimization algorithm for multi-objective Economic/Emission Dispatch", *Electrical Power and Energy Systems*, Vol.46, pp. 283-293, 2013.
- [9] H. Hamed, "Solving the combined economic load and emission dispatch problems using new heuristic algorithm", *Electrical Power and Energy Systems*, Vol. 46, pp. 10-16, 2013.
- [10] S. F. Brodesky and R. W. Hahn, "Assessing the Influence of Power Pools on Emission Constrained Economic Dispatch", *IEEE Transaction On Power Systems*, Vol. 1, pp. 57-62, 1986.
- [11] G. P. Granelli, M. Montagna, G. L. Pasini and P. Marannino, "Emission Constrained Dynamic Dispatch", *Electric Power Systems Research*, Vol. 24, pp. 56-64, 1992.
- [12] A. Farag, S. Al-Baiyat and T.C. Cheng, "Economic Load Dispatch Multi-objective optimization Procedures Using Linear Programming Techniques", *IEEE Transaction on Power Systems*, Vol. 10, pp. 731-738, 1995.
- [13] J. Zahavi and L. Eisenberg, "Economic-Environmental Power Dispatch", *IEEE Trans. on Power Systems*, Vol. 5, pp. 485-489, 1995.
- [14] C. S. Chang, K. P. Wong and B. Fan, "Security-Constrained Multi-objective Generation Dispatch Using Bicriterion Global Optimization", *IEE Proceedings on Generation, Transmission & Distribution*, Vol. 142, pp. 406-414, 1995.
- [15] N. Kumarappan, M. R. Mohan and S. Murugappan, "ANN approach applied to combined economic and emission dispatch for large scale systems", in *Proc. Int. Joint Conf. Neural Networks*, Honolulu, Vol. 1; pp. 323-327, 2002.
- [16] N. Ghorbani, P. Farhadi, "Combined Heat and Power Economic Dispatch Problem Solution using Particle Swarm Optimization with Unique Inertia Factor", *Majlesi Journal of Energy Management*, in press, 2016.
- [17] N. Ghorbani, S. Vakili, E. Babaei, A. Sakhavati, "Particle Swarm Optimization with Smart Inertia Factor for Solving Nonconvex Economic Load Dispatch Problems", *Int Trans on Elect Energy Syst*, Vol. 24, pp. 1120-1133, 2014.
- [18] J. Dhillon and S.K. Jain, "Multi-Objective Generation and Emission Dispatch Using NSGA-II", *International Journal of Engineering and Technology*, Vol. 3, pp. 200-211, 2011.
- [19] R. T .F. AhKing and H. C. S. Rughooputh, "Multi-objective Evolutionary Algorithm for Environmental/Economic Dispatch", *Congress on Evolutionary computation*, Vol. 2, pp. 1108-14, 2003.
- [20] A. Bhattacharya, P. K. Chattopadhyay, "Oppositional Biogeography-Based Optimization for Multi-objective Economic Emission Load Dispatch", *Annual IEEE India Conference (INDICON)*, pp. 1-6, 2010.
- [21] C. A. Roa-Sepulveda, E. R. Salazar-Nova, E. Gracia-Caroca, U. G. Knight, "A Coonick, Environmental Economic Dispatch via Hopfield Neural Network and Tabu Search", *UPEC'96*, pp. 1001-1004, 1996.
- [22] P. S. Kulkarni, A. G. Kothari, D. P. Kothari, "Combined economic and emission dispatch using improved back propagation neural network", *Int. J. Electr. Machines Power System*, Vol. 28, pp. 31-44, 2000.
- [23] R. Kumar, A. Sadu, R. Kumar, S.K. Panda, "A novel multi-objective directed bee colony optimization algorithm for multi-objective emission constrained economic power dispatch", *Electrical Power and Energy Systems*, Vol. 43, pp. 1241-1250, 2012.
- [24] A. Bhattacharya, P.K. Chattopadhyay, "Solving economic emission load dispatch problems using hybrid differential evolution", *Applied Soft Computing*, Vol. 11, pp. 2526-2537, 2011.
- [25] A. Bhattacharya, P. K. Chattopadhyay, "Application of biogeography-based optimization for solving multi-objective economic emission load dispatch problem", *Electric Power Components & Systems*, Vol. 38, pp. 826-850, 2010.
- [26] C. Yasar, "A pseudo spot price of electricity algorithm applied to environmental economic active power dispatch problem", *Turk J Elec Eng & Comp. Sci.*, Vol. 20, pp. 990-1005, 2012.
- [27] L. Kanagasabai, B. Ravindranath Reddy, M. Surya Kalavathi, "Improved Hybrid Particle Swarm Optimization-Based of Fish School Search Algorithm for Solving Optimal Reactive Power Dispatch Problem", *Majlesi Journal of Energy Management*, Vol. 3, No. 1, pp. 51-57, 2014.
- [28] L. Slimani, T. Bouktir, "Economic power dispatch of power systems with pollution control using artificial bee colony optimization", *Turk J. Elec. Eng. & Comp. Sci.*, Vol. 21, pp. 1515-1527, 2013.
- [29] M. Basu, "Dynamic economic emission dispatch using nondominated sorting genetic algorithm-II", *Electrical Power and Energy Systems*, Vol.30, pp. 140-149, 2008.
- [30] M. Basu, A. Bhattacharya, S. Chowdhury and S.D. Chowdhury, "Planned Scheduling for Economic Power Sharing in a CHP-Based Micro-Grid", *IEEE Trans. on Power Systems*, Vol. 27, pp. 30-38, 2012.