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A FSDT model for vibration analysis of Nano rectangular FG plate based on Modified Couple Stress Theory under moving load

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Abstract: In present paper, vibration of Nano FGM plate based on modified couple stress and First Order Shear Deformation Theories (FSDT) under moving load has been developed. Basic equations and linear strains are introduced by first order shear deformation theory and Mori Tanaka's model is used for the plate. The module of elasticity and density are assumed to vary only through thickness of plate. Governing Equations are derived according to the modified couple stress theory and Hamilton's principle. Constitutive equations are also derived based on modified couple stress and finally, analytical solution for simply supported Nano rectangular FG plate is obtained by using of Navier solution. Examples of length scales parameter and power law index are presented to show effect of this parameter on plate behaviors.

Results show that plate's deflection enhances with power law index increasing and by increasing of length scale parameter, deflection decreases, and for frequencies, the deflection with both raising of power law index and length parameter scales, are reduced.

Keywords: Modified couple stress, First Order Shear Deformation Theory, Mori Tanaka's Model, Functionally Graded Materials.

1. Introduction

Functionally Graded Materials (FGMs) are a type of composites which have continuous properties variation from one surface to another, so that they can be used for a specific goal. The main advantage of these materials to conventional composites is elimination of stress concentration which is appeared in laminated composites. FGMs are isotropic and non-homogenous which are a mixture of metal and ceramic. Thanks to smooth and continuous gradient, they have been used for specific stiffness and strength. By increasing of FGMs application accurate models are needed in order to prediction of their response. Shear deformation theories have been employed to take response because of the admissible response to FG plates. HSDT and FSDT have more application. FSDT has privilege to HSDT, owing to less

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complication and also accessible results [1-4]. By increasing of engineering fields and attention to micro/Nano structures, size dependent methods should take into consideration. Classical plate models are according to the classical continuum which could not calculate of size effect, because it has lack of length scale parameter, hence, size dependent models based on continuum length dependent deployed. Among size dependent theories, modified couple stress theory is more advantageous to the others, because it has a length scale parameter [1, 5, 6]. There are many researches done based on theories which are introduced. Van vu et al. investigated simple FSDT based meshfree method for analyses of fg plates[7]. Jooybar et al. researched thermal effect on free vibration of fg truncated conical shell panel[8]. Reddy et al studied nonlinear finite element analysis of FG circular plates with modified couple stress theory[9]. Mirsalehi et al. studied the stability of thin FG micro-plates subjected to mechanical and thermal loading using modified couple stress and spline finite strip method[10]. Lei et al. studied A size dependent FG micro-plates model incorporating higher order shear and normal deformation effects based on a modified couple stress theory[11]. Tai et al. studied the behavior of size dependent FG Sandwich micro beams based on modified couple stress theory[12]. Belabed et al. studied an efficient simple higher order shear and normal deformation theory for FGM plates[13]. Mantari et al. conducted a research concerning bending and free vibration analysis of multi-layer and isotropic plates and shells by new accurate HSDT[14]. Therefore, this article aims to study the Nano vibration of functionally graded rectangular plates under moving load by using FSDT and modified couple stress theories. The study consists of introducing of displacement fields based on FSDT, then strain and curvature function are extracted. Equations of motion are derived based on Hamilton's principle. At the end numerical results are presented and compared with those introduced to validate the accuracy of the model. Fig. A shows the characteristics of the plate.

1. Displacement Fields

In this method, FSDT theory, displacement field is as follows [15]:

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) + z\,\phi_x \\ u_2(x,y,z,t) &= v(x,y,t) + z\,\phi_y \\ u_3(x,y,z,t) &= w(x,y,t) \end{aligned} \tag{1}$$

In Eq. (1), u, v, w, ϕ_x , ϕ_y are the unknown displacement field of the midplane of the plate. ϕ_x , ϕ_y

Are also defined as: $\phi_x = -\frac{\partial \theta}{\partial x}$, $\phi_y = -\frac{\partial \theta}{\partial y}$

Theoretical strains are defined linearly and based on Eq. 1, as shown in Eq. 2[4]:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) \\ \gamma_{xz} &= \phi_x + \frac{\partial w}{\partial x} \\ \gamma_{yz} &= \phi_y + \frac{\partial w}{\partial y} \\ \varepsilon_z &= 0 \end{aligned} \tag{2}$$

2. Curvature function

The curvature function is as Eq. 3 [1]:

$$\chi_{ij} = \frac{1}{2}\left(\frac{\partial\theta_i}{\partial x_j} + \frac{\partial\theta_j}{\partial x_i} \right) \qquad (3)$$
$$i, j = 1, 2, 3$$

In Eq. 3, θ shows rotation vector. Eq. 4 shows rotation vectors according to displacement fields[16]:

$$\theta_1 = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3} \right)$$
$$\theta_2 = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \right) \qquad (4)$$
$$\theta_3 = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \right)$$

Substituting Eq. 1 in Eq. 4, Eq. 5 shows rotation vector according to displacement fields.

$$\theta_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \phi_y \right)$$
$$\theta_y = \frac{1}{2}\left(\phi_x - \frac{\partial w}{\partial x} \right) \qquad (5)$$
$$\theta_z = \frac{1}{2}\left(\left(-\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + z\left(-\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) \right)$$

