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Velocity Distribution in the 90-degree Bend based on the Probability and Entropy Concept

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ABSTRACT

Practical concept of velocity distribution of pressure flow in the bends is interesting and hence, the professional engineering design has been investigated in the current study. This paper shows that velocity distribution in the bends can be analyzed in terms of the probability distributions. The concept of entropy based on the probability is an applied and new approach to achieve velocity profiles in bends, presented by the probability distribution and tsallis entropy maximization under the importing constraints. Due to the lack of a specific equation of velocity distribution in the bends, the particular equation in the various band sections was obtained through the numerical simulation of three dimensional flow in the 90-degree bend (via Fluent). In this simulation, flow regime was laminar and the Reynolds number was in the range of 100 to 2000. Then, for m=2, the velocity distribution function was obtained. Studies showed that the proposed model for the mentioned range of Reynolds numbers is in a good agreement with other study results. The calculated error values indicated a reasonable accuracy. The model can directly calculate the velocity at each point of the spatial position. The calculated values were also compared with the results obtained from the velocity distribution based on the Shannon entropy. The results showed a reliable estimate of the velocity profiles.

Keywords

Velocity distribution; Bend; Tsallis entropy; Reynolds number

1. Introduction

Flow in bends and arcs is a main component in the water conveyance structures and structures related to the dams. Secondary and spiral flows in bends have a more complicated flow pattern than the flow in direct channels and this complexity is of utmost importance to be studied in this field. Bovendeerd et al. (1978) investigated the velocity profiles and the flow velocity contour in 90-degree bends using the finite element method. They used the laminar parabolic profile as the input condition and coherent definition of the flow field around the bend, the intensity of secondary motions and velocity axial profile presented in several sections. Van De Vosse et al. (1989) modeled the velocity profile in a three dimensional pressure flow at a 90-degree bend using the finite element method (Galerkin) considering different angles and compared the results with experimental ones and observed an acceptable agreement. Nakayama et al. (2003) carried out experimental investigation on the 180-degree duct and surveyed the results measured in separated areas and distribution of Reynolds stresses in the flow direction. Sparrow et al. (2009) examined the separation of the wedge-shaped ducts and concluded that it is affected

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by the Reynolds number and to change the separation diameter dimensionless parameter to diameter, against Reynolds number and considering the different divergence angles, they presented curves. Sadeghfam and Akhtari (2012) studied flow separation in the bends at different refraction angles by the Fluent numerical model and provided relations and graphs for the length and thickness of the separated area.

Previous studies have shown that the entropy can be an effective tool to provide a new way of hydrology and water resources research (Singh, 1997). Chiu (1978, 1988, 1989) obtained a velocity distribution equation in narrow channels by applying entropy concept and principle of maximization of entropy (Shannon, 1948). A new coordinates system, $\xi - \eta$, were formed as Isovel of primary flow and their vertical directions by Chiu et al. Tsallis (1988) provided a generalization of measurement entropy Boltzmann-Gibbs. Tsallis entropy is a generalization of Shannon entropy and includes additional parameters that can be used to create more or less sensitive to the shape of the probability distribution (Maszczyk and Dush, 2008). Araujo and Chaudhry (1998) investigated velocity distribution with measured longitudinal data and found that the entropy model acted better than the logarithmic model, not only in general terms but also in all regions of the flow, especially near the bottom of the channel. Moramarco et al. (2004) developed a simple method to reconstruct the velocity profile in a section of the river, which was based on the assumption that Chiu velocity distribution can be applied locally. They showed that the shape of the observed velocity profiles for severe flood events can be estimated using the simplified proposed method. Luo and Singh (2011) developed a new velocity distribution equation for open channel flow using the concept of tsallis entropy. To reduce many of the parameters in the coordinate system, Marini et al. (2010) developed a new method for the obtained two-dimensional velocity distribution. In their studies, the cumulative distribution function under x-y coordinate system was assumed. Bonakdari and Moazamnia (2013) predicted the velocity distribution and discharge in sewers using the Shannon entropy and measured data from a real measurement site of sewers used for testing and application of theory and showed that the model was capable of modeling and simulation of the velocity distribution from the channel bed to the free surface. In this study, a general formula for velocity in ends using the principle of entropy maximization and integration of the changes was derived. Obtained equation has fewer parameters and simple calculations and the predicted values of the cross-section velocities are in a very good agreement with the field data.

