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Lexicographic goal programming approach for portfolio optimization

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Abstract

This paper will investigate the optimum portfolio for an investor, taking into account 5 criteria. The mean variance model of portfolio optimization that was introduced by Markowitz includes two objective functions; these two criteria, risk and return do not encompass all of the information about investment; information like annual dividends, S&P star ranking and return in later years which is estimated by using data from a longer history. Thus portfolio selection is a typical multi-objective decision making (MODM) problem. It is well known that Goal Programming (GP), based on preemptive priorities and target values, has been successful in solving MODM problems. In this paper we rank objectives of the MODM model according to weights elicited from Decision Maker's (DM) preferences. Then we obtain goals from DM's opinion. As a guidance for DM, we revise these goals consistent with ranking of objectives by a Linear Programming model in a way that new goals remain as close as possible to DM's goals. After obtaining the goals we solve our MODM problem by a Lexicographic Goal Programming (LGP) model which is constructed by prioritizing objectives. Finally we illustrate our proposed LGP model by a numerical example.

Keywords: Consistent comparison matrix; Lexicographic goal programming (LGP); Multi-objective decision making (MODM); Portfolio optimization problem (POP);

1. Introduction

1.1. Portfolio optimization

The traditional mean–variance model developed by Markowitz [9] has been the basis of portfolio theory. It is the first systematic treatment of investor's conflicting objectives of high return versus low risk. On one hand, the risk of a portfolio, represented by its variance, is to be minimized, while on the other hand the expected return of the portfolio is to be maximized. This traditional portfolio theory has been applied successfully in a variety of situations in which investments are comprised of stocks, bonds, real estate, private equity, and similar instruments.

In recent years, criticism of the basic model has been increasing because of its disregard for individual investor's preferences.

Hallerbach *et al.* [4] showed how multi-attribute nature of investors can be combined into portfolios with the same attributes at the portfolio level. They found that there is a gap between objectives in Markowitz model and investor preferences. The results of research done by Schwehm [12] with several investors as well as analysts indicated that the

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expected return as used in the Markowitz model should be broken down into the criteria 12-month performance, 3-year performance and annual dividend in order to improve the possibilities of the individual investor to articulate subjective preferences. This gain of flexibility seems enough to include these 3 objectives instead of return used in Markowitz's model. The fourth objective, the Standard and Poor's star ranking, describes to what extent an investment fund follows a specific market index and is applied particularly in the case that a portfolio consists exclusively of investment funds. It evaluates the out- or under-performance divided by the tracking error over three years and rewards funds that closely follow the market index. The fifth attribute, the 3 years risk, is used as a measure of the risk of a portfolio. So we conclude that portfolio optimization problem (POP) can be considered as a classical multi-objective decision making (MODM) problem.

1.2. Proposed algorithm justification

According to the above discussion the assumption of the multi-dimensional nature of POP requires determination of priorities by DM to satisfy his/her desires well. Goal Programming (GP) introduced by Charnes and Cooper [1] is an appropriate way for solving MODM problems, like POP. There are different approaches for solving GP models like lexicographic GP (preemptive GP), weighted GP (Archimedean GP), and MINMAX GP (Chebyshev GP) (See Romero [10]). Lexicographic GP is an approach which solves GP problems with priorities on objectives. So the result obtained by this approach is suited for POP and can meet the desires of DM well when assuming POP as an MODM problem; since DM's desires and aim from investment may cause him/her prioritize the objective functions.

Another important issue here is that arbitrary selection of goals and priorities can lead to undesirable results; for example determining large goals for one high and one low priority objectives may avoid satisfying the low priority objective with its goal and cause a large deviation. So after asking goals from DM, we revise them in a way that new goals be consistent with ranking of objectives and remain as close as possible to those of DM. We do this by a linear programming problem. These new goals are as guidance for DM and he/she may or may not accept them. In this case we use DM's goals in our model. The remaining of this paper is organized as follows: In Section 2 first we describe Markowitz's model and its extensions according to the above discussion. Then we develop our model based on Lexicographic Goal Programming (LGP). In order to prioritize objectives we elicit weights of objectives from pairwise comparison matrix provided by DM using weighted least squares method. We revise DM's goals as discussed earlier by a linear programming model and propose them to DM who may or may not accept these goals. Finally, in Section 3 we illustrate our approach by a numerical example.

