pp. 215:222

Construction of Multi-Resolution Wavelet Based Mesh Free Method in Solving Poisson and Imaginary Helmholtz Problem Compare with FEM

Mohammad Reza Yousefi[*1,3], Amin Dehghani[2], Ali Asghar Amini[1,3]

[1]Department of Electrical Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Iran, mr-yousefi@iaun.ac.ir
[2] Department of Psychological and Brain Sciences, Dartmouth College, Hanover, NH, USA, amin.dehghani@dartmouth.edu
[3]Smart Micro Grid Research Center, Najafabad Branch, Islamic Azad University, Iran, amini33@yahoo.com
* Corresponding author; mr-yousefi@iaun.ac.ir

Abstract

In this paper, we propose a new multi-resolution wavelet based mesh free method for numerical analysis of electromagnetic field problems. In problems with variable object geometries or mechanical movements, the mesh free methods yield more accurate simulation results compared to the finite element approach in solving the inverse problem, because they are based on a set of nodes without using the connectivity of the elements. The wavelet based mesh free method requires effectively no local integration in the vicinity of nodes in numerical implementations. Moreover, wavelets give a more efficient approximation using multi-resolution analysis. On the other hand, boundary condition constraints are difficult to be applied on the wavelet based mesh free method. In order to apply boundary and interface conditions, we utilize a new form of jump functions in the set of basic functions. The boundary and interface conditions are applied effectively using the suggested slope jump functions. The simulation results of the proposed method using two different jump functions in solving some simple boundary problems are compared. The results are validated by analytical solutions. The results of this study can be used in future for inverse problem of Magnetic resonance electrical impedance tomography (MREIT) studies as an imaging technique for reconstructing the cross-sectional conductivity distribution of a human brain or body using EIT technique integrated with the MRI.

Keywords: Mesh Free Method, Wavelet Method, Multi-Resolution Analysis, Galerkin Method
Article history: Received 03-Nov-2021; Revised 19-Nov-2021; Accepted 20-Nov-2021. Article Type: Research paper

1. Introduction

The finite element method (FEM) is a conventional analysis approach that has been successfully applied to various problems in science and engineering [1,2]. This method relies on the primary idea of replacing a continuous function over the entire solution domain by a piecewise continuous approximation, typically using polyno-mials, over a set of sub-domains called finite elements [3,4]. FEM requires a structure of interconnected elements via nodes called finite element mesh which is considered as a constraint in some applications [5]. For example, when FEM is applied to solve partial differential equations (as a forward problem for providing simulation data in solving the inverse problem) involving moving objects or objects with changing geometrical appearance, mesh distortion is inevitable and susceptible to producing error in numerical results [6]. Moreover, in FEM, meshes must be propagated in some regions of the solution domain (e.g. near a boundary or a sharp edge) leaded to increase the computational complexity of the FEM model [7].

The mesh free (MF) methods are effective in problems with variable object geometries or mechanical movements since they are purely based on nodes distribution. This eliminates all problems related to the mesh shape or size. So far, different MF methods have been proposed to solve some field related engineering problems. The element free Galerkin (EFG) [8] and the wavelet based mesh free

(WMF) [9,10] are two well-known methods of this family which have been suggested to solve electromagnetic field problems.

The EFG is considered as a MF method, because it requires only a set of nodes distributed over the entire solution domain. For the numerical implementation of this method, it is necessary to calculate the local summation in the vicinity of nodes. In contrast to EFG, the WMF method requires no local summation in the vicinity of nodes in numerical implementations [11]. Thus nodes displacement due to variation of objects geometry results in less approximation error in wavelet based method with respect to EFG [12]. However, most of the WMF formulae have been derived from higher order differentials of the basic functions to calculate the coefficient matrix which causes large variations in these coefficients and instability in numerical implementation. Consequently, some reduced WMF formulations have been suggested in [13] and [14]. Moreover, these reduced approaches are inefficient in enforcing boundary and interface conditions in materials with different conductivity. For solving this problem, employing jump functions [14] or combination with the FEM [15] has been suggested. The jump function used in [14] was the polynomial form.

