Performance Prediction Modeling of Axial-Flow Compressor by Flow Equations

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Abstract: Design models of multi- stage, axial flow compressor are developed for gas turbine engines. Axial flow compressor is one of the most important parts of gas turbine units. Therefore, its design and performance prediction are very important. One-dimensional modeling is a simple, fast and accurate method for performance prediction of any type of compressors with different geometries. In this approach, inlet flow conditions and compressor geometry are identified and by considering various compressor losses, velocity triangles at rotor, and stator inlets and outlets are determined, and then compressor performance characteristics are predicted.

Numerous models have been developed theoretically and experimentally for estimating various types of compressor losses. In the present work, performance characteristics of the axial-flow compressor are predicted based on one-dimensional modeling approach. Firstly, the proposed algorithm for modeling and then the losses model for calculation of pressure loss coefficient in the blades cascade have been represented. In this study, models of Lieblein, Koch-Smith, Aungier, Hawell are implemented to consider the compressor losses. Finally, the model results are compared with experimental data to validate the model.

Keywords: Axial Flow Compressor, One-Dimensional Modeling, Loss Coefficient, Performance Characteristics

1. Introduction

Axial flow compressors are rotating, airfoil based compressors in which the working fluid principally flows parallel to the axis of rotation. This is in contrast with other rotating compressors such as centrifugal and mixed-flow compressors where the air may enter axially but will have a significant radial component on the exit.

Axial flow compressors produce a continuous flow of compressed gas, and have the benefits of high efficiencies and large mass flow capacity, particularly in relation to their cross-sections. They do, however, require several rows of airfoils to achieve large pressure rises making them complex and expensive relative to other designs (e.g. centrifugal compressor).

Axial compressors are widely used in gas turbines, such as jet engines, high speed ship engines, and small scale power stations. They are also used in industrial applications.

Axial compressors consist of rotating and stationary components. A shaft drives a central drum, retained by bearings, which has a number of annular airfoil rows attached. These rotate between a similar number of stationary airfoil rows attached to a stationary tubular casing. The rows alternate between the rotating airfoils (rotors) and stationary airfoils (stators), with the rotors imparting energy into the fluid, and the stators converting the increased rotational kinetic energy into static pressure through diffusion. A pair of rotating and stationary airfoils is called a stage. The cross-sectional area between the rotor drum and casing is reduced in the flow direction to maintain axial velocity as the fluid is compressed.

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Performance prediction of axial flow compressors deals with complexities and difficulties. Although wide investigations have been done on various components of axial flow compressors till now but there is no model which can exactly predict its performance characteristics. Nevertheless several theoretical and experimental models have been developed which can estimate various types of compressor losses by acceptable accuracy.

Quoting reference [1], Kesey, developed a method for calculating axial flow compressor performance with a repeating stage model. He used one dimentional analysis method for performance prediction of industrial compressors.

Vincent [1] considered blade profile and end-wall losses for a multistage axial flow compressor with repeating stage assumptions and investigated its performance. He assumed that the flow angles at stage inlet and exit are the same. He also neglected the effects of height variation through the stage.

Chen et al. [2] using one- dimensional modeling for performance prediction of single-stage axial flow compressor with changing absolute flow angle at input and output of rotor, optimized the compressor efficiency

The attempt in the present work is to predict the performance of an axial flow compressor by combination of most accurate models without assumption of repeating stage velocity triangles.

2. One- dimensional modeling

One- dimensional modeling is one of the several methods for performance prediction of axial flow compressors. In this approach, it is assumed that flow is one dimensional and changes of flow parameters in radial and tangential directions are unnoticed and values at mean diameter are considered as the average values of whole blade passage. In this method inlet flow conditions and compressor geometry are known and velocity triangles at compressor blade row inlet and exit are obtained and its performance characteristics such as pressure ratio, efficiency and mass flow are calculated [3].

