



PRACTICAL ENGINEERING APPROACH FOR GENERATING THE TORSIONAL EARTHQUAKE EXCITATION FROM TRANSLATIONAL COMPONENTS

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Worldwide networks of seismic recording instruments mostly record only translational ground motion in two or three orthogonal directions. Rotational component of earthquake is not commonly recorded. Several theories are available to estimate a torsional component from recorded translational components of ground motion. In this paper detailed investigations are made to study the relative performance of some available methods. This paper suggests a practical approach for obtaining true SH (S-wave having particle motion in horizontal direction) component from two recorded horizontal components of ground excitation. This new approach will help to obtain optimum torsional time histories from translational time histories.

Keywords: translational time history, minimum correlation coefficient, principal directions of ground excitation, true SH component, torsional response spectrum

1. Introduction

The translational components of the strong motion are always accompanied by the rotational components during a seismic event, due to travelling wave effect. But these rotational components are not commonly recorded. These components used to be considered negligible.

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But detailed analysis of structural damage during several earthquakes eg. Mexico (1985), Loma Prieta (1989), Killari (1993) and Bhuj (2001) has indicated that the damage in many cases is due to the additional stresses caused by the torsional response. Several studies such as Bielak (1978), Abdel Gaffar and Rubin (1984), Gupta and Trifunac (1989), Goel and Chopra (1994), Takeo (1998) have shown the importance of rotational components in seismic analysis and design of structures. Avad and Humar (1984) have shown that during an earthquake, even the symmetric structures can be expected to undergo substantial torsional excitation. The seismic design codes also prescribe accidental eccentricity in design force calculations to account for the effect of the torsional ground motion. The torsional response in various seismic design codes is taken in adhoc way by assigning asymmetry to even symmetric structures, which is commonly termed as accidental eccentricity. It is, therefore, essential to consider the contribution of torsional excitation to overall response of structure in rational way. As time history analysis is one of the important tools of seismic response analysis, it is essential to obtain torsional time history from recorded translational time histories.

Several theories, (Newmark (1969), Hart et. al. (1975), Nathan and Mackenzie (1975), Trifunac (1982), Niazi (1986), Olivera and Bolt (1989), Lee and Trifunac (1985,1987) ,Castellani and Boffi (1986, 1989) and Li et al (2004)), have suggested the approach to get torsional time histories from available records of translational time histories in three orthogonal directions. Detailed investigation has been carried out on two of latest theories for their relative merits. It is suggested in this paper to use the principal directions of synchronized translational excitations to obtain the torsional excitation time history. A convenient numerical approach is suggested as regard to how the recorded translational components should be used in creating the optimum torsional time history.

2. Generating the Torsional Time History

Newmark (1969) was the first to establish the simple relationship between torsional and translational components of motion. It was based on the assumption of constant velocity of wave propagation. Hart et al. (1975) confirmed the presence of this rotational ground motion by measurement of responses of several buildings during the 1971 San Fernando earthquake. Hart et al. (1975) differentiated two orthogonal translational records numerically and obtained associated free field rotational motion. Nathan and Mackenzie (1975) implemented similar technique to generate record of torsional ground motion from two components of El Centro earthquake. Niazi (1986) has shown that acceleration time histories recorded by strong motion differential arrays can be used to generate the rotational components by numerical differentiation of horizontal components. He used this technique to estimate torsion and rocking induced by differential displacements on long and rigid foundations. Olivera and Bolt (1989) pursued this idea to estimate the rotational components from five earthquakes recorded by Smart array in Taiwan.

These researchers assumed the plane wave velocity of propagation to be constant. This velocity actually depends on the frequency of the wave motion and angle of incidence. More rational procedures are developed by Trifunac (1982), Lee and Trifunac (1985, 1987) and Castellani and Boffi (1986, 1989) in which dependence of velocity on frequency of wave motion and angle of incidence is taken into account. In these studies, the dispersion and transient arrival times of waves in an elastic half space were considered. Li et al. (2004) developed a procedure to include the dependence of the angle of incidence on frequencies of the impinging harmonics of ground motion.

