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An Optimal Distributed Low Frequency Control Framework for Network-based Interconnected Power Systems Featuring Communication Network Delay

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Abstract

Nowadays, the use of networked control system (NCS) technologies in modern power systems is on the rise owing to the expansion of electric energy distribution systems and the advent and development of new communication-based technologies. However, the modern network-based energy systems are constructed from some coupled subsystems interconnected via their states and share their information through the communication networks featuring inherent time delays. In this article, an efficacious multi-agent-based cooperative distributed economic model predictive control is developed to damp the frequency fluctuations and reduce the cost of consumed electricity in networked-based smart energy systems by considering communication network inherent delays. In this regard, a buffer-based moving horizon strategy with an estimator is suggested to estimate the own states of every sub-system and the coupled states, i.e., those that their information exchange among sub-systems and their values are not accessible due to communication time delays. Moreover, the boundedness of the estimation error and the stability of the closed-loop system are established by this method. The usability and proficiency of the suggested scheme are proved by applying the developed approach for an interconnected power grid.

Keywords: Networked Control System, Distributed model predictive control, Time delay, Low frequency control, Moving horizon strategy.

1. INTRODUCTION

Recently, the development of information and communication technology (ICT), as well as increasing the complexity and expansion of industrial processes, have led to the emergence of a new field of control

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science known as networked control systems (NCSs). NCSs are network-based control systems whose components are usually spaced apart, and data are communicated via the communication channels between the control components including the controller, sensor, and actuator [1]. Today, the efficiency of NCSs is widely considered in the control of large-scale systems, such as industrial automation systems and geographically dispersed electric grids, due to the benefits of reduced cost and wiring, ease of troubleshooting, and flexibility [2].
However, the communication network-induced imperfections, such as transmission delay and packet loss, will severely affect the control system performance or even cause instability [3]. These issues will become more pronounced in the design of geographically distributed systems over communication networks. Therefore, it is necessary to develop a distributed networked control theory for wide-area systems to have proper closed-loop behavior.

Among the distributed control methods, the distributed scheme of model-based predictive control, so-called DMPC, has recently been employed in the control of wide-area systems.networked-based plants [4]-[20]. Most of the available results on DMPC-based algorithms are evolved by considering the assumption that the entire state of systems is accessible and the communication channels between subsystems are ideal with no communication inherent imperfections, for instance, time delay and packet loss. However, these assumptions may not hold in practice

Nevertheless, some DMPC-based schemes are used in handling the mentioned communication network imperfections owing to their ability to predict the system behavior during a period [10]-[16]. Among different DMPC frameworks, cooperative DMPC is an important class of DMPC-based strategies wherein a global performance criterion is optimized employing a local controller. This scheme is suitable for the stabilization of wide-area plants with intensely interacted subsystems [15, 16].

Moreover, with the expansion of power grid components and the emergence of Wide-Area Measurement System (WAMS), which includes modern digital measurement systems, such as Phasor Measurement Units (PMUs), and new telecommunication systems, it is possible to monitor and control the power systems over a wide geographical area. Accordingly, the aforesaid issues in the NCSs appear in the control of widely distributed power grids over a communication network with strongly coupled subsystems.

On the other hand, with the increase in size and complexity of modern power systems, the damping control of spatially distributed power systems with traditional dedicated point-to-point wired links became impractical. Recently, the advancements of WAMS in parallel with networked communication technologies made the prospect of damping oscillation and enhancing the performance of geographically distributed power systems through remote signals a realistic one [21,22]. However, employing the remote signals through the communication channels brings unavoidably serious difficulties, such as inevitable time varying delays and data packet loss, into the design of networked control systems which will inevitably deteriorate the control performance or may even cause entire system instability [23, 24].

In recent years, considerable researches have been done to use wireless communication networks for designing wide-area damping controllers for distributed networked power systems. But, in most of the existing literatures the effects of communication network were ignored and it is assumed that the measured signals transfer through an ideal communication network with no delays or packet dropouts. A few exceptions to this are the works reported in [25]-[33].

In [25], the authors investigate the influence of network induced delay on wide area power system stabilization, and the results showed that the delay in feedback communication channels reduces control performance. The authors in [26] try to address the communication network induced limitations by investigating the effects of packet dropouts on the oscillatory stability response of a networked controlled power system. In [27], a centralized controller based on the linear matrix inequality was proposed for the closed-loop power system. However, only delay compensators can be obtained with this approach. In [28], the time-delay compensation methods were developed to compensate for the input delay in wide area power system. In [29], a stabilizing controller is provided for the power systems multiple time delays. The authors in [30] proposed a networked linear–quadratic regulator (LQR) control scheme for a dual-machine power system by considering the communication induced delays. In [31], a centralized networked wide area damping control scheme is introduced to provide supplementary signals for FACTS devices to damp inter-area oscillations. The authors in [32] used an adaptive phasor power oscillation damping controller to compensate for the continuous time varying delays in remote feedback signal. A fuzzy logic based networked damping controller is proposed in [33].

There are a few investigations on the development of controllers for communication network-based modern power systems by considering non-ideal communication channels between the sub-systems [13]-[16]. On the other hand, with the increase of the political and commercial interests in energy systems, complex control methods are needed for responsibly generating, transmitting, and delivering energy [34]. However, designing the appropriate control schemes to operate the wide-area modern energy systems in a reliable and economical manner is a serious challenge. As a consequence, an important objective in developing new control schemes for modern networked-based energy systems is to include the economic aims, such as the cost of consumed energy, profitability, efficiency, sustainability, and capacity into control objectives [34].

As far as we know, there is no work on the design of modern power system network-based controllers considering both economic constraints and communication network non-ideal behaviors, which is the main motivation of the research in the current study.

In this article, a new distributed network-based economic model predictive control, so-called NDEMPC, scheme is suggested for the optimization of energy consumption in networked-based smart energy systems. The proposed strategy is based on the Moving Horizon Estimation (MHE) strategy, which can handle the system constraints and work as an estimator and a predictor simultaneously. To this end, the states that are coupled between sub-systems and their values are unknown due to time delays are considered as unknown disturbances. Then, each local MHE estimates these coupled states and also the local state of each subsystem by solving an optimization problem. Moreover, the performance function of the control problem is considered an economic cost function, and the distributed controller is designed to optimize some economic performances of the process. The major contributions of the paper are listed below:

(i) Proposing a NDEMPC scheme for the management of energy consumption and reducing the cost of consumed electricity in network-based smart power systems.

(ii) Developing an estimation procedure to estimate the states coupled between subsystems, the values of which are not available due to delays and the local state of each subsystem by solving an economic optimization problem.

(iii) Presenting the stability analysis of the proposed strategy.

(iv) Reflecting the proposed method for damping frequency fluctuations and optimizing the energy in a network-based interconnected power system by considering communication network-induced delays.

The rest of this article is presented as follows. The transmission network between sub-systems and the formulation of a problem is provided in Section 2, along with representing some primary assumptions. The Proposed MHE-based cooperative NDEMPC scheme is explained in Section 3. Section 4 is assigned to the convergence analysis of the proposed estimator and the stability analysis of the closed-loop system. The simulation studies to verify the obtained theoretical results are given in Section 5. Eventually, some conclusions are drawn in Section 6.

Notations. Throughout the current article, symbol $\|.\|$ stands for the induced 2-norm and Euclidean norm for matrices and vectors, respectively. The superscripts "-1" and ''T'' indicate the inverse of square matrices and the transpose operation, respectively. $\mathbb{R}^{\mathrm{n}}$ and $\mathbb{R}^{\mathrm{n}\times\mathrm{m}}$respectively give the meaning of Euclidean Space with n dimension and the collection of every $n \times m$ real matrices. Furthermore, $\mathbb{I}_M$ refers to the collection of $1, 2, \dots, M$ integers. Consider $A_i$is a vector or a matrix, then, the notation $\underset{i\in\mathbb{I}_M}{\mathrm{col}}(A_i)$ demonstrates$[A_1{}^T, \dots, A_M{}^T]^T$. Moreover, $blkdiag(A_i)$ for A_i, $i \in \mathbb{I}_M$ refers to a block-diagonal matrix, where the elements of its main diagonal are A_i, $i \in \mathbb{I}_M$, and its other components are zero. Moreover, $\underset{i\in\mathbb{I}_M}{\mathrm{row}}(A_i)$ indicates the matrix $[A_1, \dots, A_M]$, $A > 0$ gives the meaning that A is a positive definite matrix, and $\oplus$ shows the Minkowski sum. Consider $\mathbb{X}^i$ is a sequence set, we define $\prod_{i\in\mathbb{I}_M}\mathbb{X}^i \triangleq \mathbb{X}^1 \times \dots \times \mathbb{X}^M$. The symbol $r\mathbb{B}$ stands for a closed ball of radius $r > 0$ created at origin. In addition, the interior of set $\mathbb{X}$ is represented by $\mathrm{int}(\mathbb{X})$. Furthermore, x^+ predicates as a successor state for the state vector x. Moreover, if $\mathbb{X} \subseteq \mathbb{R}^n$ is a convex and compact set that the origin will be in its interior, thus, $\mathbb{X}$ is a $\mathbb{C}$-set. Consider $\subseteq \mathbb{R}^n$ if $Ax + w \subseteq \mathbb{X}$ for all $w \in \mathbb{W}$, then $\mathbb{X}$ is so-called a Robust Positively Invariant (RPI) for $x^+ = Ax + w;\ w \in \mathbb{W} \subseteq \mathbb{R}^n$ and all $x \in \mathbb{X}$.