Yang et al. [11,15,16] have used a set of orthogonal scaling basis functions, produced by the dyadic translations for realizing the WMF approximation in electromagnetic field application. This method was a single scale wavelet based method due to non-orthogonality of the scaling functions at different scales. Considering the fact that wavelets provide multi-resolution signal analysis via wavelet functions in different resolution levels, the accuracy of MF approximation may be improved by choosing the optimal number of resolution levels in each location of the solution domain. Furthermore, if the basic wavelet is properly chosen, one might be able to reduce the number of coefficients without serious approximation error [16,17]. If the basic wavelet is well localized (i.e., it approaches, zero rapidly away from the origin), then many of the coefficients with large translation will be negligible. Likewise, coefficients with large dilation will usually be small as well, since the wavelet basis function then becomes extremely narrow. In other words, very sparse coefficient matrices can be achieved by employing the multi-resolution analysis [18].

In this paper, a new multi-resolution WMF approximation is proposed using both wavelet and scaling basis functions for numerical analysis of the electromagnetic field problems.

Moreover, for boundary and interface conditions enforcement in materials with different conductivity, we utilize a new form of jump functions in a specified procedure. For this purpose, we first extract the formulation of the proposed MF method. Then the set of scaling and wavelet basis functions is presented for supporting the multi-resolution WMF approximation. Finally, in order to apply boundary condition, different polynomial and slope jump functions is added to the set of wavelet basis functions. The rest of this paper is organized as follows. In Section 2 we introduce the MF method formulation using the Galerkin approach. Section 3 presents the orthogonal set of multi-resolution wavelet basis functions. The simulation results of the proposed method using two different jump functions with the aim of enforcing boundary conditions for solving some simple boundary problems are given in Section 4 and the results are compared with analytical solution. Finally, Section 5 is devoted to concluding remarks.

2. Mesh free wavelet Galerkin method

In this section, the MF formulation is provided with reference to the Helmholtz differential equation which is usually used for modelling the electromagnetic field problems. In a one-dimensional (1-D) region enclosed by two boundary points $[x_1,x_N]$, it is defined as:

$$-\nabla.(\alpha(x)\nabla u(x)) + \beta(x)u(x) = f(x) \quad (1)$$

where α and β are the electromagnetic properties of the solution domain, f is the excitation function and u is an unknown scalar potential. Robin boundary condition (a weighted combination of Dirichlet and Neumann boundary conditions) is considered at start and end points of the solution domain:

$$\begin{aligned} \alpha(x_1)u'(x_1) + \gamma_1 u(x_1) = q_1 \\ \alpha(x_N)u'(x_N) + \gamma_2 u(x_N) = q_2 \end{aligned} \quad (2)$$

where γ_1, q_1, γ_2 and q_2 are boundary coefficients. The first step of MF approximation is a linear combination of the unknown function in terms of linearly independent basis functions [19]:

$$u^{MF}(x) = \sum_{j=1}^{M} c_j \phi_j(x) \quad (3)$$

where $\phi_j(x)$ are 1-D basis functions, M is the number of basic functions and c_j is unknown coefficients. These unknown coefficients can be obtained by the Galerkin method using the basic functions as weights of the residual for **Error! Reference source not found.**. Yang et al. [9] have proposed a polynomial jump function for applying boundary constraints in the WMF approximation. In a normalized domain, for the start and end boundary points, it is defined as:

$$\phi^s_{poly}(x) = -x^3 + 3x^2 - 3x + 1 \quad (4)$$

$$\varphi^{e}_{poly}(x) = (x+1)^3 - 3(x+1)^2 + 3(x+1) - 1$$

The jump function corresponding to start and end boundary point is illustrated in Fig. 1-a).