2. Model description

2.1. Markowitz's model

Markowitz's model is in the following form:

$$\text{Max} \quad \sum_{i=1}^{n} r_i x_i$$

$$\text{Min} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j$$

Subject to: (1)

$$\sum_{i=1}^{n} x_i = M$$

$$x_i \geq 0 \qquad i = 1, \ldots, n$$

where r_i , denotes the expected return of asset *i*. x_i represents the investment portion in asset $i \in \{1,...,n\}$, where *n* denotes the number of available assets; so the first objective function represents the expected return of portfolio which is to be maximized. σ_{ij} , denotes covariance between returns of assets *i,j*. The second objective is the risk of portfolio which is defined as the variance of the expected return. The constraint $\sum_{i=1}^{n} x_i = M$ is capital constraint where M is the total amount of capital. Without loss of generality we can assume that the capital constraint is $\sum_{i=1}^{n} x_i = 1$. From now we assume x_i denotes the percent of capital which is invested in asset *i*. For more details see Markowitz [9].

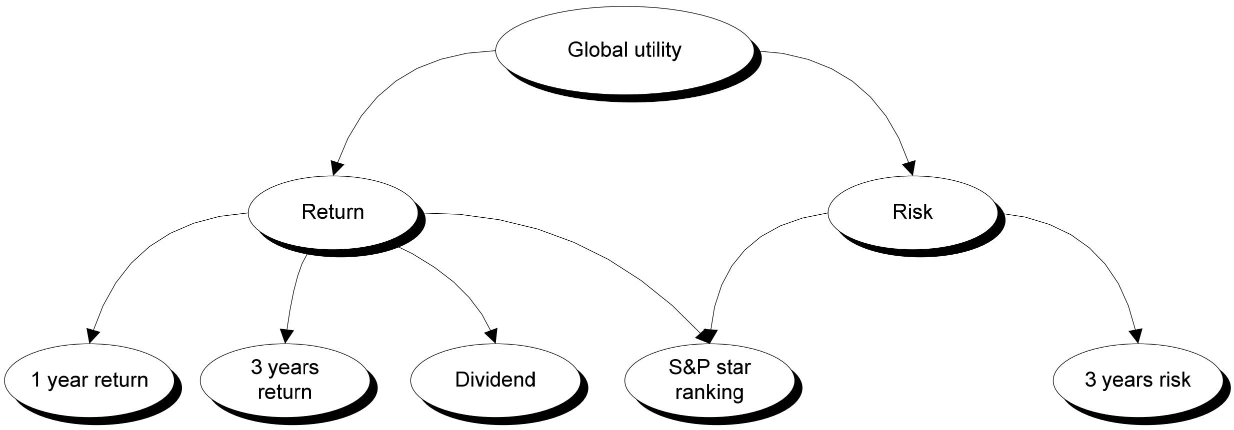

Figure 1. Objective hierarchy based on Markowitz's model.

2. 2. Objective hierarchy

An example for multi-objective POP based on Markowitz's classical model is the one that was found by Schwehm [12]. Figure 1 shows his proposed objective hierarchy extending the classical Markowitz model in t:
he sense that the two classical criteria risk and return are replaced by five more specific objective functions. Matthias Ehrgot *et al.* [3] considers individual preferences through the construction of DM specific utility functions and an additive global utility function.

2.2.1. One year return

If P_{ti} denotes the price of asset i in period t then the return rate in one year will be:

$$r_i = \frac{P_{Ti} - P_{T-1,i}}{P_{T-1,i}} \tag{2}$$

where T is present. In fact r_i is the relative change in price of asset i over the last year. This is a good approximate of expected return over future year. We do not need to know the statistical distribution of r_i.

2.2.2. Three years return

We denote the 3 years return by r_i' and define it in this way:

$$r_i' = \frac{P_{Ti} - P_{T-3,i}}{P_{T-3,i}} \tag{3}$$

Thus the objective functions for 1 year and 3 years return are:

$$f_1(x) = \sum_{i=1}^{n} r_i x_i \tag{4}$$

$$f_2(x) = \sum_{i=1}^{n} r_i' x_i \tag{5}$$

2.2.3. Annual dividend

The annual dividend of an asset is the dividend paid, relative to the highest price of the asset and is denoted by d_i.

$$d_i = \frac{d_i^a}{P_i^h} \tag{6}$$

We denote the annual dividend objective function by:

$$f_3(x) = \sum_{i=1}^{n} d_i x_i \tag{7}$$

2.2.4. Risk

As discussed earlier, the risk function is:

$$f_4(x) = -\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \tag{8}$$

where σ_{ij} is the covariance between return rate of assets i and j, and can be computed from historical data of return rates.

The minus sign shows that this objective must be maximized like all other objectives. For more details see Markowitz [9].

2.2.5 S&P star ranking

The Standard and Poor's Fund Services evaluates the performance of most investment funds contained in their data base on an annual basis which results in a performance ranking (star ranking). The ranking is based on the performance of an investment fund in comparison to the sector index and assigns between one star (for a relatively poor performance) and up to five stars (for a very good performance).

We will assume in the following that the ranking is additive in the sense that the ranking of a portfolio of investment funds can be obtained as the weighted sum of the rankings of the individual investment funds in the portfolio.

