*Journal of Industrial Engineering International* *Islamic Azad University, Tehran South Branch*
*September 2005, Vol. 1, No. 1, 50 - 58*

# An efficiency measurement model in fuzzy environments, using data envelopment analysis

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**Abstract**

Data Envelopment Analysis (DEA) is a technique used to compare efficiency in various sectors such as hospitals, chain stores, and dealerships. It represents a set of linear programming techniques and uses deterministic data (inputs and outputs), in stable conditions. The DEA technique cannot be used when there is data with indeterministic nature, or when there is an environment with dynamic conditions. To address this problem, DEA models can be developed based on linear programming in fuzzy environments (Fuzzy DEA).

Most Fuzzy DEA models introduced in the literature are parametric models based on alpha cuts. However, the model introduced in this study is non-parametric and uses fuzzy L-R numbers. From the theory point of view, the objective of this study is to develop a simple and effective Fuzzy DEA model. From the practicality point of view, this model can be applied to assess many issues associated with qualitative factors.

**Keywords:** Fuzzy DEA; Efficiency measurement; Ranking model

## 1. Introduction

The method Data Envelopment Analysis (DEA) was established in 1978 by Charnes, Cooper and Rhodes (CCR) [3], but it took some years until this technique was integrated in the fuzzy logic to introduce the first model of fuzzy data envelopment analysis. This undertaking was realized in 1992. The said concept was represented by Sengupta [12] in 1992 based on the method of Zimmermann [16]. The method of Zimmermann is designed to transform the Fuzzy Linear Programming (FLP) into crisp linear programming, in which the objective function and constraints are fuzzy. In this method, in order to transform the objective function, the decision maker shall determine the Min. (Max.) acceptable rate for the objective function in the (Min.) Max. condition and also the rate of tolerance limit from the objective function as well as the rate of the tolerance limit from each one of the constraints. In this model, the objective function is transformed to one constraint. The model Sengupta may be applied in the condition of multi-input and one output. In 1998, Girod and Triantis [14] introduced their model on the base of the method of Carlson and Korhonen [2]. These two considered the possible rates for parameters of the model Fuzzy Linear Programming (FLP) coefficient of objective function, coefficient of constraints and Right-Hand-Side in the shape of intervals, which for outputs, the lower bound of these intervals represents the

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situation of definiteness and without risk, and the upper bound represents the impossible situation. Girod and Triantis have, regarding this manner, undertaken a model-making from the model of BCC [1] in a fuzzy environment. Kao and Liu [7] represented in the 2000 a new method for finding out a membership function of the rates of the fuzzy efficiency, while the inputs and outputs are fuzzy numbers. Their method has been established on the basis of α – Cuts and extension principle. In 2001, Guo and Tanaka [6] published their model using the Possibilistic Programming Techniques. They set forth this concept, as DEA is a boundary method which is sensitive against the rates of outliers, and with respect to this matter that in most cases a part of the data has perceptual meanings and the other part has quantitative rates, therefore, the efficiency measurement with these different data in the model of DEA and its extension to the fuzzy environment is a complicated matter. In order to solve this complication, they first represented a new model named DEARA by using the concepts of Regression Analysis-RA in the model of DEA standard and then represented their model based on the combined model of DEARA and in the form of a Fuzzy Goal Programming (FGP). Saati , Memariani and Jahanshahlou [11] introduced a new model in 2000 based on the method of α – Cuts. Parallel to this model, they represented an initiative ranking method of fuzzy numbers.

Leon, Liern, Ruiz and Sirvent [8] represented their own Model "Fuzzy DEA" by using the BCC Model and the Fuzzy numbers of L-R in 2003.They changed the fuzzy constraints into crisp condition by applying the Ramik and Rimanek Principle and have enabled this model on efficiency measurement in the levels of different possibilities ($0 \le h \le 1$) by integrating a variable in the model as the possibility level (h).