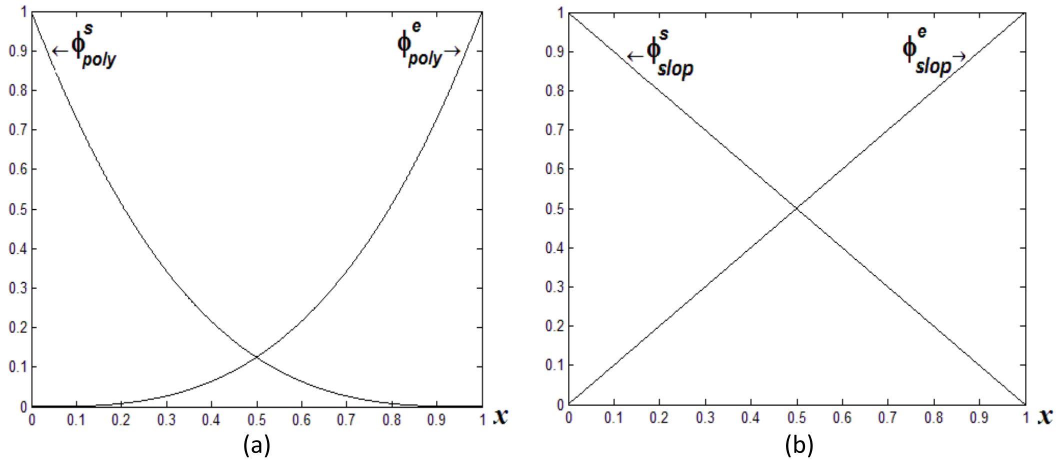

Fig. 1. (a) Polynomial and (b) slope jump functions corresponding to start and end boundary points of a normalized domain.

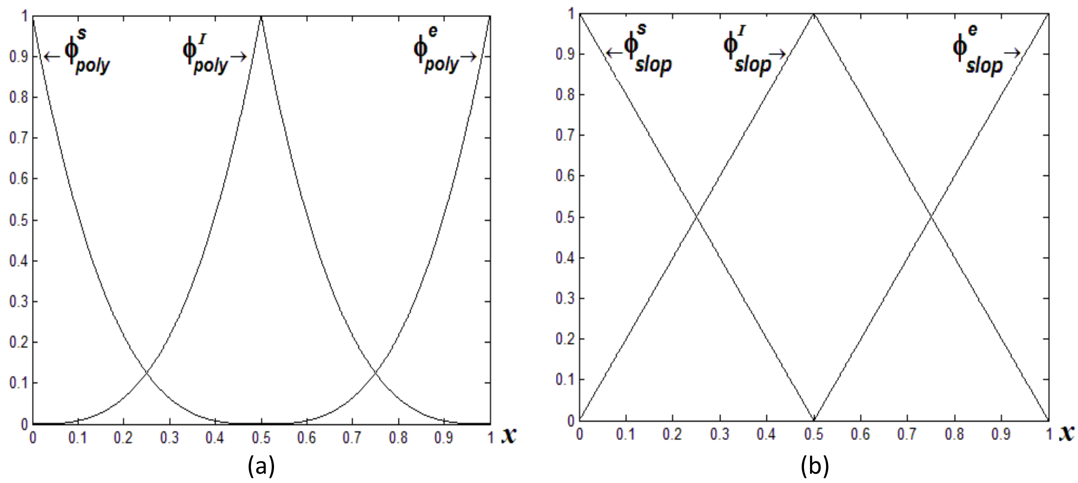

Fig. 2. (a) Polynomial and (b) slope boundary and interface jump functions of a normalized domain.

The polynomial jump function has been also suggested for applying interface conditions in [15]. The interface jump function for an interface is assumed at x=0.5 is shown in Fig. -a). This form of jump function meets the first three conditions above; however, the WMF approximation with the polynomial jump term will only approximate a derivative in the form of a Heaviside step function [20]. Moreover, employing this form of jump function into WMF approximation may cause numerical instability due to large variation of the higher order derivatives of the jump term in the medium interfaces. For eliminating this problem and supporting the derivative in the form of a Heaviside shape function exactly, we suggest using the slope jump function similar to the standard finite element shape functions for applying boundary and interface conditions:

$$\phi^{s}_{slop}(x) = 1 - x \qquad (5)$$
$$\phi^{e}_{slop}(x) = x$$

The slope jump function corresponding to start and end boundary point of a normalized domain is illustrated in Fig. -b). Fig. -b) demonstrates the slope jump functions when the interface is assumed at the midpoint of a normalized domain. Compared to polynomial jump function, we expect more accurate result when the proposed slope jump function is used to apply boundary and interface conditions; because the second derivative of this function appears as zero in (1). That means utilizing the slope jump function has less effect on the WMF approximation.

