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Stepped Spillway Performance: Study of the Pressure and Turbulent Kinetic Energy versus Discharge and Slope

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ABSTRACT

In the present study, flow over flat and pooled stepped spillways was simulated three dimensionally. The VOF method and k- ε (RNG) turbulent model was used to simulate the flow over the stepped spillway. The results indicated that the numerical model was acceptably capable of simulating the flow over the pooled and flat stepped spillways for different discharges. Following that, the effect of different discharges and slopes on the flow pattern over both stepped spillways was examined. Comparison of the results indicated that the pooled stepped spillway performed better in 10, 15, and 20-degree slopes for different discharges in comparison with the flat stepped spillway regarding decreasing the residual head at the end of the spillway. The results of turbulent kinetic energy showed that the maximum turbulent kinetic energy was greater in the pooled stepped spillway in comparison with the flat stepped spillway. Negative pressure was not formed near the steps' bed in the pooled and flat stepped spillways. However, adjacent to the vertical face, negative pressure was formed in a larger area in the flat spillway in comparison with the pooled spillway, which increases the probability of cavitation phenomenon in the flat spillway.

Keywords

Numerical model, Stepped spillway, Energy dissipation, Turbulence kinetic energy, Pressure

1. Introduction

Water flow passing over a rough or stepped surface is very turbulent. Therefore, this turbulent flow can dissipate a major part of its energy through passing over the steps (Chamani and Rajaratnam, 1999; Chanson, 2001). Numerous researches have investigated and are interested in stepped spillways since they are fundamentally used in reducing the kinetic energy of the flow passing over the spillway. This decreases the dimensions of the stilling basin (Christodoulou, 1993, Chinnarasri and Wongwises, 2006) in addition to being adaptable to roller-compacted concrete (Boes and Hager, 2003). Moreover, the stepped spillways can reduce the expenses of constructing spillways from the economic aspect (Rajaratnam, 1990). The pattern of the flow passing over a stepped spillway was divided into nappe and skimming flow regimes by Chanson (1994) and Sorensen (1985). Chanson (2001) introduced the transition flow regime, which is a transient case between the nappe and skimming flow regimes. He also demonstrated that the flow regime over a stepped spillway depends on the geometry of the step and the flow discharge. Many researches have used stepped spillways with different configurations in the past few decades to examine the energy dissipation and other hydraulic parameters. Previous researches have shown that the geometry of the stepped spillway

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seems to have an essential role in the energy dissipation. Chinnarasri and Wongwises (2006) examined the stepped spillways considering flat, pooled and inclined steps with 30, 45, and 60-degree slopes. Their results showed that the relative energy dissipation was affected by the geometry of the steps. In addition to that, for a specific discharge with a constant step height, relative energy dissipation increased by the number of steps. Felder and Chanson (2011) examined energy dissipation on the stepped spillways considering 5 types of configurations with uniform and non-uniform step heights along the spillway with a constant slope (θ= 26.6°). The results indicated that the energy dissipation of 5 different cases were slightly different. Felder et al. (2012 B) examined the flat and pooled stepped spillways with different discharges and a constant 8.9-degree slope. Their results showed that the relative energy dissipation in the stepped spillway, which was a combination of the flat and pooled steps, was greater in comparison with the pooled and flat stepped spillways. Felder and Chanson (2013) compared the flat and pooled stepped spillways with two slopes of 8.9 and 26.6 degrees. The results indicated that the maximum relative energy dissipation occurred in the pooled stepped spillway with 8.9-degree slope. Guenther et al. (2013) studied a stepped spillway with 4 types of configuration and a slope of 26.6-degree. Their results indicated that the residual head in the pooled and flat stepped spillway was smaller in comparison with the pooled stepped spillway with staggered configuration of the flat and pooled steps and the pooled stepped spillway with in-line configuration of the flat and pooled steps. Other researches studied stepped spillways with different slopes including Kökpinar (2004) (30-degree slope), Gonzalez (2005) (15.94 and 21.8-degree slopes) and Thorwarth (2008) (14.6-degree slope).

With regard to the fact that numerical models accurately simulate the flow pattern over different hydraulic structures, they have turned into an efficient method in hydraulic engineering. Using numerical models will significantly reduce the expenses in comparison to the experimental models. In addition, more data could be obtained from different parameters. There are different methods in numerical simulation that can be used for simulating the flow patterns and the flow turbulence. Cheng et al. (2006) examined the k-ε (RNG) turbulence model and the mixture method for the numerical simulation of a two-phase flow over a stepped spillway. They presented the vortex flow formed on the steps, the velocity, and pressure in their results. Dong and Hun-wei (2006) used the VOF method to simulate the skimming flow over a stepped spillway with a 10-degree slope. The presults included velocity, air concentration and pressure. Their results showed that the VOF model can precisely simulate the flow pattern over the stepped spillway. Qian et al. (2009) examined the accuracy of modeling the flow over a stepped spillway through examining different turbulence models. They came to the conclusion that the result of the k-ε (RNG) turbulence model was more accurate than other turbulence models. Bombardelli et al. (2011) used the VOF method and k- ε (RNG) turbulence model to simulate the flow over a stepped spillway with a 53-degree slope. They examined the results of the turbulent kinetic energy, velocity, and flow pattern. Zhenwei et al. (2012) obtained the hydraulic parameters of the flow including water surface profile, pressure, and velocity through 3D simulation of the flow over the stepped spillway. Their results were based on the VOF method and were fairly consistent with the experimental data. Nikseresht et al. (2013) simulated a two-phase flow over a stepped spillway with 18.8 and 28-degree slopes. They examined stepped spillways with flat and inclined steps in their research. Attarian et al. (2014) simulated a stepped spillway with a 14.036-degree slope using the *k*-*ε* turbulence model and examined the effect of the step height. Their simulation was verified by comparing the velocity, energy dissipation, and the aeration starting point results. Effects of the pool configuration of the pooled stepped spillway with a slope of 2.6-degree were presented by Morovati et al. (2015). They presented the effects of the pools on the vortex flows, standing sidewall-waves, transverse free surface profile and velocity counters in transverse direction by using *k*- *ε* (RNG) turbulence model and VOF method. Other numerical researches were conducted on the stepped spillways including Tabbara et al. (2005) and Sarfaraz and Attari (2011).

The mentioned experimental researches indicate that the spillway geometry and the discharge passing over a stepped spillway have a significant effect on the hydraulic parameters. In addition, the previously-mentioned numerical models showed that numerical models simulate the flow pattern over the stepped spillway precisely. The effect of different slopes and discharges was not examined in the previous studies. Since the spillway slope is amongst the crucial geometrical parameters on the energy dissipation and other hydraulic properties of the flow, the present research studied the flow characteristics over the flat and pooled stepped spillways with different slopes through using a commercial software. Following that, the effect of discharge was studied.

