

Estimation of Discharge over the Submerged Compound Sharp-Crested Weir using Artificial Neural Networks and Genetic Programming

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ABSTRACT

Truncated sharp crested weirs are used to measure flow rate and control upstream water surface in irrigation canals and laboratory flumes. The main advantages of such weirs are ease of construction and capability of measuring a wide range of flows with sufficient accuracy. Artificial neural networks (ANNs) and genetic programming (GP) have recently been used for estimation of hydraulic data. In this study, they were used as alternative tools to estimate flow discharge over the submerged truncated weirs. The hydraulic parameter of water flow rate, Q was determined as functions of the crest width b, upstream head h, weir height P_1 , tail water depth Y_t , and flume width B. Estimations of the ANN and GP models were in good agreement with the measured data. The ANN model results were compared with those of the GP₁, GP₂, GP₃ and GP₄ models and showed that the proposed ANN models are much more accurate than the GP models. In addition, GP₂ model has a better performance than GP₁, GP₃, GP₄ models.

Keywords

Artificial Neural Networks; Genetic Programming; submerged sharp-crested weir

1. Introduction

A weir is basically an obstruction in an open channel flow path. Weirs are commonly used for measurement of open channel flow rate. Downstream water rising above the weir crest elevation produces a submerged weir condition. When the downstream water surface is near or above the crest elevation of a sharp-crested weir, accuracy of measurement should not be expected. Because of the large loss of accuracy, designing thin-plate weirs for submergence should be deliberately avoided. However, submergence may happen unexpectedly or may temporarily necessary. range be А of measurement techniques were developed by Boss (1989) and USBR (1997). Thin-plate weirs are commonly used as measuring devices

enabling an accurate discharge measurement with simple instruments. The commonly used cross sections of sharp-crested weirs are rectangular, trapezoidal and triangular. The most important hydraulic advantages of a compound weir, in which the shape of the crest is composed of a triangular aperture (on the bottom) and a rectangular opening (top of the weir), are the ease of construction and capability of measuring a wide range of flows. (1) For low flow, as a triangular weir is found to be accurate in discharge measurements; (2) For high flow, measuring high discharges and suitable operation for preventing backwater effects that affect the structures located upstream of the weir.

Preliminary, Bos (1989), Clemmens et al. (1993) and USBR (1997) have done studies to investigate the discharge over submerged sharp-crested weirs. Wu and Rajaratnam (1996) presented the results of an experimental study on submerged flow over full-width sharpcrested rectangular weirs. Submerged flow was divided into impinging jet and surface flow regimes. A diagram has been developed to predict the occurrence of these regimes. The boundaries of these regimes depend upon the direction of the tail water change. The characteristics of a compound weir consisting of a rectangular notch and a V-notch cut into the center of the crest are discussed in the USBR (1997). However, the discontinuity in the discharge curve has been a major problem in the above stated combination of notches when measuring discharge in the transition range. Martinez et al. (2005) proposed a compound sharp-crested weir having two triangular weirs with different notch angles. They proposed a theoretical discharge equation for fully contracted flow condition. Abbaspour and Yasi (2001) experimentally investigated the flow over sharp-crested, truncated-90⁰ triangular weirs with different side construction ratios (b/B=1, 0.7, 0.5 and 0.3). A 1-D flow equation was presented from the integration of both analytical and physical based solutions. A coefficient of discharge was introduced to represent the complex functions in the equation and to overcome any uncertainties. Intensive experiments were carried out to evaluate the discharge coefficient, C_d, in terms of the most significant parameters affecting the flow over weirs with and without side contraction. The results were presented in the form of nondimensional figures from which the flow rate can be calculated in conjunction with the proposed equation (Bazargan et al. 2011). Piratheepan et al. (2006) experimentally studied a compound sharp-crested weir composed of two triangular parts with different notch angles

which proved to be accurate in measuring wide range of discharges without any discontinuity. Several methods have been also proposed to estimate the flow over the double 'V' notch compound sharp-crested weirs and one method has been experimentally validated as the most suitable one.

