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Trajectory Tracking Weeled Mobile Robot Using Backstepping Method with Connection off Axle Trailer

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Abstract

The connection of the tractor to the inactive trailer or motor vehicle causes a motion control problem when turning in the screw, forward or backward movements and high speeds. This is due to the inactive trailer being controlled by the tracking using a physical link that is not affected by the movement. Trailers usually take tracks under these conditions. This phenomenon is called Jack Knife. The problem of motion control is challenging because it is a non-holonomic 4-degree complex system (for a multi-trailer system, n + 3 degrees freedom), and the kinematics of this system are difficult to calculate due to the non-linearity and coupling of the relationships. This is an unstable dynamic system and increases the input restrictions of the robot by adding the Jack Knife phenomenon, even if we move it in a direct direction. So in this paper we are trying to use a off axial connection method to create a new idea for easy control for trajectory tracking time varying of a trailer system, or even extending it to a multi-trailer system, by turning it into a wheel mobile robot system that is very easy to control and You can create trailer inputs by creating a link through an off-axle link, with a non-linear backstepping control method of verification.

Keywords: Wheeled mobile robot, Nonholonomic systems, Trajectory tracking, Backstepping method.

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1. Introduction

Modeling and control of wheeled mobile robots is one of the important challenges in the field of mobile robotics. Therefore, based on the characteristics of these types of robots, locomotion tasks can be performed faster with high precision and low cost using nonlinear stable algorithms and require the design of nonlinear and robust controllers. Since in real engineering systems there exist unknown disturbances and uncertainties, the controller should be designed to withstand against these phenomena. In previous works, most of the researches have been used the ideal non-holonomic constraints for the system assuming pure rolling and nonslip conditions on robot wheels. However, these constraints are not always realistic and they may be violated due to the deformation of the wheels or undesirable environmental effects, such as frozen roads which are mainly inevitable and unpredictable in real systems. As a result, the stability and performance of the control algorithms, which do not include such modeling conditions or uncertainties, are not guaranteed. Therefore, WMRs control algorithms for tracking of the desired paths mainly fail taking into account anomalies such as slipping uncertainties and disturbances. These uncertainties also will be more effective and also complicated in tractor-trailer systems which are focused in this paper. References [1] and [2] have examined the design of a tractor-trailer controller with attention to slip as uncertainty. When uncertainties in the kinematic models are not taken into account, trajectory tracking control of the tractor trailer systems may go to instability or inaccurate results.

Some algorithms use features to attenuate these effects such as such as adaptive control [3, 4] and sliding mode method [5, 6] and backstepping method [7, 8]. In this paper, a nonlinear control algorithm called adaptive backstepping method is used in which adaptive rules have been used in order to estimate system uncertainties. The flexibility of the backstepping approach allows it to solve a large number of control problems with respect to other nonlinear methods. This method has been successfully applied to a wide range of nonlinear problems [9-11]. The backstepping approach follows a step by step algorithm and is used for a set of nonlinear systems called explicit feedback systems. In fact, when control plants are a subcategory of the systems that are explicit in the form of feedback, this method ensures the characteristics of the local or global stability for the closed loop system. One of the main advantages of the backstepping method is the ability to prevent omitting of nonlinear terms existing in real system dynamics. This technique can be improved using estimation of uncertainties in the control algorithm. A standard backstepping method has the stability assurance but the effects of uncertainties and disturbances can disrupt the performance of the system. Therefore, it is required to robustify the standard backstepping method to be capable of compensating these effects. One of the main source of uncertainties in wheeled robots is the slipping of wheels. Therefore, slipping can be added as parametric uncertainties to the system mathematical model and then be estimated using adaptive rules [12]. In references [13-15], by using adaptive methods the controller algorithms have been improved while the stability of the closed loop systems is assured based on Lyapunov method.

The purpose of the research is to control vehicle trailers from motion vehicle assistance systems in human transport systems to intelligent and independent systems in mobile multi-purpose robots for use on a variety of issues, including automatic transmission and guidance, and more. In terms of controlling the movement of the tractor with the N trailer, it is necessary that a trajectory tracking continuously path the speed and references of the tractor vehicle, including the angle of rotation along the trajectory, and especially the curve, so that the unstructured path created in the path is controlled by the control algorithm Planned to follow the desired path. Each passive trailer will adopt an angle. Examples can be mentioned in this attention. Controlling the trailer system using a simple linearization control for the linear model and controlling the control point is placed on the tractor's guide point [16]. However the passive trailer route can be more important than the tractor in controlling trailer system trajectory tracing [17-19].

Movement control for trailer systems is much more difficult compared to a steer wheel (without trailer). The degree of non-compliance increases with respect to the number of trailers added to it. If a strong motion control strategy for these systems is defined accordance with the principles of control and stability of the system, then there are many practical problems such as parallel parking, the Jack-knife phenomenon, and so on. Figure 1 shows this. Even with many research done in this field to control non-holonomic WMRs with a trailer, an optimized controller has not yet been suggested.