2. NETWORK AND SYSTEM DESCRIPTION

A networked large-scale smart energy system containing M linear discrete time sub-systems is considered in this work. The sub-systems interact through their states. The sub-system $i \in \mathbb{I}_M$ can exchange the data with its neighboring sub-systems N_i, where $N_i \subseteq \mathbb{I}_M$ and $N_i \neq \emptyset$. The sub-systems are supposed to have the capability for exchanging information through the communication channels featuring induced time-varying delays. For sub-system i, the time delays of the exchanged data from its neighboring sub-system j is presented by τ_k^{ij} at time step k.

Assumption 1. The time-varying communication delays τ_k^{ij} are bounded by $d_{min} < \tau_k^{ij} < d_{max}$, where $d_{min}$and d_{max} are certain positive integers displaying lower and upper bounds of time delays, respectively.

In the current paper, a buffer-based strategy described in [19] is considered for delay compensation. According to [19], all NDEMPC controllers accumulate their predictive input sequence with the length Nc into a unique package with a time-stamp and transfer them to their neighboring sub-systems via the communication channels. On the other hand, every local controller includes at most $(M-1)$ buffers, each of which is assigned to a neighboring sub-system and saves the relating packet until the entrance of the next one. Once a newer packet data is received, its time-label is compared with the time-label of the existing stored packet data in the buffer. If the newly received packet data is newer than the stored packet, the buffer will be updated, otherwise, the newly received packet data will be ignored and the buffer contents are moved one to the left, and then a zero is placed to the right side of the buffer. The buffer output data will be employed for the prediction of interacting states, which are not accessible at the current time-step, owing to the time delays of communication channels.

Assumption 2. For each subsystem, the control horizon Nc is equal to or greater than $\tilde{d}$ where, $\tilde{d} = d_{\max} - d_{\min} + 1$ [35].

Remark 1: It is worth mentioning that this work can be extended for the case of simultaneously occurring communication delays and packet dropout. In some literature, packet dropouts are considered extended delays [19, 20]. In other words, when the time delay in a packet is greater than a pre-specified upper bound of delays (i.e. d_{max} in this article), the packet is treated as a loss. An upper bound (T_p) on the number of successive time steps with packet dropouts is considered in the case of packet dropout. In this model, a subsystem will receive a new valid packet from another subsystem within $d_{max} + T_p$time steps in the worst case. This implies that we have a network with a maximum delay of $d_{max} + T_p$. Generally, the time delays induced by communication channels can include the influence of packet losses, disordering in receiving data, and network communication time delays.

The dynamic model for sub-system i is described by the linear discrete-time formulation as follows:

$$x_{k+1}^{i} = A^{ii}x_{k}^{i} + B^{i}u_{k}^{i} + \sum_{j\in\mathbb{N}_i} A^{ij}x_{k}^{p_{ij}} + D^{i}w_{k}^{i} \quad (1a)$$

$$y_{k}^{i} = C^{i}x_{k}^{i} + v_{k}^{i} \quad (1b)$$

in which $u^i \in \mathbb{U}^i \subseteq \mathbb{R}^{n_u^i}$, $x^i \in \mathbb{X}^i \subseteq \mathbb{R}^{n_x^i}$, and $y^i \in \mathbb{R}^{n_y^i}$ are the control input, state vector, and measured system output, respectively. Furthermore, $\mathbb{W}^i$ and $\mathbb{V}^i$ are $\mathbb{C}$-sets, and matrices A^{ii}, B^i, A^{ij} , D^i, and C^i are known and constant with appropriate dimensions. Besides, $\mathbb{U}^i$ and $\mathbb{X}^i$ are polytopic and polyhedral constraint sets, respectively, the origin of which is considered an interior point in these sets. Moreover, $v_k^i \in \mathbb{V}^i \subseteq \mathbb{R}^{n_y^i}$ and $w_k^i \in \mathbb{W}^i \subseteq \mathbb{R}^{n_x^i}$ stand for the unknown output and state disturbances, respectively. $\mathbb{X}^i$ is assumed as a set of all state variables where a feasible control command exists in $\mathbb{U}^i$. Besides, $x_k^{p_{ij}}$ shows the state trajectory of the jth sub-system, which is predicted and computed in ith sub-system, when the induced communication time delay occurs.

Assumption 3. For $i \in \mathbb{I}_M$, the pairs (A^{ii}, C^i) and (A^{ii}, B^i) are supposed to be detectable and stabilizable, respectively.

The whole regular system, i.e. with ignoring the communication induced time delays and disturbances, can be written as:

$$\bar{x}(k+1) = A\bar{x}(k) + Bu(k), \quad (2a)$$

$$\bar{y}(k) = C\bar{x}(k) \quad (2b)$$

with state $\bar{x} = col(\bar{x}^i) \in \mathbb{X} \subseteq \mathbb{R}^{n_x}$, control input $u = col(u^i) \in \mathbb{U} \subseteq \mathbb{R}^{n_u}$, and measured output $\bar{y} = col(\bar{y}^i) \in \mathbb{R}^{n_y}$. Furthermore, $\mathbb{U} = \prod_{i\in\mathbb{I}_M} \mathbb{U}^i$ and $\mathbb{X} = \prod_{i\in\mathbb{I}_M} \mathbb{X}^i$ are input and state constraint sets, respectively. The A matrix is introduced as below:

$$A \triangleq \begin{bmatrix} A^{11} & A^{12} & \cdots & A^{1M} \\ \vdots & \vdots & \ddots & \vdots \\ A^{i1} & A^{i2} & \dots & A^{iM} \\ \vdots & \vdots & \ddots & \vdots \\ A^{M1} & A^{M2} & \dots & A^{MM} \end{bmatrix}.$$

Also, $B \triangleq blkdiag(B^i)$ and $C \triangleq blkdiag(C^i)$.

3. THE SUGGESTED COOPERATIVE NDEMPC WITH INDUCED COMMUNICATION CHANNEL DELAYS

An efficient distributed moving horizon estimator-predictor is first proposed in this section. Then, the estimated and predicted states are used to design a cooperative NDEMPC such that the states of the closed-loop system converge to a neighborhood of the origin.

To reduce the estimation error, a pre-estimator is developed for every ith sub-system i as follows:

$$\hat{x}_{k+1}^{i} = A^{ii}\hat{x}_{k}^{i} + B^{i}u_{k}^{i} + \sum_{j\in\mathbb{N}_i} A^{ij}\hat{x}_{k}^{p_{ij}} + L^{i}(y_{k}^{i} - \hat{y}_{k}^{i}), \quad (3a)$$

$$\hat{y}_{k}^{i} = C^{i}\hat{x}_{k}^{i}, \quad (3b)$$

where $\hat{x}_k^i$ and $\hat{y}_k^i$ are the current estimation of x_k^i and current output of pre-estimator, respectively. Furthermore, L^i shows the gain matrix of the pre-estimator that should be computed in such a manner that $A_L^i \triangleq A^{ii} - L^iC^i$ is Schur. In (3), $\hat{x}_0^i \triangleq \hat{x}_{k=0}^i$ and $\hat{x}_0^{p_{ij}} \triangleq \hat{x}_{k=0}^{p_{ij}}$ are the initial states. Moreover, $\hat{x}_k^i$

denotes the current estimation of the *i*th sub-system local state trajectory and $\hat{x}_k^{p_{ij}}$ stands for the current predicted coupled state between ith and jth sub-systems.

