



A Robust Controller Design for Piezoelectric Positioning Stage with Bouc-Wen Hysteresis Compensator

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Abstract

In this paper, an inverse compensator based on the Bouc-Wen model is used to reduce the effect of hysteresis on a piezoelectric micro-actuator. This compensator is used as an open loop controller which partially eliminates the system hysteresis effect. In the last step, in order to increase the reliability of the system and to achieve the desired goals, a H_∞ robust controller is proposed in the presence of a compensator, to detect in the presence of modeling error and compensator error. At the end, simulation results are provided by MATLAB to demonstrate the effectiveness of the proposed control scheme. These results indicate that the proposed scheme provides a robust system performance and the tracking error reaches zero with acceptable accuracy.

Keywords: Bouc-Wen model, Hysteresis, H_∞ robust controller, Piezoelectric.

1. INTRODUCTION

Piezoelectric actuators are used in micron dimension positioning. These types of operators are popular because of their fast time response, high mechanical strength, and appropriate precision [1]. This article aims to achieve desired goals in the context of the system uncertainty, in addition to its

inherent behaviors. Piezoelectric actuator system has nonlinear behavior due to the phenomenon of hysteresis, which challenges the modeling and controlling such systems. This nonlinear behavior causes a hysteresis loop. This often limits the performance of the system severely, causes in output fluctuation, reducing the performance accuracy and even can results in system

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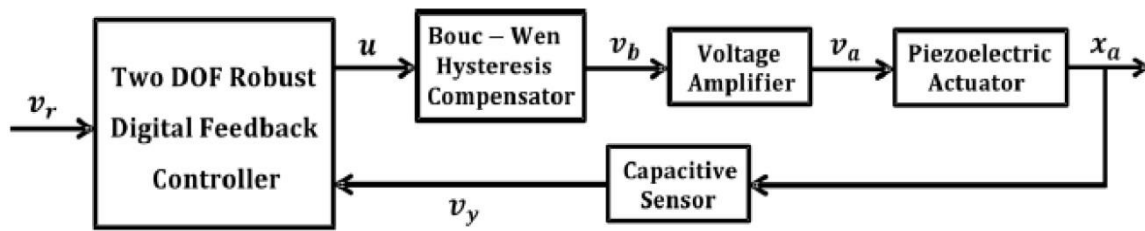


Fig. 1. Closed loop method with robust controller in presence of Bouc-Wen hysteresis compensator.

instability [2,8]. Therefore, it is necessary to find a model that can show hysteresis behavior and in accordance with that, a controller is to be designed to eliminate the destructive effects of hysteresis. In recent decades, various models have been provided for modeling the hysteresis, such as Preisach [3], KP hysteron [4], PI [5] and Bouc-Wen model [7]. Among different models, Bouc_Wen model [6,7] is preferred because of high accuracy and generalizability. Reverse Bouc-Wen compensator is used to eliminate the hysteresis loop of the model. Since piezoelectric is used for micro and nano displacement applications in accurate moving tools, finding a precise model that accurately describes the behavior of the system is very important. To compensate the modeling errors as well as uncertainties of the system, elimination of noise and detection purposes, the existence of a robust controller is very effective which it designed in this article.

2. MATERIALS AND METHODS

2.1. Principles of Piezoelectric Actuator

The block diagram of the closed loop control system is shown in Fig. 1. The control method considered is the H_∞ robust



Fig. 2. Piezoelectric actuator (P-752.21).

control in presence of perturbation and uncertainties of the system. The hysteresis compensator is designed to reduce the nonlinear hysteresis effect of the piezoelectric actuator, while the robust control is designed to enhance the tracking performance of this actuator with high stability and consolidation [6, 7].