Therefore, by applying boundary conditions at the first and last boundary point via jump functions $\phi^{s}(x)$ and $\phi^{e}(x)$ and similarly $\phi^{I}_{k}(x)$ at the kth medium interface, the expansion (3) is modified as:

$$u^{MF}(x) = \sum_{j}\sum_{i} d_{j,i}\psi^{j}_{i}(x) + \sum_{i} a_i \phi^{J}_{i}(x) + b_1\varphi^{s}(x) + b_2\varphi^{e}(x) + \sum_{k} e_k \varphi^{I}_{k}(x) \qquad (6)$$

where b_1, b_2 and e_k are unknown boundary and interface coefficients, respectively. After inserting these jump functions into the multi-resolution WMF basis set, the basis set in (6) will be remain linearly independent [21].

3. Numerical Results

In order to compare the result of enforcing boundary and interface conditions using these two different jump functions, they have been tested in solving two simple boundary problems. First, we consider the 1-D Laplace equation:

$$\nabla.(\alpha(x)\nabla u(x)) = 0 \quad (7)$$

In (7), $\alpha(x)$ is changed according to the **Error! Reference source not found.**. The Dirichlet boundary conditions considered at the start and end boundary points:

$$u|_{x=0} = 0\text{V}$$
$$u|_{x=2} = 1\text{V} \quad (8)$$

The analytical solution of this problem is:

$$u(x) = \begin{cases} \dfrac{\alpha_2}{\alpha_1+\alpha_2}x & 0 \le x \le 1 \\ \dfrac{\alpha_1}{\alpha_1+\alpha_2}x - \dfrac{\alpha_1-\alpha_2}{\alpha_1+\alpha_2} & 1 \le x \le 2 \end{cases} \quad (9)$$

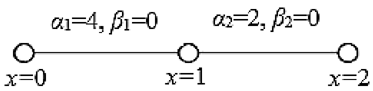

Fig. 3. The geometry of the 1-D test problem for solving by the multi-resolution WMF method.

In the second simulation, we change the equation from Laplace to Helmholtz:

$$-\nabla.(\alpha(x)\nabla u(x)) + \beta(x)u(x) = 0 \quad (10)$$

where $\alpha(x)$ is the same as the previous problem and $\beta(x)$=0.1 is a constant throughout the solution domain (β_1=β_2=0.1). The boundary conditions are assumed to be the same as the previous problem. The analytical solution of this problem is by (11).

In order to provide the set of basic functions, we decided to use Daubechies wavelets which are a family of orthogonal and compact support wavelets with highest number of vanishing moments for a given support width. The db3 wavelet and related scaling function are employed to represent the multi-resolution WMF basis set.

The numerical result of solving the Laplace and Helmholtz equations presented in (7) and (11) are compared with the analytical solution in Table 1. This result shows the effectiveness of the slope compared to polynomial jump functions. To confirm the validity of this observation, we changed the boundary condition at the end point from Dirichlet to Neumann. The result of this comparison is also given in Table 1 which shows the ability of the proposed slope jump functions in approximating the analytical solution in all presented problems. This observation supports our hypothesis that the slope jump function has less effect on the WMF approximation. The lower mean error rate resulted from solving Laplace equation compared to the Helmholtz equation has confirmed this hypothesis because the slope jump function has no effect on the WMF approximation [the variation of these jump functions and their derivation was eliminated from **Error! Reference source not found.**] in contrast to the polynomial jump functions.