2. Numerical modeling and laboratory results

In this study, in order to obtain the cumulative probability distribution function, 3D pressure flow was numerically modeled in the bends and for validating the numerical simulation, experimental results of Olsen et al. (1971) were used. The 90-degree bend in this study, had two arms. The inlet and outlet arm lengths were 300 and 150 mm, respectively with a diameter of 8 mm. in addition, the curvature radius of the bend was 24 mm. Due to the small size of the survey results and in order to eliminate the effect of the dimension, the results were presented using the dimensionless parameters. In the flow analysis, steady and laminar flow conditions were assumed. Bend geometry and its mesh was provided by pre-processor of the Gambit software. Figure 1 shows the geometry of the 90-degree bend as an example. In this figure, area A is the inlet boundary with a velocity inlet type, area B is the wall and due to pressure development, area C is the pressure outlet of the outlet section.

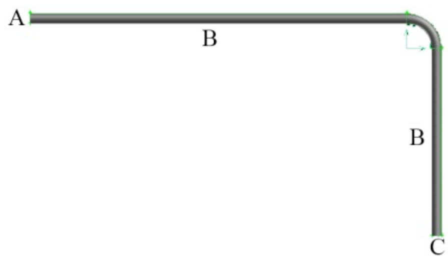

Figure 1. Boundary conditions and geometry of the 90-degree bend

The hexagonal and map mesh types were used in this study. The number of mesh grids for the 90-degree bend was 114114 that was determined using Gambit preprocessor software. The numerical solution was used to obtain the geometry of the flow field by the Fluent and Gambit softwares. In addition, the Standard scheme was used for discreting the pressure, Quick scheme was used for discreting the momentum and turbulence equations and also the Simple algorithms wee used for velocity and pressure coupling.

3. Concept and theory of the entropy

Entropy, as the second law of thermodynamics, is the macroscopic properties of a system that measures partial irregularities in the system. In information theory, Shannon (1948) has formulated the concept of the theory as a measure of information or uncertainty associated with the random variable or its probability distribution. In the next decades, principle of maximum entropy for obtaining the probability distribution of the random variable under given information in terms of constraints was expressed by Jeans (1957). The entropy can be considered as a useful property of a probability distribution and is widely applied in the environmental engineering and water, including geology, hydrology, and hydraulic. Recent application of the entropy theory in hydrology and water resources has been reviewed and discussed by Singh (1997). Application of the entropy concept in the hydraulics has been suggested by Chiu (1987, 1988, 1991) and a simple method to derive an equation for the velocity distribution using the concept of probability and entropy maximization principle was presented. This method can describe the velocity changes in the vertical and lateral directions, with maximum velocity occurs on or below the surface.

3.1. Shannon entropy

Shannon (1948) developed entropy theory to describe the uncertainty of information in the field of communication, which now is as a useful characteristic of a probability distribution of each item determined. In the probability theory, a random variable value shows possible from an experiment that is variable (uncertainty). Shannon defined a quantitative measure of the uncertainty associated with a probability distribution of the random variable in terms of entropy. Shannon entropy is called the entropy information and can be expressed as follows:

$$H = -\sum_{i}^{N} p_i \log p_i \tag{1}$$

where p_i is the probability of a random variable, and N is the total number of values.

Shannon entropy in hydraulic engineering can be developed to continuous random variables. For a random variable X, throughout the rang $(-\infty, +\infty)$ is continuous, Shannon entropy is expressed as follows:

$$H = -\int_{-\infty}^{+\infty} \sum_{i}^{N} p(x) \log p(x) dx \tag{2}$$

where $p(x)$ is the probability density function of a continuous random variable x.

3.2. Tsallis entropy

Entropy of a system can be related to its energy production. Since the energy is directly proportional to the energy in a system, when the energy is minimized, energy production is minimal. Entropy principles show that an open system in which matter and energy can enter or leave the system, minimum entropy production can be achieved at equilibrium (Chiu, 1988). Tsallis (1988) proposed a general form of the Shannon entropy as follows:

$$H = \frac{1}{m-1}\{1-\int_{-\infty}^{+\infty}[f(x)]^m ds\} = \frac{1}{m-1}\int_{-\infty}^{+\infty} f(x)\{1-[f(x)]^{m-1}dx\} \quad (3)$$

where *m* is a real number, and when *m>0*, entropy becomes a convex function. For *m=1*, the above equation converts to the Shannon entropy. Similar to the Shannon entropy, tsallis entropy can be combined with principle of maximum entropy to achieve the probability distribution of a given random variable and is possible in many cases to give more accurate results than the Shannon entropy.