Consequently, the fifth objective function can be written as:

$$f_5(x) = \sum_{i=1}^{n} sr_i x_i \tag{9}$$

where sr_i denotes the number of stars assigned to investment fund i. Therefore our MODM model is in this form:

Max $[f_1(x), \ldots, f_5(x)]$

Subject to:

$$\sum_{i=1}^{n} x_i = 1 \tag{10}$$

$$x_i \geq 0; \quad i = 1,\ldots,n$$

2.3. Determining priorities

In order to determine priorities we must obtain the degree of importance of each objective (weights). We also use these weights when revising DM's proposed goals; so there are two reasons that show the necessity of obtaining weights of objectives: determining priorities and using in revision DM's proposed goals.

The weight vector $W = (w_1,\ldots,w_5)^T$ must be elicited from DM's judgments on objectives. Usually, it can be estimated subjectively or objectively. Objective approaches such as the relative entropy method, (see Hwang and Yoon [5] and Wang and Fu [13]) and the factor analysis, (see Wang and Fu [13]) determine the weights of attributes of a multi-attribute decision making problem using decision matrix information, but we don't have any alternative here for forming a decision matrix. On the other hand these approaches take no account of DM's preferences on the relative importance of attributes. Therefore, we use a subjective approach that is extensively used so that DM's preferences can be considered in the determination of attribute weights. The most widely used subjective approach is the method of pairwise comparison matrix on the relative weights of attributes. We use this method on objectives in MODM problem. Let the relative weights of objective functions be represented by:

$$A = \begin{array}{c} \\ w_1 \\ w_2 \\ \vdots \\ w_5 \end{array} \begin{array}{c} \begin{array}{cccc} w_1 & w_2 & \cdots & w_5 \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{15} \\ a_{21} & a_{22} & \cdots & a_{25} \\ \vdots & \vdots & \ddots & \vdots \\ a_{51} & a_{52} & \cdots & a_{55} \end{bmatrix} \end{array} \tag{11}$$

where $a_{ij} = \frac{1}{a_{ji}} > 0$ and $a_{ii} = 1 \ (i, j = 1,\ldots,5)$. According to Saaty's eigenvalue method, (see Saaty [11]), weight vector $W = (w_1,\ldots,w_5)^T$ can be estimated by solving the following eigenvalue problem:

$$AW = \lambda_{\max} W \tag{12}$$

If the multiplicative preference relation A is a precise/consistent comparison matrix on the relative weights of attributes, then Eq. (12) can be simplified as:

$$AW = 5W \tag{13}$$

It is hoped that the multiplicative preference relation provided by DM should be as consistent as possible that is $a_{ik} a_{kj} = a_{ij}$; otherwise eigenvalue specifies a degree of non-consistency about information existing in matrix A (11). Saaty defines Consistency Index (CI) by (14):

$$CI = \frac{\lambda_{\max} - K}{K - 1} \tag{14}$$

where K is the dimension of matrix A, so we can write:

$$CI = \frac{\lambda_{\max} - 5}{5 - 1} = \frac{\lambda_{\max} - 5}{4} \tag{15}$$

Saaty [11] shows that $\lambda_{\max}$ for each invertible matrix is greater than or equal to $K = 5$ (dimension of matrix A) and only in case of having a full consistent matrix, $\lambda_{\max}$ equals K. Thus $\lambda_{\max} - K$ is an appropriate measure of degree of consistency of a matrix. This index after normalizing it by the dimension of matrix A, can be expressed by (14).

Saaty [11] also compares CI with a Random Index (RI). RI has been computed by generating random matrices of dimension K and computing the average of their CI. Consistency Ratio (CR) for a matrix is defined by (16):

$$CR = \frac{CI}{RI} \tag{16}$$

If $CR \le 0.1$, the consistency of matrix A is accepted, otherwise DM must revise in his/her pairwise judgments for further consistency.

If DM gives us a comparison matrix which its consistency is accepted but, not a full consistent matrix it is desirable to determine the weights w_j such that $a_{ij} \approx \frac{w_i}{w_j}$.

Chu *et al*. [2] propose the weighted least square method to obtain weights. We describe this method here. The weights can be obtained by solving the constrained optimization problem:

$$\text{Min} \;\; z = w^T D w = \sum_{i=1}^{5} \sum_{j=1}^{5} (a_{ij} w_j - w_i)^2$$

Subject to:

$$\sum_{i=1}^{5} w_i = 1 \tag{17}$$

where $W^T = (w_1, ..., w_5)$ and $D = [d_{ij}]_{5\times 5}$. The elements in matrix D are:

$$d_{ii} = 5 - 2 + \sum_{j=1}^{5} a_{ji}^{\,2} \qquad i = 1, ..., 5 \tag{18}$$

$$d_{ij} = -(a_{ij} + a_{ji}) \qquad i, j = 1, ..., 5 \quad i \ne j \tag{19}$$

Model (17) is a nonlinear programming model. In order to minimize it, the Lagrangian function is formed:

$$L = w^t D W + \lambda (e^T w - 1) \tag{20}$$

where $e = (1, 1, 1, 1, 1)$ and λ is Lagrangian multiplier.