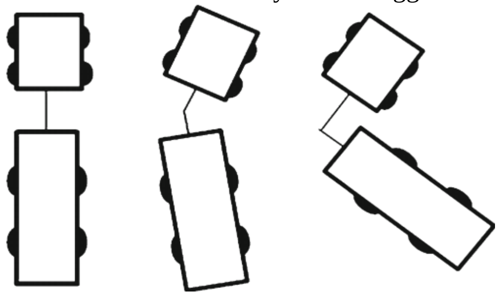

Fig. 1. Jack-knife phenomenon

Looking pervious over the past, it can be reason that most of the results relate to the trajectory tracking of wheeled mobile robots of a particular kind of car like robots and unicycle, and limits such as non-holonomic constraints for direct paths and constant velocities without slippage effects. More recently, research has been further developed to further improve the flexibility of moving, loading, and diversifying the structures of wheeled robots and bringing results to a context in which the concepts of a more general track may have include a carriage of load, in particular A tractor robot with one or more trailer has been developed and further explored to solve the new kinematics created for the problem and the design of a tracking controller for a tractor trailer robot. The main elements of these methods for this system are a non-slip motion, followed by linear paths that must be constant at constant speed, and Lyapunov's linearization is followed. This paper offers an innovative approach to the design problem of trajectory tracking reference mobile robot. This idea is, in fact, the main advantage of our design method, but it can be noted that there is no new idea in the first place, and all methods are considered on a similar and no Challenges system with some changes in the design of the controller. The highly nonlinear model of a mobile robot is divided into two parts: the space model mode, in which three inputs of two-dimensional mode are controlled by two inputs, here is a plane model that has been reviewed. First, a compatible [14] controllability design method was used to generate kinematic equations of the system that was used to solve the path tracking problem. Next, the backstepping method [22] is used to solve nonlinear algebraic equations for obtaining actual control inputs from the Lyapunov theorem function, and comparative law based on the proposed layout for dynamical error zeroing based on the Signal error send message and measurement value it is used to update controller parameters when changing robot parameters to a controller designed based on a combination of errors and uncertain parameters.

One of these nonlinear control methods is the backstepping approach, which is a powerful tool in designing control law. [9] The backstepping approach allows for a large number of design issues under wider conditions than in other ways we encountered it to solve. This method has been successfully applied to a wide range of nonlinear problems [23]. This tool is a method of designing control law based on the Lyapunov theory that is used for nonlinear systems. The idea of this method is to extend the application of the Lyapunov function from a simple system to systems with more complex structures and simultaneously ensuring sustainability [13]. The reason for naming the backstepping return for this method is to return its property to the design of the control rule, and at each stage a Lyapunov function is used to ensure system stability. The backstepping approach follows a bridging algorithm and is used for a handful of nonlinear systems. One of the most important benefits of the backstepping method It eliminates the elimination of non-linear elements in the design of the control law.

In this paper the control kinematic of tractor-trailer systems is analyzed and backstepping method. In fact, the method tries to stabilize the system around reference trajectories based on the regulation of error dynamics and attempts to control the system in several steps. In the first step, we assume an ideal controller for kinematic fixation, which is the perfect point of the problem. Next, using a Lyapunov-based algorithm the control of the system kinematic.

In the following, the kinematic model of a tractor–trailer wheeled robot (TTWR) has been obtained. Then, desired trajectories for the robot are produced. In the following, a control law is designed for the system based on adaptive backstepping method. Finally, the obtained results of the proposed method for a tractor–trailer wheeled robot (TTWR) are presented.

2. System Description and Modeling

A) Non-holonomic Constraints

Nonlinear systems are divided into holonomic and non-holonomic systems. In this paper, according to the kinematic equations of the Tractor-Trailer, a non-holonomic system is considered. But if the limits of a system are expressed as $f\ (q, \dot{q}, \ddot{q}, t) = 0$ and can not be reduced to $f\ (q, t) = 0$, it is called nonholonomic. By definition, it describes it as a set of differential equations that describe the system's movement constraint, or a general degree of freedom greater than the degree of freedom that can be controlled [24, 25].

B) Tractor-Trailer Mobile Robots

In particular, Tractor-Trailer robots have a robot with a trailer disabled. The trailer with a tractor robot has a physical link that can be off-axis or axial. Tractor-Trailer Mobile Robots (TTMR) is mostly used with a robot, since the operation and deactivation of trailer is unusual and impossible. The following is a summary of early research on non-holonomic mobile robots that has been developed for use in trailer robots. Many articles have followed the TTMR's for Trajectory Tracking in forward motion [18, 26]. An inactive trailer connected to the robot using a physical link that makes it difficult to control the trailer, this situation causes the robot to impact a high speed and also when the effects of the slip on the robot and trailer wheels are effective. This phenomenon is called as jack-knifing.

C) Mechanical structure of TTMR

Some paper have devised new TTMRs that do not come up with the jack-knifing phenomenon, while others use all-wheel drive, some look at the type of hitch between a truck and a trailer and fit the type conditions Select the optimal connection. In the reference [27, 28], the mechanical and kinematic structure has compared three types of trailer with direct pin coupling, no hooks and three points in the system equations and how they are applied. Stability analysis has shown that although the kinematics of a trailer outside the hook are complex, it has a simpler mechanical structure, and tracking performance is more comfortable and the resulting equations are algebraic if in the rest of the pin connections it creates a degree of freedom mostly, or generating generalized coordinate derivatives, which may slightly affect the equation or sustainability solution. This concludes that the trailer system outside of the hook (off axle), in spite of complex equations, makes it possible to connect the inputs through algebraic equations and is more beneficial.

D) Off-axle TTMRs

In most paper, trailers were considered to be on-axle hitching in trucks. But there are a few off-axis connections that we use in this paper. In addition, off-axle trailers take more practical operations in different industries, such as cylindrical industries, but the reality is that the system can be very unstable. As seen in [29], the trailer truck system is symmetrically flat with displacement of the axle. It can also be converted into a chainshaped system, such as a fire truck with three inputs. However, the extraction of equations for finding the Chinese form of functions is very difficult, and in the first instance, the ordering structure of the trailer must be chosen so that the derived equations of the generalized coordinates are available.[30] That a passive trailer with a trailer can be controlled in the rear direction with its treatment as a path trail problem. The obvious feature of this method is that it can be used for any desired axis to add the desired number of trailer to the vehicle.

The robot kinematic chain parameters are illustrated in Fig. 1 by a jointed joint of the off-axle (Outside Axis). The distance L_{if} and L_{ib} must be considered for connecting the tractor linkage links and the following constant or positive trailer. The relative angle of the trailer θ_0 with respect to its unit (i-1) is shown by the tilt angle of the tractor θ_1 in the Cartesian (XY) device relative to the horizon.