The basic equation of one dimensional isentropic flow along a channel with variable cross section include continuity relations, energy survival equation and governing relation on perfect gas. Using mach number relation and basic relations above and by considering

the losses, the flow equation has been extracted:

$$\frac{m\sqrt{\dfrac{RT_0}{\gamma}}}{AP_0} = \sigma\,cos(\alpha)\,M(1+\frac{\gamma-1}{2}M^2)^{\frac{\gamma+1}{2(1-\gamma)}} \qquad (1)$$

In this equation, σ is losses coefficient and α is outflow angle on blade cascade. In the left side of the equation, T_0 and P_0 are stationary temperature and pressure in blade entry and A is blade outlet cross section. In the right side of the equation, outlet mach, M, is calculated by a trial and error method.

Various sections and some geometrical dimensions of axial flow compressor are shown in Fig. 1. Sections 1, 2 and 3 denote rotor inlet, rotor outlet and stator outlet respectively [4] .

3. Algorithm of one dimensional modeling

First, the mass flow rate is calculated by determining input parameters like mach number, temperature, pressure, etc. Then according to the characteristics determination of flow input into the compressor and by using of energy survival, mass survival and governing relation on perfect gas equations, the characteristics of flow in compressor rotor and stator outlet will be determined by trial and error method. By guessing a proper mach number in blade outlet and also assuming outflow angle and issue geometry, losses coefficients will be determined and mach number be calculated from flow equation. By calculating of mach number, the outflow angle will be modified and this repetition will continue until intended precision achieved. Algorithm of present compressor modeling is shown in Fig. 2.
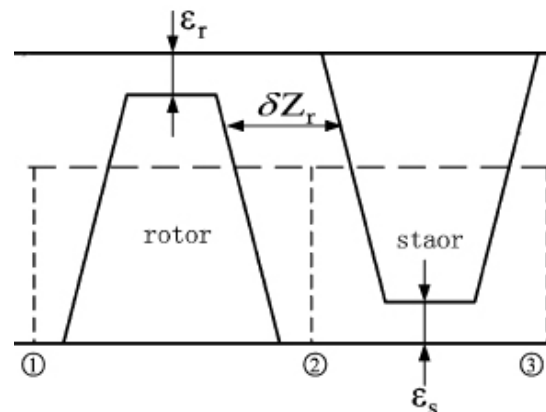