Substitution of Eq. 5 in Eq. 3, Eq. 6 shows the curvature function with displacement fields:

$$\chi_{xx} = \frac{1}{2}\left(\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial \phi_y}{\partial x} \right)$$
$$\chi_{yy} = \frac{1}{2}\left(\frac{\partial \phi_x}{\partial y} - \frac{\partial^2 w}{\partial x \partial y} \right)$$
$$\chi_{xy} = \frac{1}{4}\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_x}{\partial x} - \frac{\partial \phi_y}{\partial y} \right)$$
$$\chi_{xz} = \frac{1}{4}\left(\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y}\right) + z\left(\frac{\partial^2 \phi_y}{\partial x^2} - \frac{\partial^2 \phi_x}{\partial x \partial y}\right) \right) \qquad (6)$$
$$\chi_{yz} = \frac{1}{4}\left(\left(\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2}\right) + z\left(\frac{\partial^2 \phi_y}{\partial x \partial y} - \frac{\partial^2 \phi_x}{\partial y^2}\right) \right)$$
$$\chi_{zz} = \frac{1}{2}\left(\frac{\partial \phi_y}{\partial x} - \frac{\partial \phi_x}{\partial y} \right)$$

3.Equations of Motion

Based on the principle of Hamilton, the equations of motions are as Eq. 7[16]:

$$\int_0^T (\delta U + \delta W - \delta K)dt = 0 \qquad (7)$$

In which parameters define as: δU : virtual strain energy; δW : virtual work done by external force; and δK : virtual kinetic energy. Virtual work defined as follow[17]:

$$\delta W = \int F\delta(x - x_0(t))\delta(y - y_0(t))\, dAdw = \int F\delta(x - x_0)\delta(y - y_0)\, dAdw \qquad (8)$$

Also, virtual kinetic energy can calculated as[4] :

$$\delta k = \int_A \int_{-h/2}^{h/2} \rho(z)\left(\dot{u}_1\delta\dot{u}_1 + \dot{u}_2\delta\dot{u}_2 + \dot{u}\delta\dot{u}_3\right) dA dz =$$

$$\int_A \left[I_0\left(\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w}\right) + I_1\left(\dot{u}\delta\dot{\varphi}_x + \dot{\varphi}_x\delta\dot{u} + \dot{v}\delta\dot{\varphi}_y + \dot{\varphi}_y\delta\dot{v}\right)\right] \tag{9}$$

$$\left[+ I_2\left(\dot{\varphi}_x\delta\dot{\varphi}_x + \dot{\varphi}_y\delta\dot{\varphi}_y\right)\right] dA$$

Shear strain energy is defined as follows[12]:

$$U = \int_V (\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij})\, dV \tag{10}$$

In Eq.10: σ_{ij} are Cartesian components of the stress tensor, ε_{ij} are the strain components, m_{ij} are the components of deviatoric part of symmetric couple stress tensor and χ_{ij} are the components of the symmetric curvature tensor.

$$\delta U = \int \sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \sigma_{xy}\delta\gamma_{xy} + \sigma_{xz}\delta\gamma_{xz} + \sigma_{yz}\delta\gamma_{yz} + m_x\delta\chi_{xx} m_y\delta\chi_{yy}$$

$$+ m_z\delta\chi_{zz} + 2m_{xy}\delta\chi_{xy} + 2m_{xz}\delta\chi_{xz} + 2m_{yz}\delta\chi_{yz} =$$

$$\delta U = \int \sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij} = \int \sigma_x\delta\left(\frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x}\right) + \sigma_y\delta\left(\frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y}\right) + \sigma_{xy}\delta\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)\right)$$

$$+ \sigma_{xz}\delta\left(\frac{\partial w}{\partial x} + \phi_x\right) + \sigma_{yz}\delta\left(\phi_y + \frac{\partial w}{\partial y}\right) + \frac{1}{2}m_x\delta\left(\frac{\partial^2 w}{\partial y\partial x} - \frac{\partial \phi_y}{\partial x}\right) + \frac{1}{2}m_y\delta\left(\frac{\partial \phi_x}{\partial y} - \frac{\partial^2 w}{\partial x\partial y}\right) +$$

$$\frac{1}{2}m_{xy}\delta\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_x}{\partial x} - \frac{\partial \phi_y}{\partial y}\right) + \frac{1}{2}m_z\left(\frac{\partial \varphi_y}{\partial x} - \frac{\partial \varphi_x}{\partial y}\right) + \frac{1}{2}m_{xz}\delta\left(\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x\partial y}\right) + z\left(\frac{\partial^2 \phi_y}{\partial x^2} - \frac{\partial^2 \phi_x}{\partial x\partial y}\right)\right) +$$

$$\frac{1}{2}m_{yz}\delta\left(\left(\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial^2 u}{\partial y^2}\right) + z\left(\frac{\partial^2 \phi_y}{\partial x\partial y} - \frac{\partial^2 \phi_x}{\partial y^2}\right)\right) dA dz$$

$$\delta U = \int N_x\delta\frac{\partial u}{\partial x} + M_x\delta\frac{\partial \phi_x}{\partial x} + N_y\delta\frac{\partial v}{\partial y} + M_y\delta\frac{\partial \phi_y}{\partial y} + N_{xy}\delta\frac{\partial v}{\partial x} + Q_y\delta\frac{\partial w}{\partial y} \tag{11}$$