For sub-system i, a local constrained MHE is then designed using the constrained optimization problem as below:

$$\min_{\hat{x}_{k-Ne}^{i},\hat{x}_{k-Ne}^{p_{ij}},\dots,\hat{x}_{k-1}^{p_{ij}}} 1/2\left(\sum_{l=k-Ne}^{k}\left\|y_l^i - C^i\hat{x}_l^i\right\|^2 + \mu_i\left\|\hat{x}_{k-Ne}^i - \bar{x}_{k-Ne}^i\right\|^2\right) \quad (4a)$$

Subject to:

$$\hat{x}_{l+1|k}^{i} = A^{ii}\hat{x}_{l|k}^{i} + B^{i}u_{l}^{i} + \sum_{j\in\mathbb{N}_i} A^{ij}\hat{x}_{l}^{p_{ij}} + L^{i}\left(y_{l}^{i} - \hat{y}_{l|k}^{i}\right),$$

$$l = k - Ne, \dots, k - 1 \quad (4b)$$

$$\hat{y}_{l|k}^{i} = C^{i}\hat{x}_{l|k}^{i}, \qquad l = k - Ne, \dots, k \quad (4c)$$

$$\hat{x}_{l}^{i} \in \mathbb{X}^{i},\ \hat{x}_{l}^{p_{ij}} \in \mathbb{X}^{j}, \qquad l = k - Ne, \dots, k \quad (4d)$$

$$\hat{x}_{l+1|k}^{p_{ij}} = {A_L}^{j}\hat{x}_{l|k}^{p_{ij}} + B^{j}u_{l}^{b_{ij}} + \sum_{i'\in\mathbb{N}_j\backslash i} A^{ji'}\hat{x}_{l|k}^{p_{ii'}} + A^{ji}\hat{x}_{l|k}^{i} + L^{j}y_{l}^{j}, l = k - Ne, \dots, k - 1 \quad (4e)$$

where μ_i refers to a non-negative weight for the ith moving horizon estimator, and the estimation horizon is shown by $(Ne + 1)$. The term $\hat{x}_{l|k}^{i}$ demonstrates the $\hat{x}^{i}$ value at the time-step, which is computed at time-step k. By changing (4), the optimal solution is obtained and illustrated with $({\hat{x}^{o}}_{k-Ne}^{i}, {\hat{x}^{o}}_{k-Ne}^{p_{ij}}, \dots, {\hat{x}^{o}}_{k-1}^{p_{ij}})$ for $j \in \mathbb{N}_i$. The optimum sequence of the local states is calculated through (4b), where the current optimum state of *i*th sub-system is shown by ${\hat{x}^{o}}_{k}^{i}$. In (4a), the term $\left\|\hat{x}_{k-Ne}^{i} - \bar{x}_{k-Ne}^{i}\right\|^2$ shows the arrival criterion, and the latter estimation is employed in this work to update the $\bar{x}_{k-Ne}^{i}$ as the form

$$\bar{x}_{k-Ne|k}^{i} = {A_L}^{i}{\hat{x}^{o}}_{k-Ne-1|k-1}^{i} + B^{i}u_{k-Ne-1}^{i} + \sum_{j\in\mathbb{N}_i} A^{ij}{\hat{x}^{o}}_{k-Ne-1}^{p_{ij}} + L^{i}y_{k-Ne-1}^{i} \quad (5)$$

In sub-system i, the current *j*th predicted state is computed employing (6):

$$\hat{x}_{k|k}^{p_{ij}} = {A_L}^{j}{\hat{x}^{o}}_{k-1|k}^{p_{ij}} + B^{j}u_{k-1}^{b_{ij}} + \sum_{i'\in\mathbb{N}_j\backslash i} A^{ji'}{\hat{x}^{o}}_{k-1|k}^{p_{ii'}} + A^{ji}{\hat{x}^{o}}_{k-1|k}^{i} + L^{j}y_{k-1}^{j} \quad (6)$$

in which $u_k^{b_{ij}}$ shows the jth buffer output, $j \in \mathbb{N}_i$ is installed in the sub-system at time-step k, and $\mathbb{N}_j\backslash i$ presents the neighboring sub-systems of the *j*th sub-system without considering the *i*th sub-system. It should be noticed that the $u_k^{b_{ij}}$ input trajectory is equivalent to u_k^i, when no time delay of communication channels occurs. Moreover, $y_{k-1}^{j} = 0$ in (4e) and (6) when delays occur.

In the following, the NDEMPC is developed for system (1). In this regard, for every *i*th sub-system at the time-step k, the cooperative control criterion function is considered as:

$$V(k) = \sum_{l=0}^{Nc-1}\left[\left\|\hat{x}_{k+l}^{i}\right\|_{Q_i}^{2} + Co_{k+l}^{i}\left\|u_{k+l}^{i}\right\|_{Q_i}^{2}\right] + \left\|\hat{x}_{k+Nc}^{i}\right\|_{P_i}^{2} + \sum_{j\neq i}\sum_{l=0}^{Nc-1}\left[\left\|\hat{x}_{k+l}^{p_{ij}}\right\|_{Q_j}^{2} + Co_{k+l}^{ij}\left\|u_{k+l}^{b_{ij}}\right\|_{R_j}^{2}\right] + \left\|\hat{x}_{k+Nc}^{p_{ij}}\right\|_{P_j}^{2} \quad (7a)$$

in which $u_{\cdot}^{i}$ is the control trajectory that should be designed. Moreover, the prices of electricity consumption are considered in the optimization problem as the cost coefficients Co^{i} and Co^{ij}. For all $i \in I_M, Q_i > 0, R_i > 0$, and $P_i > 0$ are weighting matrices. It is worthwhile to note that if (A^{ii}, Q_i) are detectable, thus $Q_i \geq 0$. The parameter Nc stands for both control and prediction horizons that meet $\tilde{d} \leq Nc$. Terms $\hat{x}^{i}(.)$ and $\hat{x}_{k+l}^{p_{ij}}$ are respectively the estimated local state trajectory $x_i(.)$ and the predicted interacted state between *i*th and *j*th sub-systems, which are computed through the ith local MHE.

For the *i*th sub-system at time-step k and every iteration ρ, the optimal control command is obtained through solving (7), which is an optimal regulating control problem with constraints:

$$\begin{aligned} &\min_{u_k^i,\dots,u_{k+Nc-1}^i} V(k) \\ &\text{s.t.} \\ &u_{min} \leq u_{k+l}^{i} \in \mathbb{U}^{i} \leq u_{max}, \quad l = 0,\dots,k+Nc-1 \end{aligned} \tag{7b}$$

After ρ iterations at time-step k, the obtained optimal solution of the mentioned problem is illustrated by $(u^{*}{}_{k}^{i})^{T} = [(u^{*}{}_{k}^{i})^{T}, \dots, (u^{*}{}_{k+Nc-1}^{i})^{T}]$. The final solution results a convex combination between the current and last optimal solutions of the EMPC problem (7), i.e. $u_k^i = a_i u^{*}{}_{k}^{i} + (1 - a_i)u_{k-1}^{i}$, where a_i is the ith weighting factor of sub-systems that meets $\sum_{i \in I_M} a_i = 1$. Now, u_k^i is accumulated into one packet data and transmitted to every interconnected sub-system $j \neq i$ through a communication network featuring time delays. The first value of u_k^i is also exerted to the ith sub-system.