$$Q = C_{d} \left[\frac{8}{15} \sqrt{2g} \tan \frac{\theta}{2} h^{\frac{5}{2}} - \frac{8}{15} \sqrt{2g} \tan \frac{\theta}{2} (h - P_{2})^{\frac{5}{2}} \right]$$

if $P_{2}/h < l$ (1)

$$Q = C_d \frac{8}{15} \sqrt{2g} \tan \frac{\theta}{2} h^{\frac{5}{2}} [\frac{5}{2} (\frac{P_2}{h}) - \frac{15}{8} (\frac{P_2}{h})^2 + \frac{5}{16} (\frac{P_2}{h})^3 + \frac{5}{128} (\frac{P_2}{h})^4]$$

if $P_2/h < l$ (2)

Because of the flow complexity over compound weirs, more practical tools are required to model the flow rate. Regressions have been most commonly used to estimate the flow over submerged compound sharp-crested weirs. However, regression analysis may have large uncertainties and the computed flow can be far from the actual ones. In addition, the regression analysis has some limitations caused by predefined equations for modeling.

Recently, artificial neural networks (ANNs) and genetic programming (GP) have been used to model hydraulic processes. The methods have been used to estimate the scouring around piles by Kambekar and Deo (2003), and the scouring below spillways by Azmathullah et al. (2008). In addition, a combination of the fuzzy inference system (FIS) with ANNs, i. e. ANFIS, has been employed to estimate the wave characteristics by Mahjoobi et al. (2008). GP and ANNs have been successfully applied in maritime engineering (Kalra and Deo 2007; Singh et al. 2007; Gaur and Deo 2008). Given et al. (2009) predicted local scour downstream of hydraulic structures with genetic programing.

The purpose of this study was to investigate the flow rate over a submerged compound sharp-crested weir in a horizontal flume with different side construction ratios, b/B and upstream hydraulic head, h using the ANN and GP methods. These soft computing tools can evaluate the relative effect of input parameters, such as crest width b, upstream head h, weir height P_1 , height of triangular opening P_2 , flume width *B* and tail water depth Y_t on flow rate, *Q*.

2. Materials and methods

2.1 Experimental setup

The experimental setup consisted of two main flumes of A and B. The main flumes were 0.2 m and 0.6 m wide, respectively and 0.50 m deep with a bed slope of 0.002. A subcritical approach flow was produced in the flume. The compound sharp-crested weirs were made of Plexiglas plates. The weirs were installed perpendicular to the flow direction in the flume and attached at the middle of the open channel with the help of the water sealant to prevent leakages. The 'V' notch weirs attached to the downstream collecting tanks of A and B were used to measure the actual discharge through the channel. For this purpose, they were first calibrated accurately by the volumetric method. Water depths were measured upstream and downstream of the weir along the center line of the main channel using two point gauges with an accuracy of 0.1 mm. The flow channel section is shown in Fig. 1. A total of 242 experimental data were measured. Ranges of the variables are shown in Table 1. Reynolds number was in the range of 35000 to 64600.

The discharge over the submerged compound weir, Q is influenced by the variables characterizing the flow. The following functional relationships describe the discharge over the compound weir as a function of its independent parameters in Eqs. (3) to (6):

$$Q = f_1(h, P_1, Y_t, b, B) \tag{3}$$

(2)

$$Q = f_2(h, P_1, Y_i, b) \tag{4}$$

$$Q = f_3(h, Y, b, B) \tag{5}$$

$$Q = f_4(h, P_1, Y_t) \tag{6}$$



Fig. 1. Schematic view of the flow over the compound weir used in the experiments (Abbaspour and Yasi, 2001).