3.3. Constraints

Flow in a channel satisfies the conservation of mass, momentum and energy laws and these laws can be employed to define constraints that the velocity distribution must obey. Therefore, the constraints are defined as follows. Integration of the probability density function of velocity must always be unity, so the first constraint is expressed as follows:

$$C_1 = \int_0^{u_{\max}} f(u)du = 1 \quad (4)$$

The second constraint, C_2, is achieved by using mass conservation:

$$C_2 = \int_0^{u_{\max}} uf(u)du = \bar{u} \quad (5)$$

where $\bar{u}$ is the cross-sectional mean velocity or *Q/A*, *Q* is the discharge passing through a cross-sectional area *A*.

The third constraint, C_3, is obtained from the conservation of momentum law:

$$C_3 = \int_0^{u_{\max}} u^2 f(u)du = \beta \bar{u}^2 \quad (6)$$

where β is the momentum distribution coefficient.

The fourth constraint, C_4, is achieved from the energy conservation law:

$$C_4 = \int_0^{u_{\max}} u^3 f(u)du = \alpha \bar{u}^3 \quad (7)$$

where α is the energy distribution coefficient.

4. Probability Cumulative distribution function

Analysis of the flow in the Reynolds number range of 100 to 2000 (step 100) was performed and after reaching the convergence, flow velocity profile at angles of 90-degree (bend outlet) and 45 degrees (bend center) was extracted. It is noted that validating the numerical model by comparing the experimental results of the Olsen et al. (1971) at Reynolds number 300 has been made. The following figure shows that there is an acceptable agreement between the results of the numerical modeling and the experimental results. In this figure, the horizontal axis represents the normalized diameter of the tube so that a zero value represents the inner wall and 1 represents the outer wall. The vertical axis in Fig. 2 shows the dimensionless parameters (axial velocity/average velocity).

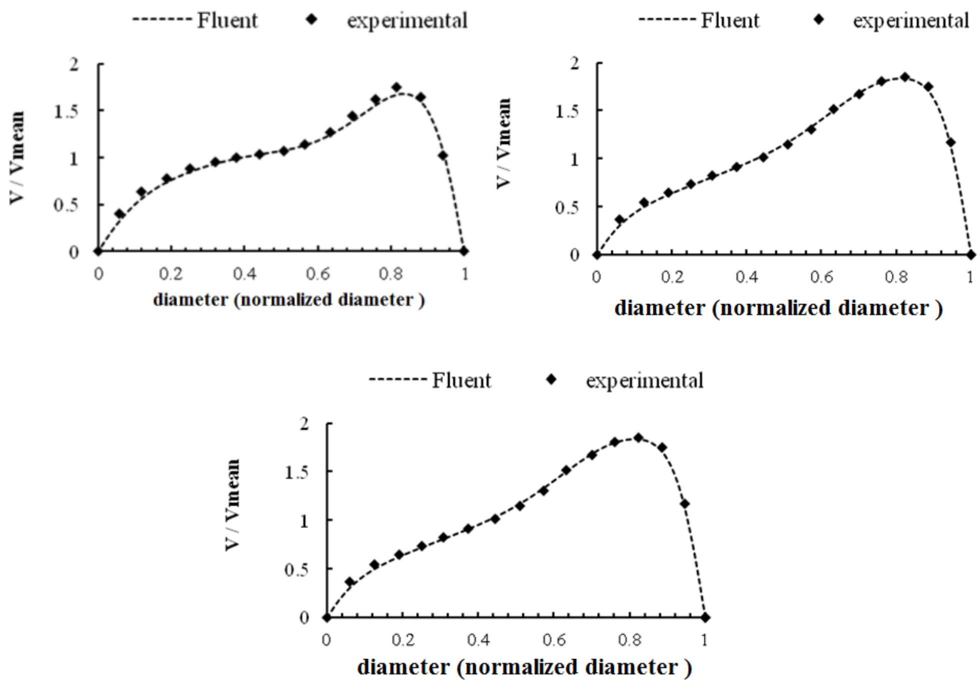

Figure 2. Comparison of numerical modeling velocity profiles and experimental results of (Olsen et al) at Reynolds number 300