4. THE ANALYSIS OF ESTIMATION ERROR AND STABILITY

4.1. The Analysis of Estimation Error

To derive an analytic expression for the estimation error, (4) is represented as a convex Quadratic Program (QP) as below:

$$\begin{aligned} &\min_{z_i} \quad \tfrac{1}{2} z_i^{T} H_i z_i + F_i^{T} z_i + r_i \\ &\text{s.t.} \quad G_i z_i \leq \xi_i \end{aligned} \tag{8}$$

in which constant matrices G_i and ξ_i, with appropriate dimensions, present the constraints of (4b)-(4e). r_i is a constant term and $z_i = col(\hat{x}_{k-Ne}^{i}, X^{j})$ shows an uncertain vector of optimization in which $X^{j} \triangleq \underset{j \in \mathbb{N}_i}{col}(col(\hat{x}_{k-Ne}^{p_{ij}}, \dots, \hat{x}_{k-1}^{p_{ij}}))$. The corresponding matrices H_i and F_i in (8) are

$$H_i = \begin{bmatrix} (A_{Ne}^{i})^{T} A_{Ne}^{i} + \mu_i & (A_{Ne}^{i})^{T} \underset{j \in \mathbb{N}_i}{\text{row}}(A_{Ne}^{ij}) \\ (\underset{j \in \mathbb{N}_i}{\text{row}}(A_{Ne}^{ij}))^{T} A_{Ne}^{i} & (\underset{j \in \mathbb{N}_i}{\text{row}}(A_{Ne}^{ij}))^{T} \underset{j \in \mathbb{N}_i}{\text{row}}(A_{Ne}^{ij}) \end{bmatrix} \tag{9a}$$

$$F_i = \begin{bmatrix} -(A_{Ne}^{i})^{T} Q_{Ne}^{i} \underline{y}^{i} + (A_{Ne}^{i})^{T} B_{Ne}^{i} \underline{u}^{i} - \mu_i \bar{x}_{k-Ne}^{i} \\ -\left(\underset{j \in \mathbb{N}_i}{\text{row}}(A_{Ne}^{ij})\right)^{T} Q_{Ne}^{i} \underline{y}^{i} + \left(\underset{j \in \mathbb{N}_i}{\text{row}}(A_{Ne}^{ij})\right)^{T} B_{Ne}^{i} \underline{u}^{i} \end{bmatrix} \tag{9b}$$

where

$$A_{Ne}^{i} \triangleq \begin{bmatrix} C^{i} \\ C^{i} A_{L}{}^{i} \\ \vdots \\ C^{i} (A_{L}{}^{i})^{Ne} \end{bmatrix},$$

$$Q_{Ne}^{i} \triangleq I - L_{Ne}^{i}, \quad \underline{y}^{i} \triangleq col(y_{k-Ne}^{i}, \dots, y_{k}^{i})$$

$$\underline{x}^{p_{ij}} \triangleq col(x_{k-Ne}^{p_{ij}}, \dots, x_{k-1}^{p_{ij}})$$

$$A_{Ne}^{ij} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C^i A^{ij} & 0 & \cdots & 0 \\ C^i {A_L}^i A^{ij} & C^i A^{ij} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C^i ({A_L}^i)^{Ne-1} A^{ij} & C^i ({A_L}^i)^{Ne-1} A^{ij} & \cdots & C^i A^{ij} \end{bmatrix}$$

$$B_{Ne}^{i} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C^i B^{i} & 0 & \cdots & 0 \\ C^i {A_L}^i B^{i} & C^i B^{i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C^i ({A_L}^i)^{Ne-1} B^{i} & C^i ({A_L}^i)^{Ne-1} B^{i} & \cdots & C^i B^{i} \end{bmatrix}$$

$$L_{Ne}^{i} \triangleq \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ C^i L^{i} & 0 & \cdots & \cdots & 0 \\ C^i {A_L}^i L^{i} & C^i L^{i} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C^i ({A_L}^i)^{Ne-1} L^{i} & C^i ({A_L}^i)^{Ne-1} L^{i} & \cdots & C^i L^{i} & 0 \end{bmatrix},$$

$$\underline{\mathrm{u}}^{\mathrm{i}} \triangleq \mathrm{col}(\mathrm{u}^{\mathrm{i}}_{\mathrm{k-Ne}}, \dots, \mathrm{u}^{\mathrm{i}}_{\mathrm{k-1}}), \quad .$$

To update matrix F_i in (9b), we need the past inputs, current and past measured outputs, predicted state, and a prior estimate of the computed state $\bar{x}^i_{k-Ne}$.

For the ith MHE, the estimation error is introduced as:

$$e^i_{k-Ne|k} \triangleq \begin{bmatrix} e^{e_i}_{k-Ne|k} \\ e^{e_{p_i}}_{k-Ne|k} \end{bmatrix} \triangleq \begin{bmatrix} x^i_{k-Ne} - {\hat{x}^o}{}^i_{k-Ne|k} \\ X^j - X^{o\,j} \end{bmatrix} \quad (10)$$

where $X^{o\,j} \triangleq \underset{j\in\mathbb{N}_i}{\mathrm{col}}\left(col\left({\hat{x}^o}{}^{p_{ij}}_{k-Ne}, \dots, {\hat{x}^o}{}^{p_{ij}}_{k-1}\right)\right)$. Note that a sequence of the estimation error is obtained using (4b) and (4e) in (10), where the current estimation error is denoted by $e^i_{k|k}$. The boundedness of e^i_k will be established with the following theorem.

Theorem 1. Consider the subsystem (1) with pre-estimator (3), considering the constrained distributed MHE problem described in (4) and the constrained NDEMPC problem presented in (7). If assumptions 1-3 are held, and when $\mu_i \geq 0$ and ${A_L}^i$ is Schur, then the current estimation error e^i_k will be bounded and there is a ℂ-set $\mathbb{E}^i$ so that for all $k \geq 0$ if $e^i_{k=0} \in \mathbb{E}^i$ then $e^i_k \in \mathbb{E}^i$.

Proof. Firstly, a dynamic model is pursued for the estimation error (10) based on the QP Active Set Strategy. The Karush–Kuhn–Tucker (KKT) conditions for (8) are as follows:

$$\mathrm{z^o}_{\mathrm{i}} = -\mathrm{H_i}^{-1}(\mathrm{F_i} + \mathrm{G_{iA}}^{\mathrm{T}}\lambda_{\mathrm{iA}}), \quad (11a)$$

$$\lambda_{\mathrm{iA}} = -(\mathrm{G_{iA}}\mathrm{H_i}^{-1}\mathrm{G_{iA}}^{\mathrm{T}})^{-1}(\mathrm{G_{iA}}\mathrm{H_i}^{-1}\mathrm{F_i} + \xi_{\mathrm{iA}}), \quad (11b)$$

$$\lambda_{iA} > 0 \quad (11c)$$

where $z^{o}{}_{i} = col(\hat{x}^{o}{}^{i}_{k-Ne}, X^{o\,j})$. λ_{iA}, G_{iA}, and ξ_{iA} stand for the active Lagrange multipliers and the relating active inequality matrices of (7), respectively.

Substituting (9a)-(9b) into (11a)-(11b), using (10), and rearranging the terms yield that

$$e^{i}_{k-Ne|k} = A^{i}_{e} e^{i}_{k-Ne-1|k} + \bar{D}^{i}_{w} w^{i}{}^{k-1}_{k-Ne-1} + \bar{D}^{i}_{v} v^{i}{}^{k}_{k-Ne-1} + H_i{}^{-1} G_{iA}{}^{T} \lambda_{iA} \quad (12a)$$

$$\lambda_{iA} - (G_{iA} H_i{}^{-1} G_{iA}{}^{T})^{-1} G_{iA} (A^{i}_{e} e^{i}_{k-Ne-1|k} + \bar{D}^{i}_{w} w^{i}{}^{k-1}_{k-Ne-1} + \bar{D}^{i}_{v} v^{i}{}^{k}_{k-Ne-1} - e^{i}_{k-Ne|k}) \quad (12b)$$

in which

$$A^{i}_{e} \triangleq H_i{}^{-1} \begin{bmatrix} \mu_i A_L{}^{i} & \underset{j\in\mathbb{N}_i}{\mathrm{row}}([\mu_i A^{ij} \quad 0 \quad \dots \quad 0]) \\ 0 & 0 \end{bmatrix},$$

$$\bar{D}^{i}_{w} \triangleq H_i{}^{-1} \begin{bmatrix} [\mu_i D^{i} & -(A^{i}_{Ne})^{T} D^{i}_{Ne}] \\ & 0 \end{bmatrix},$$

$$\bar{D}^{i}_{v} \triangleq -H_i{}^{-1} \begin{bmatrix} \mu_i L^{i} & (A^{i}_{Ne})^{T} Q^{i}_{Ne} \\ 0 & (\underset{j\in\mathbb{N}_i}{\mathrm{row}}(A^{ij}_{Ne}))^{T} Q^{i}_{Ne} \end{bmatrix},$$

$$w^{i}{}^{k-1}_{k-Ne-1} \triangleq col(w^{i}_{k-Ne-1}, \dots, w^{i}_{k-1}),$$

$$v^{i}{}^{k}_{k-Ne-1} \triangleq col(v^{i}_{k-Ne-1}, \dots, v^{i}_{k})$$

and D^{i}_{Ne} is defined as

$$D^{i}_{Ne} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C^{i} D^{i} & 0 & \cdots & 0 \\ C^{i} A_L{}^{i} D^{i} & C^{i} D^{i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C^{i} (A_L{}^{i})^{Ne-1} D^{i} & C^{i} (A_L{}^{i})^{Ne-1} D^{i} & \cdots & C^{i} D^{i} \end{bmatrix}.$$