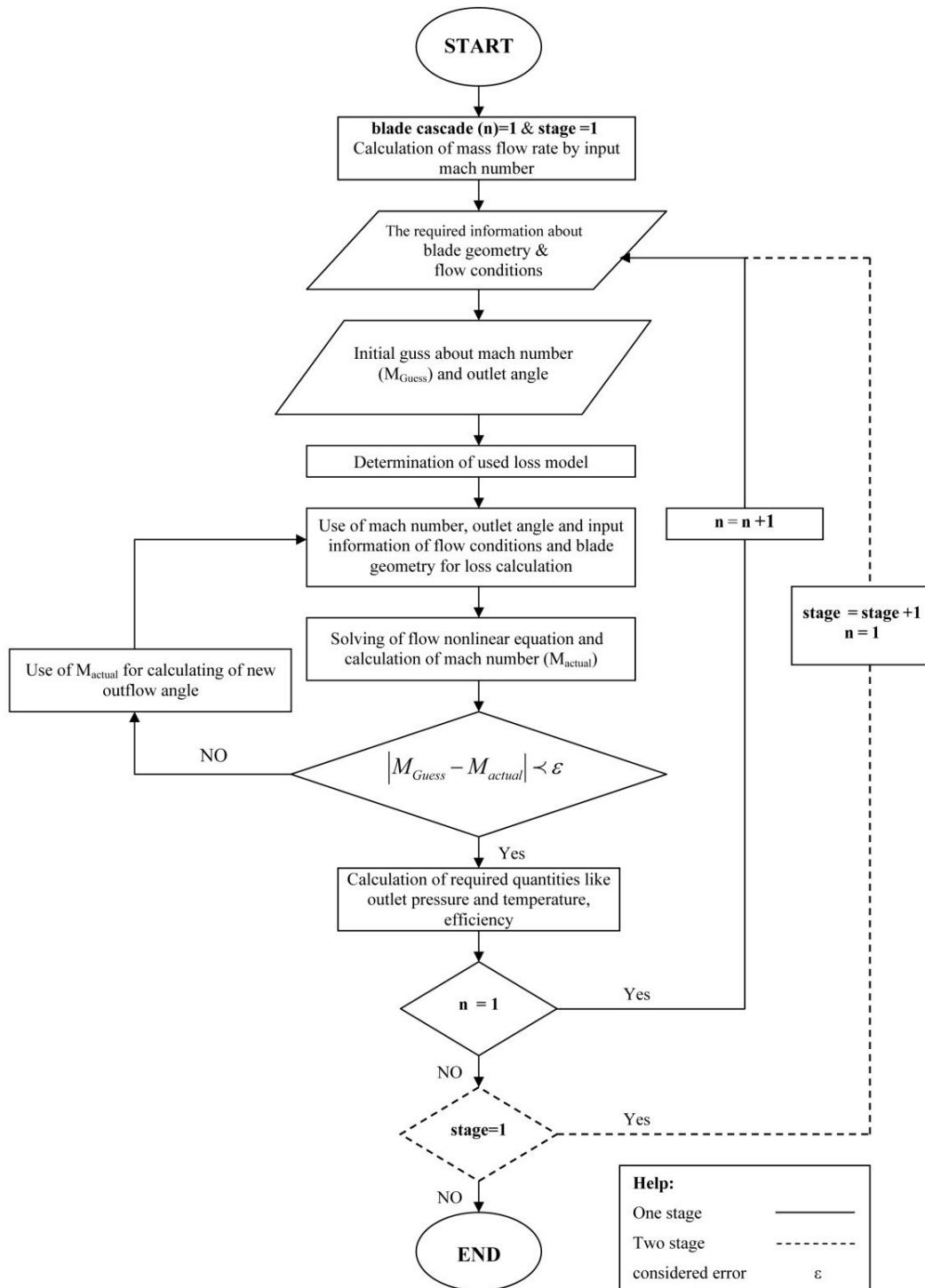**Fig. 1. Schematic view of axial flow compressor sections.**

Fig. 2. Algorithm of present compressor modeling.

The used loss coefficient in flow equation is [5]:

$$\sigma = 1 - Y\left[1 - \left(1 + \frac{\gamma-1}{2}M^2\right)^{-\gamma/(\gamma-1)}\right] \qquad (2)$$

M is the input mach to blade.

Free stream efficiency and pressure ratio are computed from the following equations:

$$\eta_{th} = \frac{(P_{03}/P_{01})^{\frac{\gamma-1}{\gamma}} - 1}{(T_{03}/T_{01}) - 1} \tag{3}$$

$$PR = \frac{P_{03}}{P_{01}} \tag{4}$$

4. Compressor geometry

In the present study, performance of a two- stage axial flow compressor without inlet guide vane is investigated. Compressor geometry is obtained from a NASA report [6].

5. Loss coefficient in compressor blades

In this section, Reynolds and Mach numbers are based on the relative gas flow velocities. The total pressure loss coefficient in the compressor is given as:

$$Y = Y_p + Y_s + Y_{EW} + Y_{TC} \tag{5}$$

Lieblein [7] expressed the blade-profile pressure loss coefficient as:

$$Y_p = 2\left(\frac{\theta_2}{C}\right) \times \frac{\sigma}{\cos\beta_2} \times \left(\frac{\cos\beta_1}{\cos\beta_2}\right)^2$$
$$\times \left(\frac{2H_{TE}}{3H_{TE} - 1}\right) \times \left[1 - \left(\frac{\theta_2}{C}\right)\frac{\sigma H_{TE}}{\cos\beta_2}\right] \tag{6}$$