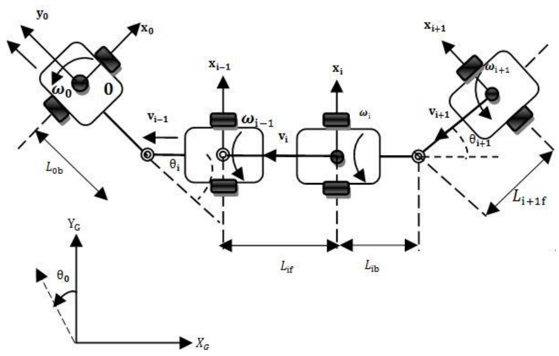

Fig. 2. Parameters of the kinematic tractor with N trailer

E) Wheeled mobile robot

A two-wheeled differential drive mobile robot with an N trailer is the best example for non-holonomic system. The kinematic model is given as equation (1). Of course, with the assumption that our goal is to control the last trailer, which in the end obtains the inputs of the linear velocity and rotation of the last trailer, and the relation between these inputs and the trailer before the link connections are inactive (off axle), the linear input of the tractor also will be achieved. The kinematic model of the vehicle is implemented by assuming a slip in the control inputs. To obtain a vehicle model in Figure 2, we describe the kinematics of each trailer separately in the form of relation (1), and then, by the relationships in the control algorithm, the relationship between the inputs $(u_i, \omega_i), (i = 0,1,\dots,N$ and the tractor $u0,\omega 0$ with the trailer entrance input equations we associate. That is the purpose of the problem(u_i, ω_0).

$$\dot{x}_i = u_i \cos\theta_i, \quad \dot{y}_i = u_i \sin\theta_i, \quad \dot{\theta}_i = \omega_i \tag{1}$$

A schematic of an N-trailer mobile vehicle with the definition of its generalized configuration coordinates is shown in Fig. 1. The vehicle consists of a guided car initially and an N trailer attached to it. The active part of the front is a tractor with control inputs ω_0 and u_0, respectively angular velocity and longitudinal axis of the axle. All remaining tracks attached to the tractor are the same passively hooked trailers, all of which are arranged by a joint at non-zero $L_{hi} \neq 0$ intervals, from the center point of the previous wheel to the end result of connecting to the next trailer link. The parameters of $L_i > 0$ represent the length of the trailer in Figure 2.

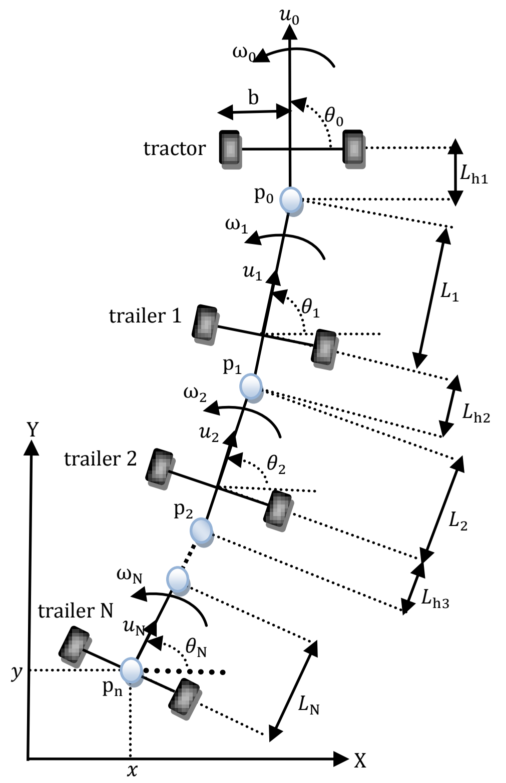

Fig. 3. Differentially driven wheeled mobile robot towing a trailer.

Reducing this equation (1), gives

$$\mathcal{A}(q)\,\dot{q} = \begin{pmatrix} \sin\theta_n & -\cos\theta_n & 0 & 0 \\ \sin\theta_{n-1} & -\cos\theta_{n-1} & -d_n\cos(\theta_{n-1}-\theta_n) & 0 \\ \sin\theta_{n-2} & -\cos\theta_{n-2} & -d_n\cos(\theta_{n-2}-\theta_n) & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \sin\theta_1 & -\cos\theta_1 & -d_n\cos(\theta_1-\theta_n) & \cdots \\ \sin\theta_0 & -\cos\theta_0 & -d_n\cos(\theta_0-\theta_n) & \cdots \end{pmatrix} \tag{2}$$

$$\begin{matrix} \cdots & 0 & 0 \\ \cdots & 0 & 0 \\ \vdots & 0 & 0 \\ 0 & \vdots & \vdots \\ -d_2\cos(\theta_1-\theta_2) & 0 & 0 \\ -d_2\cos(\theta_0-\theta_2) & -d_1\cos(\theta_0-\theta_1) & 0 \end{matrix}\Bigg) \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta}_n \\ \dot{\theta}_{n-1} \\ \vdots \\ \dot{\theta}_0 \end{pmatrix} = 0$$

This is a non-holonomic equation. If it is assumed in the system that contact with the earth (the pure rolling condition is not true) exists by the robot wheels, this indicates that the slip condition in the non-holonomic constraint equation for the N trailer does not exist, and therefore the system is uncontrollable. But from the experience and analysis of the robot's motion, we know that a two-wheel differential or car can be controlled in a 3D space, but in fact there are only two inputs. The rotational wheel limits make it a higher degree of freedom that compression the system. For a system with N trailer, two speed inputs are required, but there are N + 3 general coordinates. For this reason, the control of tractor trailer movement is uncontrollable and very complex in the usual trajectory without the necessary assumptions in equations and methods of problem solving. Because the back motion of this non-holonomic system creates an unstable loop in the stability equations of the system, since the trailer slip and the point of attachment of the trailer to the car along the path cause jack-knife to make the control a little difficult or under it effects.