Despotis and Smirlis [4] represented a new interval DEA model by using the interval arithmetic. In this model the inputs and outputs have interval values ($X_{ik} = [x_{ik}^L, x_{ik}^U]$). To measure efficiency, two models shall be solved for each unit. One results in the lower bound and the other one in the upper bound. Considering that in each one of these two models different kinds of constraint sets are applied, therefore, different frontier values shall result. Regarding this matter, there is left no comparing possibility between the efficiency values of different units. In order to remove this problem, Wang and Yang [15] represented a model by applying the interval arithmetic in 2004. Also in this model, in order to calculate the efficiency value of each unit, two models shall be solved, one of which results in the lower bound and the other one in the upper bound for the efficiency value of the relevant unit. In these models similar constraint sets are used, and the existing problem in the Despotis and Smirlis model is herewith removed and in this way there shall be the comparing possibility between the efficiencies of different units.

The model that is being represented in this study is the non-parametric model and it extends the CCR model in the fuzzy environments by applying the Fuzzy L-R numbers. In order to achieve this target, the objective function and fuzzy constraints are changed into crisp conditions, by using the Ramik and Rimanek principle and initiative methods. At the end, a method for ranking the efficient units is represented.

## 2. Data Envelopment Analysis in Fuzzy Environment

Initially, some important meanings in relation to the ranking of L-R fuzzy numbers are described and then by using these meanings, an initiative method for transforming fuzzy objective function to infinite situation is represented.

### 2.1. Some important meanings regarding the ranking of L-R fuzzy numbers

**Definition.** Take note of the $\tilde{M}$ and $\tilde{N}$ fuzzy numbers (where, '~' indicates the fuzziness. ):

$$\begin{array}{lll} \tilde{M}=(m,\alpha,\beta)_{LR} & m^L=m-\alpha & m^R=m+\beta \\ \tilde{N}=(n,\eta,\gamma)_{LR} & n^L=n-\eta & n^R=n+\gamma \end{array} \tag{1}$$

The maximum rate of the $\tilde{M}$ and $\tilde{N}$ fuzzy numbers, it means $\tilde{M} \vee \tilde{N}$, has the following membership function:

$$\mu_{\tilde{M} \vee \tilde{N}}(r) = \sup_{r=s\vee t}\left[\mu_{\tilde{M}}(s) \wedge \mu_{\tilde{N}}(t)\right] \tag{2}$$

On the basis of the definition of fuzzy maximum, Dubois and Prade [5] represented the following definition:

- If $\tilde{M}$ and $\tilde{N}$ are fuzzy numbers, the following relationship shall be established between them:

$$\tilde{M} \succ\approx \tilde{N} \quad \Leftrightarrow \quad \tilde{M} \vee \tilde{N} = \tilde{M} \tag{3}$$

Ramik and Rimanek [10] and Tanaka [13] represented, by using the above-indicated definition, a method for transforming fuzzy non-equivalence into a definite situation by applying the L-R fuzzy numbers:

- The principle of Ramik & Rimanek is as follows:

$$\tilde{M} \succ\approx \tilde{N} \quad \Leftrightarrow \quad \begin{cases} m \geq n \\ m^L \geq n^L \\ m^R \geq n^R \end{cases} \tag{4}$$

- The Tanaka principle is defined as follows:

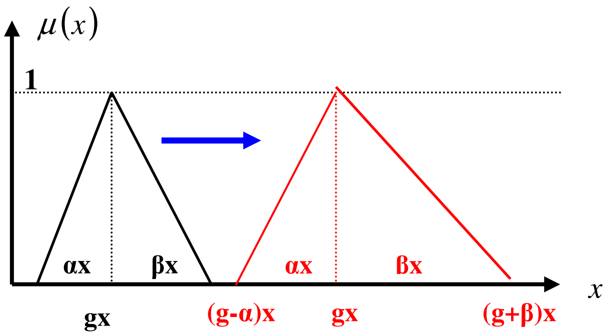
$$\tilde{M} \succ\approx h \times \tilde{N} \quad \Leftrightarrow \quad \begin{cases} m \geq h \times n \\ m^L \geq h \times n^L \\ m^R \geq h \times n^R \end{cases} \tag{5}$$

