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Solving a mathematical model with multi warehouses and retailers in distribution network by a simulated annealing algorithm

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Abstract

Determination of shipment quantity and distribution problem is an important subject in today's business. This paper describes the inventory/distribution network design. The system addresses a class of distribution network design problem, which is characterized by multiple products family, multiple warehouses and retailers. The maximum capacity of vehicles and warehouses are also known. The resulting system focuses on two key goals: minimizing the lost sales cost as a costumer's satisfaction factor and balancing sum of service distances for different warehouses. In this paper we consider the distribution network problem formulated by 0-1 mixed integer linear programming model. Due to difficulty of obtaining the optimum solution in medium and large scale problems, a simulated annealing algorithm (SA) is also applied. The efficiency of this algorithm is demonstrated by comparing its numerical experiment results with those of SA algorithm and LINGO 6. package.

Keywords: Supply chain management; Production / distribution programming; Simulated anealing

1. Introduction

Due to the advancement of technology and other supporting mechanisms, today's some new topics have been appeared in management and industry. In the recent decades, competitive pressures pose the challenge of simultaneously prioritizing the dimensions of competition: flexibility, cost, quality and delivery. The above-mentioned aspects of industrial competition developed the necessity of "supply chain management". Supply chain management (SCM) is a concept that originated and flourished in the manufacturing industry. The first visible signs of SCM were in the JIT delivery system, as part of the Toyota production system (Vrijhoef and Koskela, 2000). Related topics in SCM were proposed in the late 80's and developed in 90's (Makui, 2004).

Supply Chain Management (SCM) is the management of material and information flows both in and between facilities, such as vendors, manufacturing and assembly plants and distribution centers (DC).

One of the outcomes of the fiercely competitive business environment in the late 1990s has been the increasing attention given to supply chain networks in the manufacturing and service sectors. The customer in these business sectors have come to expect faster reaction, high reliability, and greater flexibility to ever-changing for both manufacturers and service personnel to find new and better ways to manage their material flows (Jayaraman and Pirkul, 2001). With the trend towards greater synergy between suppliers and industrial customers, most manufacturing enterprises are organized as networks of manufacturing and distribution sites that purchase raw materials, transform those materials

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into work in process and finished goods, and distribute the finished goods to customers. Management of such networks has emerged as a major topic in operations research. Improving the efficiency of these systems requires striking a balance between the various logistical function; in particular, inventory control and transportation planning need to be closely coordinated (Qu *et al.*, 1999).

Because transportation cost is a large portion of distribution cost and number of vehicles and their capacity are usually constrained, taking a suitable strategy is of a great importance. Due to these constrains, lost sales may be encountered which is one of the costumer's satisfaction factors that directly affect inventory costs.

In most given models, only shipment costs are considered and little attention is paid to the above mentioned limitation. In this research a mathematical model for distribution network is proposed including multi warehouses and retailers by taking their constraints. In assigning warehouses to retailers - on the contrary of the other given models - balancing sum of service distances for different warehouses is considered instead of minimizing transportation distances.

The paper is organized as follows: Section 2 discusses the relevant literature. Section 3 introduces the mathematical model and Section 4 presents the solution approach. Some examples are solved and the computational results are analyzed in Section 5.

2. Background

While SCM is relatively new, the idea of coordinated planning is not. The study of multi-echelon inventory/distribution systems began as early as 1960 by Clark and Scarf. They present a recursive decomposition approach to determine optimal policies for serial multi-echelon structures (Clark and Scarf, 1960).

The supply chain begins with the procurement of raw materials or subassemblies. Many traditional inventory models have focused on determining optimal order quantities for the purchaser. The model presented in this class include Single-vendor, Single-buyer [(Banerjee, 1986; Lee and Rosenblantt, 1986; Monahan, 1984), Multiple-vendors, Single-buyer (Anupindi and Akella, 1993; Lau and Lau, 1994) and Single-vendor, Multiple-buyers (Kohli and Park, 1994) all aiming to minimize the cost or maximize the profit.

Muckstadt and Thomas investigate the applicability of multi-echelon methods in low demand systems. Two approaches presented for determining stock levels in a two echelon system. Both approaches use a Lagrange relaxation technique that results in a separable problem that can be solved easily (Muckstadt and Thomas, 1980). Erkip *et al.* (1990) present an approach to determine optimal ordering policies at a depot that distributes to multiple warehouses with correlated demand.