$$+ N_{xy}\delta\frac{\partial u}{\partial y} + M_{xy}\left(\frac{\delta\partial\phi_x}{\partial y} + \frac{\delta\partial\phi_y}{\partial x}\right) + Q_x\delta\frac{\partial w}{\partial x} +$$

$$Q_y\delta\partial\phi_y + \frac{P_x}{2}\left(\frac{\partial w\delta w}{\partial x\partial y} - \delta\frac{\partial \phi_y}{\partial x}\right) + \frac{P_z}{2}\left(\frac{\partial\delta\varphi_y}{\partial x} - \frac{\partial\delta\varphi_x}{\partial y}\right)$$

$$+ \frac{P_y}{2}\left(\delta\frac{\partial \phi_x}{\partial y} - \frac{\partial w\delta w}{\partial x\partial y}\right) + \frac{P_{xz}}{2}\left(\frac{\partial v\delta v}{\partial x^2} - \frac{\partial u\delta u}{\partial x\partial y}\right) + Q_x\delta\partial\phi_x$$

$$\frac{P_{xy}}{2}\left(\frac{\partial w\delta w}{\partial y^2} - \frac{\partial w\delta w}{\partial x^2} + \frac{\partial\delta\phi_x}{\partial x} - \frac{\partial\delta\phi_y}{\partial y}\right) + \frac{R_{xz}}{2}\left(\frac{\partial\phi_y\delta\phi_y}{\partial x^2} - \frac{\partial\phi_x\delta\phi_x}{\partial x\partial y}\right)$$

$$+ \frac{R_{yz}}{2}\left(\frac{\partial\phi_y\delta\phi_y}{\partial x\partial y} - \frac{\partial\phi_x\delta\phi_x}{\partial y^2}\right) + \frac{P_{yz}}{2}\left(\frac{\partial v\delta v}{\partial x\partial y} - \frac{\partial u\delta u}{\partial y^2}\right) dA$$

According to the FSDT theory, stress resultants, mass inertia and modified couple stress resultants of rectangular Nano FG plate are defined as follows. In order to simplify Eq. 11, by replacing of Eq. 12-15 into it:

$$I_n = \int_{-h/2}^{h/2} \rho\left(z^n\right)dz \qquad n = 1,2,3 \tag{12}$$

$$N_i, M_i = \sigma_i \int_{-h/2}^{h/2} (1,z)dz, \quad i = x, y, xy \tag{13}$$

$$R_j = \int_{-h/2}^{h/2} zm_j dz,\ Q_j = k\int_{-h/2}^{h/2} \sigma_j dz \quad j = xz, yz \tag{14}$$

$$P_g = \int_{-h/2}^{h/2} m_g dz \quad g = x, y, xy, yz, xz \tag{15}$$

4.Governing Equations of Motion

Final equations of motions are defined by separation and collecting coefficients of displacement fields (δu , δv , δw , $\delta\phi_x$, $\delta\phi_y$) in Eq. 8-9-11 as shown in Eq. 16 to 20:

$$\delta u: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{\partial^2 P_{xz}}{2\partial x\partial y} - \frac{\partial^2 P_{yz}}{2\partial y^2} = I_0\ddot{u} + I_1\ddot{\phi}_x \tag{16}$$

$$\delta v: \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + \frac{\partial^2 P_{xz}}{2\partial x^2} + \frac{\partial^2 P_{yz}}{2\partial x\partial y} = I_0\ddot{v} + I_1\ddot{\varphi}_y \tag{17}$$

$$\delta w: \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial^2 P_x}{2\partial x\partial y} - \frac{\partial^2 P_y}{2\partial x\partial y} + \frac{\partial^2 P_{xy}}{2\partial y^2} - \frac{\partial^2 P_{xy}}{2\partial x^2} + F_{xy} = I_0\ddot{w} \tag{18}$$

$$\delta\phi_x: \frac{\partial M_x}{\partial x} + \partial Q_x + \frac{\partial P_y}{2\partial y} + \frac{\partial P_{xy}}{2\partial x} - \frac{\partial^2 R_{xz}}{2\partial x\partial y} - \frac{\partial^2 R_{yz}}{2\partial y^2} - \frac{\partial P_z}{2\partial y} + \frac{\partial M_{xy}}{\partial y} = I_1\ddot{u} + I_2\ddot{\phi}_x \tag{19}$$

$$\delta\phi_y: \frac{\partial M_y}{\partial y} + \partial Q_y + \frac{\partial P_x}{2\partial x} + \frac{\partial P_z}{2\partial x} - \frac{\partial P_{xy}}{2\partial y} + \frac{\partial^2 R_{xz}}{2\partial x^2} + \frac{\partial^2 R_{yz}}{2\partial x\partial y} + \frac{\partial M_{xy}}{\partial x} = I_1\ddot{v} + I_2\ddot{\varphi}_y \tag{20}$$

5. constitutive equations of Nano FG plates

constitutive equations of Nano FG plates are expressed by Eq. 21[18]:

$$E(z) = E_m + (E_c - E_m)(0.5 + \frac{z}{h})^n$$
$$\rho(z) = \rho_m + (\rho_c - \rho_m)(0.5 + \frac{z}{h})^n \tag{21}$$

