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Compare the Performance of Recovery Algorithms MP, OMP, L1-Norm in Compressive Sensing for Different Measurement and Sparse Spaces

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Abstract
In this paper, at first, compressive sensing theory involves introducing measurement matrices to dedicate the signal dimension and so sensing cost reduction, and sparse domain to examine the conditions for the possibility of signal recovering, are explained. In addition, three well known recovery algorithms called Matching Pursuit (MP), Orthogonal Matching Pursuit (OMP), and L1-Norm are briefly introduced. Then, the performance of three mentioned recovery algorithms are compared with respect to the mean square error (MSE) and the result images quality. For this purpose, Gaussian and Bernoulli as the measurement matrices are used, where Haar and Fourier as sparse domains are applied.

Keywords: Matching Pursuit, Orthogonal Matching Pursuit, Compressive Sensing, Sparse Space.

1. INTRODUCTION

Due to the high dimensions of some of the images and their storage volume in different areas such as radar and medicine, the need to provide algorithms for reducing the size of images felt. The purpose of this paper is to provide an image compression algorithm to reduce the size and so the cost of sensing and using different recovery algorithms as well. One of the activities carried out in the field of Compress Sensing is magnetic resonance imaging and magnetic resonance imaging (MRI). In the field of imaging, a lot of work has been done by compact sampling, with the main aim of reducing the number of sensors in the camera and consequently cost reduction. It should be notified that the number of sensors reduction is identical to be the size of the camera small which is an important factor in today's technology. Also, if information other than the signal bandwidth, such as the Sparse of displaying it in an appropriate space, can be used by nonlinear optimization

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methods to accurately reconstruct the signal from a far less observational method than what Shannon's theory suggests. Although, magnetic resonance imaging (MRI) is important in medical imaging, it is a time consuming process. So, using the compressive sensing for MRI can decrease the scan time.

In this paper, at first, the compressive sensing theory is reviewed. Then, three algorithms called MP, OMP, and L1-Norm are explained as the recovery algorithms. Finally, the simulation results are shown, and conclusions are given based on the recovery as the recovered images as image quality and mean square error (MSE) as the main image assessment parameter.

2. COMPRESSIVE SENSING (CS)

Consider a real-valued, finite-length, one-dimensional, discrete-time signal **x**, which can be viewed as a $N \times 1$ column vector. Any signal in R^N can be represented in terms of a basis for $N \times 1$ vectors $\{\psi_i\}_{i=1}^{N}$. For simplicity, assume that the basis is orthonormal. Using the $N \times N$ basis matrix $\psi = \psi_1|\psi_2 \dots |\psi_N$ with the vectors $\{\psi_i\}$ as columns, a signal **x** can be expressed as

$$X = \sum s_i \ \psi_i \tag{1}$$

$$X = \psi s \tag{2}$$

where s is the column vector N × 1 with weighting coefficients $s_i = \langle X, \psi \rangle$. In fact, X in the time domain and s in the ψ domain represents the signal. If in the vector s, only K component is nonzero, so that K << N, the X signal is called K-Sparse, which is compressible according to the theory of compressive sensing. Between the X and the compressed Y, there is a measurement matrix Φ with the dimension M × N, where M is much smaller than N,

$$Y = \Phi X \tag{3}$$

The obtained Y signal is a vector with dimensions M × 1. Given the Eqs. (2) and (3), we have:

$$Y = \Psi \Phi s \tag{4}$$

where $\Theta = \Psi\Phi$ is an M × N matrix. The measurement process is not adaptive, meaning that Φ is fixed and does not depend on the signal X. The problem consists of designing: a) a stable measurement matrix Φ such that the salient information in any K-sparse or compressible signal is not damaged by dimensionality reduction from x ∈ RN to y ∈ RM and b) a reconstruction algorithm to recover x from only M ≈ K measurements y (or about as many measurements as the number of coefficients recorded by a traditional transform coder) and dictionary matrix ? ? .

3. SIGNAL RECOVERY ALGORITHMS

According to the compressive sensing theory explained in Section 2, using the measurement matrix Φ with dimension M × N and the sparse matrix ψ with dimension N × N, we are going to retrieve the signal. By using three recovery algorithms called MP, OMP and L1-Norm. Features and steps for the algorithm implementation are explained in following.

