



Multi-objective Dynamic Planning of Substations and Primary Feeders Considering Uncertainties and Reliability

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Abstract

This research uses a comprehensive method to solve a combinatorial problem of distribution network expansion planning (DNEP) problem. The proposed multi-objective scheme aims to improve power system's accountability and system performance parameters, simultaneously, in the lowest possible costs. The dynamic programming approach is implemented in order to find the optimal sizing, siting and timing of HV/MV substations, feeders and distributed generations. Based on the input data, the results should be closer to the reality. So, the relevant uncertainties must well incorporate in DNEP modeling to achieve the best possible strategy. The most important uncertainties are the load forecasting, market price errors as well as the uncertainties related to the intermittent nature of the output power of renewable energy resources. Given that DNEP is a multi-objective optimization problem including several objective functions such as: cost based function, voltage deviation, voltage stability factor and measuring the amount of produced emission. NSGA-II as an appropriate alternative results several non-dominated solutions where finally fuzzy set theory is used to select the best compromise solution among them. The proposed scheme is applied to 54-bus system distribution network. The comparison study validates the efficiency of suggested method in the presence of distributed generations.

Keywords: Dynamic Expansion Planning, Feeder Routing, DG allocation, Uncertainty, Reliability, Multi-objective Optimization

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Nomenclature

| | | | |
|-------------------------------|--|----------------------------------|--|
| n_l | Number of load buses(MV/LV substations) | $load(s)$ | states of load demand |
| n_{ll} | Number of load levels | $price(s)$ | states of energy price |
| n_s | Number of states | $wind(s)$ | states of wind speed |
| n_y | Planning horizon | $states_s^{comb}$ | combination of all states |
| n_f | Number of network 's feeders | V_{rated} | magnitude of rated voltage (kv) |
| n_{ef} | Number of existing feeders | $v_{safe}^{min}, v_{safe}^{max}$ | Lower and upper limit of buses voltages for safe operating condition |
| n_{cf} | Number of candidate feeders for installation | $v_{crit}^{min}, v_{crit}^{max}$ | lower and upper limit of buses voltages for critical operating condition |
| n_{es} | Number of existing HV/MV substations | S_{ij}^{max} | Maximum apparent power of feeders between buses I and j |
| n_{cs} | number of candidate HV/MV substations for installation | $S_{i,max}^{DG}$ | capacity of DG installed in bus i |
| $n_s = n_{es} + n_{cs}$ | number of HV/MV substations | $S_{i,ll,s}^{DG}$ | apparent power of DG installed in bus i, in load level LL and state s |
| $n_n = n_l + n_{es} + n_{cs}$ | total number of network substations | R_{ij} | Resistance of feeders between buses I and j (ohm/km) |
| Y_{ij} | magnitude of admittance between buses i and j | X_{ij} | Reactance of feeders between buses I and j (ohm/km) |

| | | | |
|---------------------|--|--------------------------------------|---|
| θ_{ij} | angle of admittance between buses i and j | MPL | Maximum penetration level |
| α | Load growth rate | K_{DDG} | emission related to the power generated by DG units (Kg/MWh) |
| λ_k | failure rate of feeder k (fail/km/year) | K_{GRID} | emission related to the power received from transmission grid (Kg/MWh). |
| pw | present worth factor | $ecs_i(s)$ | expansion cost of ith existing HV/MV substation with the capacity of S (\$/kVA) |
| Int Rate, Inf Rate | Interest Rate (%), Inflation Rate (%) | $ICS_i(S)$ | installation cost of ith new HV/MV substation with the capacity of S (\$/kVA) |
| r_k | repair time of feeder k (h) | $RCF_{ij}(k)$ | replacement cost of feeder with the type of k between buses i, j (\$/km) |
| T_{ll}, dc | duration of load level LL (h) and dissatisfaction cost | $ICF_{ij}(k)$ | Installation cost of feeder with the type of k between buses i, j (\$/km) |
| $S^l_{i,ll,s}$ | apparent power of load demand in bus I, in load level LL and state s | $ICDDG_i(s)$ | installation cost of dispatchable DG with the capacity of s in bus I (\$/kVA) |
| $S^l_{i,peak}$ | apparent power of load demand in bus I, in peak condition | $ICWDG_i(s)$ | installation cost of wind DG with the capacity of S in bus I (\$/kVA) |
| $EP_{LL,s}$ | energy price in load level LL and state s | $\delta_{i,ll,s} \cdot V_{i,ll,s}$ | voltage magnitude and angle of bus I, in load level LL and state s |
| EP_{peak} | energy price in peak condition | $\mu_{i,ll,s}^V, \mu_i^V$ | degree of voltage constraint satisfaction for bus I, in load level LL and state s and degree of voltage constraint satisfaction for bus I, respectively |
| $PLF_{LL,s}$ | price level factor for load level LL and state s | μ^V, μ^I | degree of voltage constraint satisfaction for the whole network and degree of current constraint satisfaction for the whole network, respectively |
| $PLC_{ll,s}$ | Active loss cost in load level LL and state s (\$/kWh) | $P_{i,ll,s}^{DDG}, P_{i,ll,s}^{WDG}$ | Active power generated by WDG/DDG installed in bus I, in load level LL and state s |
| $QLC_{ll,s}$ | Reactive loss cost in load level LL and state s (\$/kWh) | $Q_{i,ll,s}^{DDG}$ | reactive power generated by DDG installed in bus I, in load level LL and state s |
| RC_{ll} | reliability cost of unsupplied energy in load level LL (\$/MWh) | $Q_{i,ll,s}^{WDG}$ | reactive power generated by WIND installed in bus I, in load level LL and state s |
| $P^l_{i,ll,s}$ | active load demand in bus I, in load level LL and state s | $P^loss_{i,ll,s}$ | Active loss power in bus I, in load level LL and state s |
| $P^l_{i,peak}$ | active load demand in bus I in peak condition | $Q^loss_{i,ll,s}$ | Reactive loss power in bus I, in load level LL and state s |
| $O \& MCDDG_{ll,s}$ | operation cost of DG in load level LL and state s (\$/kWh) | $P^{trans}_{i,ll,s}$ | The power imported from transmission system to distribution network through the ith HV/MV substation in load level j and state s |
| $O \& MCWDG_{ll,s}$ | operation cost of WIND in load level LL and state s (\$/kWh) | $LNS_{k,ll,s}$ | the load not supplied in load level LL and state s due to the outage of feeder k |

1. Introduction

Distribution network planning is one of the major duties of the electric power distribution companies because of yearly load increasing. Distribution network planning consists of two parts which is named: sub-transmission substation expansion planning (SSEP), and optimal feeder routing [1]. The role of Sub-transmission system is to deliver injected energy from transmission substations to distribution network. The existing sub-transmission network must be able to supply loads considering their growth rate, otherwise, expansion of network is essential task in order to not lose its adequacy [2, 3]. The goal of implementation of sub-transmission system expansion planning (SSEP) is to minimize total network cost through new installations and network reinforcement [2]. That kind of optimization problem with the mentioned purpose and its associated constraints is implemented within a specified time interval. Although, time regarding in planning process makes the procedure more complex, altering the load demand of customers makes it necessary to consider time in SSEP's computation. This type of planning is called dynamic programming technique which is

the most effective kind of planning [2]. Many mathematical techniques and algorithms have been applied to solve the distribution network planning's problem as: In [4], the placement of substations and feeder's routing is solved by genetic algorithm approach. Optimal location and sizing of HV/MV substations using pseudo dynamic methodology is presented in [5]. Also, uncertainty of load using LR fuzzy numbers is regarded in this work. An ant colony based algorithm is used to minimize investment and loss cost in [6]. A new cost function including cost of supply interruption is suggested in [7]. Moreover, genetic algorithm [8] and heuristic methods [9, 10] are used for the problem. As recent studies, in [5], loads uncertainty and load splitting is regarded in order to solve SSEP.