The boundary-layer momentum thickness at the blade outlet, θ_2, is given [8] as:

$$\frac{\theta_2}{C} = \left(\frac{\theta_2^0}{C}\right) \times \zeta_M \times \zeta_H \times \zeta_{Re} \tag{7}$$

The boundary layer trailing-edge shape factor, H_{TE}, the ratio of the boundary layer displacement thickness to the momentum thickness, θ_2, is expressed as:

$$H_{TE} = H_{TE}^0 \times \xi_M \times \xi_H \times \xi_{Re} \tag{8}$$

The values of θ_2° and H_{TE}° are for inlet mach numbers, $M_{a1} > 0.05$, no contraction in the height of the flow annulus, H, an inlet Reynolds number, $Re_{1c} = 10^6$ and hydraulically smooth blades. Based on the experimental data of Koch and Smith [8] at these conditions, the boundary-layer momentum thickness at the blade outlet is correlated accurately as:

$$\frac{\theta_2}{C} = 2.644 \times 10^{-3} \times D_{eq} - 1.519$$
$$\times 10^{-4} + \frac{6.713 \times 10^{-3}}{2.60 - D_{eq}} \tag{9}$$

The shape factor for the boundary layer trailing-edge is correlated as:

$$H_{TE}^0 = \frac{\delta_{TE}^*}{\theta_2^0} = (0.91 + 0.35 \times D_{eq})$$
$$\{1 + 0.48 \times (D_{eq} - 1)^4 + 0.21 \times (D_{eq} - 1)^6\} \tag{10}$$

A value of $H_{TE}^\circ = 2.7209$ is used when $D_{eq} > 2.0$. For conditions other than nominal, Koch and Smith developed charts for determining the correction factors ζ_m, ζ_H, ζ_{Re} in Eq. (7) and ζ_m, ζ_H, ζ_{Re} in Eq. (8). The correction factor for inlet Mach number is correlated as:

$$\zeta_M = 1.0 + (0.11757 - 0.16983 \times D_{eq}) \times M_1^n \tag{11}$$

$$n = 2.853 + D_{eq}(-0.97747 + 0.19477 \times D_{eq}) \tag{12}$$

The correction factor for the flow area contraction is given by:

$$\zeta_H = 0.53\frac{H_1}{H_2} + 0.47 \tag{13}$$

The chart presented by Koch and Smith for the Reynolds correction factor is well approximated using the approach proposed by Aungier [9]. He introduced the critical blade chord Reynolds number, $Re_{cr} = 100 \times \frac{C}{K}$, above which the effect of roughness become significant. When $Re_{1c} > Re_{cr}$, the Reynolds correction factor is expressed as:

$$\zeta_{Re} = \begin{cases} \left(\dfrac{10^6}{Re_{1C}}\right)^{0.166}, & for\ Re_{1C} \geq 2 \times 10^5 \\ 1.30626 \times \left(\dfrac{2 \times 10^5}{Re_{1C}}\right)^{0.5}, & for\ Re_{1C} < 2 \times 10^5 \end{cases} \tag{14}$$

When $Re_{1c} > Re_{cr}$, the friction losses are controlled by the surface roughness and the Reynolds correction factor may be expressed as:

$$\zeta_{Re} = \begin{cases} \left(\dfrac{10^6}{Re_{1C}}\right)^{0.166}, & for\ Re_{cr} \geq 2 \times 10^5 \\ 1.30626 \times \left(\dfrac{2 \times 10^5}{Re_{cr}}\right)^{0.5}, & for\ Re_{cr} < 2 \times 10^5 \end{cases} \tag{15}$$