In the next simulation, the slope jump functions are employed on the boundaries and interfaces without any constraint since the second derivative of this function is zero in (1). Therefore, the desired accuracy in approximation is achievable. The result of the proposed multi-resolution WMF method utilizing slope jump functions for approximating the analytical solution **Error! Reference source not found.** is shown in Fig. and for analytical solution (11) is shown in Fig. . Furthermore, by changing the boundary constraint at the end point from Dirichlet to Neumann, simulation results are compared with analytical solution in **Error! Reference source not found.**.

These results show the effectiveness of utilizing the proposed slope jump functions in accurately approximating the analytical solution in all presented problems. We are limited to use the polynomial jump functions in the medium interfaces because it causes instability in numerical implementation.

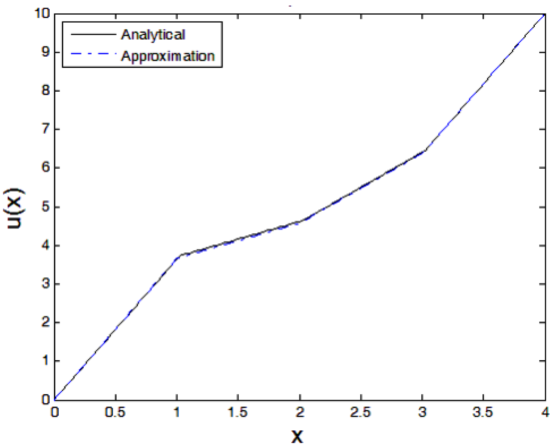

Fig. 4. The result of multi-resolution WMF approximation using slope jump functions in solving the 1-D Laplace equation with Dirichlet boundary conditions, are compared to analytical solution.

$$
u(x)=\begin{cases}\dfrac{2\sqrt{\alpha_2\beta_2}}{\zeta}\left[e^{\sqrt{\frac{\beta_1}{\alpha_1}}x}-e^{\sqrt{\frac{\beta_2}{\alpha_2}}x}\right] & 0\le x\le 1\\ \dfrac{1}{\zeta}\left[\sqrt{\alpha_2\beta_2}\left(e^{\sqrt{\frac{\beta_1}{\alpha_1}}}-e^{-\sqrt{\frac{\beta_1}{\alpha_1}}}\right)+\sqrt{\alpha_1\beta_1}\left(e^{\sqrt{\frac{\beta_1}{\alpha_1}}}+e^{-\sqrt{\frac{\beta_1}{\alpha_1}}}\right)\left(e^{\sqrt{\frac{\beta_2}{\alpha_2}}(x-1)}-e^{\sqrt{\frac{\beta_2}{\alpha_2}}(1-x)}\right)\right] & 1\le x\le 2\end{cases} \tag{11}
$$

$$
\zeta=\sqrt{\alpha_2\beta_2}\left[e^{2\sqrt{\frac{\beta_1}{\alpha_1}}}-e^{-2\sqrt{\frac{\beta_1}{\alpha_1}}}\right]+\sqrt{\alpha_1\beta_1}\left[e^{2\sqrt{\frac{\beta_2}{\alpha_2}}}-e^{-2\sqrt{\frac{\beta_2}{\alpha_2}}}\right]
$$

Table 1.
The multi-resolution WMF approximation using two different jump functions compared to analytical solution in solving the Laplace and Helmholtz equations with Dirichlet and Neumann boundary conditions.