In Eq. 21, : E_m metal elasticity modulus; E_c :ceramic elasticity modulus; n : the power; ρ_m, ρ_c: metal and ceramic density, respectively. rectangular FG plate density and elasticity modulus are in accordance with varying thickness. Eq. 22 describes constitutive linear elastic equations for rectangular FG Nano plate[3]:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix}=\frac{E(z)}{1-\upsilon^2}\begin{bmatrix}1&\upsilon&0&0&0\\ \upsilon&1&0&0&0\\ 0&0&s&0&0\\ 0&0&0&s&0\\ 0&0&0&0&s\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix} \tag{22}$$

Where $s=(1-\upsilon)/2$

Modified coupling stress is used to employ small-scale parameter in equations. (l) small-scale parameter is defined from the deviatoric section of modified couple stress (Eq. 23) as [19]:

$$m_{ij}=\frac{l^2E(z)}{1+\upsilon}\chi_{ij} \tag{23}$$

Replacing strain equation (Eq. 2) in linear elastic equation of FGM (Eq. 22), constitutive equations are written as Eq. 24:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix}=\frac{E(z)}{1-\upsilon^2}\begin{bmatrix}1&\upsilon&0&0&0\\ \upsilon&1&0&0&0\\ 0&0&s&0&0\\ 0&0&0&s&0\\ 0&0&0&0&s\end{bmatrix}\begin{bmatrix}\varepsilon_x=\frac{\partial u}{\partial x}+z\frac{\partial \phi_x}{\partial x}\\ \varepsilon_y=\frac{\partial v}{\partial y}+z\frac{\partial \phi_y}{\partial y}\\ \gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+z\left(\frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x}\right)\\ \gamma_{xz}=\phi_x+\frac{\partial w}{\partial x}\\ \gamma_{yz}=\phi_y+\frac{\partial w}{\partial y}\end{bmatrix} \tag{24}$$

Replacing Eq. 6 in Eq. 23, the deviatoric section of modified couple stress is written as Eq. 25.

$$m_{ij}=\frac{l^2E(z)}{1+\upsilon}\begin{bmatrix}\frac{1}{2}\left(\frac{\partial^2 w}{\partial x\,\partial y}-\frac{\partial \phi_y}{\partial x}\right)\\ \frac{1}{2}\left(\frac{\partial \phi_x}{\partial y}-\frac{\partial^2 w}{\partial x\,\partial y}\right)\\ \frac{1}{2}\left(\frac{\partial \phi_y}{\partial x}-\frac{\partial \phi_x}{\partial y}\right)\\ \frac{1}{4}\left(\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 w}{\partial x^2}+\frac{\partial \phi_x}{\partial x}-\frac{\partial \phi_y}{\partial y}\right)\\ \frac{1}{4}\left((\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x\,\partial y})+z(\frac{\partial^2 \phi_y}{\partial x^2}-\frac{\partial^2 \phi_x}{\partial x\,\partial y})\right)\\ \frac{1}{4}\left((\frac{\partial^2 v}{\partial x\,\partial y}-\frac{\partial^2 u}{\partial y^2})+z(\frac{\partial^2 \phi_y}{\partial x\,\partial y}-\frac{\partial^2 \phi_x}{\partial y^2})\right)\end{bmatrix} \tag{25}$$

6.Equations of motion according to displacement

Stress and modified couple stress resultants express as generalize displacement by substuting of Eq.24&25 into Eq. 12 -15:

$$N_x = \int_{-h/2}^{h/2} \sigma_x dz = \frac{E(z)}{1-\upsilon^2}\int_{-h/2}^{h/2}\left(\varepsilon_x + \upsilon\varepsilon_y\right)dz = A\int_{-h/2}^{h/2}\left(\frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x}\right) + \upsilon\left(\frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y}\right)dz$$
$$= A\int_{-h/2}^{h/2}\left(\frac{\partial u}{\partial x} + \upsilon\frac{\partial v}{\partial y}\right) + \int_{-h/2}^{h/2} B\left(\upsilon\frac{\partial \phi_y}{\partial y} + \frac{\partial \phi_x}{\partial x}\right)dz \tag{26}$$

$$N_y = \int_{-h/2}^{h/2} \sigma_y dz = A\int_{-h/2}^{h/2}\left(\varepsilon_y + \upsilon\varepsilon_x\right)dz = A\int_{-h/2}^{h/2}\left(\frac{\partial v}{\partial y} + \upsilon\frac{\partial u}{\partial x}\right)dz + B\int_{-h/2}^{h/2}\left(\frac{\partial \phi_y}{\partial y} + \upsilon\frac{\partial \phi_x}{\partial x}\right)dz \tag{27}$$

$$N_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} dz = A\left(\frac{1-\upsilon}{2}\right)\int_{-h/2}^{h/2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + 2z\frac{\partial^2 w}{\partial x\,\partial y}\right)dz =$$
$$J_1\int_{-h/2}^{h/2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + 2z\frac{\partial^2 w}{\partial x\,\partial y}\right)dz \tag{28}$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x z dz = B\int_{-h/2}^{h/2}\left(\frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x}\right)dz + \upsilon\int_{-h/2}^{h/2}\left(\frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y}\right)dz =$$
$$B\int_{-h/2}^{h/2}\left(\frac{\partial u}{\partial x} + \upsilon\frac{\partial v}{\partial y}\right)dz + C\int_{-h/2}^{h/2}\left(\frac{\partial \phi_x}{\partial x} + \upsilon\frac{\partial \phi_y}{\partial y}\right)dz \tag{29}$$