Then, the cumulative probability distribution function for each Reynolds number in the mentioned range has been calculated by using the results of the numerical modeling. Finally, the polynomial equation (following equation) with an acceptable regression coefficient was used for both 90 and 45-degree sections. In this equation, r represents the radius of the pipe section (variable parameters) and r_0 is the radius of the cross section. Thus, r/r_0 is a dimensionless parameter and ranges from +1 to -1. *F(u)* is the velocity cumulative probability distribution function. In Table 1, the coefficients of the equation and the regression coefficient for the velocity cumulative distribution function are given.

$$F(u) = p_1(x)^8 + p_2(x)^7 + p_3(x)^6 + p_4(x)^5 + p_5(x)^4 + p_6(x)^3 + p_7(x)^2 + p_8(x)^1 + p_9 \quad (8)$$

$$x = (r / r_0)$$

Table 1.Pproperties of the velocity cumulative probability distribution function

Equation constants	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	Regression coefficient
45 degrees Section (bend center)	-4.06	-2.313	7.117	3.219	-5.094	-1.9	1.606	1.001	0.431	0.957
90-degree Section (bend outlet)	-4.147	-1.504	6.519	0.999	-4.346	0.3065	1.498	0.2161	0.4771	0.9697

5. Probability density function

In the following part, the probability density function *f(u)*, which can be completed by Tsallis entropy was determined. Using the principle of maximum entropy and the constraints imposed by Eqs. (4) to (7) using the Lagrange multipliers for *m>0*, the Tsallis entropy expressed by equation (1) is maximum as follows:

$$L=\int_0^{u_{\max}}\frac{f(u)}{m-1}\{1-[f(u)]^{m-1}\}du+\lambda_0\left(\int_0^{u_{\max}}f(u)du-1\right)+\lambda_1\left(\int_0^{u_{\max}}uf(u)du-\bar{u}\right)+\lambda_2\left(\int_0^{u_{\max}}u^2f(u)du-\beta\bar{u}^2\right)+\lambda_3\left(\int_0^{u_{\max}}u^3f(u)du-\alpha\bar{u}^3\right) \quad (9)$$

$$L=\int_0^{u_{\max}}f(u)\{\frac{1-[f(u)]^{m-1}}{m-1}+\lambda_0+\lambda_1u+\lambda_2u^2+\lambda_3u^3\}du-(\lambda_0+\lambda_1\bar{u}+\lambda_2\bar{u}^2+\lambda_3\bar{u}^3) \quad (10)$$

where u is the velocity at a given point, $u_{\max}$ is the maximum velocity of the cross section, $\bar{u}$ is mean velocity, *f(u)* is the probability density function, *m* is a real number, and λ_i is the Lagrange multiplier. By considering $\frac{\partial L}{\partial f(u)}$ equal to zero, the probability density function *f(u)* is obtained as follows:

$$f(u)=[\frac{m-1}{m}\{\frac{1}{m-1}+(\lambda_0+\lambda_1u+\lambda_2u^2+\lambda_3u^3)\}]^{\frac{1}{m-1}} \quad (11)$$

$$\lambda_*=\lambda_0+\frac{1}{m-1}\Rightarrow f(u)=[\frac{m-1}{m}\{\lambda_*+\lambda_1u+\lambda_2u^2+\lambda_3u^3\}]^{\frac{1}{m-1}} \quad (12)$$

Eq. (12) is the probability distribution of the minimum power and satisfies Eqs. (4) to (7) and is defined based on Tsallis entropy. According to the theory of Brabe et al. (1991), two constraints are sufficient for the accurate description of the velocity distribution:

$$f(u)=[\frac{m-1}{m}\{\lambda_*+\lambda_1u\}]^{\frac{1}{m-1}} \quad (13)$$

Tsallis entropy function is obtained by substituting the probability density function in Tsallis entropy equation as Eq. (3) as follows:

$$H=\frac{1}{m-1}-\frac{1}{m}(\lambda_*+\lambda_1\bar{u}) \quad (14)$$

6. Two-dimensional velocity distribution model based on Tsallis entropy

By assuming the cumulative function, the next step is to calculate the distribution of the velocity profiles using a probability density function based on entropy. To obtain a general case, consider density function in the two-dimensional form as (*z,y*), where *y* is the depth of the channel and *z* is the width distance from the center line. Because velocity is a function of *z* and *y*, *f(u)* can be written as *f(u(z,y))*. Since *f(u)* is derivation of the cumulative distribution function *F(u)*, by accepting the partial derivative of *F(u)* with regard to *z* and *y*, the following equations are obtained:

$$\frac{\partial F(u)}{\partial z}=\frac{\partial F(u)}{\partial u}\frac{\partial u}{\partial z}=f(u)\frac{\partial u}{\partial z}=[\frac{m-1}{m}(\lambda_*+\lambda_1u)]^{\frac{1}{m-1}}\frac{\partial u}{\partial z} \quad (15)$$

$$\frac{\partial F(u)}{\partial y}=\frac{\partial F(u)}{\partial u}\frac{\partial u}{\partial y}=f(u)\frac{\partial u}{\partial y}=[\frac{m-1}{m}(\lambda_*+\lambda_1u)]^{\frac{1}{m-1}}\frac{\partial u}{\partial y} \quad (16)$$

Equations are simplified by defining a new variable as follows:

$$K=[\frac{m-1}{m}(\lambda_*+\lambda_1u)]^{\frac{m}{m-1}} \quad (17)$$

Partial derivatives of K calculated by considering z and y are as follows:

$$\frac{\partial K}{\partial z}=\frac{\partial K}{\partial u}\frac{\partial u}{\partial z}=\lambda_1[\frac{m-1}{m}(\lambda_*+\lambda_1 u)]^{\frac{1}{m-1}}\frac{\partial u}{\partial z} \qquad (18)$$

$$\frac{\partial K}{\partial y}=\frac{\partial K}{\partial u}\frac{\partial u}{\partial y}=\lambda_1[\frac{m-1}{m}(\lambda_*+\lambda_1 u)]^{\frac{1}{m-1}}\frac{\partial u}{\partial y} \qquad (19)$$

By comparing Eqs. (18) and (19) with Eqs. (15) and (16), the relation between $F(u)$ and K can be formed as follows:

$$\frac{\partial K}{\partial z}=\lambda_1\frac{\partial F(u)}{\partial z} \qquad (20)$$

$$\frac{\partial K}{\partial y}=\lambda_1\frac{\partial F(u)}{\partial y} \qquad (21)$$

Eqs. (20) and (21) are as a system of liner differential equations that can be solved using the Leibnitz rule:

$$\int_{(0,0)}^{(z,y)}\frac{\partial K}{\partial z}dz+\frac{\partial K}{\partial y}dy=K(z,y)-K(0,0) \qquad (22)$$

Since the point with the coordinates (0,0) is located on the bottom of the channel, its velocity is $u=0$, $K(0,0)$ on the right hand side of Eq. (22) is equal to $[\frac{m-1}{m}(\lambda_*)]^{\frac{m}{m-1}}$ in Eq. (17). Therefore, the right hand side of Eq. (22) is as follows:

$$K(z,y)-K(0,0)=K(z,y)-[\frac{m-1}{m}(\lambda_*)]^{\frac{m}{m-1}} \qquad (23)$$

Definite integrals on the left hand side of Eq. (23) can be calculated in a general point of coordinates $(\bar{z},\bar{y})$ that can be identified by a polygonal curve that starts from the origin of axes (0, 0), passes through the point $(\bar{z},0)$ and ends at $(\bar{z},\bar{y})$. The cumulative distribution function $F(u)$ is constant and equal to 0 at the point (0,0). Thus, using Eqs. (20) and (21) and integration of Eq. (22) we have:

$$\int^{(\bar{z},\bar{y})}\lambda_1\frac{\partial F(u)}{\partial z}dz+\lambda_1\frac{\partial F(u)}{\partial y}dy=$$

$$\int_0^{\bar{y}}\lambda_1\frac{\partial F(u)}{\partial y}dy=\lambda_1 F(u) \qquad (24)$$

By combining Eqs. (23) and (24) results in the following:

$$\int_{(0,0)}^{(z,y)}\frac{\partial w}{\partial z}dz+\frac{\partial w}{\partial y}dy=\int_{(0,0)}^{(\bar{z},\bar{y})}\lambda_1\frac{\partial F(u)}{\partial z}dz$$
$$+\lambda_1\frac{\partial F(u)}{\partial y}dy \qquad (25)$$

Hence $K(z,y)$ can be obtained as follows:

$$K(z,y)=\lambda_1 F(u)+[\frac{m-1}{m}(\lambda_*)]^{\frac{m}{m-1}} \qquad (26)$$

By replacing K defined by Eq. (26) in Eq. (17), the velocity distribution function is obtained as follows:

$$u=-\frac{\lambda_*}{\lambda_1}+\frac{1}{\lambda_1}\frac{m}{m-1}[\lambda_1 F(u)+(\frac{m-1}{m}\lambda_*)^{\frac{m}{m-1}}]^{\frac{m-1}{m}} \qquad (27)$$

Therefore, the general velocity distribution is obtained using the tsallis entropy. Using Eq. (27) the velocity distribution can be obtained for both 1D and 2D cases. To describe the velocity distribution on the y axis of the channel, considering that u_{max} occurs below the water surface, $F(u)$ can be obtained with regard to Eq. (8).