Qu *et al.* (1999) present a multi-item joint replacements problem, in a stochastic setting, with simultaneous decisions made on inventory and transportation policies.

Buffa and Reynolds (1977) developed a model to include a number of transport-related variables. They also have shown that transportation cost clearly influence on the inventory costs.

Constable and Whybark (1978) proposed an alternative version of the inventory-theoretic model that explicitly included both carrying and back-order cost. The model jointly determined the inventory reorder points, order quantities, and transportation choices that provide minimum total transportation and inventory costs.

Williams (1981) presented a dynamic programming algorithm for simultaneous determination of production batch sizes in an assembly network and distribution batch sizes in a conjoined distribution network. Dynamic programming algorithms were applied to solve the model.

Benjamin (1990) considered the choice of transportation mode in a production-distribution network with multiple supply and demand points and a single product class. The problem was formulated as a nonlinear program, and a heuristic solution procedure was presented along with a procedure for computing a lower bound on the global minimum.

Haq *et al.* (1991) develop a mixed integer program to determine production and distribution batch sizes that minimize system costs in a multi-stage production-inventory-distribution system.

Pyke and Cohen (1993) presented a Markov chain model of a single product three-level supply chain, consisting of a factory, a finished goods stockpile and a retailer. Near-optimal algorithms were presented to determine the expedite batch size, the normal replenishment batch size, the normal reorder point, the expedite reorder point and the order-up-to level at the retailer.

Jayaraman (1998) considered the relationship between the management of inventory, location of facilities and the determination of transportation policy simultaneously in a distribution network design environment.

Jayaraman and Pirkul (2001) studied two echelon distribution problems with multiple plants and multiple capacitated DCs. For solving the model they used a heuristic based on Lagrange relaxation and sub-gradian optimization that obtained good results too.

Dasci and Verter (2001) provided a mixed integer linear programming model for facility location which focuses on setting up a number of new facilities in an area, so that each of facilities should service a portion of demand. Demand portion in their research are not as discrete points because the model is designed in a continuous solution space.

Hwang (2002) provided a logistics system design which optimizes the performance of logistics system subject to required service levels both in a number of warehouse or distribution center and vehicle routing schedule. He formulated this problem using stochastic set-covering problem to determine the minimum number of warehouse/distribution centers among a discrete set of location sites and solved this problem using 0-1 programming method. Then he formulated a vehicle routing problem using an improved genetic algorithm.

Syarif *et al.* (2002) considered the logistic chain network problem formulated by 0-1 mixed integer linear programming model. The design tasks of that model involve the choice of the plants and distribution centers to be opened and the distribution network design to satisfy the demand with minimum cost. As the solution method, they proposed the spanning tree-based genetic algorithm by using prüfer number. The efficacy and the efficiency of this method are demonstrated by comparing its numerical experiment results with that traditional matrix-based genetic algorithm.

Syami (2002) made a research on developing traditional facility location problem considering logistic cost. For solving the constructed model two different heuristics, one based on Lagrange relaxation and the other simulated annealing were used.

Jolaymi and Olorunniwo (2004) provided a deterministic model for planning production quantities in a multi-plant, multi-warehouse environment with extensible capabilities. When the production cannot meet demand the model allows shortfalls to be met through subcontracting or the use of inventory.

Wang *et al.* (2004) proposed a just-in-time distribution requirements planning system under the limited supply capacity. The aim is to establish an optimal distribution requirements planning model to minimize the total cost of manufacturing and transportation under limited warehouse capacity. The model can be translated in to a linear programming problem and solved by simplex procedure.

Chan *et al.* (2005) developed a hybrid genetic algorithm for production-distribution problems in a supply chain with multi-plants. Their mathematical model is proposed in linear programming form. They have used GA and AHP to solve that.

Gen and Syarif (2005) proposed a production/distribution problem to determine an efficient integration of production, distribution and inventory system in order to minimize system wide costs while satisfying all demand required. This problem can be viewed as an optimization model that integrates facility location decisions, distribution costs, and inventory management for multi-products and multi-time periods. To solve the problem, they proposed a new technique called spanning tree-based genetic algorithm.

Liang (2006) developed an interactive fuzzy multi-objective linear programming method for solving the fuzzy multi-objective transportation problems with piecewise linear membership function.

Geoffrion and Graves (1974) presented a mixed integer programming formulation of multi-commodity distribution system design. A solution procedure based on Benders' decomposition is presented. This decomposition separates the problem at each iteration into several easily solved LPs. Computational results show that Benders' decomposition performs remarkably well on this class of problems.

