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A Method of Performance Estimation for Axial Flow Turbines Based on Losses Prediction

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Abstract: The main objective in this paper is to create a method for one-dimensional modeling of multi-stage axial flow turbine. The calculation used in this technique is based on common thermodynamics and aerodynamics principles in a main stream line analysis. In this approach, loss models have to be used to determine the entropy increase across each section in the turbine stage. Finally, the analysis and comparisons between the calculated results and experimental data are performed.

Keywords: Two-Stage Axial Flow Turbine, Modeling, Total Loss Coefficient, Loss Model, Performance Curve

1. Introduction

Turbo machines are devices within which conversion of total energy of a working fluid into mechanical energy takes place [1]. Turbomachines are widely used today through the world as power generating units, they are and also successfully used in space, aviation, marine, nuclear, rail-road, vehicular, cryogenic and petro-chemical applications [2]. The demand for turbomachines in the future will certainly increase.

Gas turbines are more reliable in high mass flow rate. Moreover they present a higher power to mass flow in comparison with reciprocating machines [3].

Because of cost and lack of reliable experimental information, the modeling methods have been developed [4]. According to flow pattern complexity in gas turbines, different calculating and modeling methods are required in order to design them effectively. Meanwhile, one-dimensional modeling is a simple and fast method due to the performance prediction and optimization of initial design.

During the last years, several important work has been reported on the one-dimensional modeling technique which can estimate turbine performance and various types of losses by acceptable accuracy.

Ning Wei [3], studied the significance of loss models and their applications in simulation and optimization of axial turbines. He presented useful guides for applying the models properly in turbines aerothermodynamics simulation and optimization.

Tournier and Genk [5], used one-dimensional modeling that was based on a mean-line flow analysis for performance prediction of axial flow turbines. They developed the latest loss model that was proposed by Benner *et al.*

In this work an attempt has been made to simulate the performance characteristic of the axial turbine to confirm the efficiency of suggested method for one-dimentional modeling and also the performance of different loss models was investigated. These loss models were introduced by Soderberg, Ainley, Mathieson, Tournier and Genk.

2. Flow field and loss mechanisms in axial turbine blades

The flow in turbine blades is characterized by a three-dimensional, highly unsteady motion with random fluctuations due to the interactions between the stator and rotor rows [1]. It may be incompressible or compressible, with subsonic,

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transonic, and supersonic regimes which may be presented simultaneously in different regions. Because of this, different losses are created in turbine cascade. Various losses that are often considered in turbine include profile, secondary flow, tip clearance and annulus losses.

1- Profile loss, the loss due to blade boundary layers or separation which will take place with a uniform two-dimensional flow across a cascade of blades [6].
2- Secondary loss, secondary flows are always when a wall boundary layer is turned through an angle by an adjacent curved surface [7].
3- Tip clearance loss, The pressure difference between the pressure side and suction side of the blade drives a leakage flow through the rotor tip/casing clearance gap. Typically, this flow is ejected as a strong jet which mixes with the main stream on the suction side, usually rolling up to form a vortex. This strong jet and vortex cause entropy change [8].

Annulus loss, this loss is associated with boundary layer growth on the inner and outer walls of the annulus [6].

3. Principles of one-dimensional analysis in turbine

Nowadays different computational and modeling methods are needed for simulating and optimizing turbomachines in order to design turbines more effectively and efficiently. Among these approachs, one-dimensional modeling is a simple and fast method for obtaining gas turbine performance condition. In this method, the average of aerodynamic and thermo-dynamic properties are considered on midstream line.

In the present research the principles of flow analysis in turbine are based on continuity equation, energy equation, state relation for a perfect gas and isentropic relation over every blade row, that are the basic equations for one-dimensional modeling.

It worths noting that usage of modeling through the mentioned method has specific importances that the following features can be pointed out:

1- Much less information is required from the turbine geometry.
2- In this method, computational volume is much less than other methods like CFD [4,9,10].

4. One dimensional modeling by using of suggested algorithm

The suggested algorithm is based on flow field equation in turbine blades.