Despite, deniable advantages of distributed generation (DG) incorporated in sub-transmission substation expansion planning, but, few works have been reported on the allocation of DGs in distribution networks. So, it is needed to consider different kinds of new technologies in SSEP in order to get the associated benefits [11–13]. The first research about expansion of sub-transmission

system, considering the use of DG is solved by successive elimination algorithm (SEA) [14] which leads to optimal capacity of substations, the placement and sizing of DGs, and the sub-transmission lines expansion. The optimized procedure is based on the assumption that the load of each substation is known. In [15], static programming method is applied for sub-transmission expansion planning at the presence of distributed generation. The optimal capacity of substations and DGs are obtained using genetic algorithm [16]. However, the annual load variation and DGs' operation cost and the loss of the substations and lines [17] are important options for SSEP that should be taken into consideration which is regarded in less number of researches and are taken into consideration as main parts of cost function of suggested method.

On the other hand, the renewable resources are the best energy producers because of their clean feature and permanent existence. Also, wind-based distributed generation (WDG) can attract planners' attentions which cause noticeable reduction in costs and more reliability improvement in comparison with other possible renewable power generations [18]. Varying speed of wind leads to the inconstant output power of wind turbine that should be considered as one of the possible environment uncertainties [19].

Several methods have been modeled these uncertainties such as: Minimum active loss is obtained through combination of different renewable technologies. Here, different scenarios are generated using a probability distribution function (PDF) of uncertain values [20]. Monte Carlo Simulation (MCS) is another uncertainty modeling for location and penetration level of DG units used in [21]. On the whole, distribution network expansion planning (DNEP) involves following options which is incorporated in this paper:

Different types of objective functions in order to calculate related costs and other important options and solve them through efficient methods and algorithms

Considering renewable and non-renewable DGs

Applying uncertainties related to output power of renewable DGs', load demand, and electricity price in the planning procedure.

Reliability modeling.

In this paper, dynamic programming method is used to solve distribution network expansion planning. Two types of distributed generations including renewable and non-renewable ones are used as an alternative for DNEP. An expansion planning of distribution networks is presented which solves the weakness of the previous researches. The possibilities of expansion existing substations and

feeders or installation new ones make the planning procedure more adequate. Multi-objective functions have been evaluated consisting cost function, voltage deviation, voltage stability factor and emission. Here, cost function involves different efficient costs like: sub-transmission substations' expansion and installation cost, medium voltage feeders' installation and replacement cost, DGs installation and operation cost (simultaneously determines the optimal sitting, sizing and timing of both DG units and network components), purchased energy from the transmission network's cost, network loss cost and reliability cost. DNEP considers load demand, electricity price and output power of WDG's uncertainties using scenario based modeling. The non-dominated sorting genetic algorithm (NSGA-II) has been employed to optimize the DNEP's process then best compromised solution is found to put into practice. The DNEP is applied to the 54-bus test system and results are obtained via two states called as presence and non-presence of distributed generations. The provided comparison in section 4 shows the most significant role of DG units in total reduction in costs. Other parts are organized as:

Problem formulation in Section 2, proposed solution method in Section 3, application study and numerical results in Section 4, and the conclusion in Section 5.

2. Problem formulation

Distribution network expansion planning's cost based objective which ensures standard voltages and power quality is based on a reliable service to consumers. Following decision variables can demonstrate the solution of DNEP :

- _ Existing high voltage/Medium voltage (HV/MV) substations' expansion capacity;
- _ New (HV/MV) substations' location and capacity in order to be installed;
- _ Existing medium voltage (MV) feeders' upgrading;
- _ New MV feeders' routing and type in order to be installed;
- _ Location and sizing of non-renewable DGs (DDG) and renewable DGs (here WDG)
- _ Obtaining the optimal output power of DDG units in each load level

A) Uncertainty Modeling

The uncertainty modeling associated with load-price options which are dependent to each other and wind speed considering with its independent relation with previous options are modeled based on following descriptions. This modeling method is adapted from [22]. Fig. 1, consisting of N_{ll} levels in each year, indicates the price and load duration

curves. The load/price level factors as the results of dividing load/price to the peak value of them are defined by vertical axis in Fig. 1. Also t_{ll} shows the duration of each level. According to Fig. 1, a normal distribution curve which is distributed around their special expected values is divided into five states with definite value of probability. This normal distribution curve is applied for load/price level factors (LLF, PLF). So, the modeling of electricity price and electric load can be described as:

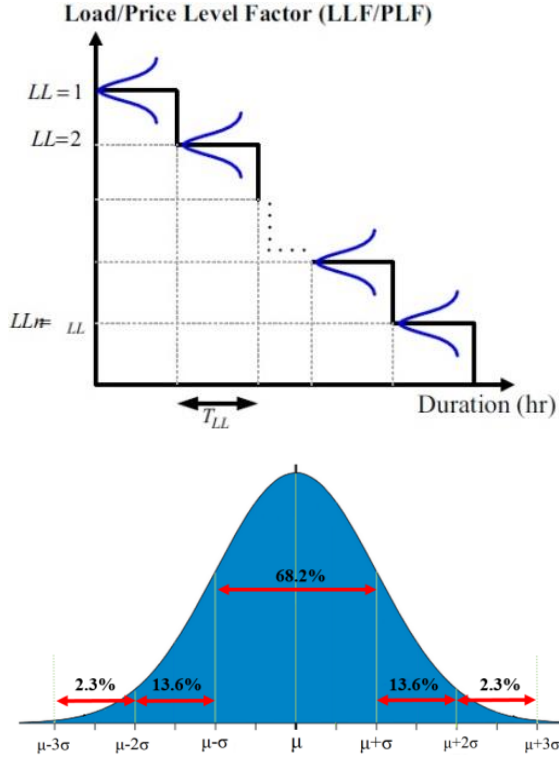


Fig. 1. Load and price level factor in uncertainty modeling

B) Wind speed modeling [23]

Rayleigh probability density function (pdf), is a kind of Weibull pdf is applied as an efficient approximation of wind speed profile (1)

$$f(v) = \left(\frac{2v}{c^2}\right) \exp\left[-\left(\frac{v}{c}\right)^2\right] \quad (1)$$

The scale index is shown by c that can be obtained using the mean value of the wind speed of a site, as:

$$v_m = \int_0^{\infty} v f(v) dv = \int_0^{\infty} \left(\frac{2v^2}{c^2}\right) \exp\left[-\left(\frac{v}{c}\right)^2\right] dv = \frac{\sqrt{\pi}}{2} c \quad (2)$$

$$c \approx 1.128 v_m \quad (3)$$

Several states of the pdf with limited steps of 1 m/s and definite limits of wind speed to form multi-state output power of the wind-based DG units are used. Table 1 shows related information in detail.