The kinematic robot is defined as for N trailer (3).

$$\dot{q}_N = \mathcal{S}(q_N)\mathrm{v_i} \tag{3}$$

The N index is used for the coordinates of the trailer system.Therefore, we can write the following relation:

$$\mathcal{S}^{\mathrm{T}}(q_N)\mathcal{A}^{\mathrm{T}}(q_N) = 0 \tag{4}$$

$\mathcal{S}(q_N)$ is the natural orthogonal complement of constraint matrix, which is specified for the N trailer.

$$\mathcal{S}(q_N) = \begin{pmatrix} \cos\theta_n & 0 \\ \sin\theta_n & 0 \\ \frac{1}{d_n}\tan(\theta_{n-1}-\theta_n) & 0 \\ \frac{1}{d_{n-1}}\frac{\tan(\theta_{n-2}-\theta_{n-1})}{\cos(\theta_{n-1}-\theta_n)} & 0 \\ \frac{1}{d_{n-2}}\frac{\tan(\theta_{n-3}-\theta_{n-2})}{\cos(\theta_{n-2}-\theta_{n-1})\cos(\theta_{n-1}-\theta_n)} & 0 \\ \vdots & \vdots \\ \frac{1}{d_{n-k}}\frac{\tan(\theta_{n-k-1}-\theta_{n-k})}{\prod_{i=n-k}^{n-1}\cos(\theta_i-\theta_{i+1})} & 0 \\ \vdots & \vdots \\ \frac{1}{d_1}\frac{\tan(\theta_0-\theta_1)}{\prod_{i=1}^{n-1}\cos(\theta_i-\theta_{i+1})} & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

The vehicle configuration is determined on a two-dimensional plane with N + 3 independent variables that can be selected as follows, $q_N = (\mathrm{x_n}, \mathrm{y_n}, \theta_0, \theta_1 \cdots \theta_n)^{\mathrm{T}}$ is system configuration vector. (x_n, y_n) Is the coordinate in the inertial frame, θ_0 and $\theta_1, \ldots, \theta_n$ represent the orientation of the tractor and trailer with respect to the inertial frame, respectively. The last trailer is called the guiding section with respect to the point (x_n, y_n), which is our point of interest. Also, $\mathcal{A}(q_N)$ is system constraint matrix.

To consider virtual input vector

$$\mathrm{v}_i = \begin{bmatrix}\omega_i \\ u_i\end{bmatrix} \tag{6}$$

A general formula can be used for virtual inputs of both sections of a tractor attached to a trailer or two trailer backward trains for $i = 1 \ldots, N$ wrote:

$$\mathrm{v}_i = J_i(\theta_i)\mathrm{v}_{i-1} \tag{7}$$

Where

$$J_i(\beta_i) = \begin{bmatrix} -\frac{L_{hi}}{L_i}\cos\beta_i & \frac{1}{L_i}\sin\beta_i \\ L_{hi}\sin\beta_i & \cos\beta_i \end{bmatrix} \tag{8}$$

is the transformation matrix with the inverse

$$J_i^{-1}(\beta_i) = \begin{bmatrix} -\frac{L_i}{L_{hi}}\cos\beta_i & \frac{1}{L_{hi}}\sin\beta_i \\ L_i\sin\beta_i & \cos\beta_i \end{bmatrix} \tag{9}$$

the result of the matrix determination is $det(J_i(\beta_i)) = -\frac{L_{hi}}{L_i}$, which is necessary for reversibility $L_i \neq 0$.

inverse relation to (3) can be written as

$$\mathrm{v}_{i-1} = J_i^{-1}(\beta_i)\mathrm{v}_i \tag{10}$$

Equations (2) and (6) make it possible to sum up the sum of the nth trailer attached to the tractor, which has the task of conducting along the vehicle chain.

$$\mathrm{v}_i = \prod_{j=i}^{1} J_i(\beta_i)\,\mathrm{v}_0, \quad i = 1, \ldots, N \tag{11}$$

but if we start with the tractor input equations and get the connection between the inputs of the last tractor, then the relation must be changed by equation (8).

$$\mathrm{v}_{i-1} = \prod_{j=i}^{N} J_i^{-1}(\beta_i)\,\mathrm{v}_N, \quad i = 1, \ldots, N \tag{12}$$

The connecting angle of the tractor to the trailer and the trailer to the trailer can be shown as follows

$$\beta_i = \theta_{i-1} - \theta_i \tag{13}$$

From the derivative of the connecting angle

$$\dot{\beta}_i = \omega_{i-1} - \omega_i \tag{14}$$

Using combining Equations (6), (11) and (14) for kinematic $\dot{q}_c = \mathcal{S}(q_c)\mathrm{v}_0$ connecting angles a general formula can be obtained without the need to calculate the single-link connection between the inputs . The C index is the connection sign.