### 2.2. Transformation of fuzzy objective function into definite situation

$Max\ \tilde{G}x = Max(g, a, \beta)_{LR} \times x = Max(gx, ax, \beta x)_{LR}$

In order to maximize the L-R fuzzy number, we should maximize its rate of suprimum, namely gx, and the rate of its right-side interval, namely $\beta x$. In other words we should transfer the $\tilde{G}x$ fuzzy number to the right side. This means that we shall consider the minimum acceptable decision against $\tilde{D}_0 = (L_1, a = 0, \beta = L_2)_{LR}$. In this way, the Max $\tilde{G}x$ objective function shall be transformed to the definite situation as follows:

$$\begin{aligned} & \max\ L_1 + L_2 \\ & \text{Subject to:} \\ & \quad gx \geq L_1 \\ & \quad \beta x \geq L_2 \\ & \quad \alpha x \geq 0 \\ & \quad L_1, L_2 \geq 0 \end{aligned} \tag{6}$$




### 2.3. Transformation of fuzzy constraints into definite situation

In order to transform the non-equal constraints, we apply the principle of Ramik and Rimanek as well as the principle of Tanaka. Now the mode of transforming of fuzzy equal constraints into definite ones is defined:

Fuzzy numbers $\tilde{N} = (n, \eta, \gamma)_{LR}$ and $\tilde{M} = (m, \alpha, \beta)_{LR}$ are at disposal:

$$\tilde{M} \approx \tilde{N} \quad \Rightarrow \quad \left| \tilde{M} - \tilde{N} \right| \approx 0 \tag{7}$$

When we say that two variables are equal, they are exactly equal in the definite situation, but we say they are approximately equal in the fuzzy situation. In other words the difference between two equal fuzzy numbers can be not zero. According to the definition, the difference of $\tilde{M}$ and $\tilde{N}$ fuzzy numbers is equal to $\tilde{M} - \tilde{N} = (m, a, \beta)_{LR} - (n, \eta, \gamma)_{LR} = (m - n, a + \gamma, \beta + \eta)_{LR}$.

Therefore, for transformation of ($\approx$) of fuzzy equal into (=) definite situation in (7), we define the variable($\psi$) as the maximum permissible deviation and equal to the support of fuzzy number resulting from the difference between $\tilde{M}$ and $\tilde{N}$ fuzzy number:

$$\psi \leq (\alpha + \beta) + (\eta + \gamma) \tag{8}$$

As the variable ($\psi$) with the title of maximum permissible deviation is defined, therefore, (=) is changed to ($\leq$). Also by transforming of the definite quantity into two smaller or equal constraints, we shall have:

$$\left| \tilde{M} - \tilde{N} \right| \approx 0 \Rightarrow \left| \tilde{M} - \tilde{N} \right| \leq \psi \Rightarrow \begin{cases} \tilde{M} - \tilde{N} \leq \psi \\ \tilde{N} - \tilde{M} \leq \psi \\ \psi \leq (\alpha + \beta) + (\eta + \gamma) \end{cases} \tag{9}$$

$$\Rightarrow \begin{cases} \tilde{M} \leq \tilde{N} + \psi \\ \tilde{M} \geq \tilde{N} - \psi \\ \psi \leq (\alpha + \beta) + (\eta + \gamma) \end{cases}$$

Now with respect to the Ramik and Rimanek principle, the Formula (9) is transformed as per the following:

$$\begin{cases} \tilde{M} \le \tilde{N} + \psi \quad \Rightarrow \begin{cases} m^L \le n^L + \psi \\ m \le n + \psi \\ m^R \le n^R + \psi \end{cases} \\ \tilde{M} \ge \tilde{N} - \psi \quad \Rightarrow \begin{cases} m^L \ge n^L - \psi \\ m \ge n - \psi \\ m^R \ge n^R - \psi \end{cases} \\ \psi \le (\alpha + \beta) + (\eta + \gamma) \end{cases} \tag{10}$$

## 3. Proposed model

The model that is being represented in this study is a model in which the inputs and output(s) of the units of this study have a fuzzy nature, which is shown by the L-R fuzzy numbers.

