

# Load Controlled Fatigue of AZ31B

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**Abstract:** With the expanding demand on application of magnesium alloys in automotive and aerospace industries, robust methods in fatigue characterization of commercially available magnesium alloys with high specific strength is anticipated. In this paper, rotating bending load controlled tests has been studied on specimens machined from an extrusion piece of AZ31B. Due to asymmetric and anisotropic behavior of AZ31B, methods of transferring the load controlled results into stress-life results are lacking. Using the approach of variable material property (VMP), the load-stress relation in fully reversed test is derived. To apply the proposed method to AZ31B, the cyclic behavior of the material is required. Fully reversed step-loading of AZ31B over a wide range of strain amplitude was carried out on a servo hydraulic fatigue rig and was reported in the literature. The stabilized cyclic behavior at half-life was obtained for different load levels. Also, the cyclic tension and the cyclic compression curves were obtained. Using the proposed load-stress model and the actual cyclic behavior of AZ31B, an elastic-plastic solution over the whole domain of cross section was obtained. An energy based fatigue model was then employed to predict the life. The predicted and experimental lives agree well. The proposed method is recommended for correlating the load controlled test with stress and/or strain controlled tests.

**Keywords:** Elastic-Plastic Solution, Bending, Magnesium AZ31B, Variable Material Property Method

## 1. Introduction

Considering the dramatic expansion of human needs and technology advances, the risk of reduction in energy resources, especially fossil ones and the need for reducing weight in a large part of industrial products, use of lightweight materials with high strength is evident. Low density, good strength, good weld ability and corrosion resistance in air, make Magnesium and its alloys one of the most suitable options for use in the space and automotive industries [1].

Compared to metals with cubic structure, textured magnesium alloys show unsymmetrical behavior and also the Bauschinger effect plays an important role in the response of the material. This behavior which comes from the hexagonal closed pack form of the material crystals, have an important role in finite element method (FEM) analysis and forecasting of the Mg-alloys behavior [2]. In magnesium alloys, the

amount of tensile yield strength is larger than the compressive strength (yield asymmetry). Also, these alloys show a flow asymmetry behavior [3].

There are two different approaches to investigate the plastic behavior of magnesium alloys. The first approach is “crystal plasticity” approach which the crystal structure of the material and microscopic considerations are taken to account. The second approach is continuum phenomenological approaches which interpret the behavior of material using the macro mechanic observations with the help of yield surfaces and their translation, and the hardening rules [1]. In the present study, the second type of approach has been employed.

Though there are many experiments on cyclic and fatigue behavior of Mg AZ31B in recent years [4], elastic-plastic solutions and fatigue characteristics of the Mg alloys in pure reversed bending loading have not been reported. Hasegawa et al. [5] performed stress- and strain-controlled axial tests on AZ31 extru-

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sions. They found that the asymmetric hysteresis loop that is considered a characteristic feature in strain-controlled tests disappears at half-life in a stress-controlled test. Park et al. [6] suggested the use of energy as a fatigue damage parameter for two reasons. They performed low cycle fatigue tests for both rolling and normal directions. They found that while completely reversed straining in the rolling direction results in a positive mean stress due to twinning in compression, applying the same load in the normal direction results in a negative mean stress due to twinning in tension. This negative mean stress was seen to cause a beneficial effect on the fatigue life. Albinmousa et al. [7] investigated both cyclic axial and torsional behavior of AZ31B extrusion. They also investigated the multi-axial fatigue of AZ31B extrusion. The authors used total energy, the sum of plastic strain and positive elastic energies, as a damage parameter and found that fatigue life in monotonic and multi-axial cyclic tests results could be correlated to the total energy.

Although the S-N curves of the AZ31 is available now, use of this curve is limited to the elastic range and the fatigue life prediction curves for the plastic strains must be obtained. In this paper, for the first time, an elastic-plastic solution over the whole domain of the cross section of a beam under pure bending, has been introduced using the variable material properties (VMP) method (Jahed et al. [8]). Using the stabilized hysteresis curves from a wide range of experiments performed on tubular samples of AZ31B, a

complete elastic-plastic solution has been obtained leading to a closed loading cycle. The result is then considered with the Albinmousa et al. [4,7,9] method to determine the fatigue life of sample. The results show a very good agreement with experimental data.