Table.1.
Selected wind speed states

| Wind speed state(s) | Wind speed limits(m/s) |
|---------------------|------------------------------|
| 1 | 0-1 |
| 2 | 1-2 |
| ⋮ | ⋮ |
| ⋮ | ⋮ |
| ⋮ | ⋮ |
| Last state | $v_{\max} - 1$ to v_{\max} |

The generated power of the wind turbine is calculated based on following formulations:

$$p_{i,t}^w(v) = \sum_{i=1}^t \xi_{i,t}^{dg} \begin{cases} 0 & \text{if } v \leq v_{in}^c \text{ or } v \geq v_{out}^c \\ \frac{v - v_{in}^c}{v_{rated} - v_{in}^c} \times p_{i,r}^w & \text{if } v_{in}^c \leq v \leq v_{rated} \\ p_{i,r}^w & \text{else} \end{cases} \quad (4)$$

where $p_{i,r}^w$ and $p_{i,t}^w$ are the rated and generated power of wind turbine related to bus i . Cut-out speed, cut-in speed and rated speed of the wind turbine are described by v_{out}^c , v_{in}^c and v_{rated} consequently. Fig. 2 shows the associated speed-power curve of a typical wind turbine.

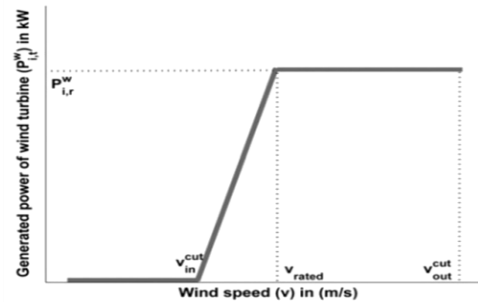


Fig. 2. The power curve of a wind turbine

The probability of each state is formulated as:

$$\pi_s^w = \int_{v_{1,s}}^{v_{2,s}} f(v) dv = \int_{v_{1,s}}^{v_{2,s}} \left(\frac{2v}{c^2} \right) \exp \left[- \left(\frac{v}{c} \right)^2 \right] dv \quad (5)$$

$$v_s = \frac{v_{2,s} + v_{1,s}}{2}$$

The generated power of wind turbine is calculated using equations (4) and (5).

The annual average power and capacity factor (CF) can be obtained by (6) and (7)

$$P_{ave} = \sum_s p_{i,r}^w(v) \times \pi_s^w \quad (6)$$

$$CF = \left[\frac{P_{ave}}{P_{i,r}^w} \right] \quad (7)$$

Table 2 and Table 3 show more details about the wind speed and power probabilities:

Table.2.
Wind speed probabilities

| Wind speed limits, m/s | Hour/year | probability |
|------------------------|-----------|-------------|
| 0-4 | 1804 | 0.205936 |
| 4-5 | 579 | 0.066096 |
| 5-6 | 984 | 0.112329 |
| 6-7 | 908 | 0.103653 |
| 8-9 | 799 | 0.9121 |
| 9-10 | 677 | 0.077283 |
| 10-11 | 439 | 0.050114 |
| 11-12 | 395 | 0.045091 |
| 12-13 | 286 | 0.032648 |
| 13-14 | 219 | 0.025 |
| 14-25 | 687 | 0.078425 |
| More than 25 | 0 | 0 |

$$states_s^{comb} = [load(s) \quad price(s) \quad wind(s)] \quad (10),(11)$$

$$prob_s^{comb}(\pi_s^c) = prob_s^l \times prob_s^p \times prob_s^w = \pi_s^l \pi_s^p \pi_s^w$$

The probability of each combined state is presented by $prob_s^{comb}(\pi_s^c)$.

Finally, a useful method known as scenario reduction technique proposed in [22] is used here to generate less number of states to have simple and swift computational process (see Appendix 1 for more details).

2.2. Constraints

Two sorts of constraints named hard and soft are considered in the planning problem. Hard constraints have to be satisfied during the DNEP,

Table.3.
Wind power probabilities

| State no | Rated power% | probability |
|----------|--------------|-------------|
| 1 | 100 | 0.078425 |
| 2 | 94.9696 | 0.025 |
| 3 | 84.9728 | 0.032648 |
| 4 | 74.976 | 0.045091 |
| 5 | 64.9792 | 0.050114 |
| 6 | 54.9824 | 0.077283 |
| 7 | 44.9856 | 0.09121 |
| 8 | 34.9888 | 0.112215 |
| 9 | 19.9936 | 0.103653 |
| 10 | 14.9952 | 0.112329 |
| 11 | 4.9984 | 0.066096 |
| 12 | 0 | 0.205936 |

C) Electricity Prices modeling

The uncertainty of electricity price in load level LL and state s is modeled as follows:

$$EP_{LL,S} = EP_{peak} PLF_{LL,S} \quad (8)$$

D) Electric Load modeling

The value of load uncertainty in bus i, year t, load level ll and state s can be calculated as:

$$S_{i,t,ll,s}^l = S_{i,peak}^l LLF_{LL,S} (1 + \alpha)^t \quad (9)$$

2.1. Combined states model:

The combination of related states is applied to produce the whole set of states, as follows:

however soft ones can be disturbed that it should be noticed and minimized.

A) Hard constraints

Network radiality constraint

In this section one of the most important network's constraints known as radiality structure is investigated and the topology of distribution network is compared with a tree based on graph theory. A tree consisting of m nodes and n (n=m-1) arcs is assumed as a connected graph without any loops. For checking the radiality of networks with a definite number of HV/MV substations, the forest

structure of network containing many trees is considered and applied.

Power Flow equation [24, 25]

$$\sum_{i=1}^{n_l} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} P_{i,ll,s}^{Trans} = \sum_{i=1}^{n_l} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} (P_{i,ll,s}^l + P_{i,ll,s}^{loss} - P_{i,ll,s}^{DDG} - P_{i,ll,s}^{WDG}) \quad (12)$$

$$\sum_{i=1}^{n_l} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} Q_{i,ll,s}^{Trans} = \sum_{i=1}^{n_l} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} (Q_{i,ll,s}^l + Q_{i,ll,s}^{loss} - Q_{i,ll,s}^{DDG} - Q_{i,ll,s}^{WDG}) \quad (13)$$

$$P_{i,ll,s}^{Trans} = V_{i,LL,S} \sum_{J=1}^{n_n} V_{j,ll,s} Y_{ij} \cos(\delta_{i,ll,s} - \delta_{j,ll,s} - \theta_{ij}) \quad (14)$$

$$Q_{i,ll,s}^{Trans} = V_{i,LL,S} \sum_{J=1}^{n_n} V_{j,ll,s} Y_{ij} \sin(\delta_{i,ll,s} - \delta_{j,ll,s} - \theta_{ij}) \quad (15)$$

Operating constraint of DG units.

The output power of DG units' must be less than its maximum capacity.

$$S_{i,ll,s}^{DG} \leq S_{i,max}^{DG} \quad (16)$$

Maximum penetration of DG units.

The reverse power flow from the distribution network to the upward grid is prevented by maximum penetration level (here 40%) used for DG units [26].

$$\sum_{i=1}^{n_l} P_i^{DDG} + \sum_{i=1}^{n_l} P_{i,rated}^{WDG} \leq MPL \times \sum_{i=1}^{n_l} P_{i,peak}^L \quad (17)$$

$$\mu_{i,ll,s}^V = \begin{cases} \frac{V_{i,ll,s} - V_{crit}^{\min}}{V_{safe}^{\min} - V_{crit}^{\min}} & V_{crit}^{\min} \leq V_{i,ll,s} \leq V_{safe}^{\min} \\ 1 & V_{safe}^{\min} \leq V_{i,ll,s} \leq V_{safe}^{\max} \\ \frac{V_{i,ll,s} - V_{crit}^{\max}}{V_{safe}^{\max} - V_{crit}^{\max}} & V_{safe}^{\max} \leq V_{i,ll,s} \leq V_{crit}^{\max} \\ 0 & \text{else} \end{cases} \quad (18)$$

(18) describes the voltage constraint satisfaction for bus i in state s. However there are more satisfaction levels for a specific bus because of more number of states in genuine network. Hence, (19) is used as the weighted average of voltage satisfaction of ith bus:

$$\mu_i^V = \frac{1}{8760} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} prob_s^{comb} T_{ll} \mu_{i,ll,s}^V \quad (19)$$

And, the average value of μ_i^V is formed to indicate the whole network buses' voltage condition as (20).