P-waves induce volumetric deformations but not shear deformations in the materials through which they travel, thus they are irrotational. When the SH-wave (S-wave having particle motion in horizontal direction), interacts with the free boundary surface it is reflected as SH-wave with a reflection angle equal to incidence angle θ_0 . Due to this, torsional component ϕ_{gz} is produced.

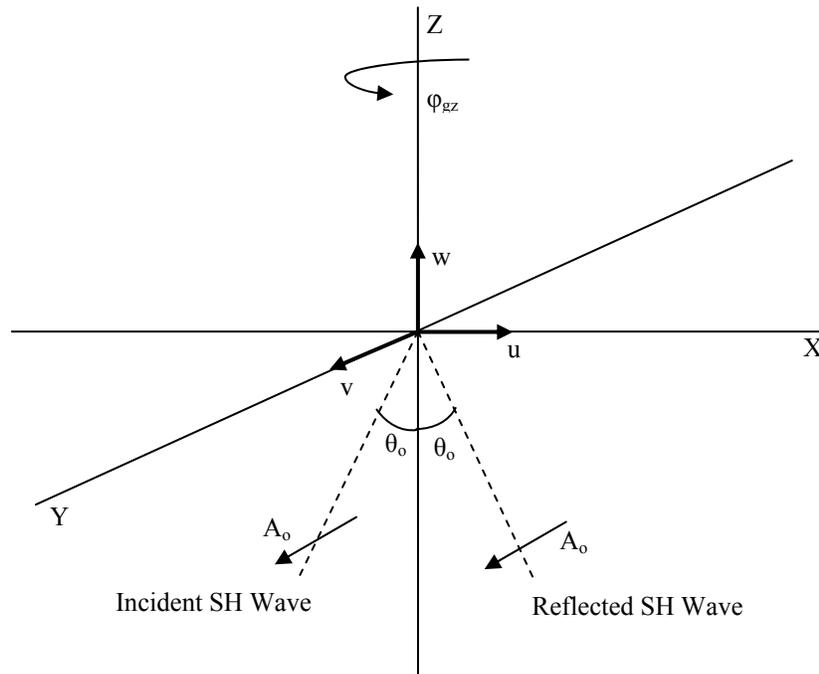


Figure .1 Reflection of SH-wave at free ground surface and resulting torsional component

2.1. Procedure Developed by Lee and Trifunac

Applying the elastic wave theory, the relation between seismic torsional component ϕ_{gz} and translational components of seismic motion can be obtained as (Lee and Trifunac, 1985):

$$\varphi_{gz} = \left(\frac{i\omega \text{Sin}\theta_0 v}{2\beta} \right) e^{-i\pi/2} = \left(\frac{i\omega v}{2c_x} \right) e^{-i\pi/2}$$

$$\frac{\varphi_{gz}}{v} = \frac{\omega}{2c_x}$$

$$\frac{|\varphi_{gz}(\omega_n)|}{|v(\omega_n)|} = \frac{\omega_n}{2\beta_{\min}} \quad \text{as } \omega_n \rightarrow \infty$$

$$\frac{|\varphi_{gz}(\omega_n)|}{|v(\omega_n)|} = \frac{\omega_n}{2\beta_{\max}} \quad \text{as } \omega_n \rightarrow 0 \tag{1}$$

$$G = \frac{2x(1 - K^2x^2)^{1/2} G}{K(1 - 2x^2)}, \quad \theta_0 < \theta_c$$

$$G = \frac{2x(K^2x^2 - 1)^{1/2} G}{K(1 - 2x^2)}, \quad \theta_0 > \theta_c$$

$$\varphi_{gz} = \left(\frac{i\omega \text{Sin}\theta_0 v}{2\beta} \right) e^{-i\pi/2} = \left(\frac{i\omega v}{2c_x} \right) e^{-i\pi/2} \tag{2}$$