## 2. Elastic-Plastic modeling

The elastic-plastic solution consists of two main steps, loading and unloading. In the first step, the data extracted from the strain control tests for a wide range of strains on the tubular specimen has been used to obtain the cyclic loading curve for the extruded AZ31B Mg alloy. The cross section is discretized to a finite number of elements. Considering a hypothetical pure elastic behavior for a beam under pure bending (Eq. 1), an elastic solution over all cross section has been performed.

$$\sigma = My/I, \quad \varepsilon = \sigma/E \quad (1)$$

Where  $M$  is the applied moment,  $y$  is the distance from the neutral axis,  $I$  is the second moment of area,  $\varepsilon$  is the strain,  $\sigma$  is the stress and  $E$  is the Young's modulus. Using the VMP method and the obtained cyclic loading curves, results are then resolved to the real elastic-plastic solution. In this step, after the first solution (elastic solution), the value of the effective Young's modulus for each element has been updated using the projection method (Fig. 1).

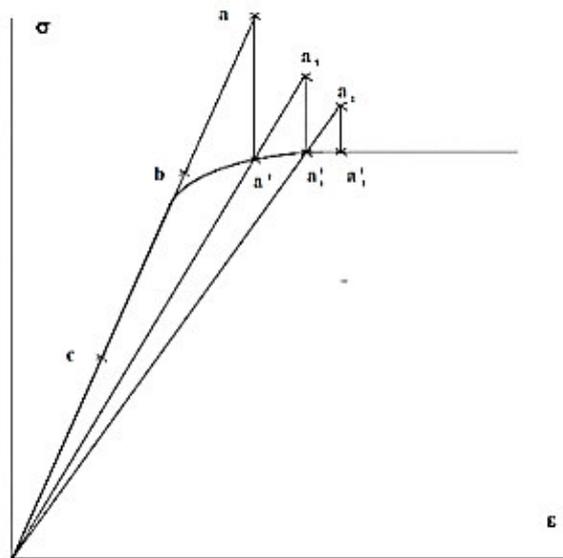


Fig. 1. Projection method used for updating the Effective Young modulus for each element [8].

The equivalent section method of composite beams has been employed to construct the transformed section and locate the new neutral axis. In this method, after determining the new effective Young's modulus ( $E_i$ ) for each element, the width of the element changes proportional to the ratio of  $E_i / E$ .

The second moment of the area and the position of the neutral axis will then be calculated using Eq. 2, where  $A_i$  and  $y_i$  are the elements' area and vertical distance from neutral axis respectively. Now all of the elements effective Young's moduli set to the new values and again a hypothetical elastic solution is obtained (Fig. 2).

Neutral axis location from previous

$$\text{maxis} = \frac{\sum A_i y_i}{\sum A_i} \quad (2)$$

The whole process is repeated until the stress and strain in each element matches a point on the experimentally obtained material curve.

According to the unique behavior of Mg alloys, none of the available hardening rules such as kinematic or isotropic hardening are suitable to describe the unloading and loading behaviors [3,7].

Accordingly, the experimentally obtained curves [10] are employed. Each element is forced to fol-

low its unloading curve based on its location on the loading stress-strain curve form at the end of loading reversal. The real stabilized cyclic stress-strain curves on a wide range of strain amplitudes were used to force each element follow its own unloading path. This curve was obtained from fully reversed step-loading of AZ31B over a wide range of strain amplitudes [10].

The strain range from the elastic-plastic solution, and the strain amplitude associated with the obtained solution were employed to predict the life using the Coffin-Manson equation with corresponding coefficient for AZ31B alloy from fatigue model of Albinmousa et al [4,7,9].

### 3. Material and experiment

All experimental results are from specimens that were machined from an air-quenched AZ31B extrusion section, Fig. 3, as reported in [4,7]. This extruded section was from a 177.8 mm diameter, 406.4 mm long billet, with an extrusion ratio of 6. The extrusion temperature was between 360 and 382 °C, with an extrusion exit speed of 50.8 mm/s.

Hysteresis of a typical fully reversed cyclic axial loading is shown in Fig. 4 as a sample.

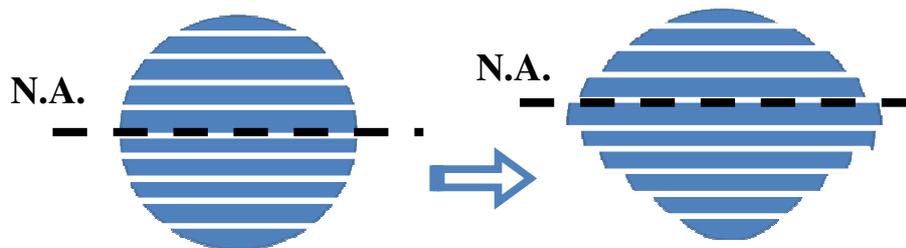


Fig. 2. Changes in elements' shape and the position of the neutral axis.