$\mathrm{v}_0 = [\omega_0 \quad u_0]^T$ is the control input of the tractor, in which $\mathrm{c^T} = [1\ 0]$, $\mathrm{d^T} = [0\ 1]$ and $I \in R^{2\times2}$ is a one-dimensional matrix. In fact, the relation (15) for the summary of equations (1) and

(6-14) in the analysis of the kinematic model for controller design method is presented in this paper.

$$\begin{bmatrix} \dot{\beta}_1 \\ \dot{\beta}_2 \\ \vdots \\ \dot{\beta}_i \\ \vdots \\ \dot{\beta}_N \\ \dot{x}_n \\ \dot{y}_n \\ \dot{\theta}_n \end{bmatrix} = \begin{bmatrix} c^T(I - J_1(\beta_1)) \\ c^T(I - J_2(\beta_2))J_1(\beta_1) \\ \vdots \\ c^T(I - J_i(\beta_i)) \prod_{j=i-1}^{1} J_j(\beta_j) \\ \vdots \\ c^T(I - J_N(\beta_N)) \prod_{j=N-1}^{1} J_j(\beta_j) \\ d^T \prod_{j=N}^{1} J_j(\beta_j) \cos\theta_N \\ d^T \prod_{j=N}^{1} J_j(\beta_j) \sin\theta_N \\ c^T \prod_{j=N}^{1} J_j(\beta_j) \end{bmatrix} v_0 \qquad (15)$$

Kinematic equations of the mobile robot for last trailer can be written as relation (1) :

$\dot{q} = g_1(q)\, u_N + g_2(q)\, \omega_N$

Where

$$g_1 = (\cos\theta_1, \sin\theta_1, 0)^T, \qquad g_2 = (0,0,1)^T \qquad (16)$$

u_N is the linear velocity and ω_N is the angular velocity of the tractor.

F) Path planning of a TTMR

Some studies have also been conducted on TTMR system control. The combination of two elements of TTMR Motion and Control is one of the most important problems. Therefore, this paper refers to two issues: (1) controller planning, design and implementation using the structure of the error dynamic equations and the adaptive law of the backstepping control algorithm; (2) the TTMR trajectory tracking in the generated paths can be modified; and Predefined by following the robot along that path. By examining and comparing theoretical and empirical results in reality, it can be seen that the trajectory error increases with the number of trailers connected to the vehicle, because with a freedom system release degree, as well as the coupling of these independent system coordinates, and disturbances caused by Uncertainties, such as slippage in the trailer system, include more difficult robot control, as well as long computation time.

In this section, first reference trajectory is planned for the TTMR. When planning a path to a TTMR, many papers do not provide specific methods for ease of control of path planning [31]. But many other methods, similar the backstepping method, are specifically used to solve this problem because the tracking efficiency of these methods depends on the effects and disturbances on the controller, which is well-suited for certain methods, such as backpacking trajectory the reference path is controlled. In the reference [32], the first following of the reference path was proposed using the closed-form solutions of the kinematic parameters for the TTMR. Design reference path a trailer robot was suggested for a specific purpose from an initial configuration to a given goal [33-35].

We assume that the reference time functions in the Cartesian space that must be followed by the robot should be expressed as $q_r(t) = (x_r, y_r, \theta_r, \theta_{0r})^T$, [3]. Derivatives are $\dot{q}_r$, $\ddot{q}_r \in L$ bound. Which L is the same as the boundary of the reference functions. To consider the trailer of the reference path as follows $u_{2r}(t) = \theta_r(t) = \text{ATAN2}\{\dot{y}_r(t), \dot{x}_r(t)\}$ that $\dot{\theta}_r(t) = \omega_r$, $u_{1r}(t) = \pm\sqrt{\dot{x}_r^2 + \dot{y}_r^2}$. Some reference inputs $u_{1r}(t) \neq 0$, $u_{2r}(t) \in R, \forall\ t \geq 0$, The atan2 is the inverse of the tangent function in a complete round.

In Fig. 3, the input controller receives the reference and generates the input signal of the desired kinematic model. Due to the presence of slip, the variables of the system are associated with uncertainties and to compensate for these uncertainties and follow the reference path by the robot, the adaptive rules, by correcting the estimation slip at any moment, correct the controller, and by generating the appropriate control input, the real robot is directed toward the reference robot They are pushing for a chasing error to zero. Also, a signal from the output of the control inputs to the $J_i^{-1}(\beta_i)$ function is introduced to convert the inputs from the trailer into the tractor inputs, which aims to link the inputs of the last trailer using the structural equations of the external link (off axle) connections.

3. Backstepping Control Algorithm

This method of control is an analytical method based on the theory of Lyapunov. Such a controller, such as sliding modeling and other nonlinear control methods, can be combined with adaptive methods and can be more resistant to disturbances and changes in parameters as well as intrusive noise. The reason for the high efficiency of this method is the simplicity of designing and designing it step by step and also above to slippery mode in terms of not having high frequency vibrations. This method is widely used in nonlinear different systems [36-38].

To control the trajectory tracking paths, based on the process explained below, the dynamic equations of the system's tracking error are formed. If these error-equations are stable at the source, the moving tracking motions of the robot move around the reference time motion paths, thus solving the problem of tracking the time movements of the moving robot reference.

The purpose of the design of the feedback control law is for the differential wheel of the robot so that the tracking error $\varepsilon = q - q_r$ is stable at the source. This controller is a kinematic type whose inputs are linear and rotational speeds. We assume that robot status variables are measured at any moment by the sensors and that the control laws are generated and corrected by these variables. In the following, a solution to this problem is presented.

First an error vector **e** is defined in a new space as

$$e = \mathcal{T}(\theta_{nr})\varepsilon \tag{17}$$

where transformation matrix $\mathcal{T}$ maps tracking errors into the new space. Transformation matrix $\mathcal{R}$ is defined as

$$\mathcal{T}(\theta_{nr}) = \begin{pmatrix} \mathcal{R}(\theta_{nr}) & O_1 \\ O_2 & 1 \end{pmatrix} \tag{18}$$

$\mathcal{R} = \begin{pmatrix} \cos\theta_{nr} & \sin\theta_{nr} \\ -\sin\theta_{nr} & \cos\theta_{nr} \end{pmatrix}$ is the matrix of the period for last (nth) trailer and $O_1 = [0 \quad 0]^T$, $O_2 = [0 \quad 0]$.

Equation (9) can be written in following form

$$e^* = R(q - q_r) \tag{19}$$

that $e^* = (e_1 \ e_2 \ e_3)^T$.