where

$$w^{i}{}^{k-1}_{k-Ne-1} \triangleq col(w^{i}_{k-Ne-1}, \dots, w^{i}_{k-1}),$$

$$v^{i}{}^{k}_{k-Ne-1} \triangleq col(v^{i}_{k-Ne-1}, \dots, v^{i}_{k}),$$

$$A^{i}_{e} \triangleq H_i{}^{-1} \begin{bmatrix} \mu_i A_L{}^{i} & \underset{j\in\mathbb{N}_i}{\mathrm{row}}([\mu_i A^{ij} \quad 0 \quad \dots \quad 0]) \\ 0 & 0 \end{bmatrix}$$

$$\bar{D}^{i}_{w} \triangleq H_i{}^{-1} \begin{bmatrix} [\mu_i D^{i} & -(A^{i}_{Ne})^{T} D^{i}_{Ne}] \\ & 0 \end{bmatrix},$$

$$\bar{D}^{i}_{v} \triangleq -H_i{}^{-1} \begin{bmatrix} \mu_i L^{i} & (A^{i}_{Ne})^{T} Q^{i}_{Ne} \\ 0 & (\underset{j\in\mathbb{N}_i}{\mathrm{row}}(A^{ij}_{Ne}))^{T} Q^{i}_{Ne} \end{bmatrix}$$

and

$$D^{i}_{Ne} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C^{i} D^{i} & 0 & \cdots & 0 \\ C^{i} A_L{}^{i} D^{i} & C^{i} D^{i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C^{i} (A_L{}^{i})^{Ne-1} D^{i} & C^{i} (A_L{}^{i})^{Ne-1} D^{i} & \cdots & C^{i} D^{i} \end{bmatrix}$$

Substituting (12b) into (12a) follows that

$$e^{i}_{k-Ne|k} = A^{i}_{e} e^{i}_{k-Ne-1|k} + D^{i}_{w} w^{i}{}^{k-1}_{k-Ne-1} + D^{i}_{v} v^{i}{}^{k}_{k-Ne-1}$$

Rewriting (4b) for $l = k - Ne$ gives

$$\hat{x}^{i}_{k-Ne+1|k} = A^{ii} \hat{x}^{i}_{k-Ne|k} + B^{i} u^{i}_{k-Ne} + \sum_{j\in\mathbb{N}_i} A^{ij} \hat{x}^{p_{ij}}_{k-Ne} + L^{i}(y^{i}_{k-Ne} - \hat{y}^{i}_{k-Ne|k}) \quad (13)$$

Moreover, (1a) is rewritten in the following form

$$x^{i}_{k-Ne+1} = A^{ii} x^{i}_{k-Ne} + B^{i} u^{i}_{k-Ne} + \sum_{j\in\mathbb{N}_i} A^{ij} x^{p_{ij}}_{k-Ne} + D^{i} w^{i}_{k-Ne} \quad (14)$$

Subtracting (13) from (14) and Ne iterations, it is obtained that:

$$e^{e_i}_{k|k} = ({A_L}^i)^{Ne} e^{e_i}_{k-Ne|k} + G^{ij} e^{p_i}_{k-Ne|k} + G^i_w {w^i}^{k-1}_{k-Ne} - G^i_v {v^i}^{k-1}_{k-Ne} \quad (15)$$

in which $G^i_w \triangleq [D^i ({A_L}^i)^{Ne-1}, \dots, D^i]$, $G^i_v \triangleq [L^i ({A_L}^i)^{Ne-1}, \dots, L^i]$ and $G^{ij} = (\underset{j \in \mathbb{N}_i}{\text{row}}(G^{ij}_{Ne}))^T$ where $G^{ij}_{Ne} \triangleq [A^{ij} ({A_L}^i)^{Ne-1}, \dots, A^{ij}]$. (15) can be rewritten as the form:

$$e^i_{k|k} = M^i e^i_{k-Ne|k} + G^i_w {w^i}^{k-1}_{k-Ne} - G^i_v {v^i}^{k-1}_{k-Ne} \quad (16)$$

where $M^i = \begin{bmatrix} ({A_L}^i)^{Ne} & G^{ij} \\ 0 & I \end{bmatrix}$. Multiplying $(M^i)^{-1}$ by both sides of (16) yields that:

$$e^i_{k-Ne|k} = (M^i)^{-1} e^i_{k|k} - (M^i)^{-1} G^i_w {w^i}^{k-1}_{k-Ne} + (M^i)^{-1} G^i_v {v^i}^{k-1}_{k-Ne} \quad (17)$$

Combining (17) and (12) and rearrangement of the terms result in the following equation:

$$e^i_{k+1|k} = M^i A^i_e (M^i)^{-1} e^i_{k|k} + \bar{w}^i_e \quad (18)$$

in which $\bar{w}^i_e = \left(I - M^i A^i_e (M^i)^{-1}\right) G^i_w {w^i}^{k-1}_{k-Ne} + M^i D^i_w {w^i}^{k}_{k-Ne} + M^i D^i_v {v^i}^{k+1}_{k-Ne}$ and $\bar{w}^i_e$ is reflected as a disturbance that lies in the $\mathbb{C}$-set $\overline{\mathbb{W}}^i_e$ introduced by:

$$\overline{\mathbb{W}}^i_e = \left(I - M^i A^i_e (M^i)^{-1}\right) G^i_w \overline{\mathbb{W}}^i_{Ne} \oplus M^i D^i_w \overline{\mathbb{W}}^i_{Ne+1} \oplus M^i D^i_v \overline{\mathbb{V}}^i_{Ne+2}$$

where $\overline{\mathbb{W}}^i_{Ne} \triangleq (\underbrace{\mathbb{W}^i \times \dots \times \mathbb{W}^i}_{Ne})$, $\overline{\mathbb{W}}^i_{Ne+1} \triangleq (\underbrace{\mathbb{W}^i \times \dots \times \mathbb{W}^i}_{Ne+1})$ and $\overline{\mathbb{V}}^i_{Ne+2} \triangleq (\underbrace{\mathbb{V}^i \times \dots \times \mathbb{V}^i}_{Ne+2})$.

The fact that ${A_L}^i$ is Schur implies that A^i_e and $M^i A^i_e (M^i)^{-1}$ are Schur matrices and there is a $\mathbb{C}$-set $\mathbb{E}^i$, which is robust positively invariant for (18) [36]. It follows that $M^i A^i_e (M^i)^{-1} \mathbb{E}^i \oplus \overline{\mathbb{W}}^i_e \subset \mathbb{E}^i$ and if $e^i_{k=0} \in \mathbb{E}^i$, then $e^i_k \in \mathbb{E}^i$, $\forall k \geq 0$.

For the ith sub-system, the prediction error is defined as $e^{p_{ij}}_k \triangleq x^j_k - x^{p_{ij}}_k$ and $\hat{e}^{p_{ij}}_k \triangleq \hat{x}^j_k - \hat{x}^{p_{ij}}_k$. Let e^p_k and x^p_k be vectors that involve in whole prediction errors and predicted states, respectively, at time-step k. The nest corollary is implied from Theorem 1, which is needed to guarantee the feasibility of the obtained input trajectory of all sub-systems.

Corollary 1. With the existence of conditions considered in Theorem 1, the prediction errors $e^{p_{ij}}_k$ and $\hat{e}^{p_{ij}}_k$ are bounded for all $j \in \mathbb{N}_i$ and $i \in I_M$. There are also two sets $\mathbb{X}^p$ and $\mathbb{E}^p$ both including the origin so that $\mathbb{X}^p \oplus \mathbb{E}^p \subseteq \mathbb{X}$ and for every $x^p_{k=0} \in \mathbb{X}^p$ and $e^p_{k=0} \in \mathbb{E}^p$, $x^p_k \in \mathbb{X}$ for all $k > 0$. Furthermore, there are two sets $\widehat{\mathbb{X}}$ and $\mathbb{E}$ both including the origin so that $\widehat{\mathbb{X}} \oplus \mathbb{E} \subseteq \mathbb{X}$ and for every $\hat{x}_{k=0} \in \widehat{\mathbb{X}}$ and $e_{k=0} \in \mathbb{E}$, $\hat{x}_k \in \mathbb{X}$ for all $k > 0$.