By applying the continuity equation on stator outlet section and replacement of Mach number relation in it, and by using other base equations, flow field equation will be obtained by considering the losses term as Eq. (1) [1,2]:

$$\frac{m\sqrt{\dfrac{RT_0}{\gamma}}}{A_{out}P_0} = \sigma\, Cos(\alpha_{out})M_{out}(1+\frac{\gamma-1}{2}M_{out}^2)^{\frac{\gamma+1}{2(1-\gamma)}} \tag{1}$$

It can be seen that the left hand side of this equation is a dimensionless function of the mass flow, $m^{\cdot}$, the outlet flow area, A_{out}, the stagnation temperature, T_0, and the stagnation pressure, P_0. And the right hand side is a function of the outlet Mach number, M_{out} , and outlet flow angle, α_{out}. The special symbol σ is a function of entropy change of actual process (Eq. 2):

$$\sigma = e^{(-\Delta s/R)} \tag{2}$$

For calculation σ parameter, that is called total pressure loss coefficient, we used in Eq. 3 and Eq. 4 that follow immediately from its definition (Eq. 2) [10]:

$$\sigma = \left\{1+Y\left[1-(1+\frac{\gamma-1}{2}M_{out}^2)^{\frac{\gamma}{1-\gamma}}\right]\right\}^{-1} \tag{3}$$

$$\sigma = (1-\frac{\gamma-1}{2}\xi M_{out}^2)^{\gamma/\gamma-1} \tag{4}$$

In the above equations, Y is pressure loss coefficient and ζ is enthalpy loss coefficient [2].

To do the calculations, rather than turbine geometry, the amount of turbine rotational speed, stagnation temperature and pressure conditions of input flow to turbine are defined. Calculation will start from turbine inlet. The solution of the flow equation through the suggested algorithm for modeling is as follows:

First, the proper Mach number is determined in blade cascade inlet. Then by using continuity equation in turbine inlet, the mass flow rate is calculated. By guessing an initial Mach number and flow angle in blade outlet, losses coefficients and total loss coefficient will be determined by using loss model. After determination total loss coefficient, the only unknown parameter in the flow equation is outlet Mach number that is calculated. By calculating outlet Mach number, the outflow angle will be modified and this repetition continued until intended precision achieved. By determining final outlet Mach number, another parameter like outlet stagnation pressure and temperature will be achieved.

This method can be repeated for each cascade of a multi-stage turbine and total characteristics like stage pressure ratio and isentropic efficiency will be calculated.

5. Loss Coefficients

The losses are used in turbines to be expressed in terms of loss coefficients. They are manifested by a decrease in stagnation enthalpy, and a variation in static pressure and temperature, compared to the isentropic flow. Usual loss coefficients in turbines are enthalpy loss coefficient, entropy loss coefficient and pressure loss coefficient.

Pressure loss coefficient for stator and rotor blades is defined as Eqs. (5) and (6) [3,6]:

$$Y_N = \frac{P_{01} - P_{02}}{P_{02} - P_2} \tag{5}$$

$$Y_R = \frac{P_{02,rel} - P_{03,rel}}{P_{03,rel} - P_3} \tag{6}$$

Enthalpy loss coefficient for stator and rotor blades (Eqs. 7 and 8):

$$\zeta_N = \frac{h_2 - h_2'}{P_{01} - h_2} = \frac{T_2 - T_2'}{\frac{1}{2}C_2^2} \tag{7}$$

$$\zeta_N = \frac{h_3 - h_3''}{P_{02,rel} - h_3} = \frac{T_3 - T_3''}{\frac{1}{2}V_3^2} \tag{8}$$

Parameters and subscripts that were used in these equations are based on T-S diagram in Fig. 1.

Because the flow in a turbine is complex and many mechanisms of the flow losses in turbine have not been known well, loss models are needed for predicting the losses in the simulation and optimization of turbines. Most of the loss models are empirical while some of them are established by combining test data and analysis of physical origins of the losses.

There are different loss models for predicting amount of loss in axial turbines. In this section three models of these losses have been presented that the researchers used them in their modeling.