2. Materials and Methods

The numerical results were verified based on Felder (2012A) experimental data. The pooled and flat stepped spillway used in the experimental model was made of 10 steps and the height (h) and length (l) of each step was 10 and 20 cm, respectively. The height of the step pool (w) was 3.1 cm in all steps of the pooled stepped spillway. Other geometrical properties of the pooled stepped spillway are shown in Fig 1.

2.1. Meshing and Boundary Conditions

Meshing the pooled and flat stepped spillways was in accordance with Fig. 2 using non-uniform sized cells. Smaller grid cells were used near the surface of all the steps. Meshing continued 170 cm along the downstream at the spillway in order to prevent the output boundary numerical results around the spillway. The meshing was considered 50 cm higher than the spillway crest in the vertical direction as well.

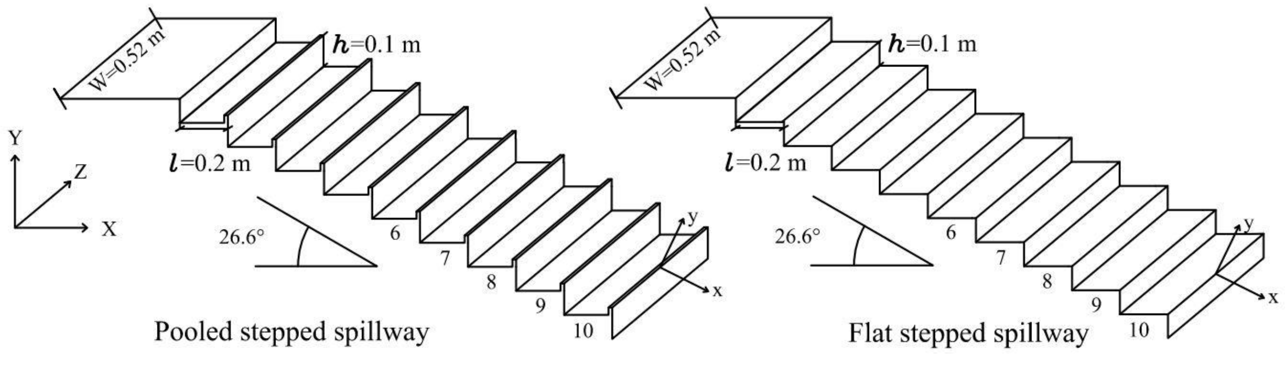

Fig. 1. Schematic view of the pooled and flat stepped spillways

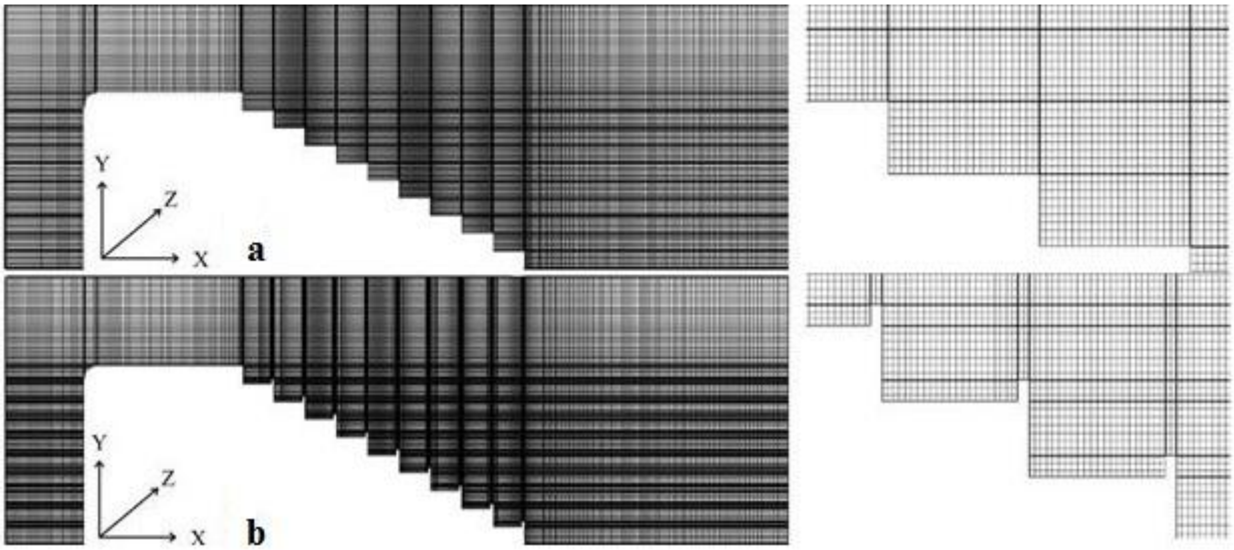

Fig. 2. Meshing pattern of the a) flat and b) pooled stepped spillways

Table 1. Number of cells in longitudinal direction

Spillway slope (degree)	10	15	20	26.6	30	35
Cell number	664	520	456	406	381	360

According to the fact that the length of the spillways with different slopes were different, the number of cells in the longitudinal direction was not the same. Table 1 shows the number of cells along the longitudinal direction, *X*, for different slopes. Dimensions of the spillways were the same in the vertical, *Y*, and the spanwise, *Z*, directions and the numbers of cells were 121 and 42, respectively.

In the experimental research, specific discharge (*Q*) was defined for the input boundary and thus, it was used in the present paper. Furthermore, the experimental results had just been presented above steps and we couldn't choose other boundary conditions including specific pressure or specific velocity for the output. Hence, outflow boundary condition was used at the output. Wall boundary condition was used for the channel bed and walls and the symmetry boundary condition was used at the upper boundary. All the results were presented after reaching to the final steady condition of the flow.