Differentiating Eq. (20) with respect to w and λ respectively, the following equations are obtained:

$$DW + \lambda e = 0$$

$$e^T w = 1 \tag{21}$$

Eq. (21) forms a set of 6 non-homogeneous linear equations with 6 unknowns. If the minimum of model (17) is $z = 0$ then it is obvious that A is a full consistent matrix and we can obtain weights from Eq. (13). Otherwise, we have $z > 0$ for all w. This shows that D is a positive definite matrix. By the property of positive definite matrix, D is invertible. By solving Eq. (21), we have:

$$W^* = \frac{D^{-1} e}{e^T D^{-1} e}$$

$$\lambda = \frac{-1}{e^T D^{-1} e} \tag{22}$$

Now these W_k 's can be ordered analogously. Without loss of generality suppose that the ranking is:

$$w_1 > w_2 > w_2 > w_4 > w_5 \tag{23}$$

This gives an ordering, i.e. a priority structure among the objectives.

2.4. Deriving LGP model

As discussed earlier, we prioritize objective functions as they can form L prioritized classes $(1 \le L \le 5)$. The LGP model is:

$$\mathrm{Min} V = \left(\sum_{k \in K_1} W_{k1}(\lambda_k^+ d_k^+ + \lambda_k^- d_k^-), \ldots, \sum_{k \in K_L} W_{kL}(\lambda_k^+ d_k^+ + \lambda_k^- d_k^-) \right)$$

Subject to:

$$f_k(x) - d_k^+ + d_k^- = g_k; \quad k = 1,\ldots,5$$

$$\sum_{i=1}^{n} x_i = 1 \tag{24}$$

$$d_k^+ d_k^- = 0; \quad k = 1,\ldots,5$$

$$d_k^+, d_k^-, x_i \ge 0; \ k = 1,\ldots,5; \ i = 1,\ldots,n$$

Let d_k^+, d_k^- be respectively the over attainment and under attainment deviation variables of the k th objective which can not be simultaneously unequal to zero (see last two constraint sets). We also assume λ_k^+ , λ_k^- are deviation weights and take them $\lambda_k^+ = \dfrac{1}{|g_k|}$ and $\lambda_k^- = \dfrac{1}{|g_k|}$ where g_k is the goal of objective f_k . If there is a priority class l in which only one objective lies, then the use of λ_k^+ and λ_k^- is meaningless and we can discard them. These deviation weights make deviation variables of the same scale and dimensionless. In order to show the priorities, the objective of model (24) is shown as a vector V which its l th component denotes the total deviation of l th priority class that is $\sum_{k \in K_l} W_{kl}(\lambda_k^+ d_k^+ + \lambda_k^- d_k^-)$; where K_l is the index set of objectives in l th priority class. Here w_{k1}s denote the differential weights of different objectives in the same class. Since in this paper we specify priorities according to the weights of objectives, those objectives which lie in the same class have equal weight, so the use of differential weights here is redundant and we can drop them from our model. Now we can simplify model (24) as:

$$\mathrm{Min} V = \left(\sum_{k \in K_1} (\lambda_k^+ d_k^+ + \lambda_k^- d_k^-), \ldots, \sum_{k \in K_L} (\lambda_k^+ d_k^+ + \lambda_k^- d_k^-) \right)$$

Subject to:

$$f_k(x) - d_k^+ + d_k^- = g_k; \quad k = 1,\ldots,5$$

$$\sum_{i=1}^{n} x_i = 1 \tag{25}$$

$$d_k^+ d_k^- = 0; \quad k = 1,\ldots,5$$

$$d_k^+, d_k^-, x_i \ge 0; \ k = 1,\ldots,5; \ i = 1,\ldots,n$$

We solve model (25) in L stages. In stage l we optimize only l th component of V and put the constraints $f_k(x) - d_k^+ + d_k^- = g_k \ (k \in K_l)$ in the constraint set of model (25). By using these constraints, we try to satisfy objectives $f(x)$ with attainment values $g_k \ (k \in K_l)$. In subsequent stage we put obtained values of deviation variables, in previous constraint set. This avoids deviating optimized objectives of l th class. Then we optimize $l+1$ th component of V and add new constraint set $f_k(x) - d_k^+ + d_k^- = g_k \,(k \in K_{l+1})$ of $l+1$ th class to our model and we stop this procedure when all of the components of V are optimized or reach an infeasible model.

Further details concerning the algorithm, extensions, and applications can be found in the text of Lee [8], Ignizio [6] and Romero [10].