B) Soft constraints

Voltage limitation

The fuzzy modeling is applied to maintain bus voltages and thermal limits of feeders in their standard range. In order to prevent the violation of voltage buses, (18) is formulated based on a penalization function [22, 27], to maintain buses voltage in safe operation state limited by $[V_{safe}^{\min}, V_{safe}^{\max}]$. Illegal increase or decrease in the amount of voltage magnitude leads the value of satisfaction falls until it becomes zero.

$$\mu^V = \frac{\sum_{i=1}^{n_l} \mu_i^V}{n_l} \quad (20)$$

Feeders' thermal limits

The same procedure with just upper limit (instead of upper and lower limits) is done to calculate the satisfaction value of feeder currents (μ^I).

2.3. Objective Functions

The implemented multi objective functions for DNEP which are optimized using efficient algorithm NSGAI can be mentioned as follows:

- $F_1 =$ Cost minimization
- $F_2 =$ Voltage deviation minimization
- $F_3 =$ Voltage stability maximization
- $F_4 =$ Emission Reduction

$$F_1 = NPW_{ECS} + NPW_{ICS} + NPW_{RCF} + NPW_{ICF} + NPW_{ICDDG} + NPW_{O\&MCWDG} + NPW_{ICWDG} + NPW_{O\&MCDDG} + NPW_{PLC} + NPW_{PPTC} + NPW_{PPTC} + NPW_{CENS} \quad (21)$$

Expansion and Installation cost of HV substations:

$$NPW_{ECS} = \sum_{t=1}^{n_y} PW^t \sum_{i=1}^{n_{cs}} ecs_i(s) \quad (22)$$

$$NPW_{ICS} = \sum_{t=1}^{n_y} PW^t \sum_{i=1}^{n_{cs}} ICS_i(s) \quad (23)$$

Installation and replacement cost of MV feeders

A) Cost Minimization

The appropriate cost function includes all associated significant costs such as: HV/MV substations' expansion and installation cost, MV feeders' installation and upgrading cost, WDG and DDG units' installation and operation cost, the cost of purchased energy from transmission network, power loss cost and reliability cost. The mentioned terms are formulated as follow:

$$NPW_{RCF} = \sum_{t=1}^{n_y} PW^t \sum_{i,j=1,i \neq j}^{n_{cf}} [RCF_{ij}(k_1) - RCF_{ij}(k_2)] \quad (24)$$

$$NPW_{ICF} = \sum_{t=1}^{n_y} PW^t \sum_{i,j=1,i \neq j}^{n_{cf}} ICF_{ij}(k) \quad (25)$$

Installation, Operation and Maintenance Costs of WDGs and DG units

The cost functions of WDGs and DGs are formulated as follow:

$$NPW_{ICDDG} = \sum_{t=1}^{n_y} PW^t \sum_{i,j=1,i \neq j}^{n_l} ICDDG_i(S) \quad (26)$$

$$NPW_{O\&M CDDG} = \sum_{t=1}^{n_y} PW^t \sum_{i=1}^{n_l} \sum_{ll=1}^{n_l} \sum_{s=1}^{ns} O \& M CDDG_{ll,s} \times T_{ll} \times \pi_s^c \times P_{i,ll,s}^{DDG} \quad (27)$$

$$NPW_{ICWDG} = \sum_{t=1}^{n_y} PW^t \sum_{i,j=1,i \neq j}^{n_l} ICWDG_i(S) \quad (28)$$

$$NPW_{O\&M CWDG} = \sum_{t=1}^{n_y} PW^t \sum_{i=1}^{n_l} \sum_{ll=1}^{n_l} \sum_{s=1}^{ns} O \& M CWDG_{ll,s} \times T_{ll} \times \pi_s^c \times P_{i,ll,s}^{WDG} \quad (29)$$

Power Loss Cost

Following equation is used to formulate the cost of losses:

$$\sum_{i=1}^{n_l} \sum_{ll=1}^{n_l} \sum_{s=1}^{ns} P_{i,ll,s}^{loss} + jQ_{i,ll,s}^{loss} = \sum_{i=1}^{n_l} \sum_{ll=1}^{n_l} \sum_{s=1}^{ns} I_{i,ll,s}^2 \times (R_{ij} + jX_{ij}) \quad (30)$$

$$NPW_{PLC} = \sum_{t=1}^{n_y} PW^t \sum_{i=1}^{n_l} \sum_{ll=1}^{n_l} \sum_{s=1}^{ns} (T_{ll} \times Q_{i,ll,s}^{loss} \times \pi_s^c \times QLC_{ll,s} + T_{ll} \times P_{i,ll,s}^{loss} \times \pi_s^c \times PLC_{ll,s}) \quad (31)$$

Cost of Purchased Active Power from Transmission line:

$$\sum_{i=1}^{n_l} \sum_{ll=1}^{n_l} \sum_{s=1}^{ns} P_{i,ll,s}^{Trans} = \sum_{i=1}^{n_l} \sum_{ll=1}^{n_l} \sum_{s=1}^{ns} (P_{i,ll,s}^l + P_{i,ll,s}^{loss} - P_{i,ll,s}^{DDG} - P_{i,ll,s}^{WDG}) \quad (32)$$

$$NPW_{PPTC} = \sum_{t=1}^{n_y} PW^t \sum_{i=1}^{n_l} \sum_{ll=1}^{n_l} \sum_{s=1}^{ns} P_{i,ll,s}^{Trans} \times T_{ll} \times EP_{ll,s} \times \pi_s^c \quad (33)$$

$$PW^t = \left(\frac{1 + \text{Inf Rate}}{1 + \text{Int Rate}} \right)^t \quad (34)$$

Reliability Cost

It is possible for loads to not get supply because of failure and interruption outages which has high probability in distribution networks. Therefore, in order to consider the disadvantages of system outage which cause noticeable reduction in obtained benefits as the reliability of network decreases, EENSC parameter can be calculated from the following equation using the failure rate in each branch and the amount of the loads not supplied for the sake of failure occurring for all customers:

$$NPW_{CENS} = \sum_{t=1}^{n_y} PW^t \sum_{k=1}^{n_l} \lambda_k \times r_k \sum_{ll=1}^{n_{ll}} RC_{ll} \times T_{ll} \sum_{s=1}^{ns} LNS_{k,ll,s} \quad (35)$$

B) Minimizing the Voltage Deviation

The value of voltage deviation should be defined and minimized in order to enhance the security and power quality of system according to the below formulation [28]:

$$VD = \sum_{i=1}^{n_l} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} \pi_s^c \times (V_{i,ll,s} - V_{rated})^2 / n_l \quad (36)$$

C) Maximizing Voltage Stability

The ability of a system for controlling power and voltage in order to have voltages in standard levels is computed by voltage stability factor formulated as (37) [29]. Voltage stability factor for each bus 'm+1' is obtained using (41) as shown in Fig. 4.

$$EM = E_{grid} + E_{DDG}$$

$$E_{grid} = \sum_{t=1}^{n_y} PW^t \sum_{i=1}^{n_l} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} \pi_s^c \times T_{ll} \times P_{i,ll,s}^{Trans} \times K_{GRID} \quad (40)$$

$$E_{DDG} = \sum_{t=1}^{n_y} PW^t \sum_{i=1}^{n_l} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} P_{i,ll,s}^{DDG} \times \pi_s^c \times T_{ll} \times K_{DDG} \quad (41)$$

calculates technical dissatisfaction cost, as:

$$VD = \sum_{i=1}^{n_l} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} \pi_s^c \times (V_{i,ll,s} - V_{rated})^2 / n_l \quad (42)$$