$$M_y = \int_{-h/2}^{h/2} \sigma_y z dz = B\int_{-h/2}^{h/2}\left(\frac{\partial v}{\partial y} + \upsilon\frac{\partial u}{\partial x}\right)dz + C\int_{-h/2}^{h/2}\left(\frac{\partial \phi_y}{\partial y} + \upsilon\frac{\partial \phi_x}{\partial x}\right)dz \tag{30}$$

$$M_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} z dz = \frac{zE(z)}{2(1+\upsilon)}\int_{-h/2}^{h/2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)dz + \int_{-h/2}^{h/2}\frac{z^2E(z)}{(1+\upsilon)}\frac{\partial^2 w}{\partial x\,\partial y}dz$$
$$= U_1\int_{-h/2}^{h/2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)dz + 2L\int_{-h/2}^{h/2}\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)dz \tag{31}$$

$$Q_x = \int_{-h/2}^{h/2} \sigma_{xz} dz = \frac{E(z)k}{2(1+\upsilon)}\int_{-h/2}^{h/2}\left(\frac{\partial w}{\partial x} + \phi_x\right)dz = O\int_{-h/2}^{h/2}\left(\frac{\partial w}{\partial x} + \phi_x\right)dz \tag{32}$$

$$Q_y = \int_{-h/2}^{h/2} \sigma_{yz} dz = \frac{E(z)k}{2(1+\upsilon)}\int_{-h/2}^{h/2}\left(\frac{\partial w}{\partial y} + \phi_y\right)dz = O\int_{-h/2}^{h/2}\left(\frac{\partial w}{\partial y} + \phi_y\right)dz \tag{33}$$

$$P_x = \int_{-h/2}^{h/2} m_x dz = \frac{l^2E(z)}{2(1+\upsilon)}\int_{-h/2}^{h/2}\left(\frac{\partial^2 w}{\partial x\,\partial y} - \frac{\partial \phi_y}{\partial x}\right)dz = D\int_{-h/2}^{h/2}\left(\frac{\partial^2 w}{\partial x\,\partial y} - \frac{\partial \phi_y}{\partial x}\right)dz \tag{34}$$

$$P_y = \int_{-h/2}^{h/2} m_y dz = D\int_{-h/2}^{h/2}\left(\frac{\partial \phi_x}{\partial y} - \frac{\partial^2 w}{\partial x\,\partial y}\right)dz \tag{35}$$

$$P_{xz} = \frac{D}{2}\int_{-h/2}^{h/2}\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x\,\partial y}\right)dz + \frac{X}{2}\int_{-h/2}^{h/2}\left(\frac{\partial^2 \phi_y}{\partial x^2} - \frac{\partial^2 \phi_x}{\partial x\,\partial y}\right)dz \tag{36}$$

$$P_{yz}=\frac{D}{2}\int_{-h/2}^{h/2}\left(\frac{\partial^2 v}{\partial x\,\partial y}-\frac{\partial^2 u}{\partial y^2}\right)dz+\frac{X}{2}\int_{-h/2}^{h/2}\left(\frac{\partial^2\phi_y}{\partial x\,\partial y}-\frac{\partial^2\phi_x}{\partial y^2}\right)dz \tag{37}$$

$$R_{xz}=\int_{-h/2}^{h/2}zm_{xz}dz=\frac{X}{2}\int_{-h/2}^{h/2}\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x\,\partial y}\right)dz+\frac{F_1}{2}\int_{-h/2}^{h/2}\left(\frac{\partial^2\phi_y}{\partial x^2}-\frac{\partial^2\phi_x}{\partial x\,\partial y}\right)dz \tag{38}$$

$$R_{yz}=\frac{X}{2}\int_{-h/2}^{h/2}\left(\frac{\partial^2 v}{\partial x\,\partial y}-\frac{\partial^2 u}{\partial y^2}\right)dz+\frac{F_1}{2}\int_{-h/2}^{h/2}\left(\frac{\partial^2\phi_y}{\partial x\,\partial y}-\frac{\partial^2\phi_x}{\partial y^2}\right)dz \tag{39}$$

In Eq.26-38, parameters are defined as follow:

$$\begin{aligned}&A=\frac{E(z)}{1-\upsilon^2},B=\frac{E(z)}{1-\upsilon^2},C=\frac{E(z)}{1-\upsilon^2},J_1=\frac{E(z)}{2(1+\upsilon)},X=\frac{l^2E(z)}{2(1+\upsilon)},\\&F_1=\frac{l^2E(z)}{2(1+\upsilon)},U_1=\frac{E(z)}{2(1+\upsilon)},L=\frac{E(z)}{2(1+\upsilon)},O=\frac{E(z)k}{2(1+\upsilon)},D=\frac{l^2E(z)}{2(1+\upsilon)}\end{aligned} \tag{39}$$