where $c_x = \beta / \text{Sin}\theta_0$ (phase velocity in horizontal direction), ω is frequency of harmonic wave, β is velocity of S-wave, v is component of the ground motion in Y direction, and θ_0 is angle of incidence. So, the ratio of amplitudes of φ_{gz} and v can be written as:

$$\frac{\varphi_{gz}}{v} = \frac{\omega}{2c_x} \tag{3}$$

At the high frequency end the phase velocities of all modes of surface waves approach β_{\min} . Thus:

$$\frac{|\varphi_{gz}(\omega_n)|}{|v(\omega_n)|} = \frac{\omega_n}{2\beta_{\min}} \quad \text{as } \omega_n \rightarrow \infty \tag{4}$$

At the low frequency end essentially only first mode is present and its phase velocity approach β_{\max} . Thus:

$$\frac{|\varphi_{gz}(\omega_n)|}{|v(\omega_n)|} = \frac{\omega_n}{2\beta_{\max}} \quad \text{as } \omega_n \rightarrow 0 \tag{5}$$

The value of the ratio $|\varphi_{gz}(\omega_n)| / |v(\omega_n)|$ can be obtained at each ω_n applying the straight line approximation. (Lee and Trifunac, 1985). Then torsional component is obtained at each discrete frequency. The IFFT of this spectrum provide the time history of torsional component.

2.2. Procedure Suggested by Li et al.

Lee and Trifunac (1985) assumed the angle of incidence of incoming SH wave to be constant. Thus, θ_0 is equal to θ_c , critical incident angle which is $\sin^{-1}(\beta/\alpha)$.

To take into account the dependence of angle of incidence on the frequencies of impinging harmonics of ground motion, Li et al. (2004) suggested the following procedure:

SH-wave need to satisfy two boundary conditions: (1) free shear stress condition at the ground surface; and (2) free normal stress condition at the ground surface.

The relations between incidence angle, θ_0 , and G are obtained by applying these boundary conditions as follows (Li et al, 2004):

$$G = \frac{2x(1 - K^2x^2)^{1/2}G}{K(1 - 2x^2)}, \quad \theta_0 < \theta_c \quad (6)$$

$$G = \frac{2x(K^2x^2 - 1)^{1/2}G}{K(1 - 2x^2)}, \quad \theta_0 > \theta_c \quad (7)$$

where $G = w/u$, $x = \sin \theta_0$, θ_c is incident critical angle which is $\sin^{-1}(\beta/\alpha)$, $K = \alpha/\beta$, α is velocity of P-wave, u is component of the ground motion in X direction, and w is component of the ground motion in Z direction.

To obtain the rotational component using the above Equations, the translational time histories (u,v,w) are first resolved into their harmonics by applying FFT. Then the value of the ratio, $G = w/u$ is obtained at each discrete frequency. By using the above equations the value of angle of incidence, $\theta_0 = \sin^{-1}x$, is obtained at each frequency. By substituting the values of β , θ_0 and v in Equation (2):

$$\varphi_{gz} = \left(\frac{i\omega \sin\theta_0 v}{2\beta} \right) e^{-i\pi/2} \quad (8)$$

torsional component is obtained at each discrete frequency. The IFFT of this spectrum provides the time history of torsional component.

3. Comparative Study of the Methods

To compare the outcome of the two methods, graphs of the ratio of torsional and translational fourier spectrum amplitudes i.e. $\varphi_{gz}(\omega_n)/v(\omega_n)$ are plotted for earthquakes in India in Uttarkashi, Indo Burma border, Kangra & North-east India (Figure 2).

From Figure 2 it is clear that in method properties of the only top layer (β_{min}) are taken into account, properties of the lower layer (β_{max}) are not considered. For example this method cannot

take into account variation in values of S-wave velocity in various lower layers i.e. $\beta_{\max} = 270, 560, 1130, 2000$. In the first method (Lee and Trifunac, 1985) only one horizontal translational component of the strong motion is used to obtain the torsional time history. Whereas in the method suggested by Li (Li et al, 2004) three synchronized components of strong ground motion are required. But in many cases such synchronized components of same duration (i.e. same number of acceleration readings) are not readily available.