The piezoelectric actuator like nonlinear hysteresis, has variable dynamics. The output displacement (X_a) is measured by a capacitive sensor. The measured output voltage (v_y) is sent to the closed loop control. Another input for closed loop control is the reference voltage (v_r) as shown in Fig.1, which describes the desired displacement. The closed loop control provides a control signal (u) for the hysteresis compensator. The output voltage (v_b) of the Bouc-Wen hysteresis compensator is gained by a voltage amplifier and it is applied to the piezoelectric

actuator. This is called the input voltage (v_a). The piezoelectric actuator considered in this thesis is a uniaxial piezoelectric actuator (P752.21) with capacitive sensor (D015). This actuator is capable of measuring up to a range of 35 μm with a precision of 0.2 nm. The installed sensor with this actuator also is able to measure up to a range of 45 μm with a precision of 0.01 nm with a bandwidth of 10 kHz. Fig. 2 shows this piezoelectric actuator [6,7].

2.2. Hysteresis Model and Open Loop Compensator

In this section the Bouc-Wen hysteresis model is provided. Then using the parameters identified for this actuator, the linear system of the actuator with nonlinear hysteresis is proposed and finally an open loop compensator is provided for reducing the hysteresis effect.

2.3. Providing a Hybrid Model Based on the Bouc-Wen Equation

The main reason of using the Bouc-Wen model for modeling the hysteresis actuator is its simplicity and high precision. The model is expressed based on differential equations [6,7],

$$\begin{aligned} \dot{\Phi}(t) = & \alpha \dot{v}_a(t) - \beta |\dot{v}_a(t)| |\Phi(t)|^{n-1} \Phi(t) \\ & - \gamma \dot{v}_a(t) |\Phi(t)|^n \end{aligned} \quad (1)$$

where Φ is the nonlinear hysteresis term. $\dot{\Phi}$ is the first derivative of the nonlinear hysteresis term, v_a shows the applied voltage to α, β, γ actuator and n is the hysteresis loop parameters [12].

$$x_a(t) = g_p v_a(t) - \Phi(t) \quad (2)$$

where X_a is the piezoelectric actuator displacement and g_p is the piezoelectric coefficient. All the piezoelectric loop parameters (α, β, γ, n) like the piezoelectric coefficient value, are identified using practical data. These data are extracted from reference [6] and are as follow:

$$\alpha = 0.7091$$

$$\beta = 2.0476$$

$$\gamma = 0.1949$$

$$n = 1$$

$$g_p = 1.143$$

Fig. 3 shows the simulated hysteresis loop based on identified parameters under voltage of 4 volt and frequency of 10π radian/second which used in reference [6, 7]. The identified linear dynamics are of order 4 and are discrete time dynamic model, see equation (3):

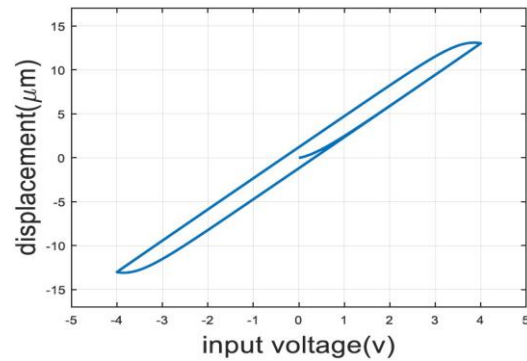


Fig 3. Hysteresis loop with identified parameters for Bouc-Wen hysteresis model.

$$G(z^{-1}) = \frac{N(z^{-1})}{D(z^{-1})} \quad (3)$$

$$= \frac{n_1 z^{-1} + n_2 z^{-2} + n_3 z^{-3} + n_4 z^{-4}}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} + d_4 z^{-4}}$$

where D and N are polynomials for the numerator and denominator of the system. The identified parameters of these polynomials are as follow:

$$\begin{aligned} n_1 &= 0.652 & d_1 &= -1.646 \\ n_2 &= -0.806 & d_2 &= 1.658 \\ n_3 &= 0.368 & d_3 &= -1125 \\ n_4 &= 0.0003 & d_4 &= 0.387 \end{aligned}$$

The identification algorithm is provided in [11]. It should be noted that the discrete model is transformed into a continuous model and it is expressed as linear state space model.

$$\begin{cases} \dot{x} = Ax + Bu \\ y = cx \end{cases} \quad (4)$$

$$A = 10^4 * \begin{bmatrix} -0.9493 & -1.4897 & -0.9671 & -0.4740 \\ 1.6384 & 0 & 0 & 0 \\ 0 & 0.8192 & 0 & 0 \\ 0 & 0 & 0.8192 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 223.9720 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [14.5205 \quad 23.9193 \quad 36.9198 \quad 65.5268]$$