6.1. Velocity distribution equation based on the Tsallis entropy

Velocity distribution for different m values and different data was calculated and for m=2, the best matching result was obtained. In general, considering of the difficulty in solving the equations and compared with the experimental data, m=2 is the best choice with a reasonable accuracy in the calculation of the velocity distribution. For m=2, two parameters of λ_1 and λ_V with a simple analytical expression can be obtained as follows:

$$\lambda_1 = -\frac{12}{u_{\max}^3}(u_{\max} - 2\bar{u}) \quad (28)$$

$$\lambda_* = \frac{4 - \lambda_1 u_{\max}^2}{2u_{\max}} \quad (29)$$

With the known values $u_{\max}$ and $\bar{u}$, the two parameters are obtained easily by substitution Eqs. (26) and (27). The velocity distribution equation is as follows:

$$u = -\frac{\lambda^*}{\lambda_1} + \frac{2}{\lambda_1}[\lambda_1 F(u) + \frac{1}{4}(\lambda^*)^2]^{1/2} \quad (30)$$

where *F(u)* is the cumulative distribution function.

6.2. Velocity distribution model based on the Shannon entropy

Chiu et al. (2006) used principles of probability and entropy as the basis for their model in estimating the velocity distribution for flowing in open channels and with its development presented the model as follows:

$$u = \frac{u_{\max}}{M}\ln\left[1 + (e^M - 1)F(u)\right] \quad (31)$$

where $u_{\max}$ is maximum velocity, M is entropy parameter and *F(u)* is the cumulative distribution function that for the desired bend, Eq. (8) is substituted. The cross sectional average velocity, u_{mean}, based on the maximum velocity parameter was defined as follow:

$$u_{mean} = \phi(M)u_{\max} \quad (32)$$

where

$$\phi(M) = \frac{e^M}{e^M - 1} - \frac{1}{M} \quad (33)$$

Using Eq. (31) and the experimental results it was shown that M is constant for any bend and any Reynolds number. Figures 3 and 4 show a linear relation between $u_{\max}$ and u_{mean} in the channel. The $\Phi(M)$ for Reynolds numbers 100 to 2000 in both 90 and 45-degree bends is constant and obtained as 0.56 and 0.5, respectively. M value based on the Shannon entropy for 90 and 45-degree bends obtained as 0.8 and 0.1, respectively. Velocity profile at the 90-degree bend using the obtained M was discussed and compared to the proposed model based on tsallis entropy.

7. Evaluation of the proposed model results

In non-dimensional Figs. 5 and 6, the horizontal axis presents the normalized pipe width (diameter) (*r/R*) and the vertical axis represents the ratio of the average velocity to the inlet average velocity (u/u_{mean}).

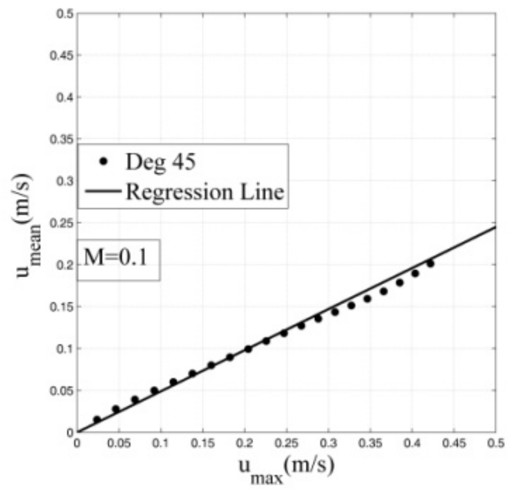

Fig. 3. Liner relation between average velocity and maximum velocity at the 45-degree bend