5.1. Soderberg's model

This model is useful for obtaining quick and preliminary estimates of turbine performance. Soderberg gave the total loss coefficient as Eqs. (9) and (10):

$$\zeta_N = \left(\frac{10^5}{Re}\right)^{1/4}\left[(1+\xi^*)\left(0.993 + 0.075\frac{l}{H}\right) - 1\right] \tag{9}$$

$$\zeta_R = \left(\frac{10^5}{Re}\right)^{1/4}\left[(1+\xi^*)\left(0.993 + 0.075\frac{l}{H}\right) - 1\right] \tag{10}$$

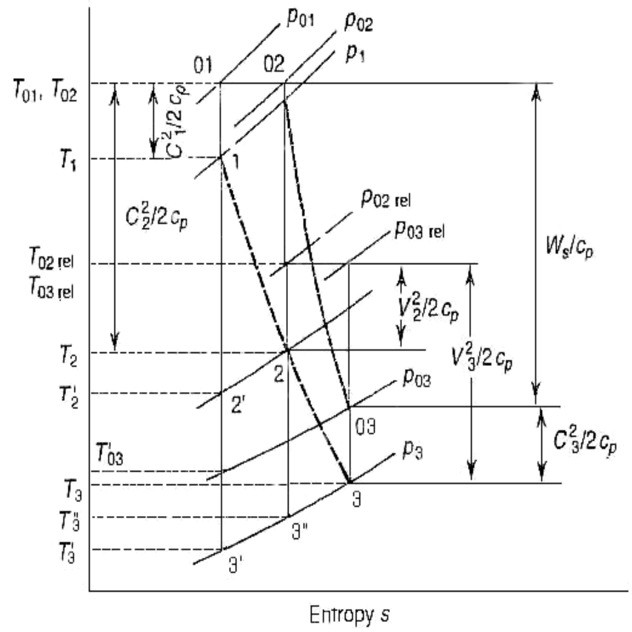

Fig. 1. T-S diagram for one stage turbine [6].

In these equations, ζ^* is the nominal loss coefficient and the main function of blade deflection (Eq. 11).

$$\zeta^* = 0.04 + 0.06\left(\frac{\varepsilon}{100}\right)^2 \quad (11)$$

Soderberg's model only includes profile and secondary flow loss but not the tip clearance loss. The profile loss was considered mainly as a function of blade deflection. The secondary loss in this model depends mainly on the blade aspect ratio, *l/H*.

Due to neglecting some parameters like inlet boundary layer and the blade geometry renders Soderberg's loss model are open to criticism [3].

5.2. Ainley & Mathieson's model:

This model is based on assumptions and experimental data that can be used to predict the performance of axial flow turbines with conventional blades over a wide part of their full operating range.

The total losses coefficient in a turbine cascade by Ainley and Mathieson consists of profile, secondary and tip leakage losses (Eq. 12).

$$Y = (Y_p + Y_s + Y_{Tl})X_{TE} \quad (12)$$

In this equation, X_{TE} is the trailing edge coefficient that can be obtained from a figure by Ainley and Mathieson [7].

Ainley and Mathieson gave profile loss model based on a series of experimental graphs of total pressure losses versus pitch/chord ratio for nozzle and impulse blades (Eq. 13).

$$Y_{p(i=0)} = \left\{Y_{p(\alpha'_{in}=0)} + \left(\frac{\alpha'_{in}}{\alpha_{out}}\right)^2 \left[Y_{p(\alpha'_{in}=\alpha_{out})} - Y_{p(\alpha'_{in}=0)}\right]\right\}\left(\frac{t_{max}/l}{0.2}\right)^{\alpha'_{in}/\alpha_{out}} \quad (13)$$

The value of $Y_{p(i=0)}$ in Eq. (13) refers to blades operating at zero incidence (i=0). The final profile loss is equal to the profile loss at zero incidence, multiplied by an incidence coefficient, χ_i (Eq. 14).

$$Y_p = x_i Y_{p(i=0)} \quad (14)$$

χ_i can be obtained from figures by Ainley and Mathieson.

The secondary loss coefficient in this model is calculated based on the blade loading which is considered as a main function of the blade turning. This loss equation was given by Ainley and Mathieson as Eq. (15):

$$Y_s = \lambda\left(\frac{c_l}{s/l}\right)^2\left(\frac{cos^2\alpha_{out}}{cos^3\alpha_{out}}\right) \quad (15)$$

In this equation, λ is a parameter that is a function of the flow acceleration through the blade row and was given in a figure by Ainley and Mathieson [7].