Differentiation from equation (10) yields

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} \omega_N e_2 - u_N + u_{1r}\cos e_3 \\ -\omega_N e_1 + u_{1r}\sin e_3 \\ u_{2r} - \omega_N \end{pmatrix} \tag{20}$$

The above-mentioned error equations can be written in general terms $\dot{e} = f\left(e, q_r, u_r, u\right)$. now, for the stability of the system, we use the Lyapunov function theory.

$$e^* = \mathcal{R}(q - q_r) \tag{21}$$

A) Stability Analysis

Let's consider the following Lyapunov function candidate for the stability analysis of the system.

$$\mathcal{V}_1 = \frac{1}{2}{e_1}^2 + \frac{1}{2}{e_2}^2 \tag{22}$$

Therefore the time derivative of $\mathcal{V}_1$ can be found as

$$+e_2(-\omega_N e_1 + u_{1r}\sin\ e_3) \tag{23}$$

In the above assumption u_x from the non-slip condition $\varphi = 0$ is obtained from the slip equations as follows.

$$\begin{cases} u_x = u_N\cos\theta_N \\ u_y = u_N\sin\theta_N \\ \varphi = \tan^{-1}\left(\frac{u_y}{u_x}\right) \end{cases} \tag{24}$$

From the values $\omega_N e_2$ and $\omega_N e_1$ we use equations (16) to simplify solving and obtaining input u_N. We also consider $\xi = sine_3$ that this value is not a control input and is a non-variable variation.

$$\xi_d = -\frac{k_2 e_2}{u_{1r}} \tag{25}$$

We assume that the optimal value of ξ is ξ_d. Now for the negative of the function $\dot{\mathcal{V}}_1$, u_1 can be chosen as follows.

$$u_N = k_1 e_1 + u_{1r}cose_3 \tag{26}$$

The terms $k_1 e_1$ and $k_2 e_2$ are termed sentences and have been used to stabilize e_1 and e_2, and other sentences have been used to transform the derivatives of the Lyapunov function into a negative semisecond function. With these choices for system inputs, the derivative of the Lyapunov candidate function $\mathcal{V}_1$ will be as follows.

$$\dot{\mathcal{V}}_1 = -k_1{e_1}^2 - k_2{e_1}^2 \tag{27}$$

Where k_1 and k_2 are positive. Now, to define the error value ξ and the optimal value of ξ_d, we define the error $\tilde{\xi}$ as follows.

$$\tilde{\xi} = \xi - \xi_d = \sin e_3 + \frac{k_2 e_2}{u_{1r}} \tag{28}$$

We obtain the derivative of the above function in relation to time as follows.

$$\dot{\tilde{\xi}} = \cos e_3\,(u_{2r} - \omega_N) + \frac{k_2}{u_{1r}}(-\omega_N e_1 + u_{1r}\sin e_3) \tag{29}$$

In the next step, we define the second-order Lyapunov function according to the backstepping method.

$$\mathcal{V}_2 = \mathcal{V}_1 + \frac{1}{2}\tilde{\xi}^2 \tag{30}$$

We compute the derivative of the above function in relation to time as follows.

$$\dot{\mathcal{V}}_2 = e_1(-u_N + u_{1r}\cos e_3) + e_2(u_{1r}\sin e_3) + \tilde{\xi}\left((u_{2r} - \omega_N)\cos e_3\right) + \tilde{\xi}\left(\frac{k_2}{u_{1r}}(-\omega_N e_1 + u_{1r}\sin e_3)\right) \tag{31}$$

By placing equations (26) and (29) in (31) we have:

$$\dot{\mathcal{V}}_2 = -k_1{e_1}^2 - k_2{e_1}^2 + \tilde{\xi}(e_2 u_{1r} + (u_{2r} - \omega_N)\cos e_3) + \tilde{\xi}\left(\frac{k_2}{u_{1r}}(-\omega_N e_1 + u_{1r}\sin e_3)\right) \tag{32}$$

For the negative of the function $\dot{\mathcal{V}}_2$ in the above we select u_2 as follows.

$$\omega_N = \frac{e_2 u_{1r} + u_{2r}\cos e_3 + \frac{k_2 u_{1r}\sin e_3}{u_{1r}} + k_u\tilde{\xi}}{\cos e_3 + \frac{k_2 e_1}{u_{1r}}} \tag{33}$$

We input the equation ω_N in (32), therefore:

$$\dot{\mathcal{V}}_2 = -k_1{e_1}^2 - k_2{e_1}^2 - k_\xi\tilde{\xi}^2 \tag{34}$$

k_ξ is the control system and is positive.

4. Results

In this section, the results obtained from the comparison of the controller with the slip estimator as well as the controller without slip estimator are presented and analyzed for performance evaluation, error convergence, reliability in the presence of disturbances, and so on. Reference paths are considered in different ways and with the following equations. Two paths for comparing the reference model and the application of controllers designed in each section, which include the presence and absence of a slip with the estimator in the controller or the effects of non-comparative rules, are considered in the other comparisons, in which the equation of these paths is expressed in relations (36) and (37).

$$\begin{cases} x_r(t) = \left(5 + \cos\left(\frac{18\,t}{50}\right)\right)\left(\cos\left(\frac{6\,t}{50}\right) - \frac{\pi}{6}\right) \\ y_r(t) = \left(5 + \cos\left(\frac{18t}{50}\right)\right)\left(\sin\left(\frac{6\,t}{T}\right) - \frac{\pi}{6}\right) \end{cases} \quad (35)$$

and

$$\begin{cases} x_r(t) = \left(5 + \cos\left(\frac{24\,t}{50}\right)\right)\cos\left(\frac{6\,t}{50}\right) \\ y_r(t) = \left(5 + \cos\left(\frac{24t}{50}\right)\right)\sin\left(\frac{6\,t}{T}\right) \end{cases} \quad (36)$$

Gone, and the robot follows the direction of reference movement.