To achieve the desired goals, a piezoelectric model, based on the identified dynamic parameters and the identified nonlinear parameters, is presented. This model includes all the linear and nonlinear features of the system and it stimulates the real system appropriately. Choosing this

model is in a way that the linearized equations in state space are considered as base system equations and equation (1) is considered as nonlinear term, then the value of this nonlinear equation is applied to the base system as an input. According to equation (5) this system is described as follow:

$$\begin{cases} \dot{x} = Ax + Bu + B_\Phi \Phi \\ \dot{\Phi}(t) = \alpha \dot{u}_a(t) \\ \quad - \beta |\dot{u}_a(t)| |\Phi(t)|^{n-1} \Phi(t) \\ \quad - \gamma \dot{u}_a(t) |\Phi(t)|^n \\ y = cx \end{cases} \quad (5)$$

In the above equation, B_Φ matrix is chosen similar to B matrix but with an opposite sign. The signs of these two matrices are opposite because of increasing the input of the system results in a positive effect of hysteresis closed loop, and reducing the input has results in a negative effect. Therefore, the value of B_Φ matrix is,

$$B_\Phi = [-256 \quad 0 \quad 0 \quad 0]$$

This model is nearly acceptable for the real system, it includes all the identified linear and nonlinear features, and it is used to design the closed loop compensator and robust controller.

2.4. Hysteresis Compensator Based on Bouc-Wen Model

In order to compensate the nonlinear hysteresis of a piezoelectric actuator, a simple Bouc-Wen hysteresis compensator according to the identified hysteresis model of Φ is designed by considering the

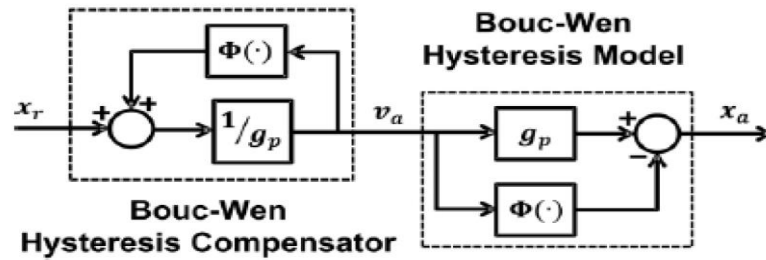


Fig. 4. Diagram block of Bouc-Wen hysteresis model with compensator.

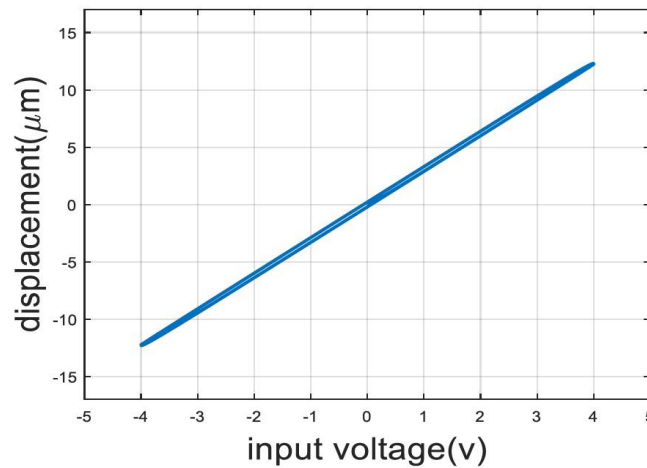


Fig. 5. Open loop hysteresis compensation with Bouc-Wen hysteresis compensator.

identified piezoelectric coefficient (g_p) as shown in Fig.4. Unlike the classical methods in which reversing a hysteresis model is used for compensator, in this approach only the inverse of one term, which is the piezoelectric coefficient (g_p) is used [6,7,9,10].

For a Bouc-Wen hysteresis model, if X_a is the real displacement of the piezoelectric actuator and v_a is the applied voltage, then the goal is achieving the equality $X_a = X_r$. According to Fig. 4 X_r is the input displacement for the hysteresis compensator.