2.5 Computing degrees of individual optimalities

We know that arbitrary selection of goals may not be feasible and lead to undesirable results when prioritizing objectives. In order to define individual optimality we first obtain the best and worst solution for each objective:

$$\begin{cases} f_k^* = \max\limits_{x \in X} f_k(x) \\ f_k^- = \min\limits_{x \in X} f_k(x) \end{cases} \quad k = 1,\ldots,5$$

where X is the feasible space of our LGP model. As Lai and Hwang, 1994 said linear membership functions expressing degrees of individual optimalities can be defined:

$$\mu_k(f_k)=\frac{f_k(x)-f_k^-}{f_k^*-f_k^-} \tag{26}$$

2.6. Revising proposed goals of DM

We must obtain some goals that are more compatible and consistent with ranking of objectives than DM's goals and as close as possible to those of DM; since goals could hardly be determined without meaningful supporting data. So DM's goals may not be feasible even when we do not prioritize objectives. In order to obtain goals consistent with ranking structure they must satisfy (27):

$$\mu_1(g_1)>\ldots>\mu_5(g_5) \tag{27}$$

We can write constraint (27) in the form of 4 separate constraints:

$$\begin{aligned}&\mu_1(g_1)-\mu_2(g_2)\geq 0\\&\vdots\\&\mu_4(g_4)-\mu_5(g_5)\geq 0\end{aligned} \tag{28}$$

By using these fuzzy membership functions as defined by (26) we must make individual optimalities of revised goals as close as possible to individual optimalities of DM's goal while preserving higher priority objective with higher individual optimality of revised goal. That is:

$$\text{Min}\ \sum_{i=1}^{n} w_i\left|\mu_i(g_i)-\mu_i(g_i^{DM})\right| \tag{29}$$

Subject to:

$$\mu_i(g_i)-\mu_{i+1}(g_{i+1})\geq 0 \qquad i=1,\ldots,4$$

In the objective function of model (29) we try to minimize the difference between individual optimality of DM's goals and obtained goals. Since μ_i makes its parameter dimensionless, we use it in the objective function of model (29). The constraints 1 through 4 tell us, the higher priority objective, the larger its individual optimality. w_i s are the weights obtained from Eq. (22). Model (29) is a non-linear one, so we construct an equivalent linear model here. Let $\varepsilon_i=\mu_i(g_i)-\mu_i(g_i^{DM})$ for $i=1,\ldots,5$, $\varepsilon_i^+=\frac{\varepsilon_i+|\varepsilon_i|}{2}$ and $\varepsilon_i^-=\frac{-\varepsilon_i+|\varepsilon_i|}{2}$. Based on ε_i^+ and ε_i^-, ε_i and $|\varepsilon_i|$ can be expressed as:

$$\begin{aligned}&\varepsilon_i=\varepsilon_i^+-\varepsilon_i^-, i=1,\ldots,5\\&|\varepsilon_i|=\varepsilon_i^++\varepsilon_i^-, i=1,\ldots,5\end{aligned} \tag{30}$$

where $\varepsilon_i^+.\varepsilon_i^-=0$ for $i=1,\ldots,5$. Now model (29) by formula $\mu_i(g_i)=\varepsilon_i^+-\varepsilon_i^-+\mu_i(g_i^{DM})$ and Eq. (30) can be rewritten as:

$$\text{Min}\ \sum_{i=1}^{5} w_i(\varepsilon_i^++\varepsilon_i^-)$$

Subject to:

$$\varepsilon_i^+-\varepsilon_i^-+\mu_i(g_i^{\text{DM}})-(\varepsilon_{i+1}^+-\varepsilon_{i+1}^-+\mu_{i+1}(g_{i+1}^{\text{DM}}))\geq 0$$

$$i=1,\ldots,4$$

$$\varepsilon_i^+\varepsilon_i^-=0 \qquad i=1,\ldots,5$$

$$\varepsilon_i^+,\varepsilon_i^-\geq 0 \quad i=1,\ldots,5 \tag{31}$$

Solving model (31) will provide goals for model (25). These new goals make guidance for DM; however they may not be acceptable by DM while they are consistent with DM's preferences on objectives. In this case we can use DM's goals in model (25). Any way DM can choose one of these two sets of goals arbitrarily. Figure 2 shows the flowchart of this algorithm. In the next section, an example will be used to illustrate the functioning and behavior of the algorithm.

3. Numerical illustration

We test our approach, by solving a POP. Let us use the set of historical annual data of 10 assets. We also assume that an investor wants to allocate one unit of wealth among some of these assets on which historical data of objectives have been calculated using the formulas described earlier.