The initial conditions of the system for trajectory tracking reference is $x_0 = (-6 -$ 6 π3 π3 π3 π3T.

In order to evaluate the efficiency of the proposed controller, the results are presented and the results of using and not using the slip estimator are compared to the system control. In the simulations, the gains used are set as $k_x = 2.6$, $k_y = 2.2$, $k_\xi = 15.14$, $L_h = 0.17$, $L = 0.05$. Regarding the control parameters presented, it is worth noting that the control benefits are assumed to be positive values in order to satisfy the closed loop system stability. Therefore, for proper operation at the same time and reasonable control inputs, the

Control gain has been selected using the trial and error method and the simulations evaluation of the closed loop system function and the amount of control inputs. Since the stability of the problem of tracking the rotational paths of the robot has been proven, it is expected that, starting with different initial conditions and with limited time, the robot tracking errors are convergent around zero and the transient responses of the system It's.

For the last trailer we assume here is N = 2 and the extraction of the governing equations of the problem, the relationship between the input of the third trailer and the tractor, which is actually the aim of obtaining the linear velocity of the third trailer ω_0 and the angular velocity of the tractor u_2, which is the relationship of the obtained relations This can be achieved. For simplicity, we will use the control workflows obtained in the previous step for the last trailer, assuming that the control inputs of the third trailer (u_N=u_2, ω_N=ω_2) are used to reach the tractor control inputs.

$$v_2 = \begin{bmatrix} \omega_2 \\ u_2 \end{bmatrix} \quad (37)$$

That

$$v_2 = \begin{cases} \omega_N = \dfrac{e_2 u_{1r} + u_{2r}\cos e_3 + \dfrac{k_2 u_{1r}\sin e_3}{u_{1r}} + k_u \tilde{\xi}}{\cos e_3 + \dfrac{k_2 e_1}{u_{1r}}} \\ u_N = u_{1r}\cos e_3 + k_1 e_1 \end{cases} \quad (38)$$

from equation (9), we can write off the equation of connection off axle to obtain v_2, v_1 inputs:

$$\begin{bmatrix} \omega_2 \\ u_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{L_3}{L_{h3}}\cos\beta_3 & \dfrac{1}{L_{h3}}\sin\beta_3 \\ L_3\sin\beta_3 & \cos\beta_3 \end{bmatrix} \begin{bmatrix} \omega_3 \\ u_3 \end{bmatrix} \quad (39)$$

$$\begin{bmatrix} \omega_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} -\dfrac{L_2}{L_{h2}}\cos\beta_2 & \dfrac{1}{L_{h2}}\sin\beta_2 \\ L_2\sin\beta_2 & \cos\beta_2 \end{bmatrix} \begin{bmatrix} \omega_2 \\ u_2 \end{bmatrix} \quad (40)$$

And inputs tractor

$$\begin{bmatrix} \omega_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} -\dfrac{L_1}{L_{h1}}\cos\beta_1 & \dfrac{1}{L_{h1}}\sin\beta_1 \\ L_1\sin\beta_1 & \cos\beta_1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ u_1 \end{bmatrix} \quad (41)$$

In summary, from Equation (8) and (11), we can derive the relationship between the traverses of the third trailer with the tractor in summary form from the product of the following multiplication

$$v_2 = J_1(\beta_1)J_2(\beta_2)v_0 \Rightarrow (\omega_0 \quad u_2)^T \quad (42)$$

The control benefits are assumed to be positive values in order to satisfy the closed loop system. Therefore, in order to have good performance at the same time and reasonable control inputs, the control gain has been selected using the trial and error method and the simultaneous review of the function of the closed loop system and the amount of control inputs. Because of the sustainability of tracking issues the robot's momentum movement has been proven. Therefore, it is expected that, starting with different initial conditions and with limited time, the robot tracking errors are convergent around zero and the transient responses of the system are eliminated, and the robot follows the reference path directions.

In Fig. 4 and Fig. 5, the path of the robot and the reference path are represented by equation (36) and (37) on the motion screen to control backstepping method.

It is observed that the reference path on the Cartesian screen is followed by a robot with ideal kinematics (without the presence of slip) starting from an initial out-of-boundary condition well. In Fig. 7, 8, the control error signals are plotted to trajectory tracking the path of the robot.

In Figure 9, control inputs are shown tractor in mode by backstepping control. In Figure 10, control inputs are provided in tow trailer mode backstepping.Controlled inputs are also of good value and are within reasonable range.

5. Final Remarks

In this paper, the backstepping method for controlling the tarjectory of wheel mobile robot paths as a nonlinear and non-linear system is presented. This method is a recursive method by choosing the right control law for an equation of one of the system modes, which results in the derivation of the Lyapunov function negatively, it stabilizes the mode, and this is repeated to stabilize other systems of the system, so that the input signal is finally selected so that the whole system is stable.