Typical ratios of the blade chord to surface roughness are: $c/K = 10,000$ to $20,000$. When $\text{Re}_{1c} > 10^6$ and $c/K = 10^4$, $\text{Re} = 10^6$ and $\zeta_{\text{Re}} = 1$. The correction factor for the inlet mach number is accurately fitted by:

$$\xi_M = 1.0 + \left[1.0725 + D_{eq} \times (-0.8671 + 0.18043 \times D_{eq}) \right] \quad (16)$$

The correction factor for the flow area contraction is calculated as:

$$\xi_H = 1.0 + \left(\frac{H_1}{H_2} - 1.0 \right) \times (0.0026 \times D_{eq}^8 - 0.024) \quad (17)$$

The correction factor for the inlet Reynolds number is given by:

$$\xi_{\text{Re}} = \begin{cases} \left(\dfrac{10^6}{\text{Re}_{1C}} \right)^{0.06}, & \text{When } \text{Re}_{1C} < \text{Re}_{cr} \\[4mm] \left(\dfrac{10^6}{\text{Re}_{cr}} \right)^{0.5}, & \text{When } \text{Re}_{1C} \geq \text{Re}_{cr} \end{cases} \quad (18)$$

The equivalent diffusion ratio, D_{eq} is given by [8]:

$$D_{eq} = \frac{W_1}{W_2} \times \left[1 + K_3 \frac{t_{max}}{C} + K_4 \Gamma^* \right]$$
$$\times \sqrt{\left(\sin \beta_1 - K_1 \sigma \Gamma^* \right)^2 + \left(\frac{\cos \beta_1}{A_{throat}^* \times \rho_{throat} / \rho_1} \right)^2} \quad (19)$$

In Eq. (19), the contraction ratio is given by:

$$A_{throat}^* = \left[1 - K_2 \sigma \left(\frac{t_{max}}{c} \right) / \cos \overline{\beta} \right] \left(1 - \frac{A_{throat}}{A_1} \right) \quad (20)$$

$$\overline{\beta} = \frac{\beta_1 + \beta_2}{2} \quad (21)$$

The throat area is assumed to occur at one-third of the axial chord, thus:

$$A_{throat} = A_1 - \frac{1}{3}(A_1 - A_2) \quad (22)$$

The gas density at the throat is calculated as:

$$\frac{\rho_{throat}}{\rho_1} = 1 - \frac{M_x^2}{1 - M_x^2} \left(1 - A_{throat}^* - K_1 \frac{\tan \beta_1}{\cos \beta_1} \sigma \Gamma^* \right) \quad (23)$$

The obtained constants in these equations from the experimental data of Koch and Smith are: $K_1 = 0.2445$, $K_2 = 0.4458$, $K_3 = 0.7688$ and $K_4 = 0.6024$. The dimensionless blade circulation parameter in Eqs. (19) and (23) is given by:

$$\Gamma^* = \frac{r_{1m} C_{1\theta} - r_{2m} C_{2\theta}}{\sigma W_1 \times (r_{1m} - r_{2m})/2} = (\tan \beta_1 - \tan \beta_2) \times \frac{\cos \beta_1}{\sigma} \quad (24)$$

The secondary flow loss coefficient is given by the correlation proposed by Howell [10] as:

$$Y_s = 0.018 \times \sigma \times \frac{\cos^2 \beta_1}{\cos^3 \beta_m} \times C_L^2 \quad (25)$$

The theoretical compressor blade lift coefficient, C_L, is expressed as:

$$C_L = \frac{2}{\sigma} \times \cos(\beta_m) \times [\tan(\beta_1) - \tan(\beta_2)] \quad (26)$$

The mean velocity vector angle is given by:

$$\tan(\beta_m) = 0.5 \times [\tan(\beta_1) + \tan(\beta_2)] \quad (27)$$

Based on a modified Howell's model [10], Aungier [9] developed the following expression for calculating the end wall loss coefficient, as:

$$Y_{EW} = 0.0146 \times \frac{C}{H} \times \frac{\cos^2 \beta_1}{\cos^2 \beta_2} \quad (28)$$