2.2. Governing Equations

In the present study, 3D simulation over the flat and pooled stepped spillway with different slopes and discharges was done by using FLOW-3D numerical code. The governing equations were discretized by the Finite-volume method on a Cartesian grid system. The continuity and Navier-Stokes equations were applied to simulate incompressible flow over the stepped spillway. The continuity equation defined as below:

$$V_F \frac{\partial(\rho)}{\partial t} + \frac{\partial}{\partial x}(uA_x) + \frac{\partial}{\partial y}(vA_y) + \frac{\partial}{\partial z}(wA_z) = \frac{R_{SOR}}{\rho} \quad (1)$$

$$\frac{\partial u}{\partial t} + \frac{1}{V_F}\left(uA_x \frac{\partial u}{\partial x} + vA_y \frac{\partial u}{\partial y} + wA_z \frac{\partial u}{\partial z}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + G_x + f_x \quad (2)$$

$$\frac{\partial v}{\partial t} + \frac{1}{V_F}\left(uA_x \frac{\partial v}{\partial x} + vA_y \frac{\partial v}{\partial y} + wA_z \frac{\partial v}{\partial z}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial y} + G_y + f_y \quad (3)$$

$$\frac{\partial w}{\partial t}+\frac{1}{V_F}\left(uA_x\frac{\partial w}{\partial x}+vA_y\frac{\partial w}{\partial y}+wA_z\frac{\partial w}{\partial z}\right)=-\frac{1}{\rho}\frac{\partial p}{\partial z}+G_z+f_z \quad (4)$$

where V_F is volume ratio of the flow, ρ and R_{SOR} are fluid density and flow resource, respectively. In addition, *u, v, w* are the flow velocity components in *x, y, z* directions, respectively. G_x, G_y, G_Z are body acceleration and f_x, f_y, f_Z are the accelerations caused by viscosity in *x, y, z* directions, respectively.

VOF method introduced by (Hirt and Nichols, 1981) was used in the simulation. In this method, the advection equation of fluid fraction is as below:

$$\frac{\partial F}{\partial t}+\frac{1}{V_F}\left(\frac{\partial}{\partial x}(FuA_x)+\frac{\partial}{\partial y}(FvA_y)+\frac{\partial}{\partial z}(FwA_z)\right)=0.0 \quad (5)$$

where *F* is the volume occupied by air in each computational cell and changes in the range of 0 to 1. If *F* is zero, there is only water in that cell. In addition, in the cells that are only occupied by the air, *F* is 1. When F changes between 0 and 1, part of the computational cell is filled with air and another part is filled with water.

3. Results and Discussion

The discharges presented in Table 2 were used to verify the numerical model. In this table, *dc* is the critical depth and *h* is the step height.

Velocity distribution was presented in this research in terms of the dimensionless functions V/V_c and $(y+w)/dc$. For the pooled stepped spillway and V/V_c and y/dc for the flat stepped spillway. *w* is the step pool height which is equal to 3.1 cm in the pooled stepped spillway and is equal to zero in the flat stepped spillway. In addition, *y* is the flow depth from the step pool to the flow surface in the vertical direction to the spillway slope. *V* is the velocity and *Vc* is the critical velocity of the flow, which is obtained as follows (Felder et al. 2012 A).

$$V_c=(g\times dc)^{1/2} \quad (6)$$

Figures 3 and 4 show a comparison of the velocity over the flat and pooled stepped spillways between the numerical and experimental models for 0.09 m^3/s discharge. It could be seen in these figures that the numerical results were fairly consistent with the experimental data. The greatest consistency can be seen near the middle of the flow depth in the flat stepped spillway. However, the greatest consistency was obtained in the flow surface in the pooled stepped spillway.

Table 2. Experimental discharges used for the numerical model verification (Felder et al. (2012A))

Configuration	Q (m^3/s)	*dc /h*
Pooled stepped spillway	0.075	1.29
And	0.09	1.45
Flat stepped spillway	0.113	1.7

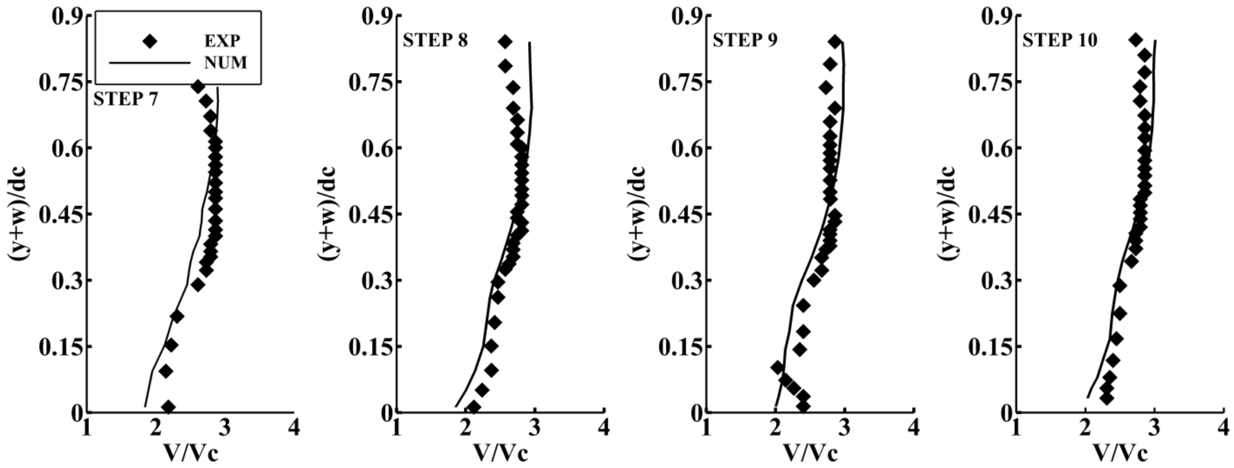

Fig. 3. Velocity distribution on steps 7, 8, 9, and 10 for Q= 0.09 m^3/s in the flat stepped spillway

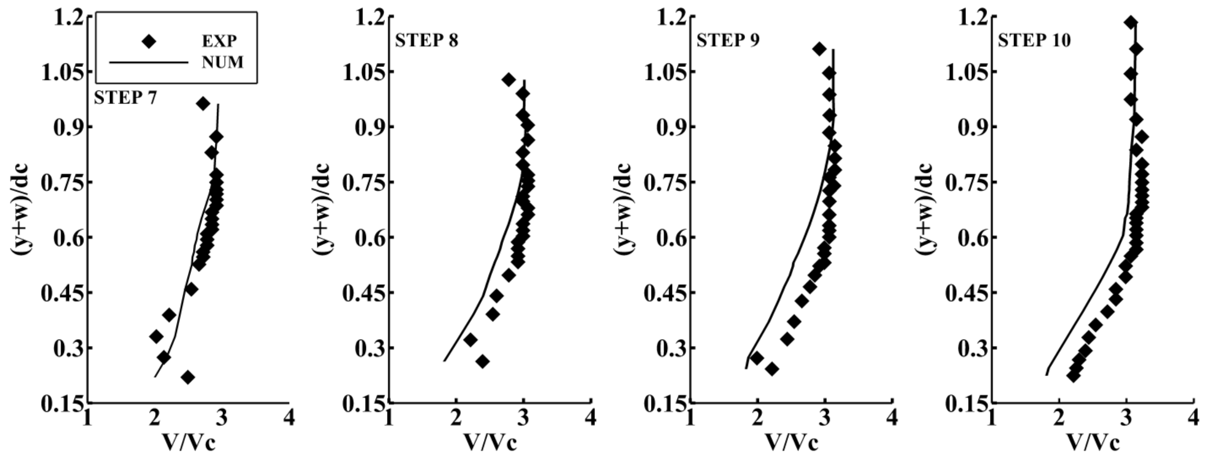

Fig. 4. Velocity distribution on steps 7, 8, 9, and 10 for Q= 0.09 m^3/s in the pooled stepped spillway

Table 3 shows the mean relative error percentage in both velocity and residual head for different discharges in the flat and pooled stepped spillways. The values presented in this table indicate that the numerical results were fairly consistent with the experimental results. In addition, the numerical model was able to simulate the flow patterns with different discharges with an acceptable level of accuracy.