7.Governing equations of motion in terms of displacement

equations of motion in terms of displacement are defined by using Eq. 26-38, and, replacing

$$\begin{aligned}&\delta u: A\left(\frac{\partial^2 u}{\partial x^2}+\upsilon\frac{\partial^2 v}{\partial x\,\partial y}\right)+B\left(\frac{\partial^2\phi_x}{\partial x^2}+\upsilon\frac{\partial^2\phi_y}{\partial x\,\partial y}\right)+J_1\left(\frac{\partial^2 v}{\partial x\,\partial y}+\frac{\partial^2 u}{\partial y^2}\right)+2U_1\frac{\partial^3 w}{\partial x\,\partial y^2}\\&-\frac{D}{4}\left(\frac{\partial^4 v}{\partial y\,\partial x^3}-\frac{\partial^4 u}{\partial y^2\partial x^2}\right)-\frac{X}{4}\left(\frac{\partial^4\phi_y}{\partial x^3\partial y}-\frac{\partial^4\phi_x}{\partial y^2\partial x^2}\right)-\frac{D}{4}\left(\frac{\partial^4 v}{\partial x\,\partial y^3}-\frac{\partial^4 u}{\partial y^4}\right)\\&-\frac{X}{4}\left(\frac{\partial^4\phi_y}{\partial x\,\partial y^3}-\frac{\partial^4\phi_x}{\partial y^4}\right)=I_0\ddot{u}+I_1\ddot{\phi}_x\end{aligned} \tag{40}$$

$$\begin{aligned}&\delta v: A\left(\frac{\partial^2 v}{\partial y^2}+\upsilon\frac{\partial^2 u}{\partial x\,\partial y}\right)+B\left(\frac{\partial^2\phi_y}{\partial y^2}+\upsilon\frac{\partial^2\phi_x}{\partial x\,\partial y}\right)+J_1\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial x\,\partial y}\right)+2U_1\frac{\partial^3 w}{\partial x^2\partial y}\\&+\frac{D}{4}\left(\frac{\partial^4 v}{\partial x^4}-\frac{\partial^4 u}{\partial x^3\partial y}\right)+\frac{X}{4}\left(\frac{\partial^4\phi_y}{\partial x^4}-\frac{\partial^4\phi_x}{\partial x^3\partial y}\right)+\frac{D}{4}\left(\frac{\partial^4 v}{\partial x^2\partial y^2}-\frac{\partial^4 u}{\partial x\,\partial y^3}\right)+\\&\frac{X}{4}\left(\frac{\partial^4\phi_y}{\partial x^2\partial y^2}-\frac{\partial^4\phi_x}{\partial x\,\partial y^3}\right)=I_0\ddot{v}+I_1\ddot{\phi}_y\end{aligned} \tag{41}$$

$$\delta w\ :2J_1\left(\frac{\partial\phi_x}{\partial x}+\frac{\partial\phi_y}{\partial y}\right)+\frac{D}{2}\left(\frac{\partial^4 w}{\partial x^2\partial y^2}-\frac{\partial^3\phi_y}{\partial x^2\partial y}\right)-\frac{D}{2}\left(\frac{\partial^3\phi_x}{\partial x\,\partial y^2}-\frac{\partial^4 w}{\partial x^2\partial y^2}\right)$$
$$+\frac{D}{4}\left(\frac{\partial^4 w}{\partial y^4}-\frac{\partial^4 w}{\partial x^2\partial y^2}-\frac{\partial^3\phi_y}{\partial y^3}-\frac{\partial^3\phi_x}{\partial x\,\partial y^2}\right)-\frac{D}{4}\left(\frac{\partial^4 w}{\partial x^2\partial y^2}-\frac{\partial^4 w}{\partial x^4}-\frac{\partial^3\phi_y}{\partial x^2\partial y}-\frac{\partial^3\phi_x}{\partial x^3}\right) \quad (42)$$
$$A\left(\frac{\partial Q_x}{\partial x}+\frac{\partial^2 w}{\partial x^2}+\frac{\partial Q_y}{\partial y}+\frac{\partial^2 w}{\partial y^2}\right)+F_{xy}=I_0\ddot{w}$$

$$\delta\phi_x\ :B\left(\frac{\partial^2 u}{\partial x^2}+\upsilon\frac{\partial^2 v}{\partial x\,\partial y}\right)+C\left(\frac{\partial^2\phi_x}{\partial x^2}+\upsilon\frac{\partial^2\phi_y}{\partial x\,\partial y}\right)+2J_1\left(\partial Q_x+\frac{\partial w}{\partial x}\right)+\frac{D}{2}\left(\frac{\partial^2\phi_y}{\partial y\,\partial x}-\frac{\partial^2\varphi_x}{\partial y^2}\right)+$$
$$\frac{D}{4}\left(\frac{\partial^3 w}{\partial y^2\partial x}-\frac{\partial^3 w}{\partial x^3}-\frac{\partial^2\phi_x}{\partial x^2}-\frac{\partial^2\phi_y}{\partial x\,\partial y}\right)-\frac{X}{4}\left(\frac{\partial^4 v}{\partial x\,\partial y^3}-\frac{\partial^4 u}{\partial y^4}\right)-\frac{F_1}{4}\left(\frac{\partial^4\phi_y}{\partial x\,\partial y^3}-\frac{\partial^4\phi_x}{\partial y^4}\right)- \quad (43)$$
$$\frac{X}{4}\left(\frac{\partial^4 v}{\partial x^3\partial y}-\frac{\partial^4 u}{\partial y^2\partial x^2}\right)+\frac{F_1}{4}\left(\frac{\partial^4\phi_y}{\partial x^3\partial y}-\frac{\partial^4\phi_x}{\partial y^2\partial x^2}\right)+X\left(\frac{\partial^2\varphi_x}{\partial y^2}-\frac{\partial^3 w}{\partial x\,\partial y^2}\right)=I_1\ddot{u}+I_2\ddot{\phi}_x$$