The tip clearance (leakage) loss factor, Y_{TC} is calculated [9] as:

$$Y_{TC} = Y_{tip} + Y_{gap} \quad (29)$$

$$Y_{tip} = 1.4 K_E \sigma \times \frac{\tau}{H} \times \frac{\cos^2 \beta_1}{\cos^3 \beta_m} \times C_L^{1.5} \quad (30)$$

and

$$Y_{gap} = 0.0049 K_G \sigma \times \frac{C}{H} (\sqrt{C_L} / \cos \beta_m) \quad (31)$$

For mid-loaded compressor blades, ($Z/C = 0.5$), $K_E = 0.5$ and $K_G = 1.0$.

6. Results and Discussion

Fig. 3 shows variation of the compressor pressure ratio versus mass flow at 11230 rpm comparing with experimental data of NASA report [6]. It shows good agreement with 1.69 % maximum difference.

Fig. 4 shows variation of compressor pressure ratio versus mass flow at different speeds. It is observed that at constant speeds, compressor pressure ratio decreases as mass flow increases. These curves are limited by surge condition at the left hand side and choke condition at the right hand side of the curves. The overall trend of
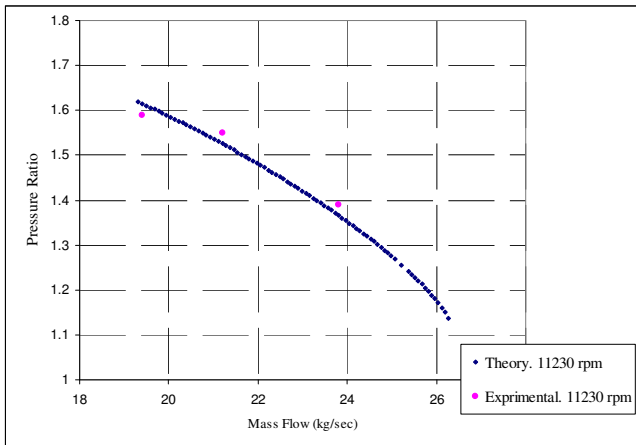
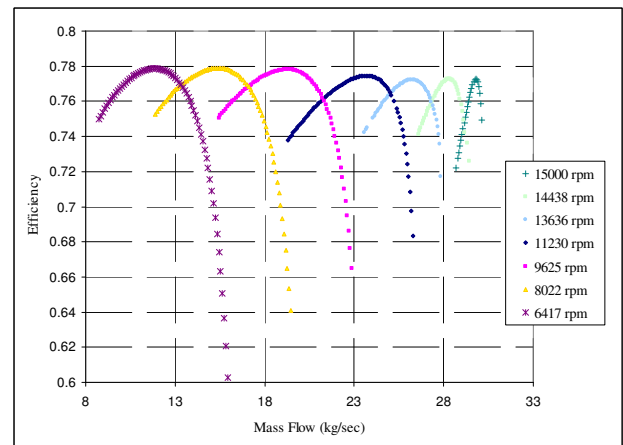Fig. 3. Pressure ratio vs. mass flow at 11230 rpm.

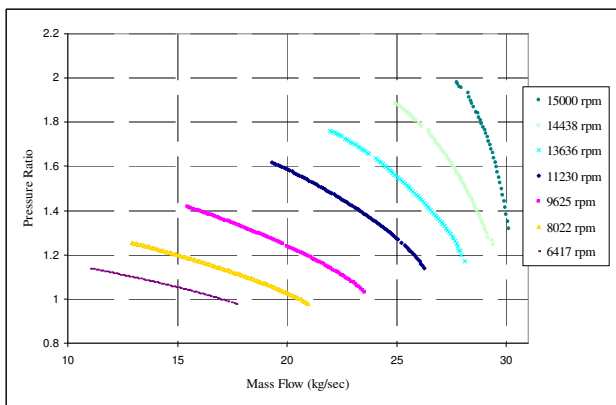Fig. 5. Compressor efficiency vs. mass flow at different rpm.