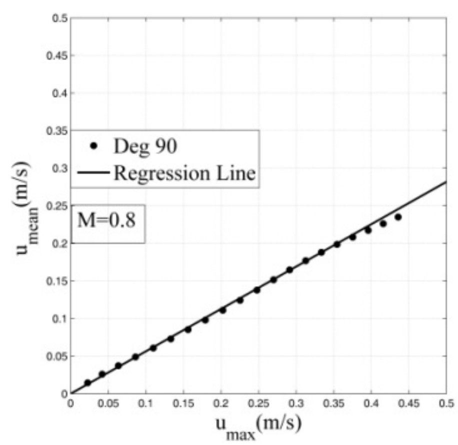

Fig. 4. Liner relation between average velocity and maximum velocity at the 90-degree bend

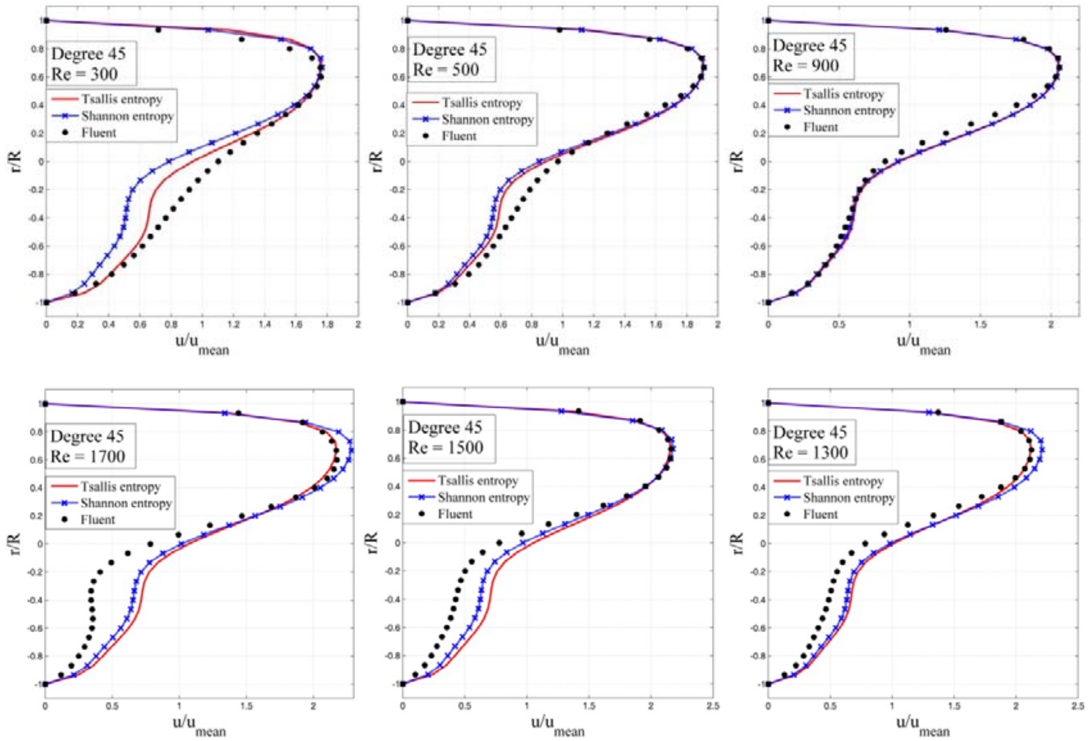

Fig. 5. Velocity distribution in deviation angle of 45 degrees

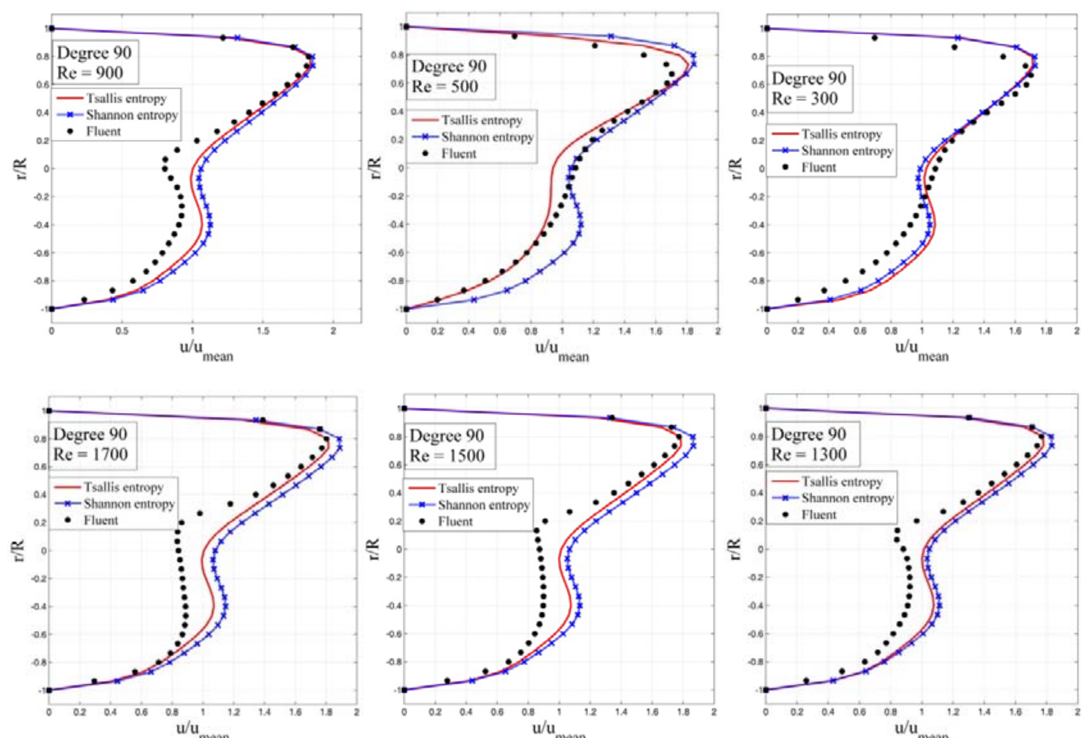

Fig. 6. Velocity distribution in deviation angle of 90 degree

Based on the 90-degree bend model at the angles of 90 degree (bend outlet) and 45 degrees (bend center) to the horizon in the Reynolds number range of 100 to 2000 (in step 100), there was numerical solution and Figs. 5 and 6 show samples of these calculations for the desired angles. According to the figures, by increasing the section angle, velocity profile tends toward the outer wall. A general comparison between the measured and calculated velocities for the model is shown in the figures, too. As can be seen from the figures, there is a good agreement between the numerical and experimental results. Tsallis model results fitted better to the experimental data, but the proposed model based on Tsallis entropy has the advantage that it does not require experimental data. As the results show, the velocity distribution model shows the location of the maximum velocity well.

7.1 Estimating the accuracy of the model

To assess the accuracy of the developed model, deviation of the measured data should be calculated. To calculate the difference between the model results and the experimental data, the following equation was used:

$$\varepsilon = \frac{u_{es} - u_{ob}}{u_{ob}} \tag{34}$$

where u_{ob} and u_{es} are the measured velocities in bend and the calculated velocities based on the proposed model, respectively. Table 2 shows the error model based on the measured values at 90-degree bend under the pressure. Both models show a high accuracy.