Equation	**Electromagnetic properties**		**Boundary conditions**		**Number of basic functions**							**Mean error rate between approximation and analytical solution in:**		**Error rate between approximation and analytical solution at end boundary point in:**	
					Wavelet functions	**Scaling functions**	**Boundary jump functions**		**Interface jump functions**		**Total**				
	α	***β***	**Start point**	**End point**			**Polynomial**	**Slope**	**Polynomial**	**Slope**		u^{MF}	$\frac{\partial u^{MF}}{\partial x}$	$u^{MF}\big\|_{x=2}$	$\frac{\partial u^{MF}}{\partial x}\big\|_{x=2}$
Laplace	*	0	$u\|_{x=0}=0\text{V}$	$u\|_{x=2}=1\text{V}$	7	1	2	-	-	-	10	16.42%	37.29%	0.000%	125.0%
					7	1	-	2	-	-	10	8.433%	22.12%	0.000%	25.00%
					7	1	2	-	1	-	11	26.17%	76.47%	0.000%	350.9%
					7	1	-	2	1	-	11	9.761%	26.18%	0.000%	50.00%
					7	1	-	2	-	1	11	0.000%	0.005%	0.000%	0.000%
			$u\|_{x=0}=0\text{V}$	$\frac{\partial u}{\partial x}\big\|_{x=2}=1\text{V/m}$	7	1	2	-	-	-	10	30.44%	39.99%	55.00%	0.000%
					7	1	-	2	-	-	10	25.13%	26.39%	33.33%	0.000%
					7	1	2	-	1	-	11	38.22%	55.99%	77.77%	0.000%
					7	1	-	2	1	-	11	20.39%	24.00%	33.33%	0.000%
					7	1	-	2	-	1	11	0.000%	0.005%	0.000%	0.000%
Helmholtz	*	0.1	$u\|_{x=0}=0\text{V}$	$u\|_{x=2}=1\text{V}$	7	1	2	-	-	-	10	15.95%	36.30%	0.000%	116.6%
					7	1	-	2	-	-	10	8.904%	23.11%	0.000%	27.78%
					7	1	2	-	1	-	11	25.72%	75.65%	0.000%	333.2%
					7	1	-	2	1	-	11	9.345%	25.38%	0.000%	44.43%
					7	1	-	2	-	1	11	0.126%	0.604%	0.000%	2.705%
			$u\|_{x=0}=0\text{V}$	$\frac{\partial u}{\partial x}\big\|_{x=2}=1\text{V/m}$	7	1	2	-	-	-	10	29.54%	38.87%	53.84%	0.000%
					7	1	-	2	-	-	10	28.17%	29.34%	38.47%	0.000%
					7	1	2	-	1	-	11	37.62%	55.54%	76.92%	0.000%
					7	1	-	2	1	-	11	19.14%	22.29%	30.76%	0.000%
					7	1	-	2	-	1	11	1.267%	2.007%	2.780%	0.000%

* According to **Error! Reference source not found.**.

Table 2.
The effect of increasing number of interfaces jump function (the slope form) on reduce error in solving the Helmholtz equations with Dirichlet and Neumann boundary conditions.

Electromagnetic properties α	Electromagnetic properties β	Boundary conditions: Start point	Boundary conditions: End point	Wavelet functions	Scaling functions	Boundary jump functions	Interface jump functions	Total	Mean error rate between approximation and analytical solution in: u^{MF}	Mean error rate: $\frac{\partial u^{MF}}{\partial x}$	Error rate between approximation and analytical solution at end boundary point in: $u^{MF}\|_{x=2}$	Error rate at end boundary point: $\frac{\partial u^{MF}}{\partial x}\|_{x=2}$
*	0.1	$u\|_{x=0}=0\text{V}$	$u\|_{x=2}=1\text{V}$	7	1	2	1	11	0.126%	0.604%	0.000%	2.705%
				7	1	2	3	13	0.023%	0.246%	0.000%	1.515%
				7	1	2	7	17	0.005%	0.116%	0.000%	0.799%
				7	1	2	15	25	0.002%	0.069%	0.000%	0.443%
				7	1	2	31	41	0.001%	0.042%	0.000%	0.226%
				7	1	2	63	73	0.001%	0.024%	0.000%	0.116%
		$u\|_{x=0}=0\text{V}$	$\frac{\partial u}{\partial x}\|_{x=2}=1\text{V/m}$	7	1	2	1	11	1.267%	2.007%	2.780%	0.000%
				7	1	2	3	13	0.657%	1.110%	1.538%	0.000%
				7	1	2	7	17	0.337%	0.582%	0.806%	0.000%
				7	1	2	15	25	0.185%	0.321%	0.445%	0.000%
				7	1	2	31	41	0.094%	0.164%	0.227%	0.000%
				7	1	2	63	73	0.048%	0.084%	0.116%	0.000%

* According to **Error! Reference source not found.**.