Table. 1. Ranges of the experimental data

Flume	<i>P1</i> (m)	<i>P2</i> (m)	<i>b</i> (m)	B (m)	Yt (m)	h (m)	Q(m3/s)	Re
А	0.1-0.16	0.1	0.2	0.2	0.13-0.33	0.1-0.38	0.002-0.03	10000 -61000
В	0.1-0.2	0.1-0.2	0.2-0.4	0.6	0.1-0.35	0.1-0.28	0.003-0.05	11000 -64600

2.2 Artificial neural network (ANN)

An artificial neural network (ANN) is an information processing paradigm that is inspired by the way biological nervous systems, such as the brain, process information. It is composed of a large number of highly interconnected processing elements (neurons) working in unison to solve specific problems. Neurons are arranged in layers, including an input layer, hidden layers, and an output layer. There is no specific rule that dictates the number of hidden layers. The function is established largely based on the connections between the elements of the network. In the input layer, each neuron is designated for one of the input parameters. The network learns by the back-propagation applying algorithm, which compares the neural network simulated values with the actual values and calculates the estimation errors. The data set in the network is divided into a learning data set, which is used to train the network, and a validation data set, which is used to test the network performance. In the present study, the neural network fitting tool (nftool) of MATLAB 7.5 was used.

After training the network, verification is conducted until the success of the training can be established. In the simulation of discharge over weirs, characteristic data were investigated with the neural network using the Levenberg-Marquardt algorithm, which is an approximation of Newton's method. In order to check the sensitivity of the neural networks, a simulation study was carried out with hidden nodes of different numbers of 5, 10, 15, and 20.

The parameters considered in the study are hydraulic head h, crest width b, weir height P_1 , height of the triangular opening P_2 , and flume width B. The parameters of h, b, P_1 , Y_t and Bwere used as inputs to the ANN model to estimate the discharge over the submerged compound weir. Two hundred and forty two experimental data sets were used for the ANN simulations. They were divided into three parts, i.e. 80% for training, 10% for validation, and 10% for testing.



Fig. 2. Feed-forward neural network model (Anonymous, 2007)

The correlation coefficient (R), the root mean square error (RMSE), the mean absolute error (MAE), and the Nash-Sutcliffe efficiency coefficient (NSE) statistics were used to evaluate the model accuracy. R shows the degree to which two variables were linearly related. Different types of information about the predictive capabilities of the model are measured through RMSE MAE. and An efficiency of 1 (NSE = 1) corresponds to a perfect match of the modeled values to the observed data.

$$R = \frac{\sum_{i=1}^{n} (x_i - \overline{x}) (y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})^2 \sum_{i=1}^{n} (y_i - \overline{y})^2}}$$
(7)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \left(X_i - Y_i\right)^2}{n}}$$
(8)

$$MAE = \frac{\sum_{i=1}^{n} (X_i - Y_i)}{n}$$
(9)

$$NSE = 1 - \frac{\sum_{i=1}^{n} (X_i - Y_i)^2}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$
(10)

where X_i is the observed value, \overline{X} is the mean of X_i , Y_i is the estimated value, \overline{Y} is the mean of Y_i , and *n* is the number of data sets.

From the simulation study that was carried out on different numbers of hidden nodes, it was found that good estimation accuracy was achieved with 10 neurons in the hidden layer in four trials. The sigmoid, $f(x)=1/(1+e^{-x})$ and linear activation functions were used for the hidden and output nodes, respectively.

2.3 Genetic programming (GP)

In artificial intelligence, genetic programing (GP) is an evolutionary algorithm-based methodology inspired by biological evolution to find computer programs that perform a userdefined task. GP initializes a population consisting of random members known as chromosomes. and the fitness of each chromosome is evaluated with respect to a target value. The principle of Darwinian natural selection is used to select and reproduce fitter programs. GP creates computer programs that consist of variables and several mathematical functions sets as the solution. The function set of a system can be composed of arithmetic operations (+, -, \times , \div), function calls (such as e^x , x, sqrt, and power), even relational operators (>, <, =) or conditional operators, and a terminal set with variables and constants (x_1, x_2, \dots, x_n). An initial population is randomly created with a number of individuals formed by nodes (operators, variables, and constants) and previously defined according to the problem domain. An objective function must be defined to evaluate the fitness of each individual. Selection, crossover, and mutation operators are then applied to the best individuals and a new population is created. The whole process is repeated until the given generation number is reached [11].

The fitness of a GP individual may be computed using Eq. (11):

$$f = \sum_{j=1}^{n} \left| X_{j} - Y_{j} \right|$$
(11)

where X_j is the value returned by a chromosome for the fitness case *j*, and Y_j is the expected value for the fitness case *j*.