With regard to the previously mentioned issues, it can be seen that the numerical model desirably simulated the flow pattern over the flat and pooled stepped spillways. Therefore, the performance of the flat and pooled stepped spillways with different slopes and discharges will be examined here after. The slopes used for the flat and pooled stepped spillways were the same. Figure 5 shows the schematic view of the pooled stepped spillway. The step pool height, the width, and the height of the steps were remained the same in all the cases.

Table 3. Mean relative error percentage of the flat stepped spillway and the pooled stepped spillway with different discharges

Spillway type	Q (m^3/s)	d_c/h	STEP 6	STEP 7	STEP 8	STEP 9	STEP 10	Residual head errors
pooled stepped spillway	0.075	1.29	-	6.7	7.16	11.87	10.4	4.89
	0.09	1.45	-	5.66	6.38	8.4	9.37	5.09
	0.113	1.7	-	-	8.18	9.16	9.38	6.36
Flat stepped spillway	0.075	1.29	6.38	6.31	7.09	7.59	7.7	7.86
	0.09	1.45	-	6.38	5.2	6.62	4.33	7.54
	0.113	1.7	-	-	6.2	5.01	7.83	8.06

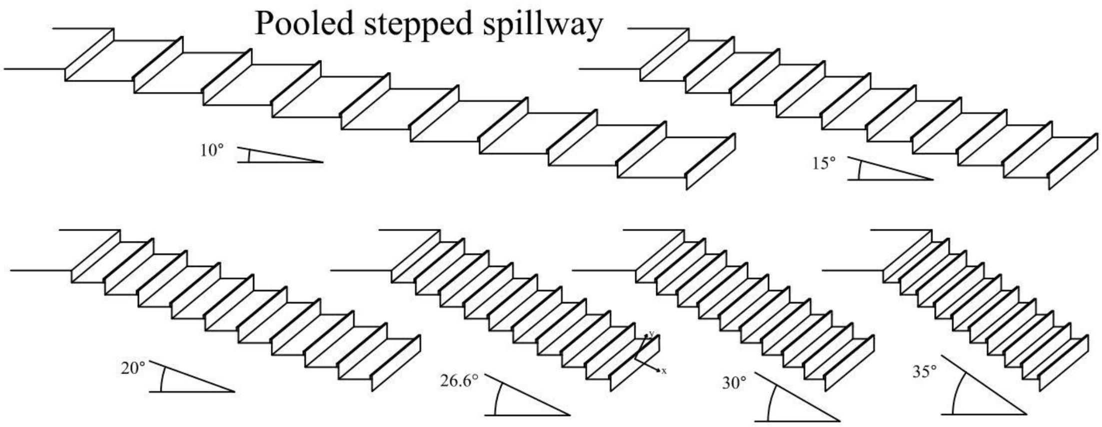

Fig. 5. Schematic view of the pooled stepped spillways with different slopes

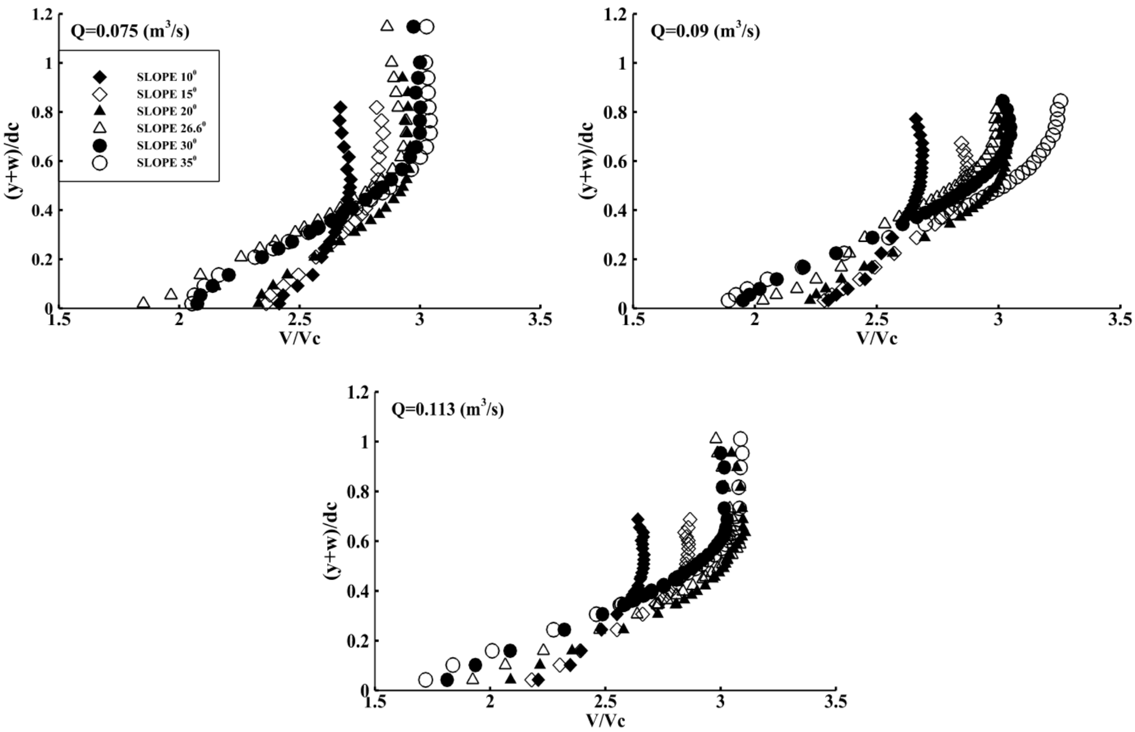

Fig. 6. Velocity distribution for different discharges in the flat stepped spillway

3.1. Flow velocity

Velocity distribution along the slope on the flat stepped spillway for different discharges is shown in Fig. 6. For all three discharges, it can be seen that the minimum and maximum velocities at the flow surface are obtained for the stepped spillways with 10 and a 35-degree slopes, respectively. In addition, near the step bed, the minimum velocity for 0.075 m^3/s discharge was obtained in the stepped spillway with a 26.6-degree slope and for 0.09 m^3/s and 0.113 m^3/s discharges, it was obtained in the stepped spillway with a 35-degree slope.