Table 3.
The multi-resolution WMF approximation compared to analytical solution in solving the Poisson equation (equation).

Level of resolution	Wavelet functions	Scaling functions	jump functions	Total	Mean error rate between approximation and analytical solution in: u^{MF}	Mean error rate: $\frac{\partial u^{MF}}{\partial x}$
-	-	-	33	33	0.354%	1.971%
1	1	1	33	35	0.352%	1.970%
1-2	3	1	33	37	0.354%	1.958%
1-3	7	1	33	41	0.346%	1.934%
1-4	15	1	33	49	0.262%	1.630%
1-5	31	1	33	65	0.210%	1.411%
1-6	63	1	33	97	0.208%	1.409%

1-7	127	1	33	161	0.207%	1.402%

Table 4.
The multi-resolution WMF approximation compared to analytical solution in solving the Helmholtz equation (equation).

Number of basic functions				Mean error rate between approximation and analytical solution in:	
Wavelet functions	Scaling functions	jump functions	Total	u^{MF}	$\frac{\partial u^{MF}}{\partial x}$
1	1	1	2	0.126%	0.604%
7	1	3	13	0.023%	0.246%
7	1	7	17	0.005%	0.116%
7	1	15	25	0.002%	0.069%
7	1	31	41	0.001%	0.042%
7	1	63	73	0.001%	0.024%

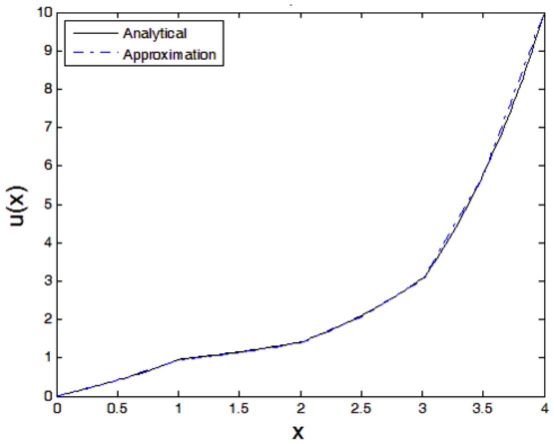

Fig. 5. The result of multi-resolution WMF approximation using slope jump functions in solving the 1-D Helmholtz equation with Dirichlet boundary conditions are compared with the analytical solution.

4. Conclusion and future work

This paper presented a detailed description of the formulation and enforces boundary conditions in the proposed multi-resolution WMF method for numerical analysis of the Laplace and Helmholtz boundary problems. These differential equations are usually used for modeling the electromagnetic field problems. To apply boundary and interface conditions, two different jump functions are added to the set of wavelet basis functions. The simulation results show the effectiveness of the proposed slope jump functions for enforcing the boundary and interface conditions compared to polynomial jump functions. As future work, the proposed method could be used for numerical analysis especially for inverse problem of impedance tomography studies as non-invasive imaging technique for reconstructing the cross-sectional conductivity distribution of a human brain or body.

References

[1]. S. Samadinasab, F. Namdari, M. Bakhshipoor, “A novel approach for earthing system design using finite element method”, Journal of Intelligent Procedures in Electrical Technology, vol. 8, no. 29, pp. 54-63, June 2017.

[2]. P. Zuo, Y. Geng, J. Li, J. Yuan, "An approach for eddy-current calculation in railguns based on the finite-element method", IEEE Trans. on Plasma Science, vol. 43, no. 5, pp. 1592-1596, May 2015.

[3]. A. Ahmadpour, S. Seyed Shenava, A. Dejamkhooy, E. Mokaramian, “Electromagnetic force analysis of transformer on the ferroresonance due to consecutive 3–phase short–circuit faults using finite element method (FEM)”, Journal of Intelligent Procedures in Electrical Technology, vol. 11, no. 41, pp. 47-60, June 2020.

[4]. Y.J. Hwang, J.Y. Jang, H. Jeon, "Overhang effect analysis of a homopolar HTS synchronous generator using 3D finite element method", IEEE Trans. on Applied Superconductivity, vol. 30, no. 4, pp. 1-5, June 2020.