$\tilde{x}_{ik} = (x_{ik}, a = x_{ik} - x^l_{ik}, \beta = x^R_{ik} - x_{ik})_{LR}$

$\tilde{x}_{ik}$: Fuzzy input i for the unit k

$\tilde{y}_{jk} = (y_{jk}, a = y_{jk} - y^l_{jk}, \beta = x^R_{jk} - y_{jk})_{LR}$

$\tilde{y}_{jk}$: Fuzzy output j for the unit k

We know that the initial and nonlinear DEA model is as follows ($v_i$ = weight assigned to input $i$ and $u_j$ = weight assigned to output $j$):

$$\max \frac{\sum_{j=i}^{m} u_j y_{jc}}{\sum_{i=1}^{n} v_i x_{ic}}$$

Subject to: (11)

$$\begin{aligned} &\frac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{jk}} && \le 1 && k = 1,\dots,r \\ &u_j && \ge \varepsilon && j = 1,..,m \\ &v_i && \ge \varepsilon && i = 1,..,n \end{aligned}$$

Therefore, the (11) fuzzy model shall be as per the following:

$$\max \frac{\sum_{j=i}^{m} u_j y_{jc}}{\sum_{i=1}^{n} v_i x_{ic}}$$

Subject to: (12)

$$\begin{aligned} &\frac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{jk}} && \prec= 1 && k = 1,\dots,r \\ &u_j && \ge \varepsilon && j = 1,..,m \\ &v_i && \ge \varepsilon && i = 1,..,n \end{aligned}$$

With assumption of $\sum_{i=1}^{n} v_i \tilde{x}_{ic} \approx \tilde{1}$ the model (12) shall be as per the following:

$$\max \sum_{j=1}^{m} u_j \tilde{y}_{jc}$$

Subject to: (13)

$$\begin{aligned} &\sum_{i=1}^{n} v_i \tilde{x}_{ic} && \approx \tilde{1} && \\ &\frac{\sum_{i=1}^{n} u_j \tilde{y}_{jk}}{\sum_{i=1}^{n} v_i \tilde{x}_{ik}} && \prec\approx 1 && k = 1,\dots,r \\ &u_j && \ge \varepsilon && j = 1,..,m \\ &v_i && \ge \varepsilon && i = 1,..,n \end{aligned}$$

With respect to (6), the objective function shall be transformed as per the following:

$$\max\ L_1 + L_2$$

Subject to: (14)

$$\begin{aligned} &\sum_{j=1}^{m} u_j y_{jc} && \ge L_1 \\ &\sum_{j=1}^{m} u_j (y^R_{jc} - y_{jc}) && \ge L_2 \\ &\sum_{j=1}^{m} u_j (y_{jc} - y^L_{jc}) && \ge 0 \\ &L_1, L_2 && \ge 0 \end{aligned}$$

Now the mode of transformation of the fuzzy constraints of the model (13) into the definite situation is being defined:

$$I)\ \sum_{i=1}^{n} v_i\ \tilde{x}_{ic} \approx \tilde{1} \qquad (15)$$

With respect to the definition of the section (2-3), the equal constraint (15) is transformed into the following:

$$\begin{cases} \sum_{i=1}^{n} v_i\ \tilde{X}_{ic} \le \tilde{1}+\psi_c \ \Rightarrow \begin{cases} \sum_{i=1}^{n} v_i x_{ic}^L \le 1^L+\psi_c \\ \sum_{i=1}^{n} v_i x_{ic}^L \le 1+\psi_c \\ \sum_{i=1}^{n} v_i x_{ic}^L \le 1^R+\psi_c \end{cases} \\ \psi_c \le \sum_{i=1}^{n} v_i\left(\alpha_{ic}+\beta_{ic}\right)+\left[\left(1-1^L\right)+\left(1^R-1\right)\right] \\ \sum_{i=1}^{n} v_i\ \tilde{X}_{ic} \ge \tilde{1}-\psi_c \ \Rightarrow \begin{cases} \sum_{i=1}^{n} v_i x_{ic}^L \ge 1^L-\psi_c \\ \sum_{i=1}^{n} v_i x_{ic}^L \ge 1-\psi_c \\ \sum_{i=1}^{n} v_i x_{ic}^L \ge 1^R-\psi_c \end{cases} \end{cases} \qquad (16)$$

With assumption of $\tilde{1}=(1,0,0)_{LR}$, the relations of (16) shall be transformed as per the following:

$$\begin{cases} \sum_{i=1}^{n} v_i\ \tilde{X}_{ic} \le 1+\psi_c \ \Rightarrow \begin{cases} \sum_{i=1}^{n} v_i x_{ic}^L \le 1+\psi_c \\ \sum_{i=1}^{n} v_i x_{ic} \le 1+\psi_c \\ \sum_{i=1}^{n} v_i x_{ic}^R \le 1+\psi_c \end{cases} \\ \psi_c \le \sum_{i=1}^{n} v_i\left(\alpha_{ic}+\beta_{ic}\right) \\ \sum_{i=1}^{n} v_i\ \tilde{X}_{ic} \ge 1-\psi_c \ \Rightarrow \begin{cases} \sum_{i=1}^{n} v_i x_{ic}^L \ge 1-\psi_c \\ \sum_{i=1}^{n} v_i x_{ic} \ge 1-\psi_c \\ \sum_{i=1}^{n} v_i x_{ic}^R \ge 1-\psi_c \end{cases} \end{cases} \qquad (17)$$

In the relations of (17) it is obvious that in the first three non-equation, if it be $\sum_{i=1}^{n} v_i x_{ic}^R \le 1+\psi_c$, so the other non-equations shall be convinced, and in the three second non-equations, if it be $\sum_{i=1}^{n} v_i x_{ic}^L \ge 1-\psi_c$, so the other non-equations shall also be evaluated, therefore we shall have:

$$\begin{cases} \sum_{i=1}^{n} v_i\ \tilde{X}_{ic} \le 1+\psi_c \ \Rightarrow\ \sum_{i=1}^{n} v_i x_{ic}^R \le 1+\psi_c \\ \sum_{i=1}^{n} v_i\ \tilde{X}_{ic} \ge 1-\psi_c \ \Rightarrow\ \sum_{i=1}^{n} v_i x_{ic}^L \ge 1-\psi_c \\ \psi_c \le \sum_{i=1}^{n} v_i\left(\alpha_{ic}+\beta_{ic}\right) \end{cases} \qquad (18)$$

Finally, the constraint (15) shall be transformed into the constraints (18).

II ) (19)

$$\left(\frac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{ik}}, \frac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{ik}} - \frac{\sum_{j=1}^{m} u_j y_{jk}^L}{\sum_{i=1}^{n} v_i x_{ik}^R}, \frac{\sum_{j=1}^{m} u_j y_{jk}^R}{\sum_{i=1}^{n} v_i x_{ik}^L} - \frac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{ik}}\right)_{LR} \preceq\approx 1$$

$$\left\{ m=\frac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{ik}},\ \alpha=\frac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{ik}}-\frac{\sum_{j=1}^{m} u_j y_{jk}^L}{\sum_{i=1}^{n} v_i x_{ik}^R},\ \beta=\frac{\sum_{j=1}^{m} u_j y_{jk}^R}{\sum_{i=1}^{n} v_i x_{ik}^L}-\frac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{ik}} \right\}$$