Figure 7 shows the velocity distribution for different discharges on the pooled stepped spillway. It can be seen from this figure that the minimum velocity was obtained in many points of the flow depth in the pooled stepped spillway with a 10-degree slope. For the 0.075 m^3/s discharge, the maximum velocity occurred near the flow surface on the stepped spillway with a 35-degree slope and the minimum velocity occurred near the step bed on the stepped spillway with a 20-degree slope. For the 0.09 m^3/s discharge, the maximum velocity obtained near the flow surface on the stepped spillway with a 30-degree slope and the minimum velocity obtained near the step bed in the stepped spillway with a 20-degree slope. For the 0.113 m^3/s discharge, in the pooled stepped spillway with a 35-degree slope, the maximum and minimum velocities occurred at the flow surface and at the step bed, respectively.

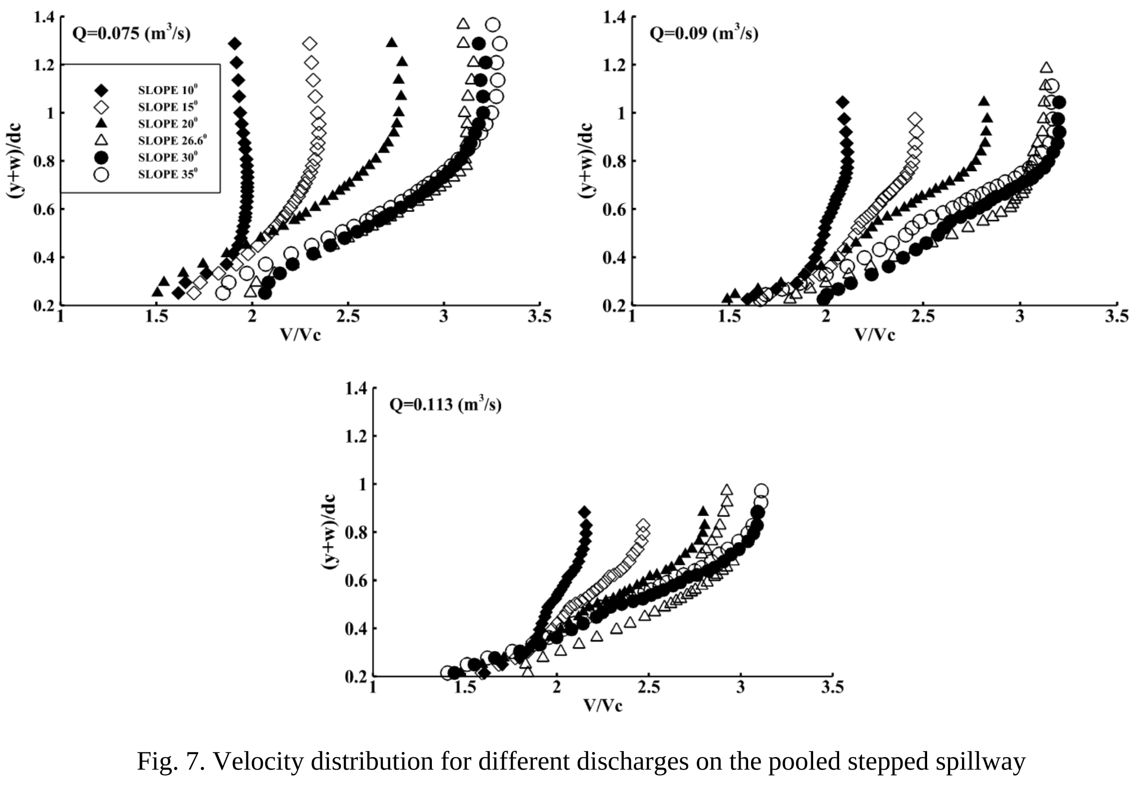

Fig. 7. Velocity distribution for different discharges on the pooled stepped spillway

3.2. Residual head at the end of the spillways

Figure 8 shows the residual head at the end of the spillway. By comparing the results of the residual head on the last step of the pooled and flat stepped spillways it can be concluded that the residual head reduced more in the pooled stepped spillway in comparison with the flat stepped spillway for all the three discharges and 10, 15, and 20-degree slopes. The change trends were, however, different for 26.6, 30, and 35-degree slopes.

The residual head reduced more in the flat stepped spillway with a 26.6-degree slope, in comparison with the pooled stepped spillway under all three discharges. For the 0.075 m^3/s and 0.09 m^3/s discharges, the flat stepped spillway with a 30-degree slope reduced the residual head more in comparison with the pooled stepped spillway and the results were very close to each other for 0.113 m^3/s discharge. In the 35-degree slope for 0.075 m^3/s discharge, the residual head was less in the flat stepped spillway and they were almost equal for the other discharges.

3.3. Turbulence kinetic energy

Velocity gradients on the steps of the stepped spillways create turbulence kinetic energy (Bombardelli et al., 2011). The turbulence kinetic energy over the flat stepped spillway is shown in Fig. 9. According to the fact that the kinetic energy changing trend is similar in all the three discharges, only the figures related to 0.0113 m^3/s discharge is displayed in Fig. 9. It can be seen from this figure that as the spillway slope increased up to 26.6-degree, the turbulence kinetic energy decreased over the spillway. However, the turbulence kinetic energy slightly increased for larger slopes in comparison to the 26.6-degree slope. The maximum turbulence kinetic energy on each step almost occurred in its middle.

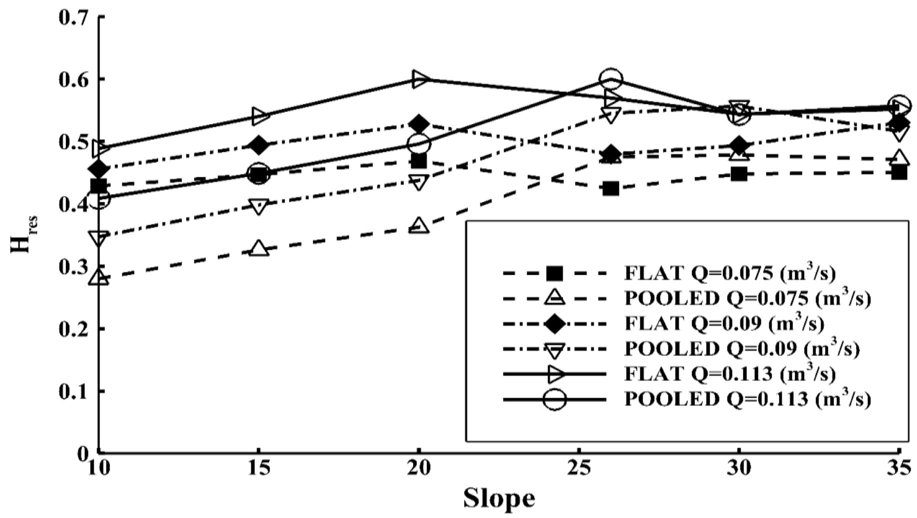

Fig. 8. Residual head at the end of the spillway with different discharges and slopes in the flat and pooled stepped spillways