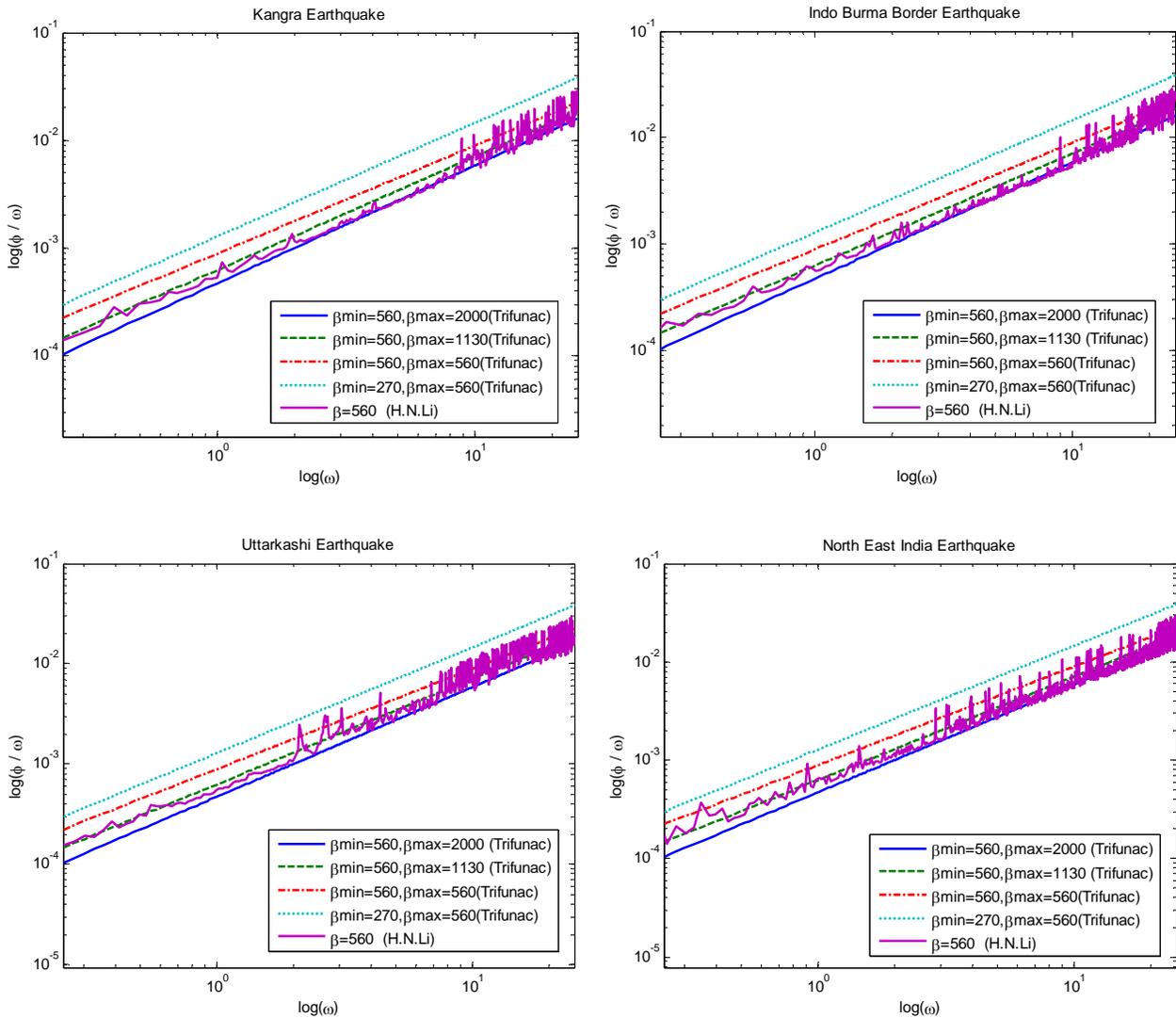


Figure 2. Ratio of torsional to translational fourier spectrum amplitudes

In this study both methods are applied to obtain the torsional components of various earthquakes. The results indicate that consideration of dependence of angle of incidence on frequencies of the impinging harmonics, makes the computation more involved without any significant change in the outcome. It has been also noticed that the method developed by Lee and Trifunac (1985) for

generating torsional accelerograms is an exact analytical solution if it is accepted that the motion occurs in linear elastic, layered half space (Rotational Workshop Report, 2006). Hence, the method developed by Lee and Trifunac (1985) is adopted in this study.