$$\Rightarrow m=\frac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{ik}},\ m^L=m-\alpha=\frac{\sum_{j=1}^{m} u_j y_{jk}^L}{\sum_{i=1}^{n} v_i x_{ik}^R},$$

$$m^R=m+\beta=\frac{\sum_{j=1}^{m} u_j y_{jk}^R}{\sum_{i=1}^{n} v_i x_{ik}^L}$$

According to the Ramik and Rimanek principle, we have:

$$\begin{cases} m = \dfrac{\sum_{j=1}^{m} u_j y_{jk}}{\sum_{i=1}^{n} v_i x_{ik}} \le 1 \\ m^L = m - \alpha = \dfrac{\sum_{j=1}^{m} u_j y_{jk}^L}{\sum_{i=1}^{n} v_i x_{ik}^R} \le 1 \\ m^R = m + \beta = \dfrac{\sum_{j=1}^{m} u_j y_{jk}^R}{\sum_{i=1}^{n} v_i x_{ik}^L} \le 1 \end{cases} \qquad (20)$$

In the non-equations (20), it is obvious that, if the right side of the L-R fuzzy number be $m^R \le 1$, so the other non-equations shall be convinced, therefore, the constraint (20) shall be transformed with the following constraints:

$$\frac{\sum_{j=1}^{m} u_j y_{jk}^R}{\sum_{i=1}^{n} v_i x_{ik}^L} \le 1 \quad \Rightarrow \qquad (21)$$

$$\sum_{j=1}^{m} u_j y_{jk}^R - \sum_{i=1}^{n} v_i x_{ik}^L \le 0$$

With respect to the relations (14) and (18) and (21), the final model shall be as follows:

$$\max \quad L_1 + L_2$$

Subject to: (22)

$$\sum_{j=1}^{m} u_j y_{jc} \ge L_1$$

$$\sum_{j=1}^{m} u_j \left(y_{jc}^R - y_{jc}\right) \ge L_2$$

$$\sum_{j=1}^{m} u_j \left(y_{jc} - y_{jc}^L\right) \ge 0$$

$$\sum_{i=1}^{n} v_i x_{ic}^R - \psi_c \le 1$$

$$\sum_{i=1}^{n} v_i x_{ic}^L + \psi_c \ge 1$$

$$\psi_c - \sum_{i=1}^{n} v_i (\alpha_{ic} + \beta_{ic}) \le 0$$

$$\sum_{j=1}^{m} u_j y_{jk}^R - \sum_{i=1}^{n} v_i x_{ik}^L \le 0 \qquad k=1,\dots,r$$

$$u_j \ge \varepsilon \qquad j=1,\dots,m$$

$$v_i \ge \varepsilon \qquad i=1,\dots,n$$

$$L_1, L_2 \ge 0$$

## 4. Numerical example

In this section we solve a numerical example by the proposed model (This example has been indicated in [9]). In this example there are 10 units, each one of which has 2 inputs and 2 outputs. The information has been given in the table 1. The example has been solved by the program "DS for Windows", the solution result of which has been demonstrated by the proposed model and the model of Saati , Memariani and Jahanshahloo (comparative model) in table 2 and the details of the solution by the proposed model in table 3.

**Table 1**: Data of the numerical example for the proposed model.

| | Input 1 | Input 2 | Output 1 | Output 2 |
|---|---|---|---|---|
| unit 1 | (7,1,1) | (30,1,2) | (38,2.5,3) | (411,2,5) |
| unit 2 | (6,0.5,0.5) | (35,2,1.5) | (40,1,3) | (480,2,4) |
| unit 3 | (9,1.5,1.5) | (45,2,3) | (35,3,3) | (299,2,2) |
| unit 4 | (8,1,2) | (39,1.5,3) | (31,3,0) | (352,5,8) |
| unit 5 | (11,2,1) | (44,1,1) | (35,2,3) | (411,5,4) |
| unit 6 | (10,0,0) | (55,2,2.5) | (38,2,2) | (286,4,3) |
| unit 7 | (12,2,2) | (110,3,3) | (36,1.5,2) | (400,4,5) |
| unit 8 | (13,4,3) | (100,5,1) | (41,4,5) | (393,6,9) |
| unit 9 | (14,2,1) | (125,5,6) | (27,3,1) | (404,4,2) |
| unit 10 | (8,3,2) | (38,3,1) | (50,2,1) | (470,0,0) |