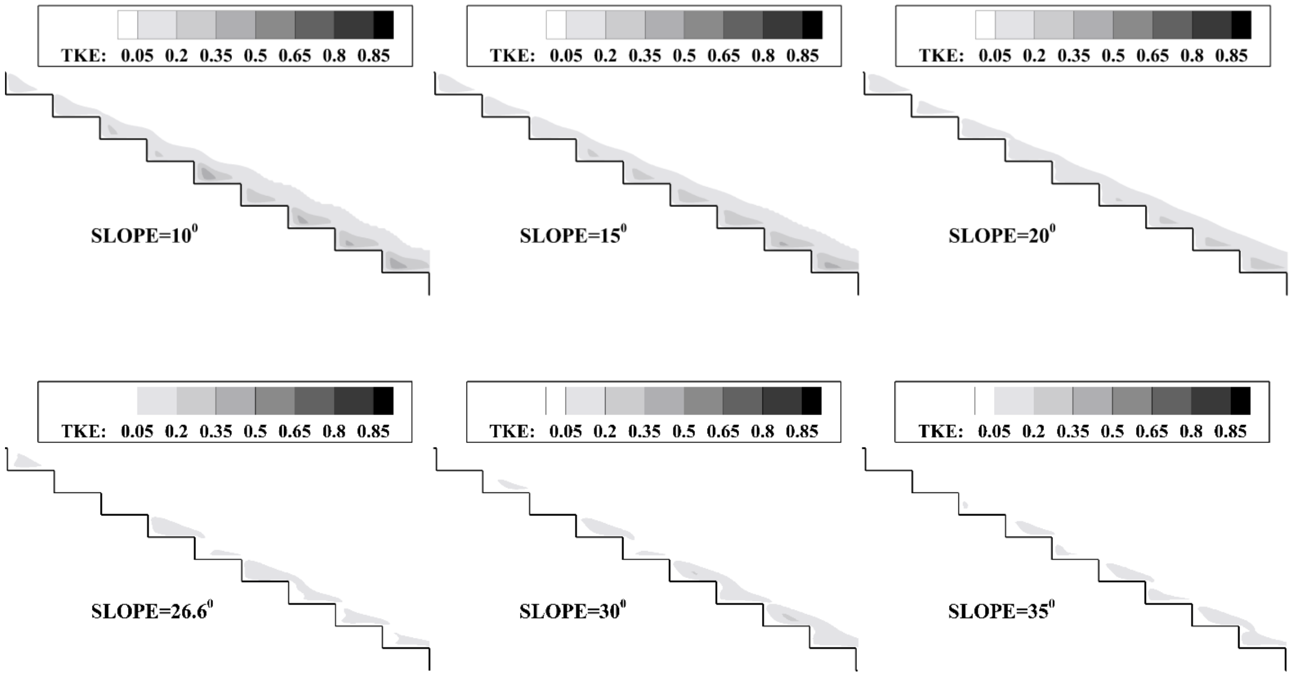

Fig. 9. Turbulence kinetic energy (in m^2/s^2) over the flat stepped spillways for Q= 0.113 m^3/s

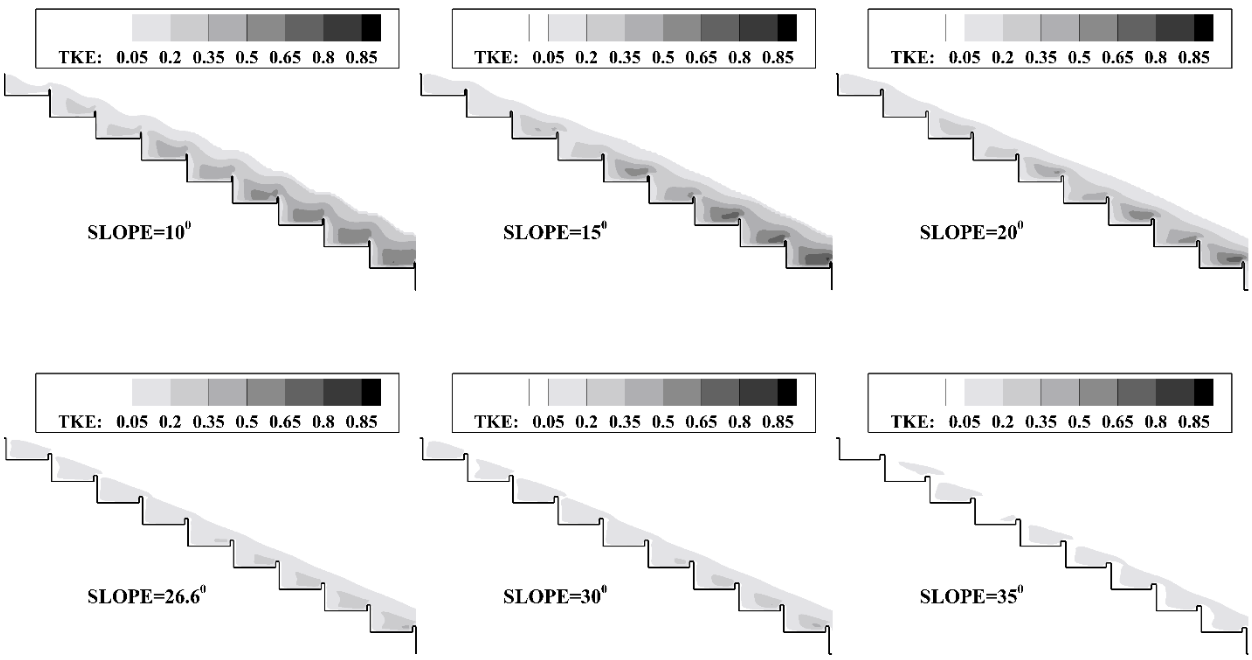

Fig. 10. Turbulent kinetic energy (in m^2/s^2) over the pooled stepped spillway for Q= 0.113 m^3/s

The turbulence kinetic energy of the pooled stepped spillway for 0.113 m^3/s discharge is shown in Fig. 10. For the slopes greater than 15-degree, the turbulence kinetic energy decreased by the slope. In contrast to the flat case, the maximum turbulence kinetic energy occurred near the step pool.