Cohen and Lee (1989) presented an integer programming model designed to support strategic resource deployment decision making in a global manufacturing and distribution network. The model is used to determine resource deployment, given a logistics structure. In practice, such a tool is useful for evaluating and supporting strategic decision making.

Ross (2000) presented a two phase approach for supply chain problem. The first phase deciding based on a strategy that selects the best set of distribution centers to be open. The second phase is an operational deciding that includes customer and resource assignments. Simulated annealing is applied for solving this problem.

Jayaraman and Ross (2003) provided a distribution network in two models focusing on two key stages; planning and implementing. Determining warehouses and cross-dock centers allocation to open warehouses and family products allocation from warehouses to cross-dock centers are all results of solving the first model. The second model is an operational model aiming to minimizing the cost of transportation to warehouses, costs of transportation from warehouses to cross-dock centers and cost of product distribution to customer's hand. Simulated annealing is used to achieve a sub-optimal solution for both models.

3. Problem formulation

In today's customer-oriented business environment, a firm's ability to assign its customers to available warehouses can be translated into a competitive advantage (Zhou *et al.,* 2003). The purpose of this research is to provide a model for distribution network in order to assign vehicles from warehouses to retailers and also determining optimum shipment quantity by considering lost sale cost as a customer satisfaction factor and balancing the sum of service distances for different warehouses.

A simple prototype of the 'distribution network 'problem investigated in this paper is provided in Figure 1.

As it is obvious in Figure 1, the problem is allocating warehouses to retailers. Instead of direct warehouse assignment to retailers, warehouse vehicle is assigned to each one. Against the previous proposed models which their aim was to minimize shipment cost, we have considered balancing sum of service distance for every warehouse as a goal in order to balance the transportation costs for every warehouse. Another assumption is predefined constant number of vehicles for every warehouse, and the last is a definite constant number of shipments for vehicles. Every retailer could meet his demand from multiple warehouses. In the following, first decision variables and required parameters are defined and then problem modeling is provided in details.

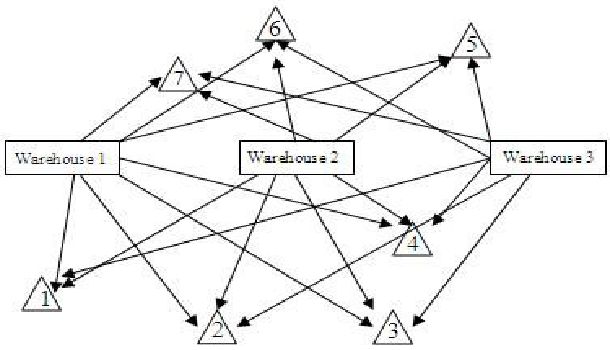

Figure1. Three central warehouses and seven retailers.

The following indices, parameters and variables are used to describe the mathematical model.

Notation of MIP model:

t The number of periods.

l The number of products group.

i The number of retailers.

j The number of warehouses.

p_j The set of vehicles that are belong to warehouse *j*.

p The elements *of* p_j set.

It could be pointed that number of vehicles for every warehouse is definite and constant. Each vehicle has a unique index that is assumed to start from first warehouse and increase by the order of next warehouses. For example if there are three warehouses that the first one has two vehicles, the second has three and the third one has two, the index of the first warehouse vehicles will be $p_1 = \{1,2\}$, the second $p_2 = \{3,4,5\}$ and $p_3 = \{6.7\}$ for the third one.

q_{tlip} Quantity of product *l* which is shipped to retailer *i* by vehicle *p* (*p* belongs to each p_j) at each service in period *t*.

z_{tpi} Binary variable, equal to 1 if vehicle *p* service retailer *i* in period *t*, 0 otherwise.

c_p Traveling cost per kilometer for vehicle *p*.

D_{tli} Maximum demand of retailer *i*, product *l* in period *t*.

n_{tpi} Number of service times for vehicle *p*, retailer *i* in period *t*.

k_{pt} Maximum available distance for vehicle *p* in period *t*.

a_{tlj} Maximum supply of warehouses *j*, product *l* in period *t*.

d_{ji} Distance between warehouse *j* and retailer *i*.

h_{li} Lost sale cost per unit of product l for retailer i.

c_p Distance cost for vehicle p.

v_p Maximum capacity for vehicle p.