4. Determination of True SH Component

In the method suggested by Lee and Trifunac (1985), the greater of two horizontal components of strong ground motion is used. But greater of two horizontal components will not be necessarily the true SH component. This is because the major and minor principal directions will not always coincide with the directions of recorded horizontal components.

A numerical technique is adopted here to establish major and minor principal directions of ground motions and to obtain true SH component. The orientation of both u and v orthogonal components in x and y directions is rotated by 1 degree at a time as shown in Figure 3.

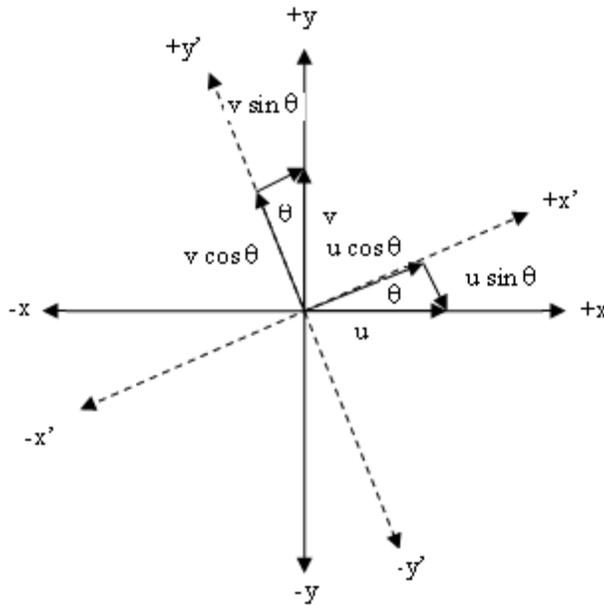


Figure 3. Orientation of x' and y' axes

For each orientation (from $\theta = 1^\circ$ through 180°) of recorded set of u and v a new set of u' and v' components is determined as follows:

$$u' = u \cos \theta + v \sin \theta, \quad v' = -u \sin \theta + v \cos \theta \quad (9)$$

The correlation coefficient is determined for each set of u' and v' . The set for which correlation coefficient is minimum correspond to major and minor principal directions. For example in case of Kangra earthquake minimum correlation is observed for the set of u' and v' corresponding to

$\theta=78^\circ$. Hence the component in that direction is major principal component. Torsional time histories are obtained from this component as well as recorded horizontal component (maximum of u and v) applying Lee and Trifunac (1985) method. Then the torsional response spectrums are obtained for these two torsional time histories for 5% damping ratio. To observe the effect of using the component in major principal direction (for obtaining torsional time history), the ratio of torsional response spectrum for the component in major principal direction to the torsional response spectrum for recorded horizontal component (maximum of u and v) is plotted. Similar procedure is followed for other earthquakes and these ratios are plotted in Figure 4. From these graphs it can be easily noticed that there is considerable increase in torsional response of the structure if the major principal component is taken in to account instead of recorded horizontal component.