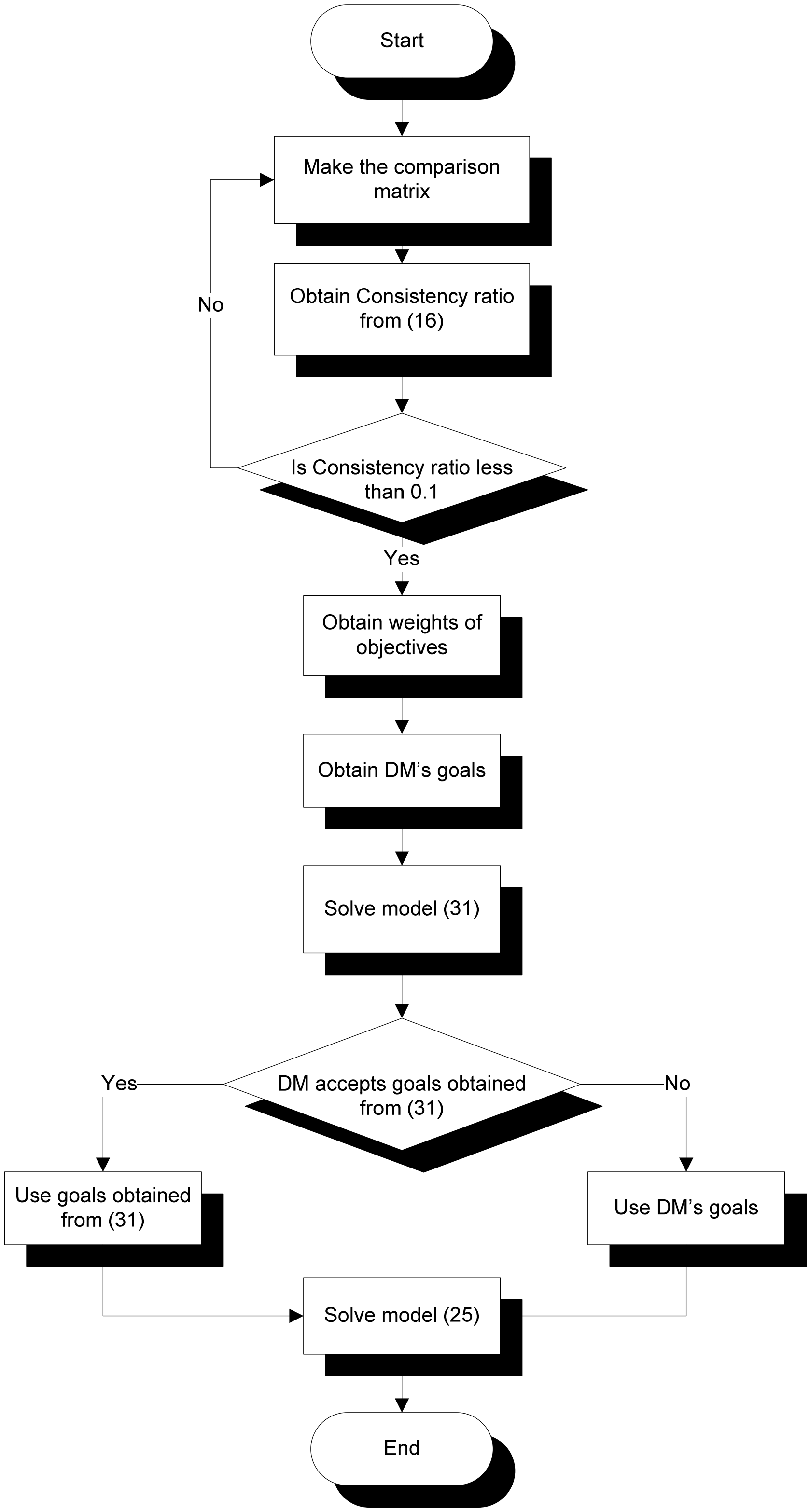

Figure 2. Flowchart for the proposed algorithm.

We assume that the pairwise comparison matrix made by DM is:

$$A = \begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} \\ 4 & 1 & 3 & 4 & 3 \\ 2 & \frac{1}{3} & 1 & 2 & \frac{3}{2} \\ 2 & \frac{1}{4} & \frac{1}{2} & 1 & \frac{1}{2} \\ 3 & \frac{1}{3} & \frac{2}{3} & 2 & 1 \end{bmatrix}$$

The maximum eigenvalue of the above matrix is $\lambda_{\max} = 5.14$ and by using Eq. (15) we have:

$$CI = \frac{\lambda_{\max} - 5}{4} = \frac{5.14 - 5}{4} = 0.04$$

and finally from (16) the consistency ratio is:

$$CR = \frac{CI}{RI} = \frac{0.04}{1.11} = 0.04 < 0.1$$

So we can conclude that DM's judgments in matrix A is consistent.