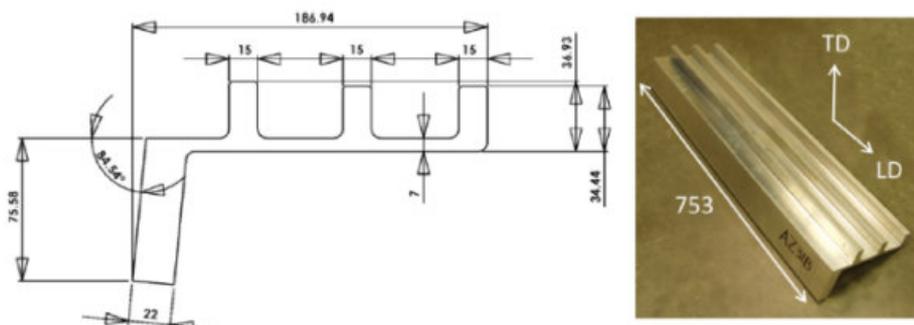


Fig. 3. AZ31B extrusion. All dimensions in mm [4].

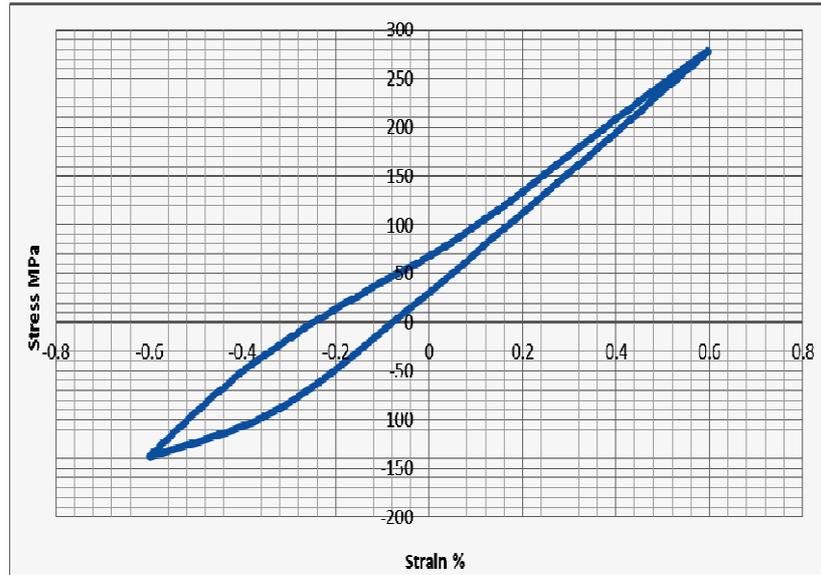


Fig. 4. Typical cyclic axial behavior of AZ31B extrusion, strain amplitude: 0.6%.

Stress relieved cylindrical specimens have been tested by a Rotating Bending Machine (RBM) to prepare S-N curves. All tests performed at standard lab condition. Tests were performed at frequencies between 50 Hz for low cycle to 100 Hz for high cycle tests. [10] Specimens used for the test are shown in Fig. 5.

In RB testing at each rotation, the specimen experiences an oscillating stress varying from maximum compressive to maximum tensile. Applying different loads on the specimen and recording the number of cycles to failure provides the data necessary for the S-N curve. Fig. 6 shows the S-N curve obtained from fatigue tests as reported in [10].

## 4. Results and discussion

In this section, an example for elastic-plastic solution is described. Loading and unloading is applied to a cylindrical sample made from AZ31B alloy and the stress-strain response is obtained. Results are then used to determine the fatigue life, using Coffin-Manson Eq. [9] and Experimental Data [10].

### 4.1. Numerical example of elastic-plastic solution

An 8 N.m moment is applied to a cylindrical shape AZ31B alloy with a diameter of 3.86 mm and the Young's modulus of 44 GPa. Using the

VMP method [8] and the proposed method, the stress-strain response after loading and unloading is obtained. Fig. 7 shows the stress-strain curve for the cross section after loading. As depicted by this figure, the responses successfully followed the experimental cyclic stress-strain curve. It is observed that the values of maximum and minimum strain and stress are not the same in the tension and compression parts which is due to the asymmetric behavior of the AZ31B.

Fig. 8 shows the loading and unloading for upper and lower point of the cross section. The figure depicted three significant points. First, the maximum stress values (in tension and compression) are different. Second, there is an obvious difference between the loading and unloading paths in each element. Third, there is some residual stress remained after unloading, especially in compression part which shows more nonlinear behavior and plastic strain in a defined applied moment in comparison with tension part.