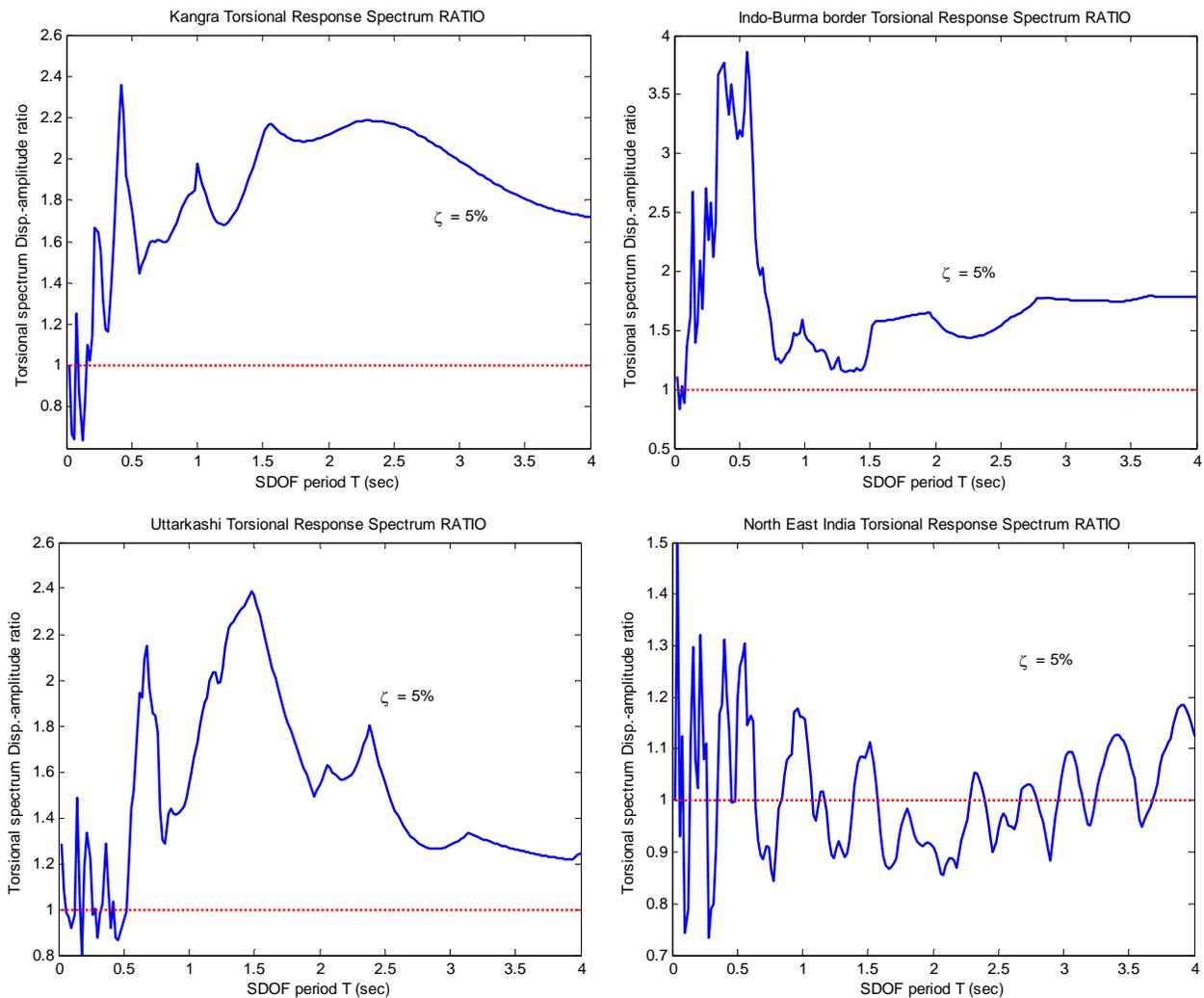


Figure 4. Ratio of torsional response spectrum for the component in major principal direction to the torsional response spectrum for recorded horizontal component

5. Conclusion

It is concluded from this study that the method developed by Lee and Trifunac (1985) is efficient as compared to that developed by Li (2004) for obtaining torsional component from recorded translational components.. It also takes into account the variation in the velocity of S-wave across the depth. The Li method (2004) is more involved without any substantial gains.

There is considerable increase in torsional response of structure if the major principal component is taken into account instead of recorded horizontal component. Hence, it is recommended to use the Lee and Trifunac method (1985), with the modification approach suggested in this paper.

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