The tip leakage loss is also considered, with the same principle as the secondary loss, as a function of blade loading supplemented with the ratio of tip clearance to the blade height. It can be calculated with Eq. (16):

$$Y_{Tl} = B\frac{\tau}{h}4(tan\,\alpha_{in} - tan\,\alpha_{out})\left(\frac{cos^2\alpha_{out}}{cos\,\alpha_{out}}\right) \quad (16)$$

h, is annulus height. The constant *B* is 0.25 for a shrouded blade with side clearance and 0.5 for a radial tip clearance blade [3,7].

5. 3. Kacker and Okapuu's developed model

This model is the latest refinements proposed by Benner et al. of Kacker and Okapuu's Model and Tournier and Genk developed it, that the researchers used in this research.

The total pressure loss coefficient in this model is given as Eq. (17):

$$Y = (Y_p + Y_s) + Y_{TE} + Y_{Tl} \quad (17)$$

Benner et al. proposed a loss scheme for the breakdown of the profile and secondary losses as Eq. (18):

$$(Y_p + Y_s) = \left(1 - \frac{Z_{TE}}{H}\right) \times Y'_p + Y'_s \quad (18)$$

The profile loss coefficient, based on recent turbine cascade data is given by Eq. (19):

$$Y_p' = 0.914\times\lfloor K_{in} Y'_{p,AM} K_p + Y_{shock} \rfloor \times K_{Re} \tag{19}$$

The factor K_p in Eq. (19) is identical to that introduced by Kacker and Okapuu[5] to account for the gas compressibility. $Y'_{p,AM}$ is the same profile loss that was presented by Ainley and Mathieson.

The shock losses, appearing in Eq. (19), are calculated as Eq. (20):

$$Y_{shock} = 0.75(M_{in,H} - 0.4)^{1.75}$$

$$\left(\frac{r_H}{r_T}\right)\left(\frac{\rho_{in}}{\rho_{out}}\right)\frac{1-\left(1+\frac{\gamma-1}{2}M_{in}^2\right)^{\gamma/\gamma-1}}{1-\left(1+\frac{\gamma-1}{2}M_{out}^2\right)^{\gamma/\gamma-1}} \tag{20}$$

In Eq. (20), $M_{in,H}$ is the inlet Mach number at the hub. It can be obtained from the correlation between the inlet average Mach number and the ratio of the hub radius to the tip radius [5].

The factor K_{in} = 2/3, used by Kacker and Okapuu, underpredicts the profile losses for blade rows with axial inflow. Zhu and Sjolander have introduced a Reynolds number correction factor based on their recent blade cascade data, as Eq. (21) and Eq. (22):

$$K_{Re} = \left(\frac{2\times10^5}{Re_{out}}\right)^{0.575}, \quad \text{for } Re_{out} < 2\times10^5 \tag{21}$$

$$K_{Re} = 1.0, \qquad \text{for } Re_{out} < 2\times10^5 \tag{22}$$

The spanwise penetration depth (Z_{TE}) of the separation line between the primary and the secondary loss regions, appearing in Eq. (18), is given by Eq. (23):

$$\frac{Z_{TE}}{H} = \frac{0.10\times|F_t|^{.79}}{\sqrt{\cos\alpha_{in}/\cos\alpha_{out}}\times(H/l)^{.55}} + 32.7\left(\frac{\delta^*}{H}\right)^2 \tag{23}$$

δ^*, is the boundary layer displacement thickness at the inlet end wall [5].

The tangential loading parameter, F_t, in Eq. (23) is given by Eq. (24):

$$F_t = 2\frac{s}{l\times\cos\phi}\times\cos^2(\alpha_m)\times(\tan\alpha_{in} + \tan\alpha_{out}) \tag{24}$$

The mean velocity vector angle is given by Eq. (25):

$$\tan(\alpha_m) = \frac{1}{2}(\tan\alpha_{in} - \tan\alpha_{out}) \tag{25}$$

The secondary loss coefficient in Eq. (18) is given by Eq. (6):

$$Y_s' = F_{AR}\times\frac{0.038 + 0.41\times\tanh(1.2\delta^*/H)}{\sqrt{\cos\phi}\times(\cos\alpha_{in}/\cos\alpha_{out})\times(l.\cos\alpha_{out}/l_x)^{0.55}} \tag{26}$$

The aspect ratio factor, F_{AR}, is a function of the blade aspect ratio (Chord/Height) [5].