The maximum turbulence kinetic energy over the flat stepped spillway for different discharges and slopes is shown in Fig. 11. The turbulence kinetic energy increased in all the slopes by discharge. It can be seen from Fig.11 that by increasing the slope up to 26.6-degree, the turbulence kinetic energy decreased for all the three discharges. However, this trend changed after that and the turbulence kinetic energy slightly increased.

The maximum turbulence kinetic energy for different discharges and slopes over the pooled stepped spillway is shown in Fig. 12. It can be seen from this figure that for slopes smaller than 15-degree, the turbulence kinetic energy increased by the slope and for the slopes greater than 15-degree, this value decreased by the slope. The maximum reduction rate occurred in the area between 20 and 26.6-degree slopes.

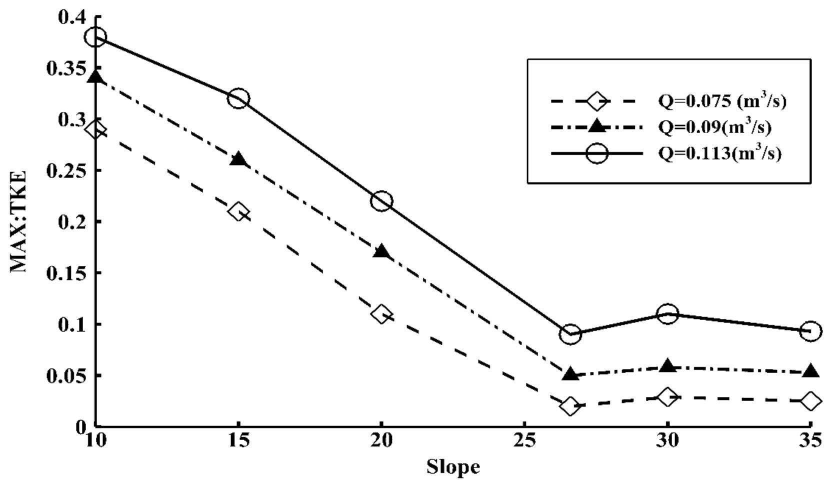

Fig. 11. Maximum turbulence kinetic energy over the flat stepped spillway for different discharges and slopes

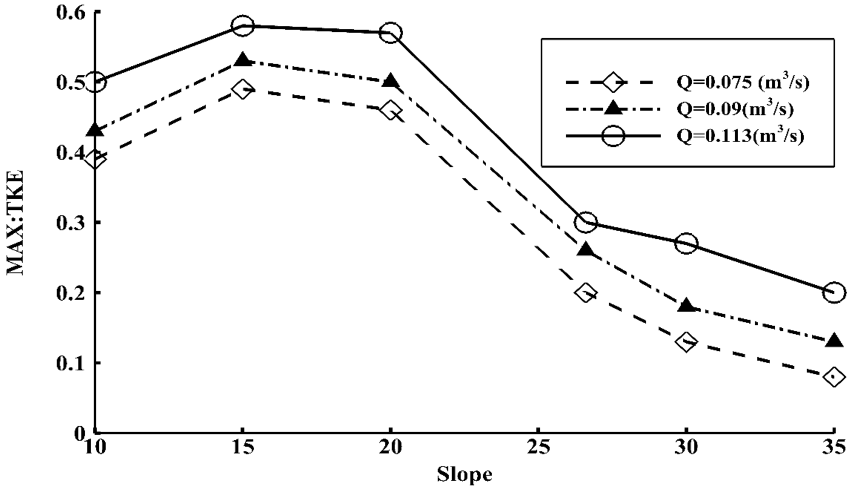

Fig. 12. Maximum turbulence kinetic energy over the pooled stepped spillway for different discharges and slopes

By comparing the obtained values for the maximum turbulence kinetic energy in the flat and pooled stepped spillways, it can be stated that the value and also the location of the maximum turbulence kinetic energy on each steps was different in both types of the spillways. The obtained values indicated that the higher velocity gradients occurred in the most parts of the pooled stepped spillway. In addition, the maximum turbulence kinetic energy occurred on the last step in most cases.

3.4. The pressure

Stepped spillways are less prone to cavitation in comparison with the smooth spillways (Frizell et al., 2012). According to Cheng et al. (2006), cavitation most likely occurs in points with negative pressures. Pressure near the step bed in the pooled and flat stepped spillways with different slopes and discharges are shown in Fig. 13. The results are only presented for step 10 taking into account the fact that the pressure changes trend is almost similar over the last steps. With regard to Fig.13, it can be stated that the pressure value at the beginning of the step in the pooled stepped spillways was larger in comparison with the flat stepped spillway. In all the cases in the pooled stepped spillway, the maximum pressure obtained at the end of the step (near the step pool). However, in the flat case, the place where the maximum pressure occurred moved from the point around the step length center towards the end of the step as the slope increased. Negative pressure occurred in a tiny range of the step length in the 15-degree slope for 0.09 m^3/s discharge over the flat stepped spillway. However, negative pressure did occur near the step bed for any other cases in the flat and pooled stepped spillways.