$$\delta\phi_y\ :B\left(\frac{\partial^2 v}{\partial y^2}+\upsilon\frac{\partial^2 u}{\partial x\,\partial y}\right)+C\left(\frac{\partial^2\phi_y}{\partial y^2}+\upsilon\frac{\partial^2\phi_x}{\partial x\,\partial y}\right)+2J_1\left(\partial Q_y+\frac{\partial w}{\partial y}\right)-\frac{D}{2}\left(\frac{\partial^3 w}{\partial x^2\partial y}-\frac{\partial^2\phi_y}{\partial x^2}\right)$$
$$-\frac{D}{2}\left(\frac{\partial^2\varphi_y}{\partial x^2}-\frac{\partial^2\varphi_x}{\partial x\,\partial y}\right)-\frac{D}{4}\left(\frac{\partial^3 w}{\partial y^3}-\frac{\partial^3 w}{\partial x^2\partial y}-\frac{\partial^2\phi_x}{\partial x\,\partial y}-\frac{\partial^2\phi_y}{\partial y^2}\right)+\frac{X}{4}\left(\frac{\partial^4 v}{\partial x^2\partial y^2}-\frac{\partial^4 u}{\partial x\,\partial y^3}\right) \quad (44)$$
$$+\frac{F_1}{4}\left(\frac{\partial^4\phi_y}{\partial x^2\partial y^2}-\frac{\partial^4\phi_x}{\partial y^3\partial x}\right)+\frac{X}{4}\left(\frac{\partial^4 v}{\partial x^4}-\frac{\partial^4 u}{\partial y\,\partial x^3}\right)+\frac{F_1}{4}\left(\frac{\partial^4\phi_y}{\partial x^4}-\frac{\partial^4\phi_x}{\partial y\,\partial x^3}\right)=I_1\ddot{v}+I_2\ddot{\phi}_y$$

8. Dimensionless Formulation

Using the following parameters, equations become dimensionless[11].

$$U_2=\frac{u}{h},\ V=\frac{v}{h},\ W=\frac{w}{h}X=\frac{x}{h},\ Y=\frac{y}{h},\ ,\ \varpi=\frac{\omega a^2}{h}\sqrt{\frac{\rho_c}{E_c}} \quad (45)$$

9.Analytical Solution

Navier solution is for simply supported nano rectangular FG plate employed to solve Nano FGM according to simplified FSDT. Eq. 46 is as follows [16]:

$$U(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}\cos(\alpha X)\sin(\beta Y)e^{i\varpi T}$$
$$V(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}\sin(\alpha X)\cos(\beta Y)e^{i\varpi T} \quad (46)$$
$$W(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin(\alpha X)\sin(\beta Y)e^{i\varpi T}$$

$$\phi_x(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}X_{mn}\cos(\alpha X)\sin(\beta Y)e^{i\varpi T}$$

$$\phi_y(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Y_{mn}\sin(\alpha X)\cos(\beta Y)e^{i\varpi T}$$

In Eq.44, the parameters are as Eq. 47[16]:

$$\alpha=\frac{m\pi}{a},\ i=\sqrt{-1}\ \text{and}\ \beta=\frac{n\pi}{b} \quad (47)$$

ω, U_{mn}, V_{mn}, W_{bmn}, W_{smn} are frequency and deflection field coefficients of Eq. 46 in dual series of analytical solution of the Navier.

Replacing Eq. 46 in dimensionless equations and simplification, matrix form of equations are as Eq. 46:

$$\left(\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15}\\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25}\\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35}\\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45}\\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55}\end{bmatrix}-\omega^2\begin{bmatrix} I_0 & 0 & 0 & I_1 & 0\\ 0 & I_0 & 0 & I_1 & I_1\\ 0 & 0 & I_0 & 0 & 0\\ I_1 & I_1 & 0 & I_2 & 0\\ 0 & I_1 & 0 & 0 & I_2\end{bmatrix}\right)\begin{pmatrix}U_{mn}\\ V_{mn}\\ W_{mn}\\ X_{mn}\\ Y_{mn}\end{pmatrix}=\begin{bmatrix}0\\0\\F_{xy}\\0\\0\end{bmatrix} \quad (48)$$