The following relation is defined for balancing the costs:

$$Min\{\underset{j}{Max}\sum_{t}^{m}\sum_{i}^{n}\sum_{p_j} c_p \times z_{tpi} \times d_{ji}\} \quad (1)$$

Obviously, the relation (1) is a nonlinear term and should be linearized. In order to making that linear, do as follows:

$$z = Max\{\sum_{t}\sum_{i}\sum_{p_j} c_p \times z_{tpi} \times d_{ji}\} \quad \forall p_j \quad (2)$$

$$Min\ z \quad (3)$$

S.t.

$$\sum_{t}\sum_{i}\sum_{p_j} c_p \times z_{tpi} \times d_{ji} \leq z \quad \forall p_j \quad (4)$$

$$z_{tpi} = \{0,1\} \quad \forall t, i, p_j \quad (5)$$

The Objective Function (2) is forcing the model to balance shipment cost in different periods based on allocating different warehouse vehicles to the set of retailers. Constraint (4) is the linearization of the Relation (1) which assures that sum of assignments in different period for different warehouses don't exceed of z and finally Constraint (5) enforces the integrality restriction on the decision variable.

Using the above-mentioned, we may formulate the problem by using the following mixed integer linear programming model:

$$Min \sum_{t}\sum_{i}\sum_{l} h_{li} \times (D_{tli} - (\sum_{j}\sum_{p_j} n_{tpi} \times q_{tlip})) + z \quad (6)$$

S.t.

$$n_{tpi} \times q_{tlip} \leq D_{tli} \times z_{tpi} \quad \forall t, l, i, p_j \quad (7)$$

$$\sum_{p_j} n_{tpi} \times q_{tlip} \leq D_{tli} \quad \forall t, l, i, p_j \quad (8)$$

$$\sum_{p_j}\sum_{i} n_{tpi} \times q_{tlip} \leq a_{tlj} \quad \forall t, j, p_j \quad (9)$$

$$\sum_{l}\sum_{i} q_{tlip} \leq v_p \quad \forall t, p_j \quad (10)$$

$$\sum_{i} n_{tpi} \times z_{tpi} \times d_{ij} \leq k_{pt} \quad \forall t, p_j \quad (11)$$

$$\sum_{t}\sum_{i}\sum_{p_j} c_p \times z_{tpi} \times d_{ji} \leq z \quad \forall j \quad (12)$$

$$z_{tpi} = \{0,1\} \quad \forall t, i, p_j \quad (13)$$

$$q_{tlip} \geq 0 \quad \forall t, l, i, p_j \quad (14)$$

3.1. Objective function

The objective function is to minimize lost sales costs and balancing sum of shipment costs for different warehouses.

3.2. Constraints

Constraint (7) represents that shipment quantity for every retailer of different products with every vehicle in each period should be less than product demand in the same period. Shipment quantity ($n_{tpi} \times q_{tlip}$) is calculated proportional to the number of times that a vehicle services a retailer and q_{tlip} could be nonzero if the vehicle is assigned to that retailer.

Since every retailer may receive its demand from multi warehouses, then different vehicles could provide retailer's demand. Constraint (7) considers maximum demand for every retailer by different vehicles. There should be a constraint to prevent sum of shipments for every retailer of a specific product in a special period from different vehicles exceed to maximum demand of that retailer for every product in each period. Constraint (8) represents the demand which transported by different vehicles to every retailer in each period should be less than or equal to maximum demand of that retailer for that product.

In practice, because of the limitations in the amount of different products supply from warehouses, a limitation in maximum supply capacity for different products in different periods is assumed for each warehouse. The warehouse capacity constraint is ensured by Constraint (9) while Constraint (10) represents the capacity constraint for vehicles. Constraint (11) restricts travel distance by an upper bound on expected accumulated kilometer for each vehicle. Constraint (12) is used to linearization the balancing relation in the model, and aims to assign warehouses to retailers with equal shipment costs. Constraint (13) represents zero-one constraint while Constraint (14) imposes non-negative decision variable in the model. The proposed model for assigning vehicle and determining shipment quantity of every retailer is a large-scale model. This problem can be viewed as determining shipment quantity and assignment problem simultaneously. So this problem is known to be NP-Hard (Gen and Chang 1997).

4. Simulated annealing approach

Simulated annealing is a computational process which attempts to solve hard combinatorial optimization problems through controlled randomization. The procedure was popularized by Kirkpatrick *et al.* (1983) and is based on work by Metropolis *et al.* (1953) (the so-called Metropolis algorithm) in statistical mechanics. Simulated annealing emulates the physical process of annealing (hence the name of the heuristic) which attempts to force a system to its lowest energy state through controlled cooling.