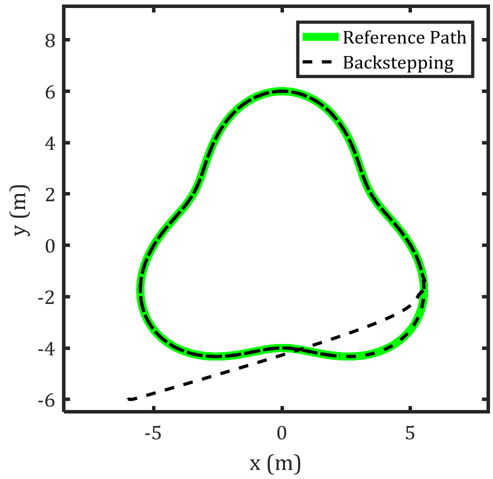

Fig. 4. The path of the robot and the reference path (36) using backstepping controller

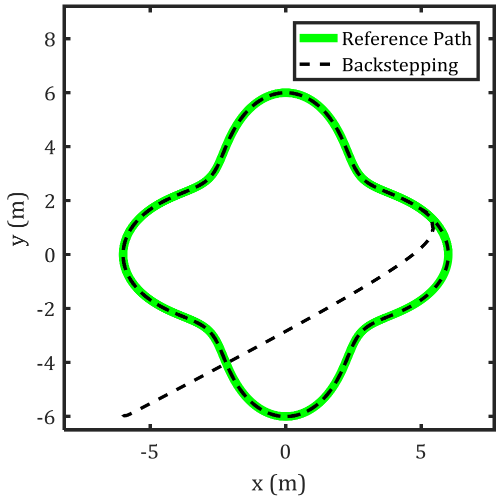

Fig. 5. The path of the robot and the reference path (37) using backstepping controller

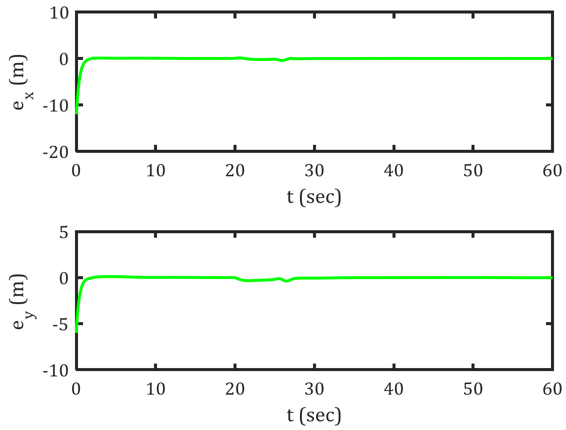

Fig. 6. Error signals of tracking control tow trailer

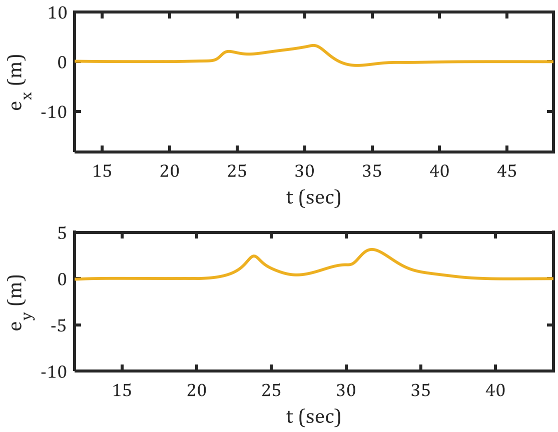

Fig. 7. Error signals of tracking control tow tractor

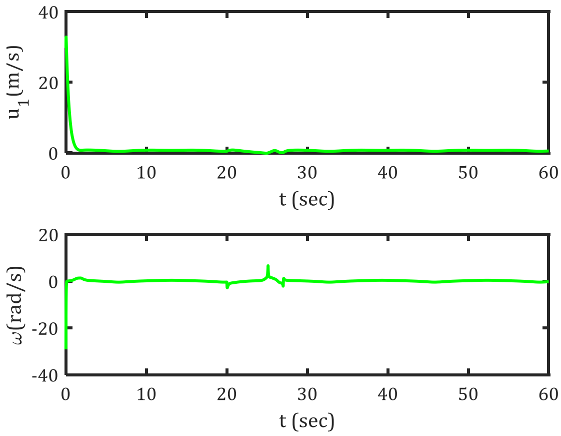

Fig. 8. Backstepping control inputs tractor

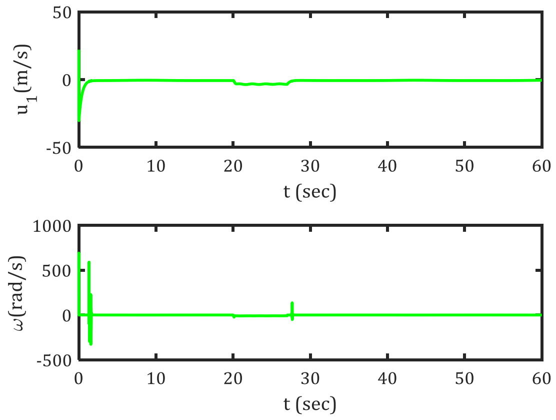

Fig. 9. Backstepping control inputs tow trailer

As in the case of feedback loci, the nonlinear system requires a behavior similar to that of a linear system, but in this method, due to the large selection of control rules, unlike the feedback law, nonlinear elements can be useful for stabilizing and It did not remove the tracking, and also kept the control signal range narrower. In this paper, the kinetic equations of the system were first extracted. Subsequently, appropriate reference paths for the robot were generated, and then the backstepping method control regimen was designed to generate control input values to minimize the robot error. In the design of the control law, the effectiveness of the proposed method for controlling the robot following the various reference time paths was confirmed by providing comparative results.

Indeed, in this paper, the concept of differential matching is used to track reference time paths, robot mobiles from backpacking algorithms and reference paths can be controlled. The trajectory tracking has been selected to satisfy the conditions based on the Lyapunov theory. The results indicate that the route error increases with the number of trailer attached to the tractor (moving vehicle), as well as the long computation time. In this paper, a review of the literature on the tractor trailers and path planning of tractor-trailer robots has been presented. It also details on the various control methods of a mobile robot, grouping them and highlighting the advantages and disadvantages of each methodology. All the existing and unresolved problems in this field, which is modeled on a system non-holonomic conjunctions of the main robot, and the limits of the off axle connection of the trailer with the car modeled in the kinematics and dynamics of the structured model it affects this problem, it is listed completely without any hypothesis to simplify the dynamics of the problem.

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