**Table 2**: Results of the solution of the example by the proposed model and comparative model.

| | α=0 | α=.2 | α=.4 | α=.6 | α=.8 | α =1 | Proposed Model |
|---|---|---|---|---|---|---|---|
| unit 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| unit 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| unit 3 | 0.84 | 0.79 | 0.75 | 0.71 | 0.66 | 0.61 | 0.6065 |
| unit 4 | 0.76 | 0.74 | 0.72 | 0.70 | 0.68 | 0.66 | 0.6545 |
| unit 5 | 0.78 | 0.75 | 0.73 | 0.71 | 0.69 | 0.68 | 0.6622 |
| unit 6 | 0.69 | 0.67 | 0.65 | 0.63 | 0.60 | 0.58 | 0.5179 |
| unit 7 | 0.63 | 0.59 | 0.55 | 0.51 | 0.48 | 0.45 | 0.4309 |
| unit 8 | 0.85 | 0.75 | 0.66 | 0.59 | 0.53 | 0.47 | 0.5011 |
| unit 9 | 0.46 | 0.44 | 0.42 | 0.40 | 0.38 | 0.66 | 0.3599 |
| unit10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Table 3**: Details of the solution of the example by the proposed model.

| | $v_1$ | $v_2$ | $u_1$ | $u_2$ | $\theta$ |
|---|---|---|---|---|---|
| unit 1 | 0 | 0.0345 | 0.0165 | 0.0008 | 1 |
| unit 2 | 0 | 0.0303 | 0 | 0.0021 | 1 |
| unit 3 | 0 | 0.0233 | 0.016 | 0 | 0.6065 |
| unit 4 | 0 | 0.0267 | 0 | 0.0018 | 0.6545 |
| unit 5 | 0 | 0.02333 | 0.037 | 0.0013 | 0.6622 |
| unit 6 | 0 | 0.0189 | 0.0129 | 0 | 0.5179 |
| unit 7 | 0.1 | 0 | 0 | 0.0011 | 0.4309 |
| unit 8 | 0.1111 | 0 | 0.0109 | 0 | 0.5011 |
| unit 9 | 0.0833 | 0 | 0 | 0.0009 | 0.3599 |
| unit 10 | 0 | 0.0286 | 0.0137 | 0.0006 | 1 |

## 5. Representation of a method for ranking the efficient units

After having solved the models DEA and fuzzy DEA and determination of the efficient units we are now confronted with one question, namely how should be the ranking among the efficient units? In order to reply this question, different methods for ranking of the efficient units have been represented.

One of the fuzzy models represented in this case, is the model which Saati, Memariani and Jahanshahloo [9] have represented. They have applied the dual model CCR for this case and have taken the duty of ranking of the efficient units (units 1, 2, 10) in the example 4 in the above-indicated items.

Now we go on with the introduction of a new method which Aryanezhad and Najizadeh [9] have represented and then we shall extend this method in the fuzzy environment and with its application we shall rank the units.

The method of Aryanezhad and Najizadeh based on an argumentation has been shaped as per the following:

- If we can introduce a virtual unit which may be absolutely more efficient rather than the studied units, then we shall reach a ranking among these units by adding this virtual unit to the model and its solution for the efficient units.
- This efficient virtual unit is named Superior Virtual Unit – SVU.

Representation of SVU in the condition that the data are definite, shall be a very simple work, but with the fuzzy data, executing of this work shall not be so easy, because verification of being smaller or bigger of the fuzzy numbers in the form of observation shall not be possible in many cases.