3. Proposed solution method

In this paper NSGA-II algorithm has been applied to solve the suggested DNEP problem. The result is several non-dominated solutions known as

$$VSF_{m+1} = 2V_{m+1} - V_m \quad (37)$$

The value of VSF in voltage collapse point becomes zero that happens when the magnitude of receiving end bus voltage is half of magnitude of sending end bus voltage. The sum of VSF of all the load buses leads to the entire value of voltage stability over the whole distribution network as:

$$VSF_{total} = \sum_{i=1}^{n_l} \sum_{ll=1}^{n_{ll}} \sum_{s=1}^{ns} (2V_{i+1,ll,s} - V_{i,ll,s}) / n_l \quad (38)$$

The higher value of VSF_{total} provides more voltage stable condition. On the other hand, in order to maximize the voltage stability factor the $1/VSF_{total}$ must be minimized.

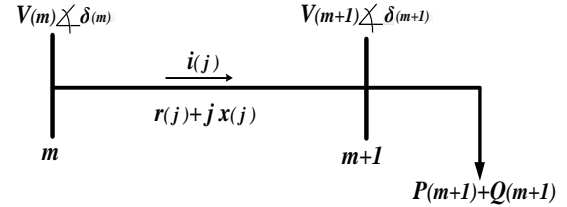


Fig. 3. Two bus section of radial distribution system

D) Environmental Impacts

Another main role of distributed generation is about decreasing the amount of pollutions as the result of greenhouse gases. One of the most effective objectives in DNEP is to calculate the amount of produced emission from grid and DG units as below formulations:

Pareto Fronts. The principle of NSGA-II in details is out of this paper's scope, that's why it has not been expressed. Complete review can be found in several papers e.g. [30]. At last, fuzzy set theory is applied to choose best compromise solution [31].

4. Example and analysis

A test system with 54-nodes illustrated in Fig. 7 is used to show the effectiveness of DNEP technique. This 33 kV network consists of 50 load

points (MV/LV buses), two existing (s1,s2) and two candidates (s3,s4) HV/MV substations to feed associated loads. Table 4, 5 show detail information about load points (MV/LV buses) [32,33] and HV/MV substation's characteristics, respectively.

Also, 17 existing feeders and 56 candidate feeders with twelve different sorts are used to form the forest structure of network. The related information of feeders is presented in Table 6.

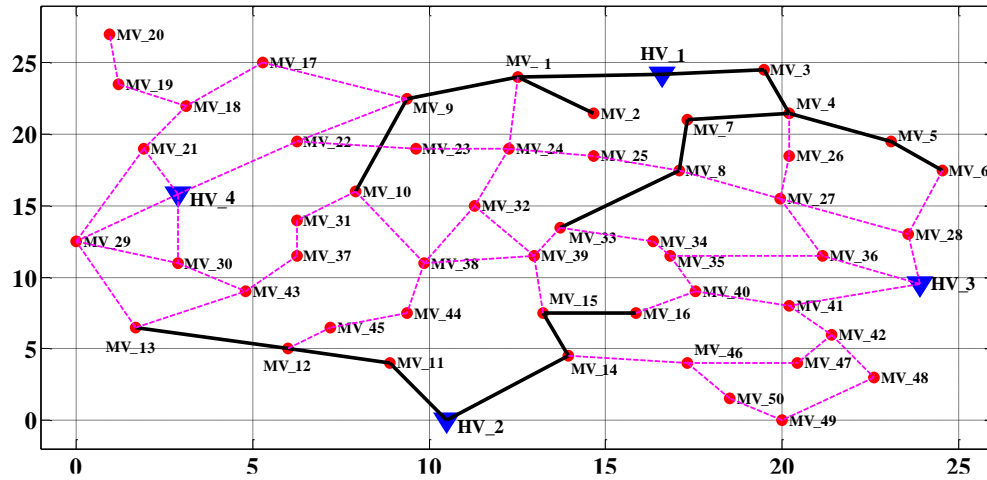


Fig. 4. The test system

Table.4. Specification of load points of Fig. 7.

| No | Load (kw) | No | Load (kw) | No | Load (kw) | No | Load (kw) | No | Load (kw) |
|----|-----------|----|-----------|----|-----------|----|-----------|----|-----------|
| 1 | 42 | 1 | 30 | 2 | 18 | 3 | 70 | 4 | 90 |
| 1 | 00 | 1 | 0 | 1 | 00 | 1 | 0 | 1 | 0 |
| 1 | 15 | 1 | 18 | 2 | 11 | 3 | 17 | 4 | 12 |
| 2 | 00 | 2 | 00 | 2 | 00 | 2 | 00 | 2 | 00 |
| 1 | 70 | 1 | 11 | 2 | 10 | 3 | 29 | 4 | 13 |
| 3 | 0 | 3 | 00 | 3 | 00 | 3 | 00 | 3 | 00 |
| 1 | 11 | 1 | 10 | 2 | 50 | 3 | 12 | 4 | 14 |
| 4 | 00 | 4 | 00 | 4 | 0 | 4 | 00 | 4 | 00 |
| 1 | 26 | 1 | 14 | 2 | 90 | 3 | 90 | 4 | 80 |
| 5 | 00 | 5 | 00 | 5 | 0 | 5 | 0 | 5 | 0 |
| 1 | 70 | 1 | 19 | 2 | 12 | 3 | 30 | 4 | 18 |
| 6 | 0 | 6 | 00 | 6 | 00 | 6 | 0 | 6 | 00 |
| 1 | 10 | 1 | 70 | 2 | 15 | 3 | 21 | 4 | 10 |
| 7 | 00 | 7 | 0 | 7 | 00 | 7 | 00 | 7 | 00 |
| 1 | 19 | 1 | 12 | 2 | 70 | 3 | 11 | 4 | 80 |
| 8 | 00 | 8 | 00 | 8 | 0 | 8 | 00 | 8 | 0 |
| 1 | 12 | 1 | 14 | 2 | 14 | 3 | 10 | 4 | 50 |
| 9 | 00 | 9 | 00 | 9 | 00 | 9 | 00 | 9 | 0 |
| 1 | 29 | 2 | 80 | 3 | 26 | 4 | 14 | 5 | 80 |
| 0 | 00 | 0 | 0 | 0 | 00 | 0 | 00 | 0 | 0 |

Moreover, two new developed distributed generation, one renewable and one non-renewable technology known as gas turbine and wind turbine are incorporated in DNEP with characteristics given in Table 7. DG units can be installed on any load buses. The probabilistic wind output power according to the data given in Tables 2 and 3 is applied to produce clean energy. Other essential parameters' values used in the planning procedure are given in Table 8.