Fig. 5. Round specimens used for extracting S-N curve of AZ31B [10].

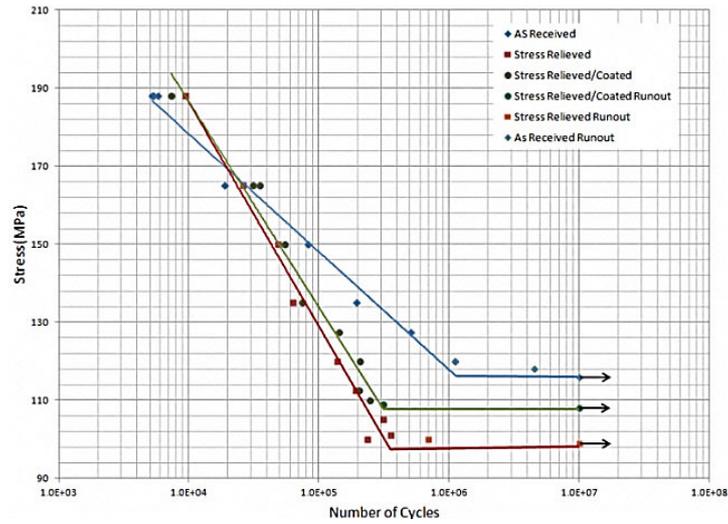


Fig. 6. S-N curves of as received, stress relieved, and stress relieved /coated specimens of AZ31B [10].

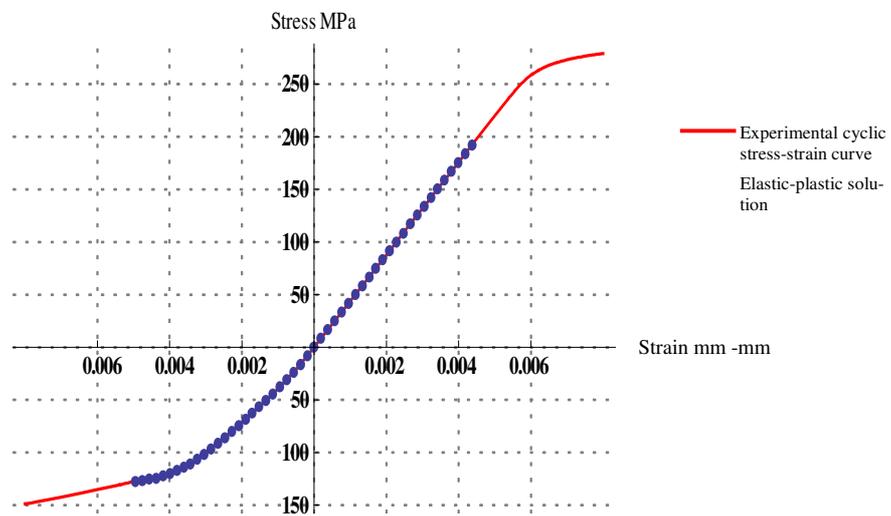


Fig. 7. Elastic-plastic solution for a. 8N.m applied moment.

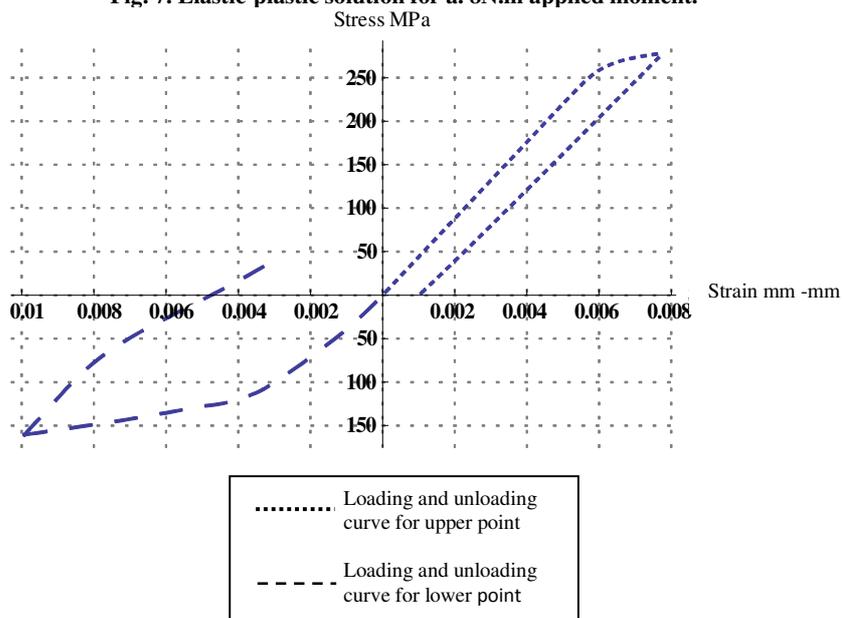


Fig. 8. Loading and unloading curves for upper and lower points of the cross section.