Now the equation for hysteresis compensator based on equation (6) is written,

$$v_a(t) = \frac{1}{g_p} (x_r(t) - \Phi(t)) \quad (6)$$

where Φ shows the nonlinear hysteresis, term used in compensator feedback. Fortunately, reversing the hysteresis model is not needed for the hysteresis compensator in this design.

Fig. 5 shows the simulated results of hysteresis compensation with Bouc-Wen hysteresis compensator. This simulation result shows the ability of Bouc-Wen model for nonlinear compensation of the hysteresis effectively.

$$\begin{cases} \dot{x} = Ax + B_r u + B_{\phi_r} \Phi \\ \dot{\Phi}(t) = \alpha \dot{u}_a(t) - \beta |\dot{u}_a(t)| |\Phi(t)|^{n-1} \Phi(t) \\ \quad - \gamma \dot{u}_a(t) |\Phi(t)|^n \\ y = cx + D_n n = y_1 + D_n n \end{cases} \quad (7)$$

According to Fig. 8, we have:

$$\begin{aligned} z_1 &= w_1(r-y) = w_1 e \\ z_2 &= w_2 u \\ z_3 &= w_3 y_1 \\ z_4 &= w_4 y_1 \end{aligned} \quad (8)$$

In equation (8), variables w_1 to w_4 are the weight functions related to the defined sensitivities and considered uncertainties in previous parts. According to Fig. 8, the state space equations of weight functions are expressed as follow:

$$\begin{aligned} w_1 &\begin{cases} \dot{x}_{z1} = A_{z1} x_{z1} + B_{z1}(r-y) \\ z_1 = c_{z1} x_{z1} + D_{z1}(r-y) \end{cases} \\ w_2 &\begin{cases} \dot{x}_{z2} = A_{z2} x_{z2} + B_{z2} u \\ z_2 = c_{z2} x_{z2} + D_{z2} u \end{cases} \\ w_3 &\begin{cases} \dot{x}_{z3} = A_{z3} x_{z3} + B_{z3} c x \\ z_3 = c_{z3} x_{z3} + D_{z3} c x \end{cases} \end{aligned} \quad (9)$$

$$z_4 = w_4 c x$$

Now for designing the generalized transform function of P , the vectors of generalized system and the inputs of its disturbance are as written in equation (10),

$$X = \begin{bmatrix} x_{z1} \\ x_{z2} \\ x_{z3} \\ x \end{bmatrix} \quad W = \begin{bmatrix} d \\ n \\ r \\ x \end{bmatrix} \quad (10)$$

$$P = \begin{bmatrix} A_{z1} & 0 & 0 & -B_{z1}c & 0 & -B_{z1}D_n & B_{z1} & 0 \\ 0 & A_{z2} & 0 & 0 & 0 & 0 & 0 & B_{z2} \\ 0 & 0 & A_{z3} & B_{z3}c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A & B_a & 0 & 0 & B \\ c_{z1} & 0 & 0 & D_{z1}c & 0 & -D_{z1}D_n & D_{z1} & 0 \\ 0 & c_{z1} & 0 & 0 & 0 & 0 & 0 & D_{z2} \\ 0 & 0 & c_{z1} & D_{z3}c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_4c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c & 0 & -D_n & 1 & 0 \end{bmatrix} \quad (11)$$

Note that the generalized system input is u . According to the above equations, the transform function of P for designing the robust controller is calculated as follow. MATLAB toolbox is used to solve this problem.

2.6. Robust Controller Design

The first step in designing a robust controller is expressing the system equations in state space and obtaining a generalized system. As stated before, the weight function w_1 at Z_1 output traces the system performance, indicates the disturbance and by choosing this function appropriately, it will result in

desired performance. Weight function W_2 applies required limitations on the control signal and its appropriate selection prevents the actuators saturation. Also, by appropriate selection of weight function W_3 at Z_3 output, the input and output noises may remove.

To avoid high dynamic controllers in H_∞ controller design, the equation which added as a hysteresis compensator is considered as uncertainty. Which means to prove the controller's robustness against the uncertainty, the equation related to the compensator is removed and it is assumed that the system is under uncertainty. Therefore, the block diagram of a system that should be designed is shown in Fig. 8.