By using formulas (18) and (19) we calculate matrix D:

$$D = \begin{bmatrix} 37 & -4.25 & -2.5 & -2.5 & -3.34 \\ -4.25 & 4.36 & -3.34 & -4.25 & -3.34 \\ -2.5 & -3.34 & 13.95 & -2.5 & -2.17 \\ -2.5 & -4.25 & -2.5 & 28.25 & -2.5 \\ -3.34 & -3.34 & -2.17 & -2.5 & 15.62 \end{bmatrix}$$

And the vector of weights from formula (22) is:

$$\frac{D^{-1}e}{e^T D^{-1}e} = \begin{bmatrix} 0.0886 \\ 0.4636 \\ 0.1754 \\ 0.1093 \\ 0.1631 \end{bmatrix} \tag{32}$$

Thus the obtained ranking is:

$$f_2 > f_3 > f_5 > f_4 > f_1$$

Now we obtain max and min of each objective in feasible space. Results are shown in Table 1.

So we can compute μ_i s from (26):

$$\mu_1(f_1) = \frac{f_1 - 2.56}{2.84} \qquad \mu_2(f_2) = \frac{f_2 - 5}{6.45}$$

$$\mu_3(f_3) = \frac{f_3}{1.6} \qquad \mu_4(f_4) = \frac{f_4 + 0.26}{0.23}$$

$$\mu_5(f_5) = \frac{f_5 - 1}{3}$$

We also assume that DM's goals are:

$$g_1^{DM} = 5 \qquad g_2^{DM} = 7 \qquad g_3^{DM} = 1.5$$

$$g_4^{DM} = 0.1 \qquad g_5^{DM} = 4$$

Now we form model (31) in order to obtain new goals. Note that our ranking is $f > f > f > f > f$. So we have:

$$\text{Min}\ [0.0886 \quad 0.4636 \quad 0.1754 \quad 0.1093 \quad 0.1631] \begin{bmatrix} \varepsilon_1^+ + \varepsilon_1^- \\ \varepsilon_2^+ + \varepsilon_2^- \\ \varepsilon_3^+ + \varepsilon_3^- \\ \varepsilon_4^+ + \varepsilon_4^- \\ \varepsilon_5^+ + \varepsilon_5^- \end{bmatrix}$$

Subject to:

$$\begin{aligned} &\varepsilon_2^+ - \varepsilon_2^- - \varepsilon_3^+ + \varepsilon_3^- + \mu_2(7) - \mu_3(1.5) \ge 0 \\ &\varepsilon_3^+ - \varepsilon_3^- - \varepsilon_5^+ + \varepsilon_5^- + \mu_3(1.5) - \mu_5(4) \ge 0 \\ &\varepsilon_5^+ - \varepsilon_5^- - \varepsilon_4^+ + \varepsilon_4^- + \mu_5(4) - \mu_4(0.1) \ge 0 \\ &\varepsilon_4^+ - \varepsilon_4^- - \varepsilon_1^+ + \varepsilon_1^- + \mu_4(0.1) - \mu_1(5) \ge 0 \\ &\varepsilon_i^+, \varepsilon_i^- \ge 0, i = 1,\ldots,5 \\ &\varepsilon_i^+ . \varepsilon_i^- = 0 \quad i = 1,\ldots,5 \end{aligned} \tag{33}$$

where w_i s are used from (32).

Solving model (33) with lingo gave us revised goals as follows:

$$g_1 = 5 \qquad g_2 = 12.29 \qquad g_3 = 1.81$$

$$g_4 = -0.01 \qquad g_5 = 4.39$$

Now model (25) is completed as described earlier. Since there is no priority class in which more than one objective lies, thus we can discard deviation weights.

$$\text{Min } (d_2^+ + d_2^-, d_3^+ + d_3^-, d_5^+ + d_5^-, d_4^+ + d_4^-, d_1^+ + d_1^-)$$

Subject to:

$$f_1(x) - d_1^+ + d_1^- = 5$$

$$f_2(x) - d_2^+ + d_2^- = 12.29$$

$$f_3(x) - d_3^+ + d_3^- = 1.81$$

$$f_4(x) - d_4^+ + d_4^- = -0.01$$

$$f_5(x) - d_5^+ + d_5^- = 4.39 \tag{34}$$

$$\sum_{i=1}^{n} x_i = 1$$

$$d_k^+ . d_k^- = 0 \quad , \quad d_k \geq 0 \; ; \quad k = 1,\ldots,5$$

$$x_i \geq 0 \; ; \quad i = 1,\ldots,10$$

We solve this model by lexicographic GP using revised goals and DM's goals. Results are shown in Table 2. We can see that using revised goals leads to less deviation than using DM's goals especially in high priority objectives which are more important from DM's point of view. In each case the first priority objective has no deviation while f_3 which is in second priority class has deviated from DM's goal and no deviation from revised goal. This is also true about f_5 which is in third priority class. It has no deviation from revised goal while has from DM's goal. Both f_4 and f_1 have deviation from two types of goals while f_1 has deviated from revised goal less than that of DM. So we see that revised goals lead to better results than use of DM's goals when solving our model with LGP; however DM can arbitrarily choose one of these two sets of goals. We also solve our model by ordinary GP and compare results with LGP approach. Results are shown in Table 3. The ordinary GP model is:

$$\text{Min } \frac{(d_1^+ + d_1^-)}{5} + \frac{(d_2^+ + d_2^-)}{12.29} + \frac{(d_3^+ + d_3^-)}{1.81} + \frac{(d_4^+ + d_4^-)}{0.01} + \frac{(d_5^+ + d_5^-)}{4.39}$$