3.1. Matching Pursuit (MP) Algorithm

The MP algorithm is one of the greedy methods seeking to find the best fit in projecting a multi-dimensional data over the dictionary called Θ where $\Theta = \Psi\Phi$. At each step, it selects a column from the dictionary that has the largest internal multiplication with the Y matrix as input at the first iteration or residual for the next steps of the algorithm. The steps for implementing the MP algorithm are listed below:

1. $s_{\mathbf{0}} = 0 \in \mathbf{R}^N$
2. Compute the inner product, $\mathbf{g}_j = \Theta^{\mathbf{T}} Y_{j-\mathbf{1}} \in \mathbf{R}^N$.
3. Find the index k where: $k = \arg\max_{i=1,\dots,N} |\mathbf{g}_j[i]|$.
4. Update $s_j[k] = s_{j-1}[k] + \mathbf{g}_j[k]$ where $s_j[k]$ and $\mathbf{g}_j[k]$ refer to the k-th element in $s_j \in R^N$ and $\mathbf{g}_j \in R^N$. Then, compute the residual $Y_j = Y_{j-1} - \mathbf{g}_j[k]\Theta_k$ where Θ_k is the k-th column of dictionary **.**
5. Consider the iteration number, j, or obtain the residual norm value, $\| Y_j \|_{\ell_2}$. Stop the algorithm if j is greater or $\| Y_j \|_{\ell_2}$ is less than the pre-defined value. Otherwise go through the second step.

3.2. Orthogonal Matching Pursuit (OMP) Algorithm

OMP algorithm is the most widely used as signal recovery algorithm in compressive sensing and is proved to be practical and easy for implementation. OMP algorithm is a classical greedy algorithm and its performance is dependent heavily on the properties of the measurement matrix. The procedure of OMP algorithm are presented in following:

1. $s_{\mathbf{0}} = 0 \in \mathbf{R}^N$
2. $\mathbf{\Lambda}_0 = \Phi$
3. Compute the inner product, $\mathbf{g}_j = \Theta^{\mathbf{T}} Y_{j-\mathbf{1}} \in \mathbf{R}^N$.
4. Find the index k where $k = \arg\max_{i=1,\dots,N} |\mathbf{g}_j[i]|$.
5. Update the index set and matrix of chosen atoms $\mathbf{\Lambda}_j = \mathbf{\Lambda}_{j-1} \cup \{k\}$, $\Theta_{\Lambda_j} = \Theta_{\Lambda_{j-1}} \Theta \bigcup_k$.
6. Obtain the new estimate $\tilde{s} = (\Theta^{\mathbf{T}}_{\mathbf{\Lambda}_j} \Theta_{\mathbf{\Lambda}_j})^{-1} \Theta^{\mathbf{T}}_{\mathbf{\Lambda}_j} \mathbf{y}$. Note that the size of $\tilde{\mathbf{x}}$ is growing while the number of iteration is increasing. Compute the coefficient vector $s_j[\mathbf{\Lambda}_j] = \tilde{s}$.
7. Update the residual $Y_j = Y - \Theta s_j$.
8. Consider the iteration number, j, or obtain the residual norm value, $\| Y_j \|_{\ell_2}$ Stop the algorithm if j is greater or $\| Y_j \|_{\ell_2}$ is less than the predefined value. Otherwise go through the second step.