2.7. Weight Functions Selection

The most important part of the design is the selection of weight functions. For choosing these functions, we face with two challenges. At first, these functions should be selected in such a way that they provide the best performance for the system and second, these functions should be selected as simple as possible. As the designed controller grade equals to the sum of number of system states and the selected weight functions state, so a simpler selection of these functions will ultimately result in a simpler controller. Obtaining weight functions needs a vast experience in classic design and LQG and optimum weight functions are obtained by iterative simulation.

Sensitivity weight function W_1 is chosen as,

$$W_1 = \frac{24}{s + 10^{-8}} \quad (12)$$

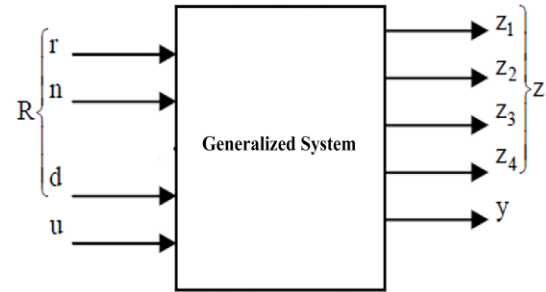


Fig. 8. Design structure of generalized system.

This function is selected according to the limitation on the controller's energy and the robust performance in such a way that the system speed is maximum and tracing error at low frequencies reaches its lowest possible value. Indeed, the constant state error is zero depending on the integral nature of plant.

The supplementary sensitivity weight function W_3 which indicates the noise attenuation at output is obtained as,

$$W_3 = \frac{2.14s + 10^{-8}}{s + 10000} \quad (13)$$

And the weight function W_2 is,

$$W_2 = \frac{45s + 45}{s + 1} \quad (14)$$

By choosing this equation the system step response will not saturate. Because the uncertainty weight function at all frequencies is very close to 1 and, given the simplicity of control, the weight function W_4 is considered to be,

$$W_4 = 1 \quad (15)$$

By choosing weight functions, the generalized plant for the design of the controller is completely identified.

2.8. H_∞ Controller Design

A generalized system for implementing a H_∞ control design algorithm is now ready. Robust controller is designed according to the stated equations in previous section, and by using MATLAB software. The obtained controller is of order 7 which is equal to the order of the generalized plant and the order

of the system cannot be reduced by considering the error to be less than 1%. Thus, the controller will be of order seven,

$$K \begin{cases} \dot{x}_k = A_k x_k + B_k u \\ y = c_k x_k + D_k u \end{cases} \quad (16)$$

where:

$$A_k = 10^{+4} * \begin{bmatrix} -0.0001 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & -0.0000 \\ -0.0000 & -0.2474 & -1.3243 & -0.1120 & -0.1445 & -0.3676 & -0.0087 \\ 0.0000 & 1.3699 & -0.2036 & 0.3881 & 0.4046 & 0.5270 & 0.0068 \\ -0.0000 & 0.4141 & -0.0886 & -0.1914 & -0.0384 & -0.4949 & -0.0046 \\ -0.0000 & -0.0317 & -0.0199 & -0.1478 & -0.8983 & 0.2067 & -0.0050 \\ 0.0000 & 0.0254 & -0.0985 & 0.4744 & 0.6481 & -0.4418 & -0.0115 \\ -0.0000 & 0.0128 & -0.2891 & 1.1951 & -0.1231 & -0.8727 & -0.0492 \end{bmatrix}$$

$$B_k = \begin{bmatrix} -0.0000 \\ -1.2562 \\ -0.4360 \\ -6.1109 \\ 2.1430 \\ 2.8399 \\ 326.0649 \end{bmatrix} \quad C_k^T = \begin{bmatrix} 0.0000 \\ 0.0002 \\ -0.0054 \\ 0.0179 \\ 0.0426 \\ 0.1063 \\ 0.4692 \end{bmatrix} \quad D_k = 0$$

3. SIMULATION RESULTS AND DISCUSSION

In the first step, it is necessary to examine the behavior of the closed loop system against nonlinear hysteresis. Fig. 9 shows the hysteresis loop in the presence of robust control.