The convexity of $\mathbb{U}_i$, corollary 1, and the initialization procedure employed in the suggested cooperative guarantee that if there is a feasible input trajectory for every sub-system, then there are feasible input trajectories for all subsystems at future times.

4.2. The Closed-Loop Robust Exponential Stability Analysis

Firstly, using (3a) and (1a) and some mathematical manipulation, the following equation can be obtained for the *i*th sub-system:

$$\hat{x}_{k+1}^{i} = A^{ii}\hat{x}_{k}^{i} + B^{i}u_{k}^{i} + \sum_{j\in\mathbb{N}_i} A^{ij}\hat{x}_{k}^{j} + t_{k}^{i}$$

where $t_k^i \triangleq -\sum_{j\in\mathbb{N}_i} A^{ij}\hat{e}_k^{p_{ij}} + L^iC^ie_k^{e_i}$. Accordingly, the closed-loop system can be presented as the form

$$\hat{x}_{k+1} = F(\hat{x}_k, u_k) + t_k, \tag{19}$$

where $F(\hat{x}_k, u_k) \triangleq A\hat{x}_k + Bu_k$ and $t_k \triangleq col(t_k^i)$.

Theorem 2. Considering the constrained distributed MHE problem (4), the constrained cooperative NDEMPC problem (7), and Assumptions 1-3, the origin of the closed-loop system presented in (19) is robustly exponential stable on $\mathrm{int}(\mathbb{X})$.

Proof: Firstly, it should be noted that the nominal system (i.e. without disturbances and communication delays) under the state feedback NDEMPC is exponentially stable [37]. Afterward, to prove the robust exponential stability of the closed-loop system similar to [37], it is required to indicate that there is $\alpha > 0$ and $0 < \gamma < 1$ so that $\mathbb{C} \subseteq \mathbb{X}$ with $0 \in \mathrm{int}(\mathbb{C})$ for all compact sets, and given every $\varepsilon > 0$, $\|\Phi_{Nc}(k,x)\| \le \alpha\gamma^k\|x\| + \varepsilon$, $\forall\ k \ge 0$. For the system presented in (19), the initial condition is indicated with $(x,u) \in \mathbb{X} \times \mathbb{U}^{Nc}$. d_k^i and d_k are bounded owing to proposition 1, presented in [13], and Theorem 1. Thus, given every $\varepsilon > 0$, there is $\Gamma > 0$ so that $\max_{k\ge0}\|d_k\| \le \Gamma$, $\max_{k\ge0}\|e_k\| \le \Gamma$, where $e_k = col(e_k^i)$. According to [38], a solution of system (19) is introduced as $\Phi_{Nc}(k,z)$ at each time step k.

Let $\hat{z} \triangleq (\hat{x}, u)$ and $H(z)$ as a difference inclusion [38]. From Lemma 30 provided in [38], for any $\upsilon > 0$ there is $\Gamma > 0$ so that for all $(\hat{z}, e, t, e^+) \in \mathbb{Z} \times \Gamma\mathrm{B} \times \Gamma\mathrm{B} \times \Gamma\mathrm{B}$, so that $\hat{x} \in \mathbb{X}$, and some $0 < \sigma < 1$, the following condition holds:

$$\max_{z^+\in H(z)} V_n(z^+) \le \max\{\sigma V_n(z), \upsilon\} \tag{20}$$

Using Lemma 1 presented in [37], there is $a > 0$ and $b > 0$ so that $a\|z\|^2 \le V_n(z) \le b\|z\|^2$. Owing to (20), we have $\|\Phi_{Nc}(k,x)\|^2 \le V_n(\Phi_{Nc}(k,z)) \le \max\{\sigma^k V_n(z), \upsilon\} \le \max\{\sigma^k b\|z\|^2, \upsilon\}$. Considering $\overline{\alpha} = (b/a)^{1/2}$, $\gamma = \sigma^{1/2}$ and $\overline{\varepsilon} = (\upsilon/a)^{1/2}$ result in $\|\Phi_{Nc}(k,z)\| \le \max\{\overline{\alpha}\gamma^k\|z\|, \overline{\varepsilon}\}$. Besides, using Lemma 1 provided in [13], we have $\|\Phi_{Nc}(k,x)\| \le \|\Phi_{Nc}(k,z)\| \le \max\{\overline{\alpha}\gamma^k\|z\|, \overline{\varepsilon}\} \le \overline{\alpha}\gamma^k\|z\|$ $+\overline{\varepsilon} \le \alpha\gamma^k\|x\| + \varepsilon$ $\quad \|u\| \le d\|\hat{x}\| \le d\|x\| + d\Gamma$ that implies $\alpha = \overline{\alpha}(1+d)$ and $\varepsilon = \overline{\varepsilon} + \overline{\alpha}d\Gamma$.

5. CASE STUDY

The method proposed in this paper is used to design damping control signals in a multi-area power system under the communication network. In this regard, we first extract the model of a power system with coupled subsystems and then use the proposed controllers to stabilize its low-frequency fluctuations. The model of a power system includes various algebraic and differential equations that describe the dynamic model of generators, controllers, loads, and networks. In this paper, a linear model of a power system, which includes generator dynamics, a voltage control excitation system, and an equivalent transmission network, is used to design a low-frequency fluctuation damper. The equivalent network model is a reduced network model in which all nodes are directly connected to each other in the generator. The generator model is considered a third-order linear model with the excitation system. The generator dynamics equations are as follows:

$$\begin{aligned} &\Delta\dot{\delta}_i(t) = \Delta\omega_i(t) \\ &M_i\Delta\dot{\omega}_i = -D_i\Delta\omega_i - \Delta P_{ei} \\ &T'_{doi}\Delta\dot{E}'_{qi} = -\Delta E'_{qi} + (X_{di} - X'_{di})\Delta I_{di} + \Delta E_{fi} \end{aligned} \quad (21)$$

where δ_i is the ith generator angle, ω_i is the relative speed of the rotor, and P is the active power delivered to the ith generator terminal. Moreover, E is the transient electromotive force on the orthogonal axis, which is assumed to be constant when the SCR controller gain is large. The parameters M_i and D_i represent the inertia constant and the damping constant of the ith generator, respectively. X_{di} and X'_{di} are synchronous reactance and transient reactance on the orthogonal axis of the ith generator, respectively. T'_{doi} is the transient time constant of the orthogonal open circuit of the ith generator. Finally, E_{fi} denotes the electromotive force. A second-order transmission function is considered for the Automatic Voltage Regulator (AVR) as shown in Fig. 1.

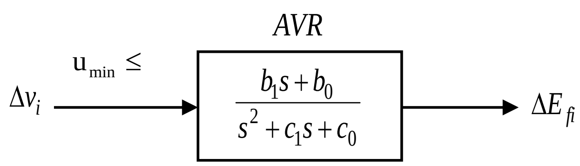

Fig. 1. The Automatic voltage Regular model.

In fact, AVR controls the excitation current and, consequently, the wet voltage of the generators. The model of automatic excitation and the voltage control system is as follows.

$$\begin{aligned} &\dot{z}_{2i} = -c_{1i}z_{2i} - c_{0i}z_{1i} + \Delta v_i \\ &\dot{z}_{1i} = z_{2i} \\ &\Delta E_{fi} = b_{1i}z_{2i} + b_{0i}z_{1i} \end{aligned} \quad (22)$$

In the above relationship, z_{2i} and z_{1i} are the internal states of AVR. We consider the dynamic state variables x_i of the ith synchronous generator as follows.

$$x_i = \begin{bmatrix} \Delta\delta_i & \Delta\omega_i & \Delta E'_{qi} & z_{2i} & z_{1i} \end{bmatrix}^T \quad (23)$$

In general, the model of a power grid is described by algebraic nodal equations that describe the relationships between injection currents, generator voltage, and loads using the admittance matrix. If we remove the load nodes, we obtain the equivalent model of the power grid in which the generator nodes are directly connected to each other. Therefore, I_{di}, which is the orthogonal axis currents of the generator and p as the active power delivered at the ith generator terminal, are obtained as follows:

$$I_{di} = \sum_{j=1, j\neq i}^{N} E'_{qi}[B_{ij}\cos\delta_{ij}(t) - G_{ij}\sin\delta_{ij}(t)] \quad (24)$$

$$P_{ei}(t) = E'_{qi}\sum_{j=1}^{N} E_{qj}\left[B_{ij}\sin\delta_{ij}(t) + G_{ij}\cos\delta_{ij}(t)\right] \quad (25)$$

where $\delta_{ij}(t) = \delta_i(t) - \delta_j(t)$ is the angle difference between generators i and j, N is the total number of generators, and $i, j \in \{1, \ldots N\}$