Table.5. Specification of HV/MV substations

| Substation | Geographical position Km | Existing capacity (MVA) | Expandable capacity (MVA) |
|------------|--------------------------|-------------------------|---------------------------|
| S1 | 6.6 | 4.2 | 2×15 |
| S2 | 10.5 | 0 | 1×15 |
| S3 | 23.9 | 9.5 | 0 |
| S4 | 2.9 | 15.8 | 0 |

Table.6. Specification of conductors used in feeders

| Conductor or type | Resistance (ohm/km) | Reactance (ohm/km) | Current capacity (A) | Cost (KS/km) |
|-------------------|---------------------|--------------------|----------------------|--------------|
| 1 | 0.7500 | 0.1746 | 61 | 17 |
| 2 | 0.4794 | 0.1673 | 84 | 22 |
| 3 | 0.3080 | 0.1596 | 114 | 30 |
| 4 | 0.1972 | 0.1496 | 156 | 42 |
| 5 | 0.1208 | 0.1442 | 208 | 54 |
| 6 | 0.0723 | 0.1262 | 303 | 85 |
| 7 | 0.0487 | 0.1217 | 400 | 125 |
| 8 | 0.0405 | 0.1196 | 453 | 140 |
| 9 | 0.0350 | 0.1180 | 500 | 165 |
| 10 | 0.0247 | 0.1140 | 645 | 220 |
| 11 | 0.019 | 0.11 | 700 | 270 |
| 12 | 0.017 | 0.09 | 850 | 310 |

Table.7.
Characteristics of DG units used in the problem

| DG technology | Size (MVA) | Installation cost (KS/MVA) | Operation cost (S/MW h) |
|-------------------|------------|----------------------------|-------------------------|
| Gas turbine (GT) | 1 | 400 | 46 |
| Wind turbine (WT) | 1 | 800 | 10 |

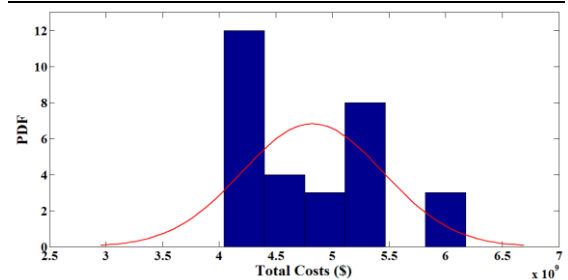
A) First state

In this state, the implementation of distribution network expansion planning through the upgrading or installation of feeders and substations is solved without incorporating distributed generation. The practicability and efficiency of the proposed method should be investigated. The problem of this state consists of multiple objective functions which are solved by using multi objective genetic algorithms (NSGA II). The output of NSGA II will be several non-dominated solutions that each of them can be chosen as the final strategy. It is important that all constraints of electrical network must be satisfied in all the obtained Pareto fronts. Due to the differences in Pareto fronts containing the structure of distribution network and the size, site and time of installed components, the values of the objective functions become different from one non-dominated solution to other ones. To analyse the efficiency and the practicality of the obtained optimal solutions, a statistical investigation on the non-dominated solutions is performed due to their values of objective functions. In order to study Pareto fronts in detail, Probability Density Function of each objective function is illustrates by Fig. 8. Applying fuzzy set theory to non-dominated solutions obtained from NSGA II in order to find the best compromise solution leads to the following result. In this regards, optimum size and time of HV/MV substation installation are provided in the Table. 9.

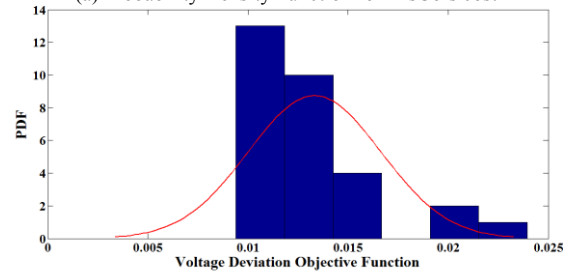
Table.8.
Some other essential values

| Parameter | Value |
|--|------------------------------|
| EP_{peak} | 60 |
| Int Rate (%) | 12 |
| Inf Rate (%) | 10 |
| V_{safe}^{max} | 0.95 |
| V_{safe}^{min} | 1.05 |
| V_{crit}^{min} | $0.95 \times V_{safe}^{min}$ |
| V_{crit}^{max} | $1.05 \times V_{safe}^{max}$ |
| $I_{crit,i}^{max}$ | $1.1 \times \text{current}$ |
| Failure rate of feeders (fail/km/year) | 0.2 |

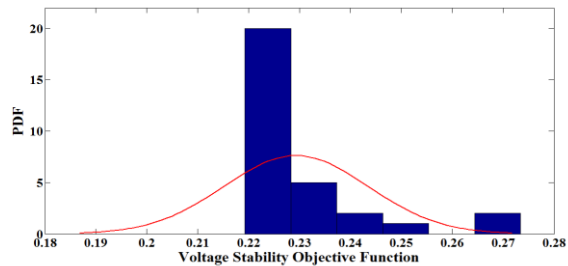
| | |
|---|-----|
| Repair time (h) | 2 |
| Emission of CO2 related to the received power from the transmission grid (kg/MWh) | 632 |
| Emission of CO2 related to the generated power by DDGs (kg/MWh) | 365 |
| Maximum number of installable DGs on each bus | 5 |



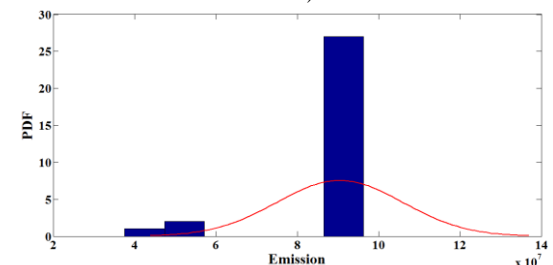
(a) Probability Density Function for DisCo's cost



(b) Probability Density Function of VD



(c) Probability Density Function of VSF (Voltage Stability Index)



(d) Probability Density Function of EM (Emission)

Fig. 5. Probability Density Function of COST, VD, VSF, and EM objective functions

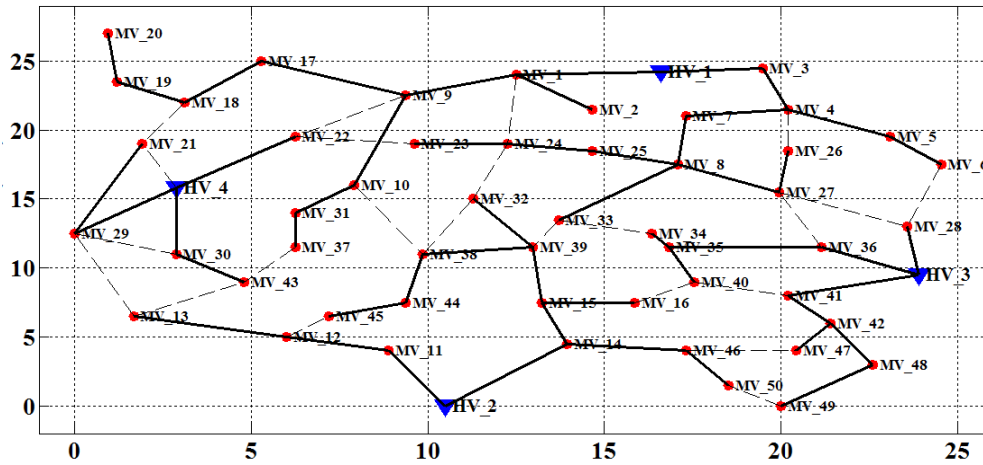
Table.9.
The size of HV/MV substations during planning period

| Year HV / | 1 | 2 | 3 | 4 | 5 |
|-----------|---|---|---|---|---|
| | | | | | |