As seen in Fig. 9, hysteresis nonlinear effect has been well eliminated at low frequencies by applying the robust controller and that is not visible in output. Removing the hysteresis effect is the main job of using the controller. Further details about other

elements of this controller is discussed in following.

In the second step, the behavior of the system is considered for pulse input and sinusoidal input. By applying the robust controller, the system output for pulse and sinusoidal inputs are shown in Fig. 10 and Fig. 12, respectively. The tracing error for these two is shown in Fig. 11 and Fig. 13, respectively.

According to the results, with a robust controller, the steady state error for the step input reaches to a very small value

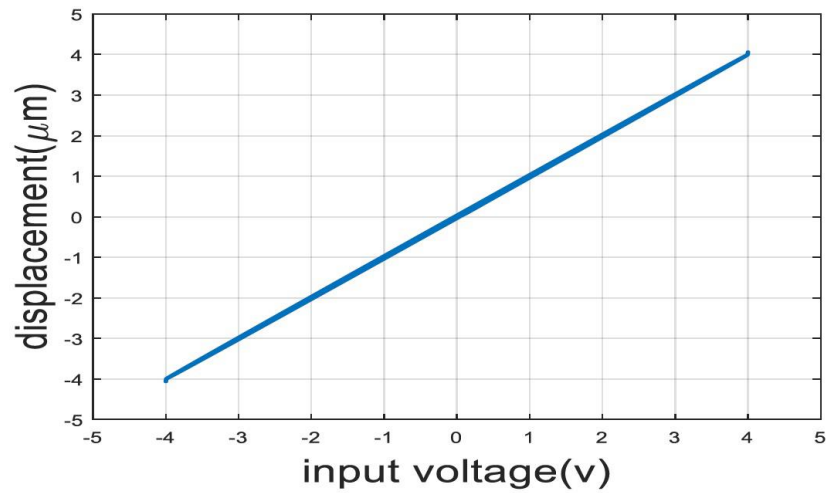


Fig. 9. Hysteresis loop after applying the robust controller.

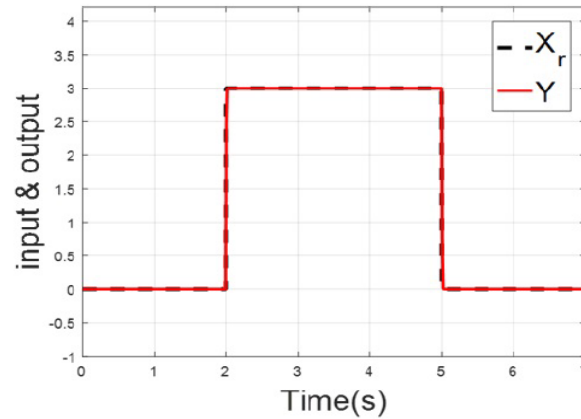


Fig. 10. Output and input for system under H_∞ robust controller and hysteresis compensator.

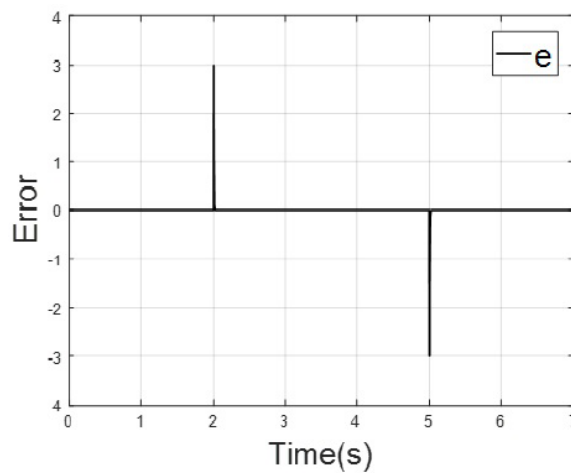


Fig. 11. Tracing error for system under H_∞ robust controller and hysteresis compensator.