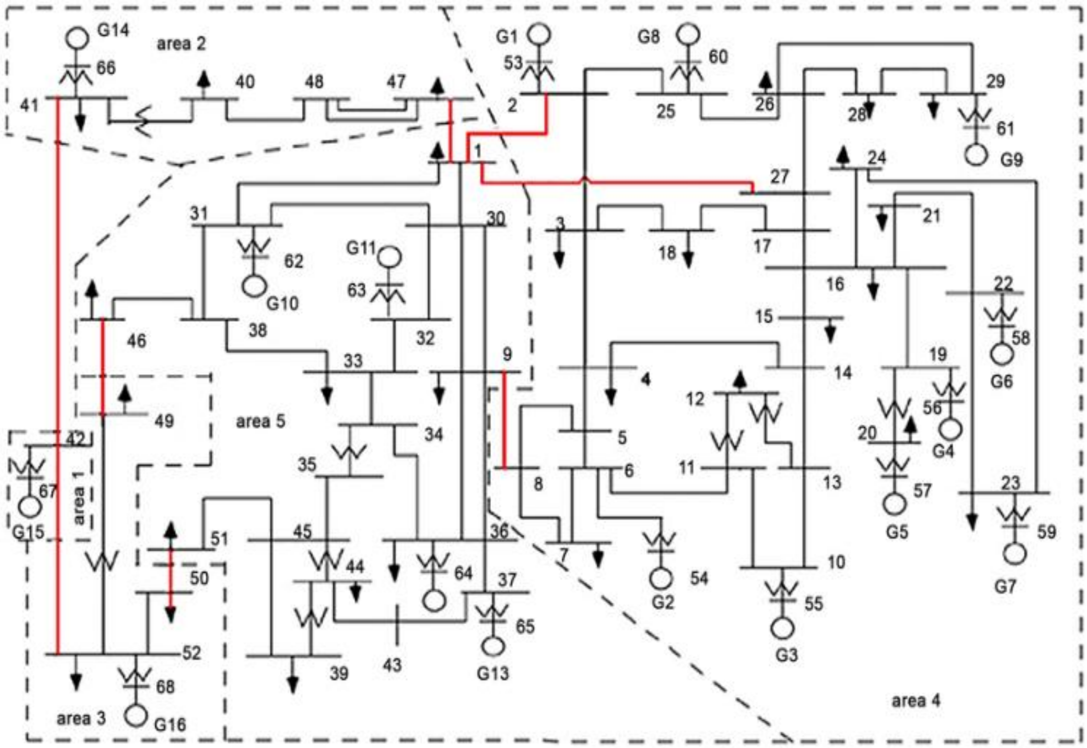

***Fig. 2. Interconnected power system schematic*.**

B_{ij} and G_{ij} are the real and imaginary parts of the y-admittance matrix Y_{ij}. By linearizing the above equations, we obtain the following:

$$\Delta P_{ei} = \begin{bmatrix} \frac{\partial P_{ei}}{\partial \delta} & \frac{\partial P_{ei}}{\partial E'_q} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta E'_q \end{bmatrix}$$
$$\Delta I_{di} = \begin{bmatrix} \frac{\partial I_{di}}{\partial \delta} & \frac{\partial I_{di}}{\partial E'_q} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta E'_q \end{bmatrix} \quad (26)$$

where $\Delta\delta$ denotes changes in the rotor angle and $\Delta E'_q$ indicates transient voltage changes of all generators. By combining the above relations, the dynamic relation of the small signal of the generator connected to the power system is obtained as the following:

$$\dot{x}_i = A_i x_i + B_i u_i + \sum_{j \in N_i} A_{ij} x_j \quad (27)$$

where N_i is the set of generators connected physically to the ith generator.

To simulate the performance of the proposed method, the simulation results for a 5-area-16-machine power system are presented in the following. Fig. 2 shows a single-line diagram of this power system, which includes 68 basses and 86 lines, and its areas are separated using a dashed line. The connecting lines between the areas are marked as red lines. Complete information about this system is given in reference [39, 40]. To show the effect of non-ideal behavior of the telecommunication network between subsystems, numerical studies are presented in ideal and non-ideal cases of the telecommunication network in different operating conditions.

In this case, it is assumed that the communication channel between the subsystems is ideal and the phenomena of data packet loss and network-induced latency do not occur; in other words, $\tau_{ij}(k)=0$. In the predictive control design based on the economic model, the control and forecast horizons are considered 5 and 8, respectively. To evaluate the proposed method, the performance of the designed stabilizers is tested in the presence of single-phase and three-phase ground faults. In this regard, once the single-phase error and then the three-phase error are exerted to the system in the bus 26 and the 26-29 line for 50 seconds, and the system is simulated for 20 seconds. The results of the system performance against single-phase and three-phase ground faults applied in bus 26 and lines 26_29 are shown in Figs. 4 and 5. These figures show the generator angles 1 and 9 relative to the reference generator (13) angle in the presence of a single-phase ground fault, respectively. The angles of these generators obtained relative to the reference generator in the presence of the three-phase ground fault are depicted in Figures 6 and 7. The figures indicate a good performance of the method when the telephone channel is ideal and there are no network delay and data packet loss phenomena.

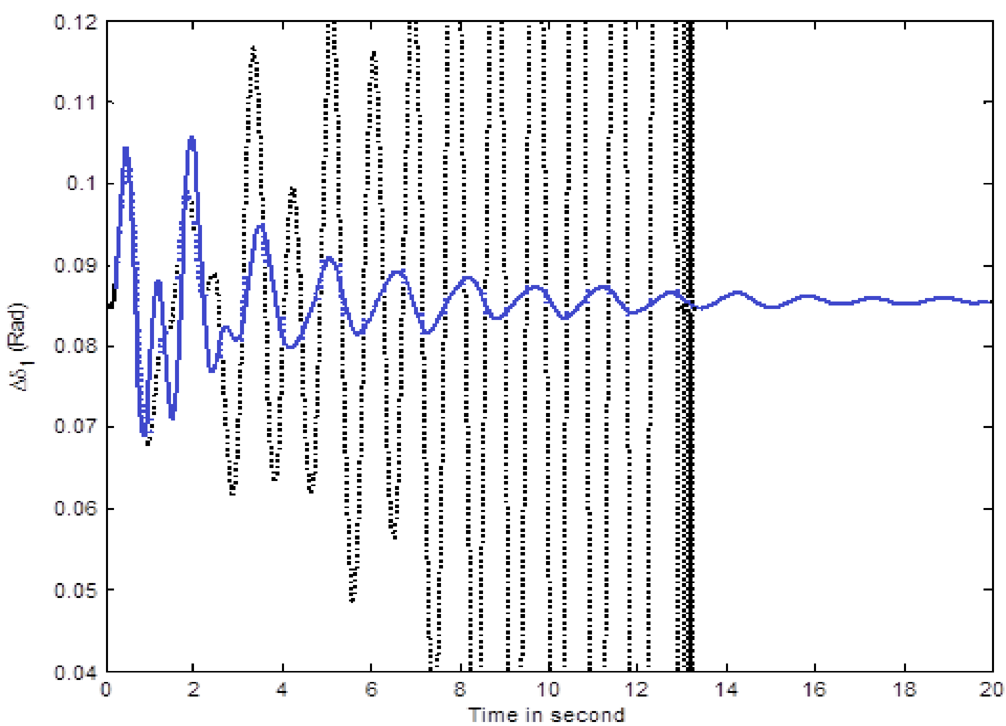

Fig 3. Generator angle 1 relative to the generator angle 13 after applying the single-phase ground fault in the system under an ideal telecommunication network using the proposed method (solid lines) and without a controller (dashed lines).