Subject to:

$$f_1(x) - d_1^+ + d_1^- = 5$$

$$f_2(x) - d_2^+ + d_2^- = 12.29$$

$$f_3(x) - d_3^+ + d_3^- = 1.81$$

$$f_4(x) - d_4^+ + d_4^- = -0.01$$

$$f_5(x) - d_5^+ + d_5^- = 4.39 \tag{35}$$

$$\sum_{i=1}^{n} x_i = 1$$

$$d_k^+ . d_k^- = 0 \quad , \quad d_k \geq 0 \; ; \quad k = 1,\ldots,5$$

$$x_i \geq 0 \; ; \quad i = 1,\ldots,10$$

As shown in Table 3, objective f_2 which lies in first priority class is fully satisfied with both goals. But f_3 which is in second priority class is deviated from both goals while it has no deviation when using LGP approach with revised goal. In Table 3, f_4 has no deviation from both goals while it is desirable for DM to reach a result with f_3 less deviated from its goal rather than f_4. This desirable result is obtained by using LGP approach with revised goals as Table 2 shows. Since DM prioritizes objectives, it's more desirable for DM to reach a solution with better satisfaction of higher priority objectives like f_2, f_3, f_5 which are fully satisfied by LGP approach with revised goals (see Table2). So we can infer here that DM is more pleased with results obtained from lexicographic approach than ordinary GP. Table 4 shows selected portfolios by using ordinary GP and lexicographic GP with revised and DM's goals.

Table 1. Maximum and minimum of each objective.

Objective	Max	Min
f_1	5.4	2.56
f_2	11.45	5
f_3	1.6	0
f_4	-0.03	-0.26
f_5	4	1

Table 2. Results obtained for model (12).

		f_1	f_2	f_3	f_4	f_5
DM's goals	d_k^+	0	0	0	0	0
	d_k^-	1.0228	0	0.5960	0.0210	1.1299
	Achieved Goal	3.9772	7	0.904	0.079	2.8701
Revised goals	d_k^+	0	0	0	0	0
	d_k^-	0.0560	0	0	0.1656	0
	Achieved Goal	4.234	12.29	1.81	-0.1756	4.39

Table 3. Results obtained by using ordinary GP.

		f_1	f_2	f_3	f_4	f_5
DM's goals	d_k^+	0	0	0	0	0
	d_k^-	1.0948	0	0.6962	0	0.9255
	Achieved Goal	3.9052	7	0.8083	0.1	3.0745
Revised goals	d_k^+	0.2348	0	0	0	0
	d_k^-	0	0	0 .2438	0	0.0681
	Achieved Goal	5.2348	12.29	1.5662	-0.01	4.3219

Table 4. Selected portfolios using different approaches and goals.

Asset	Selected portfolio			
	Ordinary GP		Lexicographic GP	
	DM's goals	revised goals	DM's goals	revised goals
x_1	0.000000	0.000000	0.000000	0.000000
x_2	0.000000	0.6190456	0.000000	0.3129872
x_3	0.000000	0.000000	0.000000	0.000000
x_4	0.000000	0.000000	0.000000	0.000000
x_5	0.000000	0.000000	0.000000	0.000000
x_6	0.000000	0.000000	0.000000	0.000000
x_7	0.4627595	0.000000	0.5649718	0.2720128
x_8	0.0560979	0.07626602	0.000000	0.1912769
x_9	0.000000	0.000000	0.000000	0.000000
x_{10}	0.4811426	0.3046883	0.4350282	0.2237231

4. Conclusions

In this paper, we interceded for preferences and interests of DM in the POP favorably. We consider the multi-attribute nature of desires, tastes and preferences of DM. In order to obtain some results which are more desirable for DM, these subjective and mental features of DM should be considered in the objective functions of model. So we considered multiple objectives to tackle the POP.

The objectives considered by DM, have different or probably same priorities from his/her point of view. So in order to fulfill DM's desires and tastes, it had better that DM specify degrees of importance of different Objectives from his/her viewpoint. Since the LGP is answerable to these types of situations, according to above discussion we utilized it in our paper.

Ultimately the solution of proposed approach is closer to the mentality of DM. In addition the approach proposed in this paper is a general one and can be applied to any POP with arbitrary objective functions.

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