[5]. L. Xuan, Z. Zeng, B. Shanker, L. Udpa, "Element free galerkin method for static and quasi-static electromagnetic field computation," IEEE Trans. on Magnetics, vol. 40, pp. 12-20, 2004.

[6]. M. Hadinia, R. Jafari, "A hybrid EFG-FE analysis for DOT forward problem", Proceeding of the IEEE/ISOT, pp. 1-6, Toronto, Canada, Oct. 2010.

[7]. S. Jaffard, "Wavelet methods for fast resolution of elliptic problems", SIAM Journal on Numerical Analysis, vol. 29, no. 4, pp. 965-986, 1992.

[8]. V. Cingoski, N. Miyamoto, H. Yamashita, "Hybrid element free galerkin-finite element method for electromagnetic field computations," IEEE Trans. on Magnetics, vol. 36, pp. 1543-1547, 2000.

[9]. G. Wang, G. Pan, B.K. Gilbert, "A hybrid wavelet expansion and boundary element analysis for multiconductor transmission line in multilayered dielectric media", IEEE Trans. on Microwave Theory Tech., vol. 43, pp. 664-675, 1995.

[10]. Y. Shiyou, N. Guangzheng, Q. Jingen, L. Ronglin, "Wavelet Galerkin method for computations of electromagnetic fields", IEEE Trans. on Magnetics, vol. 34, pp. 2493–2496, 1998.

[11]. S. Y. Yang, G.Z. Ni, J.R. Cardoso, S.L. Ho, J.M. Machado, "A combined wavelet-element free galerkin method for numerical calculations of electromagnetic fields", IEEE Trans. on Magnetics, vol. 39, pp. 1413-1416, 2003.

[12]. S. Karimi-Shahraki, M. Khezri, "Identification of Attention Deficit Hyperactivity Disorder Patients Using Wavelet-Based Features of EEG Signals", Journal of Intelligent Procedures in Electrical Technology, vol. 12, no. 47, pp. 1-11, Dec. 2021.

[13]. S.Y. Yang, G.Z. Ni, S.L. Ho, J.M. Machado, M.A. Rahman, H.C. Wong, "Wavelet galerkin method for computations of electromagnetic fields computation of connection coefficients", IEEE Trans. on Magnetics, vol. 36, pp. 644–648, 2000.

[14]. S.L. Ho, S. Yang, H.C. Wong, "Weak formulation of finite element method using wavelet basis functions", IEEE Trans. on Magnetics, vol. 37, pp. 3203-3207, 2001.

[15]. M.R. Yousefi, R. Jafari, H. Abrishami-Moghaddam, "A combined wavelet based mesh free method for solving the forward problem in electrical impedance tomography". IEEE Trans. on Instrumentation and Measurement, vol. 62, no. 10, pp. 2629-2638, Oct. 2013.

[16]. M. Karimi, H. Pourghassem, G. Shahgholian, "A novel prosthetic hand control approach based on genetic algorithm and wavelet transform features", Proceeding of the IEEE/CSPA, pp. 287-292, Penang, Malaysia, March 2011.

[17]. M. Manoochehri, H. Pourghassem, G. Shahgholian, "A novel synthetic image watermarking algorithm based on discrete wavelet transform and fourier-mellin transform", Proceeding of the IEEE/ICCSN, pp. 265-269, Xi'an, China, May 2011.

[18]. K .R. Castleman, Digital image processing, Prentice Hall, 1996.

[19]. M. N. O. Sadiku, Numerical techniques in electromagnetics, CRC Press, Second Edition, 2001.

[20]. M.R. Yousefi, R. Jafari, H.A. Moghaddam, "Imposing boundary and interface conditions in multi-resolution wavelet Galerkin method for numerical solution of Helmholtz problems", Computer Methods in Applied Mechanics and Engineering, vol. 276, pp. 67-94, 2014.

[21]. Y. Krongauz, T. Belytschko, " EFG approximation with discontinuous derivatives", International Journal for Numerical Methods in Engineering, vol. 41, pp. 1215-1233, 1998.