In a physical system with a large number of atoms, the equilibrium may be characterized as the minimal value for the energy of the system. This is accomplished by a slow cooling of the temperature. Then, the system is said to be in thermal equilibrium at temperature T if the probability of being in state i with energy E_i follows the Boltzmann distribution:

$$Prob\{x = i\} = \frac{exp\{\frac{-E_i}{K_B T}\}}{\sum exp\{\frac{-E_j}{K_B T}\}} \tag{15}$$

where K_B is the Boltzmann constant and the sum is extended to all the possible states. Moving the atoms randomly to new configurations, different energy changes are induced (ΔE). If the increment is negative, the new configuration is accepted as a new state, but if the configuration has higher 'energy' than the previous state, it is only accepted with a certain probability:

$$exp\{\frac{-\Delta E}{K_B T}\} \tag{16}$$

Repeating these steps, it is shown that the accepted configurations converge to the Boltzmann distribution after some indeterminate number of iterations at each particular temperature. The procedure may be easily applied to a large number of optimization problems where the objective function plays the role of the energy. In this context, the temperature is a control parameter to define large or small moves for the optimization variables (Marin and Salmeron, 1996).

In general, the annealing process involves the following steps:

1. The temperature of the system is raised to a sufficient level.
2. The temperature of the system is maintained at this level for a prescribed amount of time.
3. The system is allowed to cool under controlled conditions until the desired energy state is attained.

The initial temperature (Step 1), the time the system remains at this temperature (Step 2) and the rate at which the system is cooled (Step 3) are referred to as the annealing schedule. If the system is allowed to cool too fast, it may "freeze" at an undesirable, high energy state. With respect to optimization problems, the state of the system corresponds to the value of the objective function. Similarly, the freezing of a system at an undesirable energy state corresponds to an optimization problem which is "frozen" at a local optimum. Given this, in simulated annealing the problem starts at some sub-optimal solution, and a series of moves (changes of values of decision variables) are made according to a user-defined annealing schedule until either the optimal solution is attained or the problem becomes frozen at a local optimum from which it cannot improve. To avoid freezing at a local optimum, the algorithm moves slowly (with respect to the objective value) through the solution space. This controlled improvement of the objective value is accomplished by accepting non-improving moves (i.e., those which yield an objective value greater than or equal to the last accepted objective value) with a certain probability (based on the resulting change in the objective value and the current temperature) which decreases as the algorithm progresses. The algorithm's transitions can be modeled as a collection of finite-length Markov chains corresponding to each temperature level of the system. Hence, through selection of an appropriate probability distribution and through control of its parameters, the algorithm's rate of convergence is controlled. The general procedure for implementing a simulated annealing algorithm follows (Chen *et al.*, 1996):

Step1. Select an initial temperature, t, and an initial solution, X_0. Let $f_0 = f\ (X_0)$ denote the corresponding objective value. Set $i = 0$ and go to Step 2.

Step2. Set $i = i + 1$. Randomly generate a new solution, X_i, and evaluate $f_i = f\ (X_i)$.

Step3. If $f_i < f_{i-1}$, then go to Step 5. Otherwise, accept f_i as the new solution with probability $e^{(f_i - f_{i-1})/t}$.

Step4. If f_i, was rejected as the new solution in Step 3, set $f_i = f_{i-1}$. Go to Step 5.

Step5. If satisfied with the current objective Value (f_i) stop. Otherwise, adjust the temperature, t, according to the annealing schedule and go to Step 2.

4.1. Problem solving with SA

As mentioned before, using integer variables and increasing problem size in the proposed model causes NP-Hard problem. In the warehouse assignment to retailers, both of the selecting vehicles for products shipment and determining the order quantity are simultaneously performed. If we want to use SA for solving the model, because of many decision variables, it is not possible to introduce a suitable algorithm for generating a feasible neighbor from a point in the solution space. By the above-mentioned facts it seems that using a stochastic search process could search in the solution space more efficiently.

In this paper, stochastic search algorithm adapted with SA logic and is used Boltzmann function for considering the quality of convergence for the searched points toward global optimum solution. In this section, every part of the main algorithm is explained.