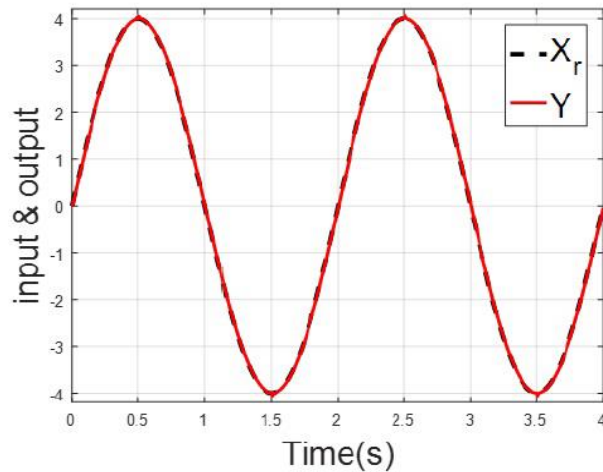


Fig. 12. Output and input for system under H_∞ robust controller and hysteresis compensator.

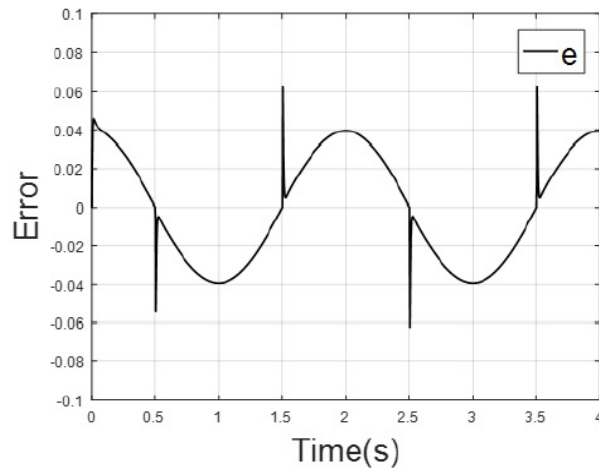


Fig. 13. Tracing error for system under H_∞ robust controller and hysteresis compensator.

(2.47×10^{-8}), and for the sinusoidal input with a frequency of 1 radian/second and the amplitude 4 reaches to 0.03 which is an acceptable value. It is observed that the control shows a quick response against rapid changes and makes the tracing error to its lowest value rapidly. In other words, in the above-mentioned system, a small jump is observed. Ripples created due to the disturbance, or the hysteresis, immediately decreases its effect on output. This property is important for actuators with lower input

domain.

In the third step, the behavior of the system in presence of uncertainties is studied. As previously mentioned, the uncertainty considered for the above system, is the removal of equation of the hysteresis compensator out of equations of the considered system. We continue the system analysis by removing the compensator block as an uncertainty action for a sinusoidal input.

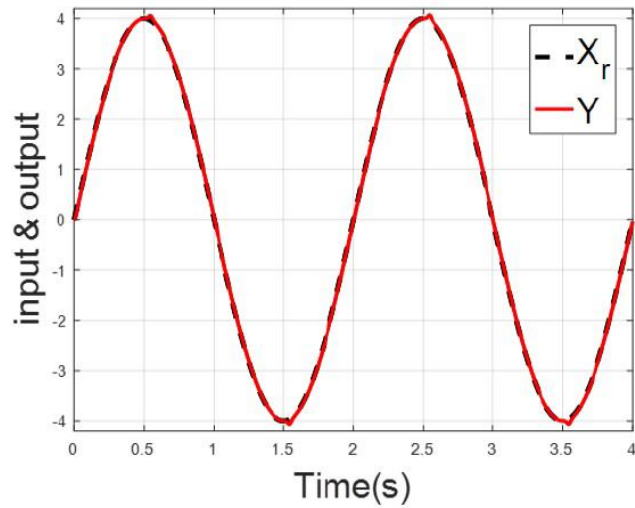


Fig. 14. Control system performance in presence of uncertainty.