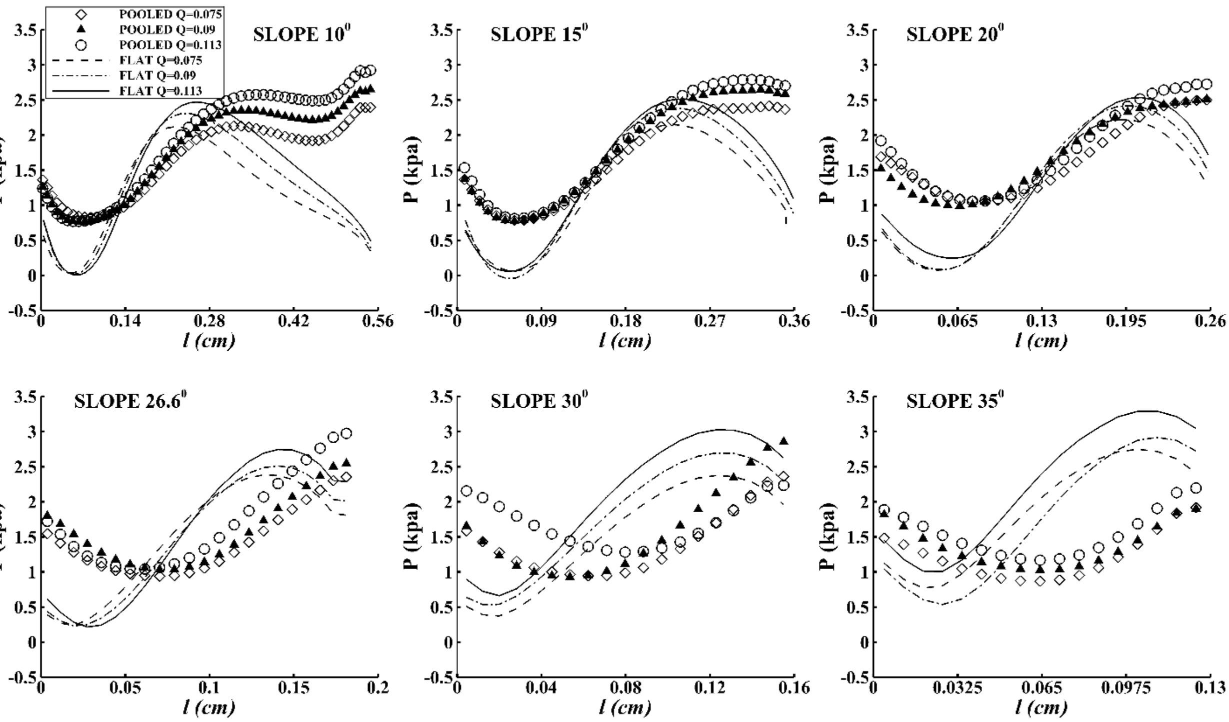

Fig. 13. Pressure distribution near the last step bed for different slopes and discharges

Figure 14 shows pressure distribution adjacent the vertical face of step 9. It can be seen from this figure that for the flat stepped spillway, negative pressure occurred in most parts of the depth. In the flat cases for 0.113 m^3/s discharge and a 20-degree slope, less negative pressure created in comparison with the rest of the other cases. In the pooled stepped spillway with 10, 15, and 20-degree slopes, negative pressure occurred in a few points located near the step pool. In the pooled case, it can also be seen that as the slope increased, pressure increased near the step pool.

The pressure distributions in Figs. 13 and 14 indicated that the negative pressure do not occur near the step bed, while the negative pressure can be seen adjacent to the vertical face of the step. Therefore, cavitation phenomenon is more likely to occur on the vertical face of the step. Moreover, cavitation phenomenon less likely occurs in the pooled case rather than in the flat state since smaller area near the step surface experiences negative pressure.

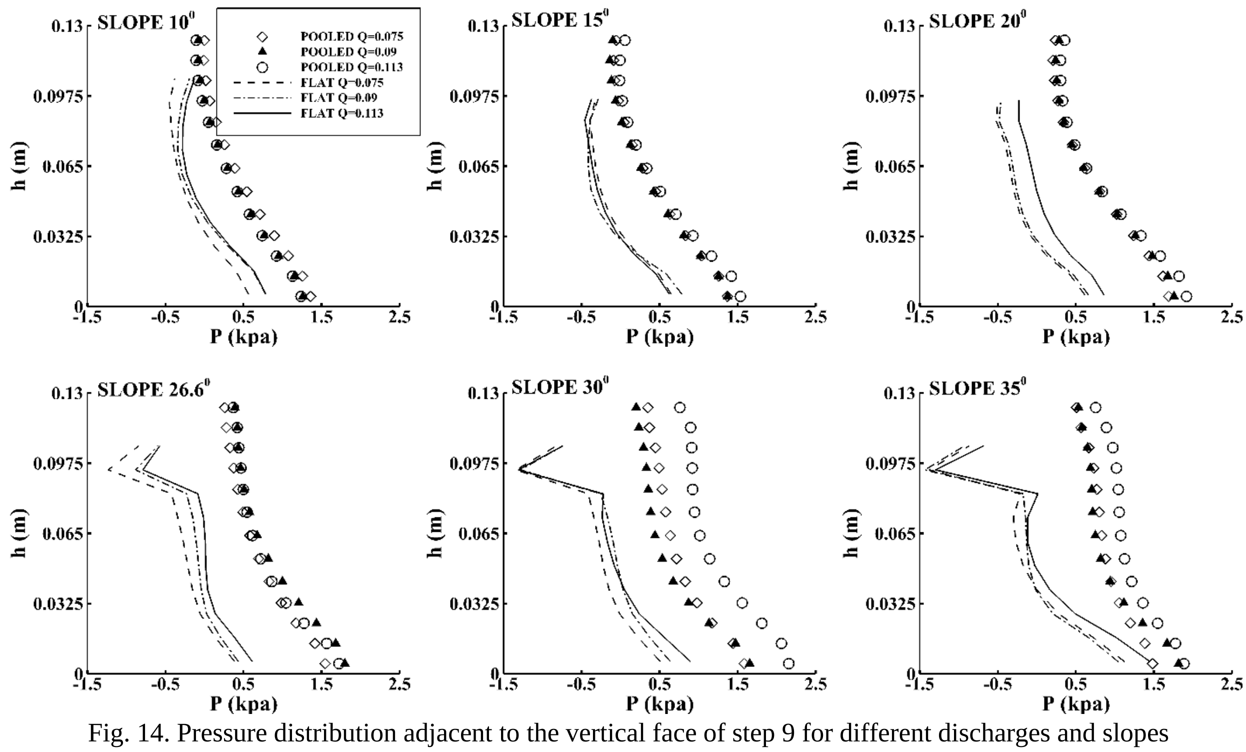

Fig. 14. Pressure distribution adjacent to the vertical face of step 9 for different discharges and slopes

4. Conclusions

In the present paper, three dimensional simulation of the flow over the flat and pooled stepped spillways with different slopes and discharges carried out using a commercial software. In the model, VOF method and *k-ε* (RNG) turbulence model employed. Verifying the numerical results with the experimental results indicated a fair consistency. Comparing the results obtained from different discharges and slopes showed that changing trend of the residual head was similar in the pooled and flat stepped spillways with 10, 15, and 20-degree slopes. The difference was that the pooled stepped spillway resulted a lower head in a similar discharge in comparison with the flat case. Turbulence kinetic energy in the pooled case was more than in the flat case for all the slopes and discharges. The maximum turbulence kinetic energy on each step occurred near the step pool in the pooled stepped spillway and almost in the middle of the step in the flat stepped spillway. Overall, the maximum turbulence kinetic energy in the entire length occurred at the end of the spillway. The pressure results near the bed of the step at the end of the spillway indicated that the negative pressure did not occur in the flat and pooled cases. Negative pressure near the vertical face occurred in most parts of the depth in the flat stepped spillway in comparison with the pooled case. In addition, the negative pressure in the pooled case eliminated for larger slopes. With regard to the fact that the negative pressure occurred in a larger area adjacent to the vertical face of the steps in the flat stepped spillways, it is more likely that the cavitation occur in this spillway rather than the pooled stepped spillway.

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