In this model, the trailing edge loss coefficient, Y_{TE}, is a function of outlet Mach number and kinetic energy loss coefficient that its equation is presented by Kacker and Okapuu [3].

The tip leakage loss coefficient, Y_{Tl}, in turbine blades cascade is calculated using Yaras and Sjolander approach [5].

6. Results and discussion

The performance of two-stage turbines that is described in this section has been simulated by different loss models. The main input data for calculations are the stage inlet stagnation pressure and temperature, mass flow, turbine speed and geometric parameters of each cascade. The turbine geometry, experimental data and flow conditions are obtained from a NASA report [11].

Figs. 2, 3 and 4 show variation of the turbine mass flow versus pressure ratio at design speed (5041 rpm) that were compared with experimental data. The conditions are the same as the experiments.

The comparison of achieved results of modeling with experimental results shows that the theoretical values agree well with the experimental values and a very good adaptation exists between these results.

The pressure ratios range is 1/17 to 3/8. This range of pressure ratio covers the large off-design region.

In Table 1, the percent errors of pressure ratio in modeling and its experimental value, in design point with rotational speed 5041 rpm and mass flow 19.95 kg/sec, are presented. Also a comparison between some off- design points is shown in Table 2.

Table 1. Percent error of pressure ratio toward experimental data in design point

	Soderberg	Ainley & Mathieson	Tournier & Genk
Percent of error	%9	%1.6	%0.1

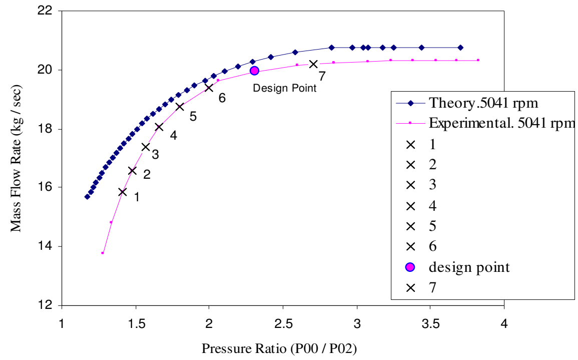

Fig. 2. Mass flow vs. pressure ratio at design speed by using Soderberg's model.

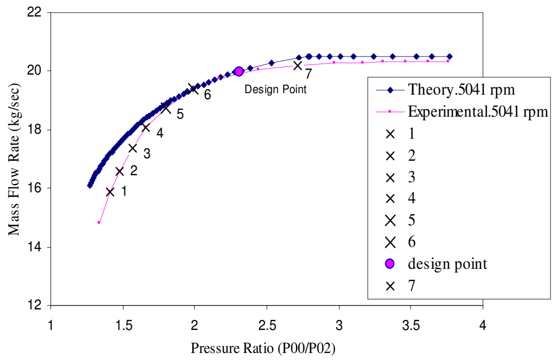

Fig. 3. Mass flow vs. pressure ratio at design speed by using Ainley and Mathieson's model.

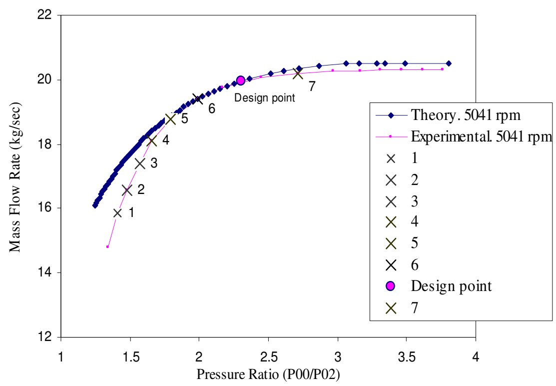

Fig. 4. Mass flow vs. pressure ratio at design speed by using Tournier and Genk's model.

Soderberg performance curve have the greatest error in respect to the experimental curve in design and off-design points. This is because of Soderberg's model estimates the loss processes lower than actual measures.

Whereas the Tournier and Genk's model has exhibited the best result, in Fig. 5 performance curves of two stage-axial flow turbine have been shown in four rotating speed by using of this model.