$$\begin{aligned}
S_{11}&=A_1\alpha^2+J_{11}\beta^2+\frac{D_1}{2}\beta^2(\alpha^2+1)+X_1\beta^2\\
S_{12}&=A_1\upsilon\alpha\beta+J_{11}\beta\alpha+\frac{D_1}{2}\left(\alpha\beta^3+\beta\alpha^3\right)\\
S_{13}&=U_{11}\alpha\beta\\
S_{14}&=B_1\alpha^2+X_1\beta^2\left(\alpha^2+\beta^2\right)\\
S_{15}&=\upsilon\alpha\beta B_1-X_1\alpha^3\beta+X_1\alpha\beta^3\\
S_{22}&=A_1\beta^2+J_{11}A\alpha^2+\frac{D_1}{2}\alpha^2(\alpha+\beta^2)\\
S_{23}&=U_{11}\alpha^2\beta\\
S_{24}&=B_1\upsilon\alpha\beta+X_1\left(\alpha\beta^3+\beta\alpha^3\right)\\
S_{25}&=B_1\beta^2+X_1\left(\alpha^2\beta^2+\alpha^4\right)\\
S_{33}&=2L_1\alpha^2\beta^2+2J_{11}\left(\alpha^2+\beta^2\right)+\frac{D_1}{2}\left(3\alpha^2\beta^2+\beta^3-\alpha^4+\beta^4\right)\\
S_{34}&=2J_{11}\alpha+\frac{D_1}{2}\left(2\alpha\beta^2+\alpha^3\right)\\
S_{35}&=2J_{11}\beta+\frac{D_1}{2}\left(\beta^3+2\alpha^2\beta\right)\\
S_{44}&=B_1\alpha^2+2J_{11}+\frac{D_1}{2}\left(\alpha^2+\beta^2\right)+\frac{X_1}{2}\left(\beta^3+\alpha^2\beta^2\right)+\frac{F_{11}}{2}\beta^2\\
S_{45}&=B_1\upsilon\alpha\beta+D_1\alpha\beta+\frac{F_{11}}{2}\alpha\beta^3+\frac{F_{11}}{2}\alpha^3\beta\\
S_{55}&=C_1\beta+D_1\alpha^2+2J_{11}+\frac{F_{11}}{2}\left(\alpha^2\beta^2+\alpha^4\right)
\end{aligned} \quad (49)$$

Some parameters in E.q (48), also defined as follows:

$$A_1=\int_{-h/2}^{h/2}A\,dz\ ,B_1=zA_1,C_1=z^2A_1,J_{11}=\int_{-h/2}^{h/2}J_1dz\ ,U_{11}=z\ J_{11},L_1=zU_{11},$$
$$O_1=\int_{-h/2}^{h/2}O\,dz\ ,\ D_1=\int_{-h/2}^{h/2}Ddz\ ,X_1=zD_1,F_{11}=zX_1 \tag{50}$$

10.Numerical results

It is assumed that plates with two simply supported ends are as follows. The effects of some parameters such as power-law index, small-scale parameter on deflection and frequency are studied, in order to evaluate the plate, parameters are demonstrated as below[15, 19]:

$$\begin{array}{ll} E_m=70\,\text{Gpa} & \rho_m=2702\,\text{kg/m}^3 \\ E_c=380\text{Gpa} & \rho_c=3800\,\text{kg/m}^3 \\ \vartheta=0.3 & l=.5\,\text{nm} \end{array} \tag{51}$$

Employing Modified Couple Stress, no results have been published for vibration of Nano rectangular FG plate based on FSDT theory under moving load, so as to results compared and evaluated which those are homogeneous cases. First part studied effects of material parameters of moving load on deflection then on frequencies. Figure1 shows the 3D deflection of Nano rectangular fg plate which is imposed of a force at the middle of plate.it is obvious that maximum deflection happened at the middle. Either of fig 2&3 is showing deflection of Nano rectangular fg plate by affecting of power law index and length parameter scales. Fig 2 represents deflection of the plate with constant power law index and different length scale parameter at x_0=x and y_0=b/2. It can seen from figure which, by increasing of length scale parameter deflection decreased. The reason of decreasing is, because with increasing of mentioned plate's factor flexibility diminished so that the plate's deflection decreased. Fig 4 shows deflection of the plate's with constant length scale parameter and various power law indexes. Despite of fig 2, increasing of power law index this factor caused increasing of deflection. Fig 3 just showed to comparing with fig 2(a>b) which is alongside of y direction and presented that with same condition, deflection of fig 3 is slightly less than of fig 2 because of less length. Fig 5 shows deflection of the plate with constant power law index and various length parameter scales during the time. It is noticeable that assumed load moves along x direction over the centerline of FG plate by a constant velocity $v_0\left(x=v_0t\right)$, also traveling time of the moving load defined as $t_f=\frac{a}{v_0}$, and k=5/6, so the T which is showed represented as $T=\frac{t}{t_f}$ [17]. by increasing of length parameter and presuming constant power law index deflection dropped because the flexibility diminish, and the other side, with constant length parameter scales and raising of power law index deflection increased through out of the time as showed in the fig 6. The reason is why, because metal purity increased than to ceramic so as to bending stiffness of elasticity module dropped which is led to more flexibility and eventually plate's deflection increased. Figure 7 and 8 investigated frequency of the plate with different length scale parameter and various power law index and vice versa, respectively. As it can seen from the picture both figure shows the same patterns, i.e. by raising of each of factor frequencies decreased this might be because of small width of plate's in nano structure which is so significant and present such a trend.

Conclusion

Vibration of Nano rectangular fg plate based on modified couple stress and FSDT theories under moving load is investigated. According to FSDT theory strain derived, curvature function introduced based on FSDT theory. Hamilton's principal and modified couple stress are used to extract the equation and finally equation solved with Navier solution for simply supported plate. With different power law index and length parameter scales results are evaluated. The results shows by increasing of power law index deflection diminished because of increasing of metal purity than to ceramic and with raising of length parameter scales results showed inversed pattern because flexibility declined, but by increasing of both factor frequencies dropped because of its significant effect on Nano structures.

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