In the GP model, many operators like sin, cos, and log as well as mathematical functions were used, and it was found that the functions of the proposed GP model were complex. In addition, the GP model using more operators has larger estimated difference. In this study, six arithmetic operators $(+, -, \times, \div, \text{ sqrt}, \text{ and power})$ were used for simplicity. The functional and operational parameter settings used in the GP model are shown in Table 2. Performance of the GP model in training and testing sets was validated in terms of the common statistical measures of *R*, *RMSE*, *MAE*, and *NSE*.

Table 2	. Parameters	of GP Model	
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Parameter	Parameter description	Setting of parameters			
p_1	Function set	$+, -, \times, \div$, sqrt, power			
p_2	Generation number	50 000			
p ₃	Mutation frequency (%)	50			
p ₄	Number of chromosomes	30			
p ₅	Number of generations	3			
\mathbf{p}_6	Linking function	Multiplication			

3. Results and discussion

In order to investigate the discharge over the submerged compound weirs, the flow and geometry characteristics, such as hydraulic head h, crest width b, weir height P_I , tail water depth Y_t and flume width B, were evaluated. During model development in this study, discharge (Q) was selected as an output and five parameters of h, B, b, P_I and Y_t as inputs for ANN and GP₁ models. Also four parameters for models of GP₂ (h, b, P_I and Y_t), GP₃ (h, b, B and Y_t) and three parameters for GP₄ (h, P_I and Y_t) were used as inputs.

3.1 Discharge estimation using ANN model

Different ANN structures were investigated in terms of hidden layer node numbers. In this study, the number of neurons in the hidden layer was obtained by trial and error. From the simulation study, which was carried out using the ANN model, it was found that with 10 neurons in the hidden layer, the estimation accuracy increased to some extent.

For a better understanding of the model performance, plots of Q simulation taken from

the training, validation, and testing data sets are given in Fig. 3. In general, an *R* and *NSE* values greater than 0.9 and 0.7, respectively indicate a very satisfactory model performance.

Comparison of the estimated values of Q with measured data (Fig. 3) showed that an excellent estimation using the ANN model could be achieved.

3.2 Discharge estimation using GP model

Figs. 4 to 7 show the estimated values of Q for the training and testing data. An almost perfect agreement between the measured values and the GP estimations was clearly observed. For the GP model, referring to Figs. 4 to 7, the GP models have a good ability in estimating Q values, as reflected in low values of *RMSE* and *MAE* and a high value of *R*.

Comparison between the estimated and measured values of Q (Fig. 8) showed that using the GP_1 model an excellent estimation could be achieved.



Fig. 3. Comparison of the measured and estimated values of *Q* using ANN model for training, validation, and testing data



Fig. 4. Comparison of the measured and estimated values of Q using GP₁ model for training and testing data



Fig. 5. Comparison of the measured and estimated values of Q using GP₂ model for training and testing data



Fig. 6. Comparison of the measured and estimated values of Q using GP₃ model for training and testing data



Fig. 7. Comparison of the measured and estimated values of Q using GP₄ model for training and testing data



Fig. 8. Comparison of the measured discharges with the estimated ones using GP1 for training and testing data

Superior performance of the GP models compared with the other methods is attributed to the powerful artificial intelligence techniques for computer learning inspired by natural evolution to find an appropriate mathematical model to fit a set of points. GP employs a population of functional expressions and also numerical constants, based on how closely they fit to the corresponding data [11]. The simplified analytical forms of the proposed GP_1 , GP_2 , GP_3 and GP_4 models for Q with different hydraulic parameters may be expressed as shown in Eqs. (12) to (15), respectively:

$$Q = \left[h\left(b + \frac{P_1}{C_0 C_1 + Y_t}\right)\right] \left[P_1 + C_2 - \frac{Y_t - h}{2P_1}\right] \left[h - b\sqrt{Y_t h}\right]$$
(12)

$$Q = \left[(P_1 - Y_t)(Y_t - h) \left(\frac{Y_t}{P_1} + Y_t \right) + h \right]$$
$$\left[h + P_1(h+b) \left(\frac{h}{b} - Y_t \right) \right] \left[\sqrt{b - (h - 2Y_t + b - C_5)^2} \right] (13)$$