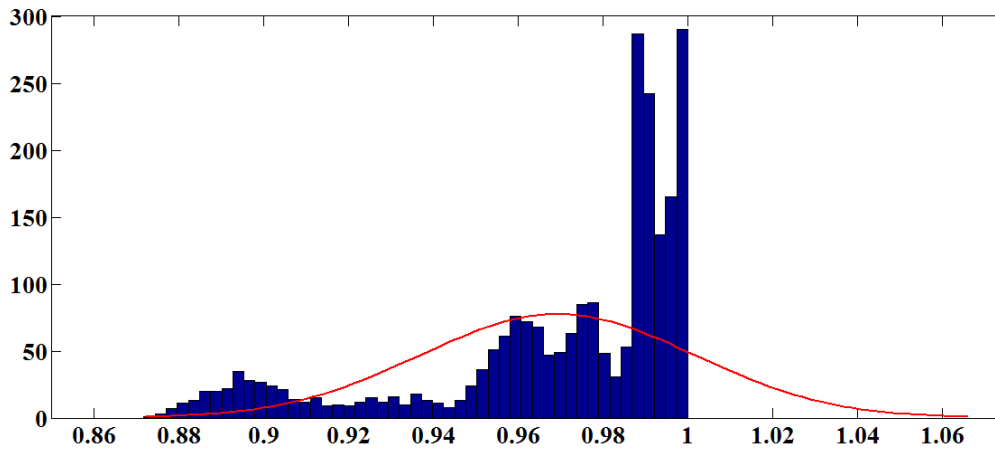
| Substation ID | | | | | |
|---------------|----|----|----|----|----|
| 1 | 45 | 45 | 45 | 60 | 60 |
| 2 | 30 | 30 | 30 | 30 | 30 |
| 3 | 15 | 15 | 15 | 15 | 15 |
| 4 | 15 | 15 | 15 | 15 | 15 |

Voltage amplitude can be assumed as an important index which affects power system in the case of power flow, power losses, power quality,

voltage stability and etc. So, any solution has to maintain the buses' voltages in acceptable level. In order to validate the fact that voltage amplitude has preserved in acceptable range, probability distribution function of all bus's voltages considering all the scenarios in planning period is provided. Also, the optimum structure of distribution network is illustrated in the following Fig. 9.



(a): Optimum structure of distribution network



(b): Probability Density Function of all bus's voltages

Fig. 6. Probability Density Function for the voltage of buses and DDG's operation

B) Second state

In this state, the DNEP is implemented at the presence of renewable and non-renewable distributed generations. Like previous state obtained results are shown as Fig.8.

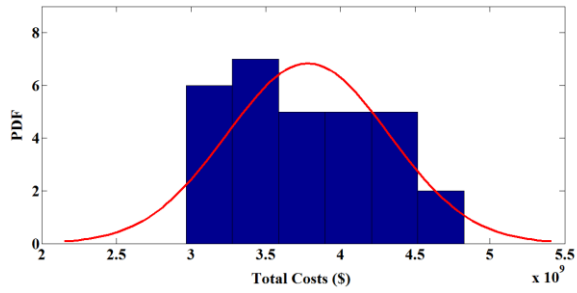
Table 11 shows the size, time, type and associated buses which DDG/WDG are installed. Provided comparisons in Table. 12, Table. 13 and Fig. 12 show the effectiveness of suggested DNEP considering the presence of DGs through significant reduction in each cost components such as: installation and upgrading costs of substations and feeders, energy loss and reliability costs and purchased energy from upward network's cost. Moreover, the amount of generated pollution and

value of voltage deviation decreases. Also, voltage stability factor improves as other advantages of presence of DGs. So, the remarkable increasing in obtained benefits means the achievement of proposed method's main goal.

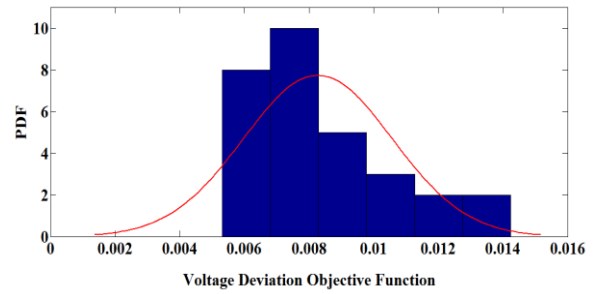
5. Conclusion

In this paper, a distribution network expansion planning at the presence of distributed generation is suggested and solved. Multi objective functions including cost function (considering most of the possible related cost components), voltage deviation, voltage stability factor and emission are put into consideration as the goal of optimization. Also, the reliability of network is calculated and

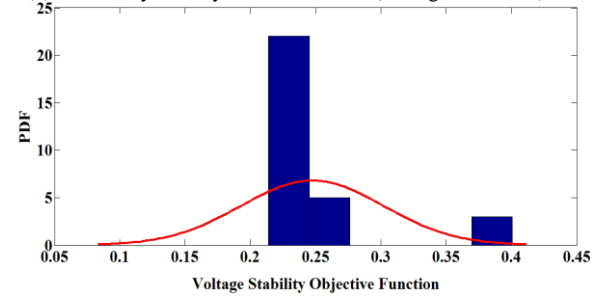
enhanced significantly by presence of DG units. The DNEP was solved considering different possible uncertainties such as: load demand, energy price and renewable DGs' output power with attention to the annual load variation. The proposed DNEP with its constraints and objective functions has been implemented using NSGAI1 algorithm. Comparing results of two different states which were applied to 54-bus network show the advantages of incorporating DG units in DNEP.



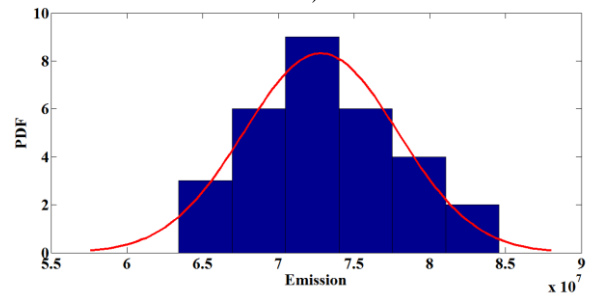
(a) Probability Density Function for DisCo's cost



(b) Probability Density Function of VD (Voltage Deviation)

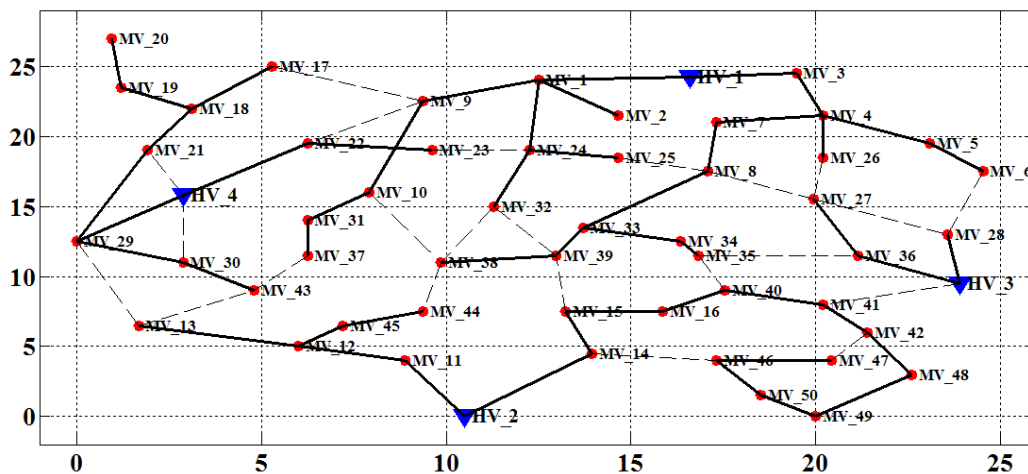


(c) Probability Density Function of VSF (Voltage Stability Index)