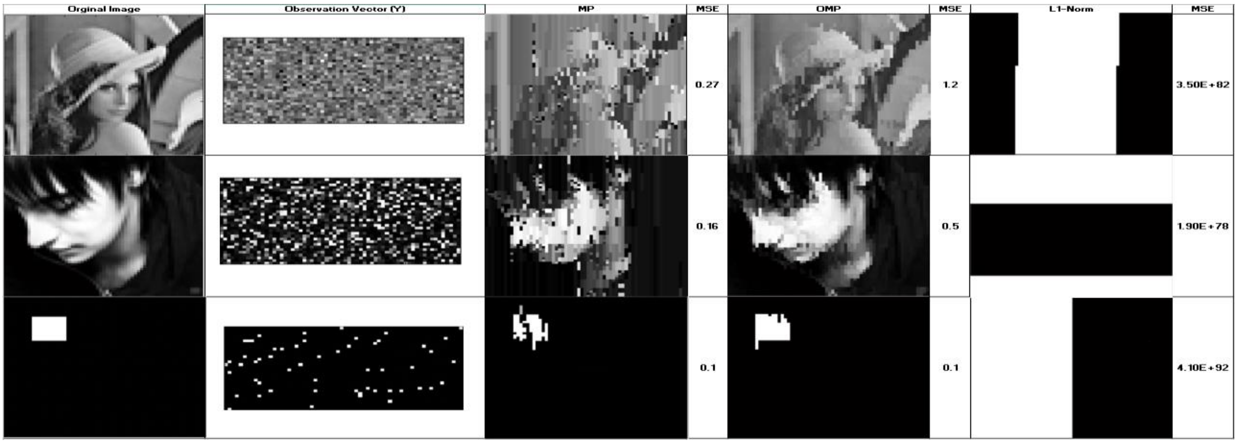
3.3. L1-Norm Algorithm

$$\hat{s} = \arg\min_{s} ||s||_1 \tag{5}$$

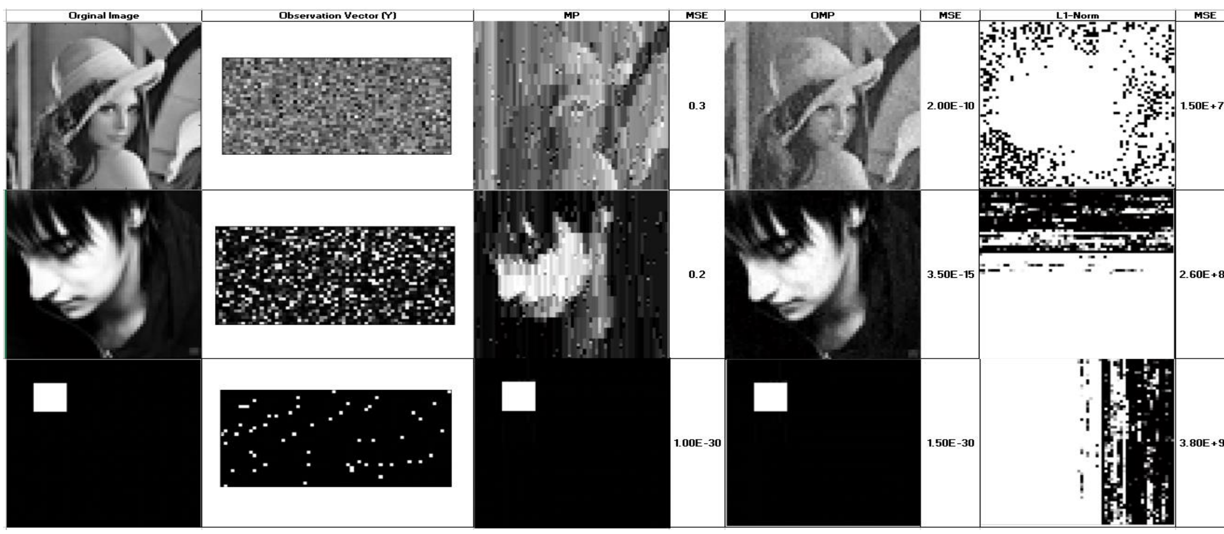
This algorithm is based on the first norm and it does not use internal multiplication. In this algorithm, the goal is to retrieve the vector s of Φ. The first approach to retrieve s is to consider the optimization problem in that which ensures s is consistent with the measurements and s can be retrieved as a Sparse matrix which is in accordance with the measurements Φ.

4. EXPERIMENTAL RESULTS

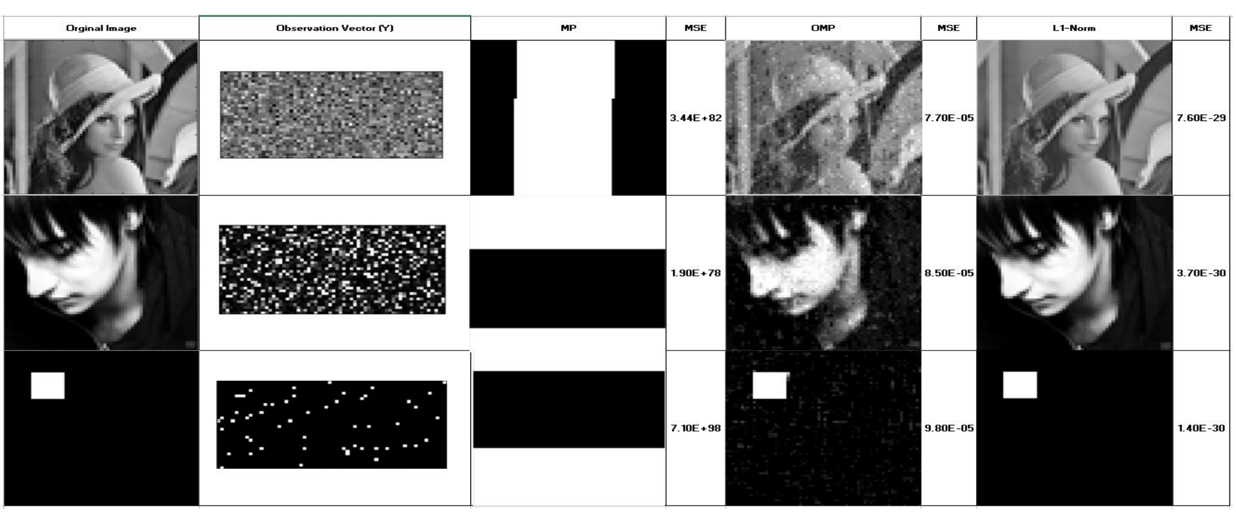
For compressive sensing, different matrices can be used as the measurement matrix Φ