In order to avoid the problems relating to the ranking of the fuzzy numbers, we determine the amounts of input and output of SVU with representing the expression of the "Max. and Min." of the triangular fuzzy numbers and its application:

- If we have the s of the $\tilde{M}_i$ fuzzy number and the $\tilde{N}$ fuzzy number:

$$\tilde{M}_i=(m_i,\alpha_i,\beta_i)_{LR}\equiv(m_i^L=m_i-\alpha_i,m_i,m_i^R=m_i+\beta_i)$$
$$i=1,\ldots..,s$$
$$\tilde{N}=(n,\eta,\gamma)_{LR}\equiv(n^L=n-\eta,n,n^R=n+\gamma)$$

We say that the $\tilde{N}$ fuzzy number is absolutely greater than the s of the fuzzy number, in case:

$$n^L=\max\ m_i^L\ ,\ n\ =\max\ m_i\ ,\qquad\qquad (23)$$
$$n^R=\max\ m_i^R\qquad \forall\, i=1,\ldots..,\ s$$

- If we have the s of the $\tilde{M}_i$ fuzzy number and $\tilde{N}$ fuzzy number:

$$\tilde{M}_i=(m_i,\alpha_i,\beta_i)_{LR}\equiv(m_i^L=m_i-\alpha_i,m_i,m_i^R=m_i+\beta_i)$$
$$i=1,\ldots..,s$$
$$\tilde{N}=(n,\eta,\gamma)_{LR}\equiv(n^L=n-\eta,n,n^R=n+\gamma)$$

We say that the $\tilde{N}$ fuzzy number is absolutely smaller that the s of the fuzzy number, in case:

$$n^L=\min\ m_i^L\ ,\ n\ =\min\ m_i\ ,\qquad\qquad (24)$$
$$n^R=\min\ m_i^R\qquad \forall\, i=1,\ldots..,\ s$$

By applying of the above-expressions to determine the SVU inputs and outputs, we should determine the smallest lower bound, smallest suprimum and smallest upper bound among the inputs and outputs of the 10 units of the example presented in section 4:

Input 1= (5, 6 , 6.5)
Input 2 = (29 , 30 , 32)
Output 1= (48,50,51)
Output 2 = (478 , 480 , 484)

Now we add this new unit to the proposed model and this causes it to be added to each one of the following constraint models:

$$-5v_1-29v_2+51u_1+484u_2\le 0$$

**Table 4:** The result of the solution of the example by the represented method of ranking and comparative model.

| | | Unit 1 | Unit 2 | Unit 10 |
|---|---|---|---|---|
| *Comparative Model* | α = 0 | 1.25 | 1.30 | 1.70 |
| | α = 0.2 | 1.19 | 1.24 | 1.48 |
| | α = 0.4 | 1.12 | 1.17 | 1.30 |
| | α = 0.6 | 1.08 | 1.11 | 1.20 |
| | α = 0.8 | 1.04 | 1.05 | 1.10 |
| | α = 1 | 1 | 1 | 1 |
| | Proposed method | 0.8595 | 0.9091 | 1 |

- On the basis of the two ranking models, the sequence of the units 10, 2 and 1 shall be reached.

## 6. Conclusion

The represented model fuzzy DEA is designed based on the L-R fuzzy numbers. Whereas, the L-R fuzzy numbers, on the basis of different right and left functions, envelop different shapes of the fuzzy numbers, therefore, the proposed model shall possess the ability of model-making by an extended spectrum of Data. The only constraint of the model is that the right and left functions should be the same (This necessity is established by the application of the Ramik and Rimanek principle).

The represented model, while being efficient, possesses less complications and capacity in comparison to many actual models.

The represented ranking method has been established on the basis of one simple and logic expression and as the ranking of the efficient ranking is implemented by adding one constraint to the proposed fuzzy DEA model, therefore it is a simple and efficient method, adding that method does not have the current complications in ranking of the fuzzy numbers.

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