$$Q = \left[\frac{h}{b(b+1)+Y_t}\right] \left[Y_t + bh^2 C_2(h+Y_t)\right] \left[\frac{bh}{Y_t}\right]$$
(14)

$$Q = \left[\left(h - \sqrt{Y_t} (P_1 - Y_t) \right) + P_1 \right] \left[\left(\frac{P_1^2}{C_3^2 Y_t} - h \right)^2 \right]$$
$$\left[Y_t \left(1 - \frac{c_5}{h} \right) + 1 \right]$$
(15)

where C_0 , C_1 , C_2 , C_3 , C_4 and C_5 are constant coefficients, which are determined by the GP models (Table 3).

3.3 Comparison of the ANN and GP models

The ANN and GP models are compared in Figs. 3 to 8. It can be seen from the fit line equations (the equations are assumed to be as y = ax + b) in the scatter plots of the GP models that the coefficients *a* and *b* with a higher *R* value are closer to 1 and 0, respectively for the ANN model than the GP model. This can be clearly observed from the ANN model fit line equation coefficients.

Table 3. Constant coefficients of the GP model

Models	Parameter	Constant coefficient							
Widdeis		C_0	C ₁	C ₂	C ₃	C_4	C ₅		
GP_1	$Q = f_1(h, P_1, Y_t, b, B)$	2.702423	1.924164	2.702423	-4.829712	2.702423	-1.922852		
GP_2	$Q = f_2(h, P_1, Y_t, b)$	4.091858	-1.061676	4.091858	-5.8219	5.150756	-0.082305		
GP ₃	$Q = f_3(h, Y_t, b, B)$	1.06784	3.484894	8.150818	-9.96933	4.008239	-5.241699		
GP_4	$Q = f_4(h, P_1, Y_t)$	-6.001678	-4.044921	-0.224731	4.098266	-9.877045	-2.272949		

Table 4. RMSE, MAE, R, and NSE statistics of the training and testing data of the ANN and GP models

Parameter	Model	RMSE	MAE	R	NSE	RMSE	MAE	R	NSE
		Training					Testing		
$Q = f_1(h, P_1, Y_t, b, B)$	ANN	0.0005	0.0002	0.999	0.99	0.0004	0.0003	0.999	0.99
$Q = f_1(h, P_1, Y_t, b, B)$	GP_1	0.0017	0.0011	0.982	0.97	0.0013	0.0009	0.993	0.99
$Q = f_2(h, P_1, Y_t, b)$	GP_2	0.0013	0.0008	0.989	0.98	0.0011	0.0008	0.996	0.99
$Q = f_3(h, Y_t, b, B)$	GP ₃	0.0017	0.0011	0.981	0.96	0.0015	0.001	0.990	0.98
$Q = f_4(h, P_1, Y_t)$	GP_4	0.0019	0.0013	0.977	0.95	0.002	0.0015	0.984	0.96

Table 4 compares the ANN and GP models, with all the statistical measures of R, RMSE, NSE, and MAE of the training and testing data. According to Table 4, the ANN model has a lower absolute error as compared with the GP models showing that the proposed ANN models are much more accurate than the GP models for water engineering purposes.

4. Conclusions

In general, performance of the ANN and GP models are superior to the statistical regression schemes. The ANN and GP models were developed to determine the discharge over the weir. The input parameters used for the ANN and GP simulations included hydraulic head h, crest width b, weir height P₁, tail water depth Y_t and flume width B. These models can be successfully used in computation of the discharge. The optimum ANN model was obtained after different structures were investigated in terms of hidden layer node numbers. Estimations of the ANN model were compared with those of the GP models. According to Table 4, model performance can be evaluated as satisfactory if NSE > 0.7 and R > 0.9, with low values of RMSE and MAE. The proposed ANN models were much more accurate than the GP models. In addition, the GP models were much more practical than the ANN models because they provide nonlinear mathematical equations. Estimated values of Q using GP_2 model were in a good agreement with the measured data compared to GP₁, GP₃ and GP₄ models.

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