One advantage of this modeling is choked flow region. By increasing the pressure ratio, the mass flow rate also rises but in a special pressure ratio, when this increase is stopped mass flow remains constant. In this situation in a section of blade (gorge), Mach number or critical velocity ratio is equal to one.

For values of Mach number greater than 1.0, a new stator exit flow angle is computed from the area required to pass the choking mass flow rate [12].

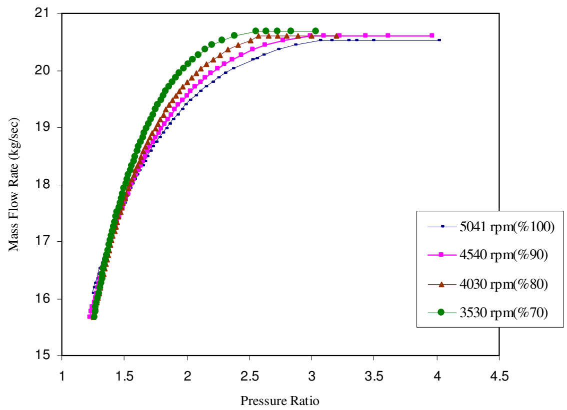

Fig. 5. Mass flow vs. pressure ratio at different rpm.

Table 2. Percent error of pressure ratio toward experimental data in off design points

	point	Loss Model Soderberg	Ainley & Mathieson	Tournier & Genk
percent of error	1	%15.3	%12	%12
	2	%12.9	%11.5	%11.2
	3	%11.2	%9	%8.7
	4	%7.1	%5	%4.6
	5	%7	%2.5	%2.7
	6	%6	%0.5	%1
	7	%15.4	%10	%3
Choking mass flow difference (Kg/Sec)		0.44	0.2	0.15

In Fig. 6, efficiency curves of modeling are introduced for different speeds. These curves are also based on Tournier and Genk's loss model. In each rotational speed, efficiency rises as pressure ratio increases until it reaches its maximum measure. The reason for these changes is that in special cases, the incidence angle and energy losses reach its minimum value consequently, so that in this condition the efficiency will maximize, and after this, the losses will increase again.

A comparison between predicted efficiencies and experimental data in 4030 rpm is shown in Fig. 7. Efficiency prediction by using Soderberg and Tournier Genk loss models has better conformity with the experimental data.

Soderberg's model underestimates the losses. There-fore, the values of predicted efficiency by using this model are higher than the reference data.

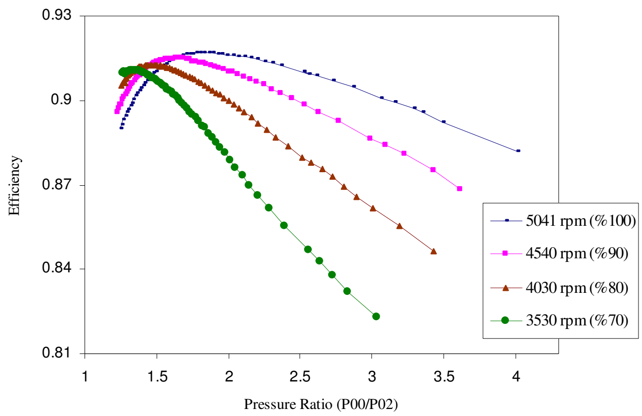

Fig. 6. Turbine efficiency vs. pressure ratio at different rpm.

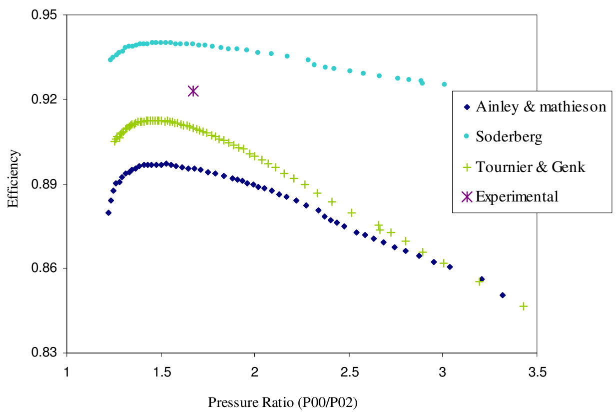

Fig. 7. comparison between predicted efficiencies and experimental data.