Table 2. Calculation error

Reynolds Numbers	Models	300	500	900	1300	1500	1700
45 degrees Error	Tsallis	0.116	0.081	0.058	0.229	0.383	0.449
	Shannon	0.197	0. 128	0.071	0.295	0.235	0.283
90-degree Error	Tsallis	0.213	0.164	0.155	0.145	0.141	0.142
	Shannon	0.284	0.298	0.332	0.254	0/261	0.271

8. Conclusions

Tsallis entropy concept, based on the probability theory was applied in this paper and a new equation for the velocity distribution in bends flow was presented. The obtained velocity equation is able to describe the velocity changes in both the vertical and lateral directions. In this study, the flow velocity profiles at 90-degree section (bend outlet) and 45-degree section (bend center) were obtained. For validating the numerical simulation, the experimental results of Olsen et al. were used. The steady and laminar flow condition was used. Based on the analysis, the velocity profiles at different Reynolds numbers (100 to 2000) were obtained. By increasing the angle of the bend sections, the velocity profile deviated toward the outer wall. There was a good agreement between the predicted and measured data. In this study, a new model for the velocity distribution was obtained using the Tsallis entropy and the probability at bend concepts. This method is effective in the view of accuracy and parameter estimation. The model was capable of modeling and simulating the velocity distribution and the obtained results showed a good agreement with the experimental data. In addition, a new equation for the cumulative distribution function at bends was achieved in this study. Both the tsallis entropy and Shannon entropy using this function are able to calculate velocity distribution profiles at different angles and Reynolds numbers.

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