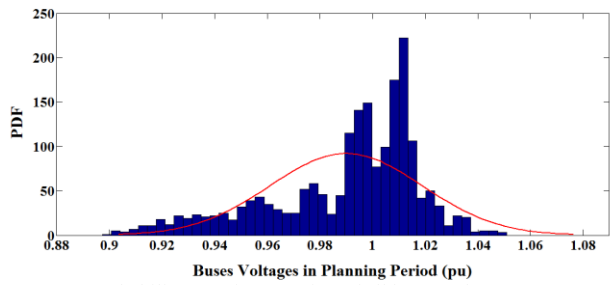


(d) Probability Density Function of EM (Emission)

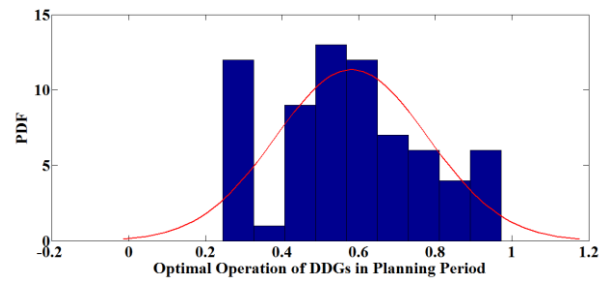
Fig. 7. Probability Density Function of COST, VD, VSF, EM objective function



(a): Optimum structure of distribution network



(b): Probability Density Function of all bus's voltages



(c): Probability Density Function of DDG's operation

Fig. 8. Probability Density Function for the voltage of buses and DDG's operation

Table.10.
The size of HV/MV substations during planning period

| Year HV / Substation ID | 1 | 2 | 3 | 4 | 5 |
|-------------------------|-----|-----|-----|------|------|
| 1 | 45 | 45 | 45 | 60 | 60 |
| 2 | 30 | 30 | 30 | 30 | 30 |
| 3 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 |
| 4 | 15 | 15 | 15 | 22.5 | 22.5 |

Table.11.
The size and type of DGs and their installed buses

| Type | Bus No. | Year | Capacity (kW) |
|------|---------|------|---------------|
| DDG | | 1 | 1 5000 |
| DDG | | 1 | 3 2000 |
| DDG | | 18 | 1 2000 |
| DDG | | 27 | 4 1000 |
| DDG | | 35 | 1 3000 |
| DDG | | 42 | 4 1000 |
| DDG | | 47 | 3 2000 |
| DDG | | 50 | 3 2000 |
| WDG | | 19 | 1 2000 |
| WDG | | 33 | 3 1000 |
| WDG | | 38 | 1 2000 |
| WDG | | 49 | 1 1000 |
| WDG | | 49 | 2 1000 |

Table.12.
Monetary Details of Optimal Planning

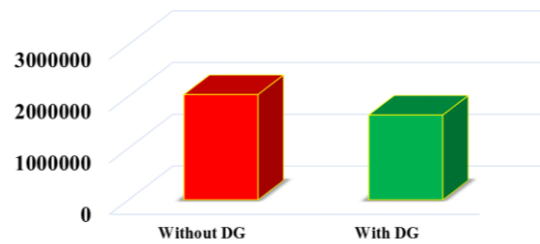
| Cost | First state | Second state |
|--|------------------|-------------------|
| Substation Installation and Expansion Costs (\$) | 2034716.1 913 | 1638394.5242 |
| Feeder Placement Costs (\$) | 1906197.0 713 | 1840169.0594 |
| DG Installation Costs (\$) | 0 | 13768085.093 3 |
| DG Operation Costs (\$) | 0 | 16623549.363 4 |
| Power Loss Costs (\$) | 242130.03 22 | 139003.4648 |

| | | |
|--|---------------------|---------------------|
| Cost of Purchased Active Power From TransCo (\$) | 5990424.7 034 | 4144322.769 |
| Energy not supplied | 43350202 89.4739 | 3330443632.9 042 |
| Total Costs (\$) | 43451937 57.472 | 3368597157.1 783 |

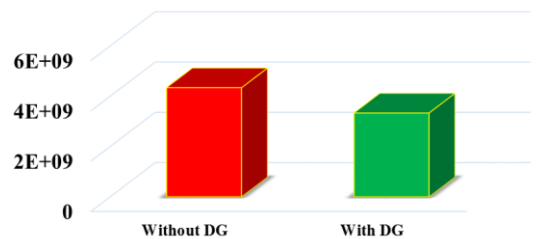
Table.13.
Technical and Environmental Aspects of Optimal Planning

| Cost(object function) | First state | Second state |
|-----------------------|-------------|-----------------|
| Emission Pollutant | 95281625.8 | 76497979.423323 |
| Voltage Deviation | 0.01297434 | 0.0054629196512 |
| Voltage Stability | 0.26771843 | 0.2428484530380 |

Substation Installation and Expansion Costs (\$)



Energy not Supplied Costs (\$)



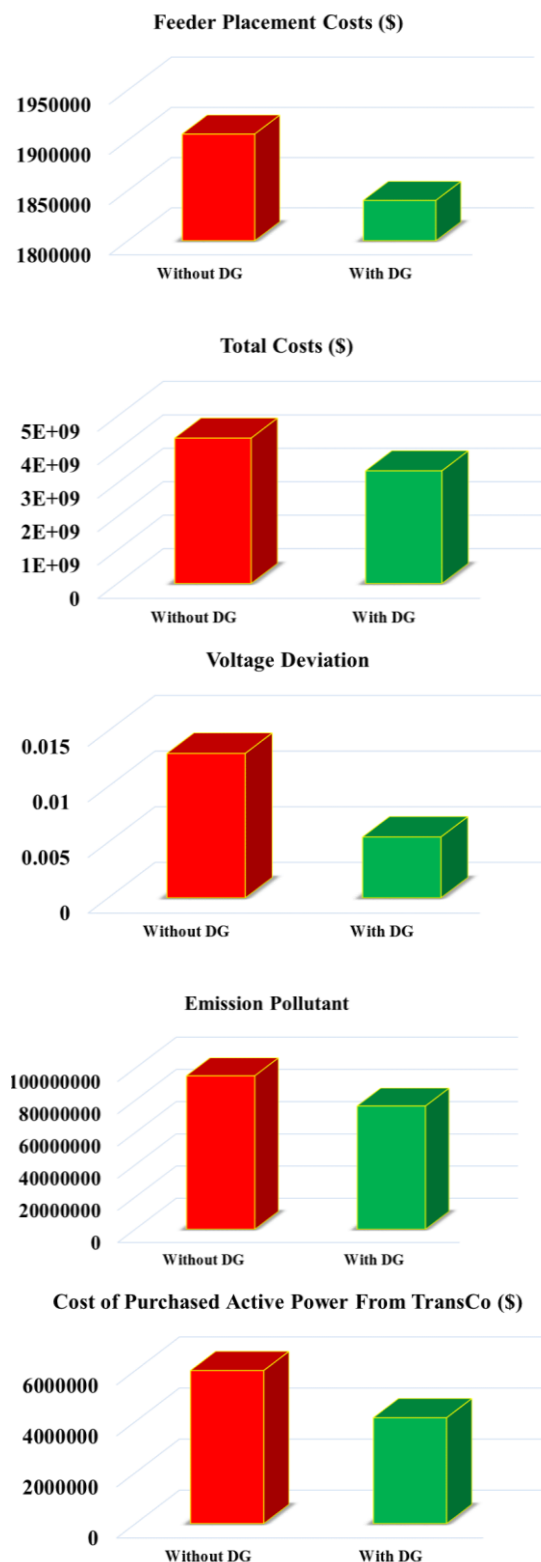


Fig. 9. Comparison results of two states

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