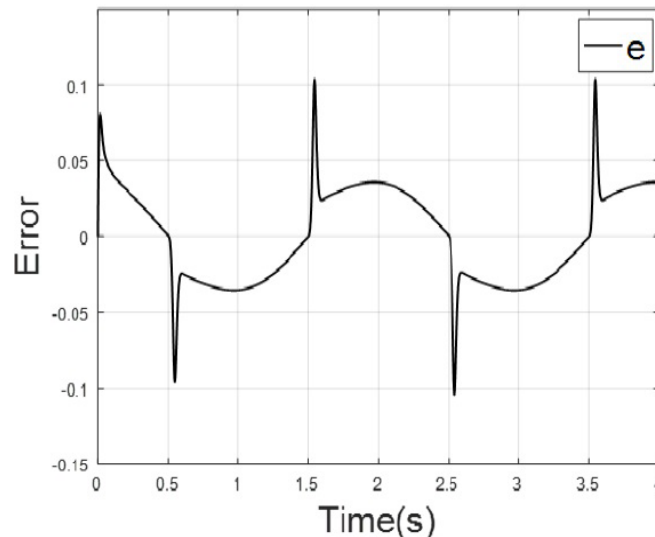


Fig. 15. System tracing error in presence of uncertainty.

As shown in Figs. 14 and 15, it is observed that by applying uncertainty to the closed loop system, the robust controller shows a really good resistance against the changes and keeps the tracing error as low as before (0.03).

In the last step, we examine the behavior of the system in the presence of noise. White noise with amplitude of 0.15 and power of

10^{-7} is added to the system input. According to Figs. 16 and 17, a robust controller has been considerably able to reduce the effect of input noise at the output of the system.

4. CONCLUSION

In this paper, a nonlinear dynamic model presented to reach the desirable goals in

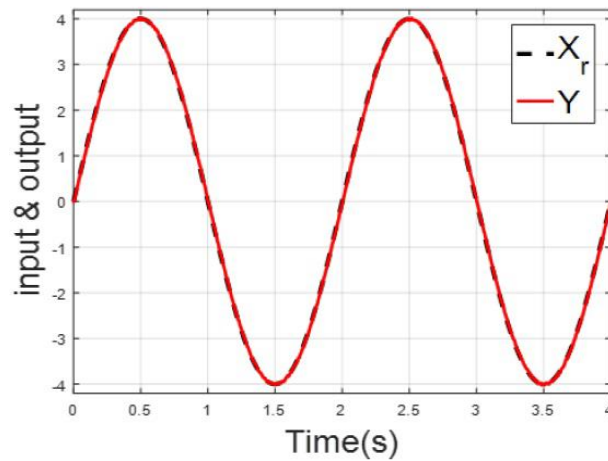


Fig. 16. H^∞ robust controller system performance in presence of noise.

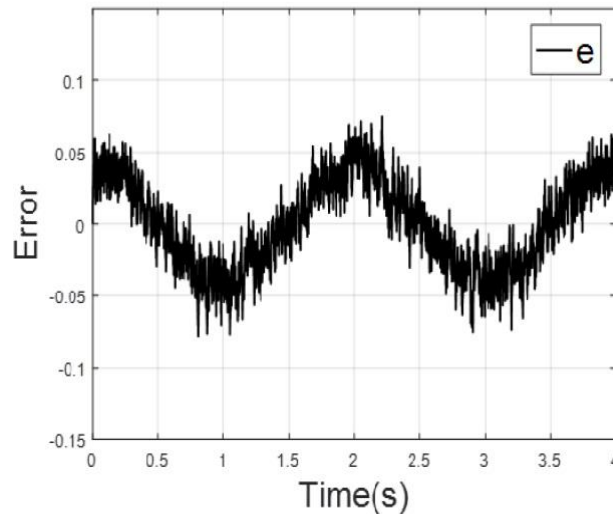


Fig. 17. Tracing error of H^∞ robust controller system in presence of noise.

presence of disturbance and uncertainties. In this regard, the nonlinear dynamic model of system was studied based on the identification of linear and nonlinear dynamics of the system. Nonlinear system parameters are set based on the Bouc-Wen compensator loop. This compensator partly eliminates the hysteresis effects of the system according to the open loop control. In addition, in order to increase the system

reliability and reach the optimal goals, H^∞ robust controller in presence of a hysteresis compensator was also studied for tracing in the presence of modeling error and the hysteresis compensator error.

Finally, the simulation results presented in detail, which showed how the robust controller minimizes the system uncertainties as well as system noise in the output and leads the actuator to its desirable goals.

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