Orginal Image
Observation Vector (Y)
MP
MSE
OMP
MSE
L1-Norm
MSE
0.27
1.2
3.50E+82
0.16
0.5
1.90E+78
0.1
0.1
4.10E+92

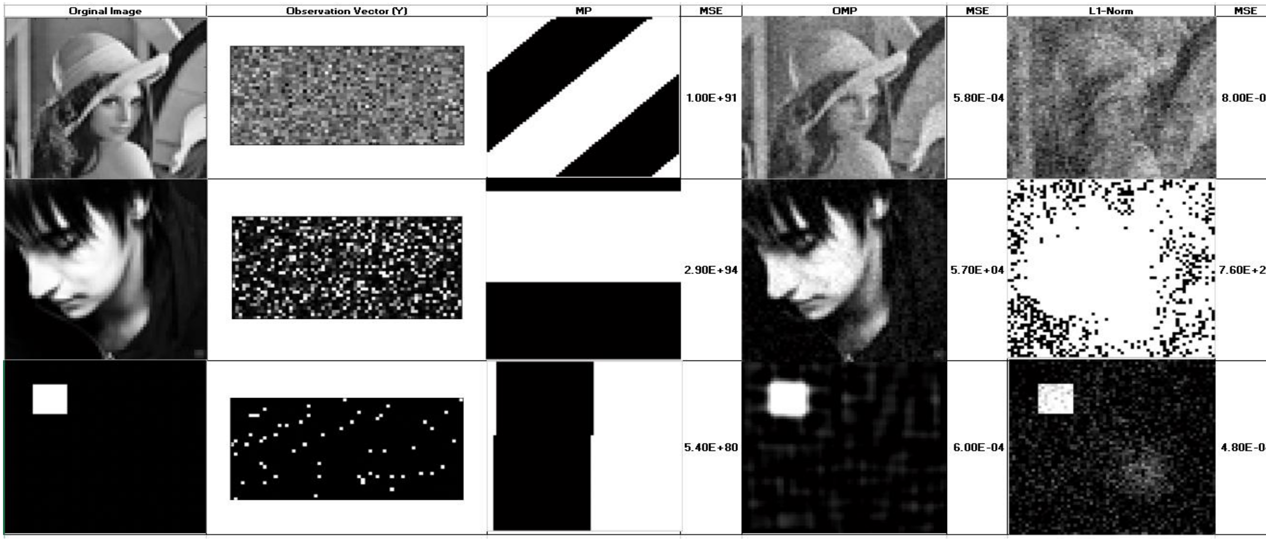
(a)


Orginal Image
Observation Vector (Y)
MP
MSE
OMP
MSE
L1-Norm
MSE
0.3
2.00E-10
1.50E+75
0.2
3.50E-15
2.60E+87
1.00E-30
1.50E-30
3.80E+90

(b)


Orginal Image
Observation Vector (Y)
MP
MSE
OMP
MSE
L1-Norm
MSE
3.44E+82
7.70E-05
7.60E-29
1.90E+78
8.50E-05
3.70E-30
7.10E+98
9.80E-05
1.40E-30

(c)



(d)

Fig. 1. Comparison the performance of recovery algorithms called MP, OMP, L1-Norm in compressive sensing for different measurement matrices and sparse domains. (a) Bernoulli- Haar, (b) Gaussian-Haar, (c) Bernoulli- Fourier, (d) Gaussian- Fourier.

and the sparse matrix ψ. In this paper, the main goal is to evaluate the performance of different matrices when we apply three introduced recovery algorithms. For this purpose, we have used three test images. In this regard, measurement matrices are Bernoulli and Gaussian and the sparse matrices are Haar and Fourier and three recovery algorithms are MP, OMP and L1-Norm. The results of simulation are shown in Fig. 1 (a- e).

In all simulations, the original size (X) is 64×64, the observed image size (Y) is 32×64, the recovered image size is 64×64, the dictionary matrix size (Θ) is 4096×2048, the spase matrix size (ψ) is 4096×4096, and finally, the measurement matrix size (Φ) is 4096×2048.

5. CONCLUSION

As seen in Fig. 1-a, for the Bernoulli measurement matrix and Haar sparse matrix, none of the three MP, OMP, and L1-Norm recovery algorithms perform well, whereas for Gaussian-Haar pairs due to the proper coherency between the Sparse matrix and the measurement matrix, the performance of OMP algorithm is appropriate. This algorithm also works well for Gaussian-Fourier as well. According to the results, the MP recovery algorithm is not recommended for all pairs of the measurement and sparse matrices. However, the L1-Norm algorithm is recommended for the Bernoulli measurement space and Fourier sparse space. Quantitative values for MSE also confirm the achieved visual results.

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