In Fig. 8 the losses predicted by using Tournier and Genk's model, over the second stage rotor of turbine and design speed, have been shown. This figure shows five different component of loss coefficients.

The profile loss has greatest value among other loss coefficients. The value of this loss, which is calculated with Eq. (19), gives the lowest value near the pressure ratio 1.75 that this pressure ratio is related to about zero incidence.

In high pressure ratio that relates to high incidence angle and large absolute value of the ratio of flow inlet to outlet angles, which imply the high turning of the blade shape, will easily induce flow separation on the blade surfaces and therefore produce a high off-design profile loss.

Another important loss shown in this figure is the secondary loss. This loss is calculated using Eq. (26). This loss is correlated to the blade loading, which is in terms of flow inlet and outlet angles, as well as the blade aspect ratio. From the predicted results in Fig. 8, the secondary loss rises with the increase of pressure ratio. Since according to the increase in pressure ratio, incidence angle also rises and gets to a positive range, whereupon the difference between flow inlet and outlet angles becomes large, flow finds a high turning and blade loading increases. But the secondary loss increase is not salient among other losses in the investigated turbine geometry.

The secondary, tip clearance and trailing edge loss as almost exhibit a linear behavior. Also this behavior has been reported by Ning Wei [3].

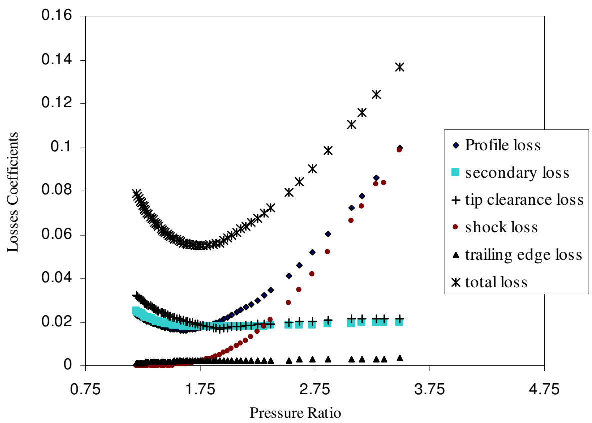

Fig. 8. Loss coefficients vs. pressure ratio.

The shock loss is a component of the profile loss that is calculated by Eq. (20). In this two stage turbine, the loss influence is observed in pressure ratios grater than 1/9. The shock loss occurs due to a local flow acceleration at the highly curved leading edges [5].

7. Conclusions

According to modeling results, it is clear that this modeling and suggested algorithm for solving flow equation, predict the turbine performance acceptably at both the design and off-design conditions. But this results in design point and as it gets near it has a greater accuracy.

Also, it was found that all these loss models give the same trend of overall performance compared with the trend of experimental results of the turbine stages.

The Tournier and Genk's Model gives close results to the reference data because this model estimated the loss coefficients with a greater accuracy, specially, the profile loss coefficient that is the main loss component in the investigated turbine geometry.

Nomenclature

A	cross-sectional flow area (m2)
C	gas absolute velocity component (m/s)
C_L	blades lift coefficient
H	height of blades (m)
h	enthalpy (j)
h_0	stagnation enthalpy (j)
i	incidence angle at blades leading edge (°)
l	actual chord length of blade (m)
M	gas Mach number
$\dot{m}$	mass flow rate (kg/s)
P	pressure (Pa)
R	gas constant
r	radius (m)
Re	Reynolds number
S	pitch or distance between blades in cascade (m)
T	temperature (K)
t	blade thickness (m)
$t_{\max}$	maximum blade thickness (m)
V	gas relative velocity vector with respect to rotor wheel(m/s)
α	flow angle (°)
α'	blade angle relative to meridional plane (°)
α_m	mean velocity vector angle
ϕ	blades stagger angle measured from axial direction (°)
γ	ratio of specific heat capacities
δ^*	boundary layer displacement thickness (m)
ΔS	entropy changes
ρ	density (kg/m3)
τ	blades clearance gap (m)

Subscripts

H	hub of impeller
in	inlet to blade cascade
N	stator
out	outlet of blade cascade
P	profile losses
R	rotor
rel	Relative to rotating blades
s	secondary losses
$shock$	shock waves
T	tip of impeller
TE	trailing edge of blades
Tl	tip leakage of blades
x	axial component

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