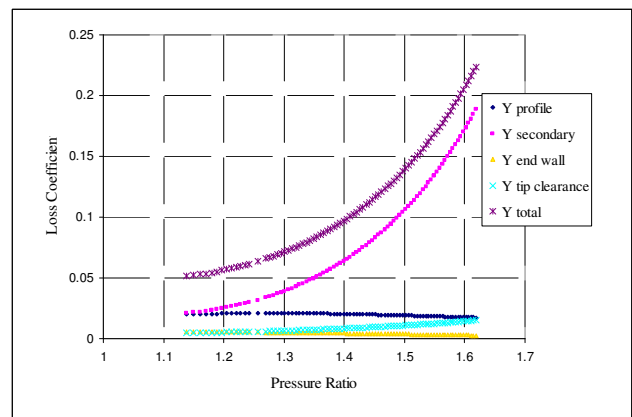Fig. 4. Pressure ratio vs. mass flow at different rpm.

Fig. 6. Loss coefficients vs. pressure ratio at 11230 rpm.

the curves is such as expected. At high mass flows, decrement in pressure ratio is more intensive due to increase of losses. At constant mass parameter, compressor pressure ratio increases as speed increase.

Variation of compressor isentropic efficiency versus mass flow at various speeds is shown in Fig. 5. As it is seen, the curves of efficiency versus mass flow at each speed have a maximum point and by departing from this this point, efficiency deficits because of loss grooving, which is more intensive at high mass flows.

Fig. 6 shows variation of profile loss coefficient, secondary loss coefficient, End Wall loss coefficient, tip clearance loss coefficient and Total loss coefficient as a function of pressure ratio at 11230 rpm . It is observed that at constant speeds, compressor loss coefficients increases as pressure ratio increases.

7. Conclusions

Performance of a two-stage axial flow compressor is investigated using one dimensional modeling approach.

In this method, it is possible to reach the intended performance features and investigate the effect of these changes on its performance, in order to optimize the compressor, while in other methods, by changing of geometrical parameters, the model should be reconstructed and reperformed, that according to the mentioned explanations, it`s so difficult.

In the present work, previous models are used to prepare a comprehensive model which is not found in the open literature.

At 11230 rpm, the results are compared with experimental data of NASA. Maximum error is about 1.69%. The results have good agreement with the

experimental data of NASA. Regarding that the one-dimensional modeling approach has some advantages such as simplicity, rapidity and applicability, this error may be neglected.

Nomenclature

A	ross-sectional flow area (m2)
C	actual chord length of blade (m)
C_L	blades lift coefficient based on mean vector velocity
D_{eq}	equivalent diffusion ratio
H	height of blades (m)
H_{TE}	boundary-layer shape factor
$\dot{m}$	mass flow rate of working fluid through turbo-machine (kg/s)
M	gas Mach number
P	pressure (Pa)
r	radius (m)
Re_{1C}	Reynolds number at the blade leading edge,
Re_{2C}	Reynolds number at the blade trailing edge,
t_{max}	maximum blade thickness (m)
T	temperature (K)
W	gas relative velocity vector with respect to rotor wheel (m/s)
C_θ	gas tangential velocity component (m/s)
Y	pressure loss coefficient of a blades
Z	position of maximum camber, measured from blade leading edge

Greek symbols

α	angle between C and meridional plane
β	angle between W and meridional plane
γ	ratio of specific heat capacities
Γ	blade circulation parameter (dimensionless)
δ^*	boundary layer displacement thickness (m)
θ	boundary-layer momentum thickness (m)
κ	peak-to-valley surface roughness (m)
ρ	density (kg/m3)
σ	blade cascade solidity
τ	blades clearance gap (m)

Subscripts

EW	end wall losses
gap	gap losses
P	profile losses
S	secondary losses
TC	tip clearance losses
x	axial component

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