## 4.2. Estimating fatigue life

Using the results from elastic-plastic solution the fatigue life is predicted. Coffin-Manson equation with coefficients suitable for AZ31B [9] is employed. The actual experimental stress-life curve [10] is then used to evaluate the results. Eq. 3 shows Coffin-Manson equation and its corresponding parameters for AZ31B are listed in Table 1. In this section  $\varepsilon_a$  is the strain range,  $\sigma'_f$  is the fatigue strength coefficient,  $\varepsilon'_f$  is the fatigue toughness,  $b$  is the fatigue strength exponent,  $c$  is the fatigue toughness exponent,  $E$  is the Young's modulus and  $N_f$  is the number of reversals to failure.

$$\varepsilon_a = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (3)$$

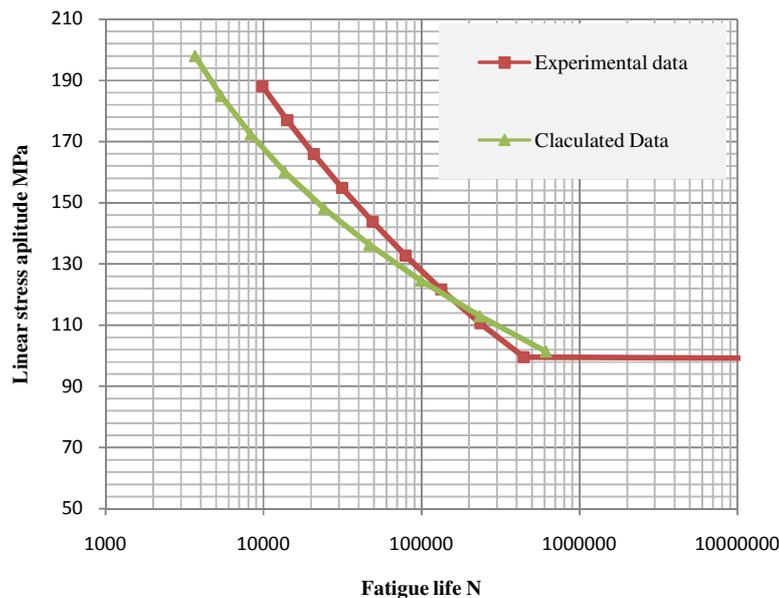
Elastic-plastic solution for a range of applied mo

ment is obtained and the predicted fatigue life using Coffin-Manson equation and experimental data are presented in Fig.9.

Fig. 9 indicates that Coffin-Manson equation shows good agreement with experimental results. It consists of two lines, the experimental S-N curve from AZ31B rotating bending test reported by Mahmoudi [10] and the obtained results from method proposed in this study. As it is shown, estimated fatigue life values at given amplitudes of stress are in a good accordance with experimental data.

**Table 1. Coffin-Manson's strain-life equation parameters for AZ31B [4].**

Fatigue strength coefficient (MPa)	$\sigma'_f = 495.82$
Fatigue toughness (mm/mm)	$\varepsilon'_f = 1.589$
Fatigue strength exponent	$b = -0.115$
Fatigue toughness exponent	$c = -0.939$



**Fig. 9. S-N curve of AZ31B alloy.**

## 5. Conclusions

Using experimental cyclic loading and stabilized hysteresis curves of AZ31B, the VMP method and a proposed method, elastic plastic solution for a cylindrical specimen made of AZ31B alloy under pure rotating bending condition is obtained. Results are implemented to predict the fatigue life using

Coffin-Manson equation. Experimental data are used to verify these results. According to the solution obtained, the following conclusions are made:

- Proposed method is able to find an asymmetric elastic-plastic response for the cylindrical specimen under applied moment which can follow the experimental cyclic curve very well.
- Asymmetric behavior of the AZ31B alloy can be observed clearly in elastic plastic response.

- The conventional linear elastic solution which is used in S-N curves differs considerably from elastic-plastic solution.
- Despite the wide use of Coffin-Manson equation in fatigue life prediction, this equation can just predict the life for AZ31B alloy without mean stress consideration. This could come from the asymmetric behavior of magnesium alloys. Another methods such as energy method (Jahed et al. [11]) may be a better estimate of the life in magnesium alloys [9].

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