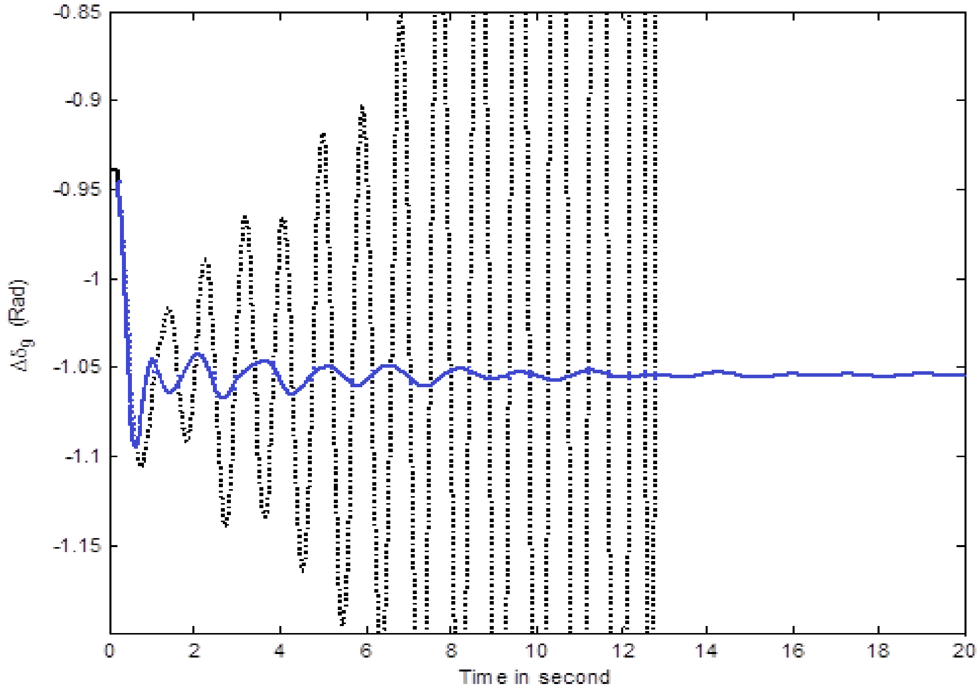

Fig. 4. Generator angle 9 relative to the generator angle 13 after applying the single-phase ground fault in the system under an ideal telecommunication network using the proposed method (solid lines) and without a controller (dashed lines).

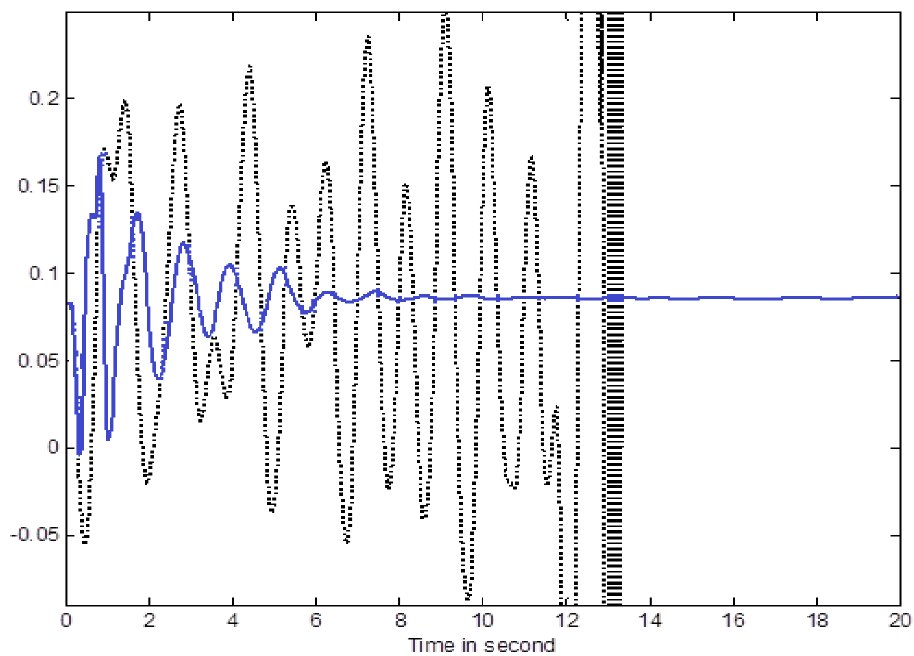

Fig. 5. Generator angle 1 relative to the generator angle 13 after applying the three-phase ground fault in the system under an ideal telecommunication network using the proposed method (solid lines) and without a controller (dashed lines).

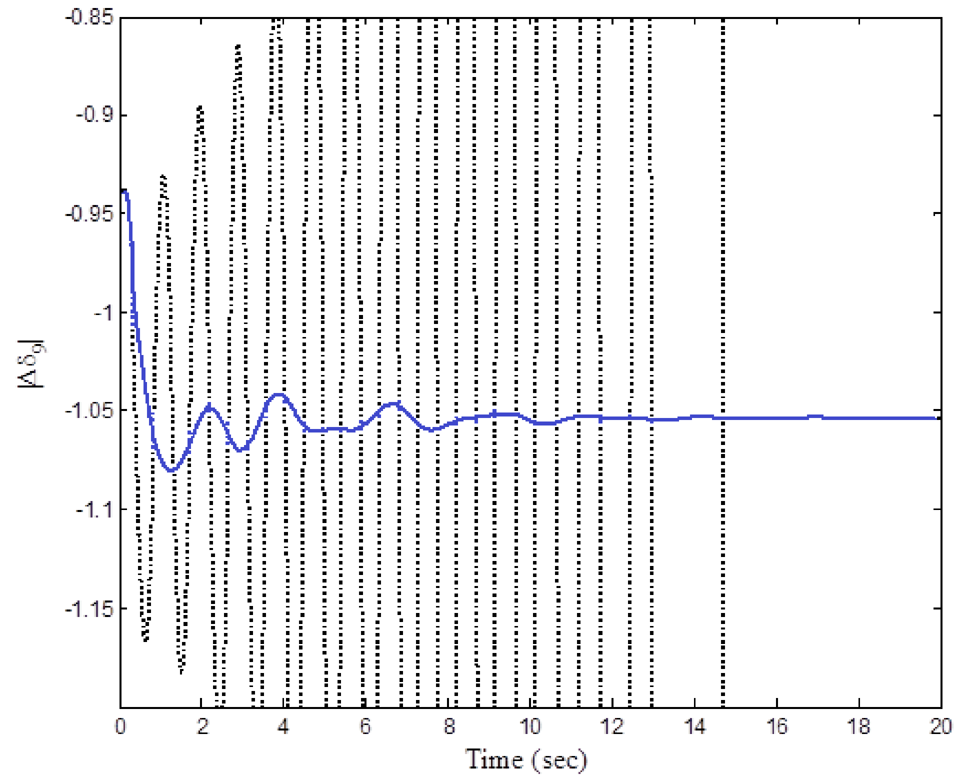

Fig. 6. Generator angle 1 relative to the generator angle 13 after applying the three-phase ground fault in the system under an ideal telecommunication network using the proposed method (solid lines) and without a controller (dashed lines).

6. CONCLUSION

In the current article, an efficient cooperative-based distributed economic model predictive control is suggested for the optimization of energy consumption in networked-based smart energy systems. The proposed approach is based on the moving horizon estimation method, which can handle system constraints and work as an estimator and a predictor simultaneously. Then, each local MHE estimates these coupled states and the local state of each subsystem by solving an optimization problem. To verify the efficiency of the designed control scheme, it is applied for damping of frequency fluctuations and the optimization of energy in an interconnected power system considering communication network-induced delays. The obtained results show the profitability of the proposed method. In this paper, despite having several advantages such as eliminating the effect of latency and loss of packets, resistance to changing system parameters and various disturbances and easy implementation, it has disadvantages such as dependence on high bandwidth induced by the network and multiple controller design parameters. Resolving this problem can be considered as a continuation of this research.

NOMENCLATURE

$\Re^{ni}$	*ni*-dimensional Euclidean space
$\Re^{ni \times mi}$	The set of all $ni \times mi$ matrices
T_s	Sampling time (*sec*)

N_c	Control horizon
d_{min}	Lower bound of τ_i (sec)
d_{max}	Upper bound of τ_i (sec)
x_i	State vector
u_i	Control input
A_i	State matrix
B_i	Input matrix
A_{di}	State delay matrix
A_{ij}	Interconnection matrix
N	Number of subsystems
$\\| \cdot \\|$	Induced 2-norm and Euclidean norm
II_M	the collection of 1, 2, . . . M integers
T	Denotes the transpose of a matrix or a vector
$Row\{A_i\}$	For $i=1, 2,...,N$ shows the matrix $max[A_1, A_2,...,A_N]$
$\underset{i\in II_M}{col}(A_i)$	$[A_1^T,...,A_M^T]^T$
$blkdiag(A_i)$	block-diagonal matrix
rB	closed ball of radius r>0 created at origin
x^+	predicates as a successor state for the state vector x

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