4.2. Objective function of SA algorithm

The objective function minimizes lost sale costs by considering different retailers, and balancing the sum of shipment costs for every warehouse. These objectives form the total cost (TC) objective function. In order to compute the quantity of TC objective function, we should compute amount of last sales and also balance sum of shipment costs. Lost sales are a portion of customer demand that because of some limitations is not met. This quantity is subtracted from the sum of shipment quantities to the retailers and the result is multiplied by the penalty rate of lost sales. By this way, the total quantity of lost sales in the TC objective function is computed. For balancing, a EQU_i variable for every warehouse is assumed. If a warehouse is assigned to a retailer the multiply transportation cost of the distance between warehouse and the retailer could be computed and saved in EQU_i. Finally these variables for each warehouse assignments are summed and compared for different warehouses. If they are balanced, corresponding assignments will be accepted and the obtained quantity will be set in the objective function.

4.3. Approach of generating random search

For generating a feasible solution by the provided algorithm, firstly the numbers of warehouse to be service, then the warehouses which will provide service are randomly determined. Number of vehicles in every warehouse and consequently vehicle of each warehouse to be serviced is randomly determined. Finally retailers receiving service will be selected and assigned. It is necessary to mention that all of the selection is completely random. For more clarity an operator is used to search randomly in the feasible solution space. This operator is defined as follows:

- Determine the number of existing warehouses (NW).
- Define variable V_j, which is the number of vehicles in every warehouse.

- Determine the whole number of retailers (NC).
- Generate an integer uniform random number between [1, NW] called RNW which presents the number of selected warehouses to provide service.
- Generate non-iterative integer uniform random numbers equal to RNW are as selected warehouses. (RNWI)
- Generate a non-iterative random number in $[1, V_j]$ for each of the selected warehouses, which are considered as the vehicles number from the warehouse RNWI selected to service (RNWIN).
- Generate non-iterative random numbers by the number of selected vehicles [1, RNWIN] as the indices of vehicles from the selected warehouse to stand by service.
- Generate an integer uniform random number in [0, NC] called RC as the number of retailers to be serviced.
- Generate non-iterative random numbers in [1, NC] for RC times as the indices of retailers that receive service.

Each of above assignments is shown by a variable:

z_{tijk} Vehicle *i* from warehouse *j* that service retailer *k* in period *t*.

V_{ij} If vehicle *i* from warehouse *j* is selected to be in service is equal to one, o otherwise.

At the beginning of the algorithm all z_{tijk} and V_{ij} are equaled to zero. In order to perform assignment, below steps should be followed:

1. Select vehicle (i.e. vehicle *i* from warehouse *j)* to service the first retailer (or retailer *k*).
2. Let L=Min {inventory of warehouse *j*, capacity of vehicle *i* from warehouse *j*, demand of the customer *k*}.
3. If L = 0, then select the next vehicle and go to 1.
4. L unit of products are shipped by the vehicle *i* from warehouse *j* to the retailer *k* ($z_{tijk} = 1$, $V_{ij} = 1$).
5. Subtract an *l* unit from inventory of warehouse *j* and capacity of vehicle *i* and demand of customer *k*.
6. If there is still any vehicle to be assigned, go to Step 1, else terminate the procedure.

Neighbor generating process is run in each temperature. In order to increase the accuracy of the SA, it is usual to generate neighbors for more times in a specific temperature. When using this algorithm an auxiliary memory is provided to record the best solution. It guarantees if a solution obtained in a specific temperature is worst than the last solution – even with a very small probability – the best solution is saved in the memory and introduced as the final solution.

In general, using this memory guarantees storing the best solution. Minimum temperature criterion is used for determining the end of runtime. In the problem solved with SA, it has been observed that usually in the temperature below 0.1 degree of Celsius, quantity of S is going to be steady-state and its deviations become smaller over the time. The best temperature for stopping criterion in the proposed SA algorithm - in this problem set - is below 0.1 degree of Celsius (i.e. 0.1 ^{0}c). According to the most examples solved, Δ is converging to the best obtained value.

5. Computational results

The proposed model is solved by different values parameters with VBA software and LINGO package. In the following, the computational results of problems randomly value parameters summarized in Tables 2 and 3.

During solving the problems the initial temperature was set to 10000 degree of Celsius and the cooling rate was set to 0.95% with the stopping criterion of reaching to the temperature of 0.1 ^{0}c (Table 1). By considering the algorithm of searched points in the solved sample problems, it can be shown that the extensive range of solution area in high temperatures, is searched by SA.

Table 1. SA parameter settings.

T_0	α	*K* (Iteration in each temperature)	*T* stop
10000	.95	20	0.1

Table 2. Summary of computational results (constant parameters).

Case	Problem Size # Retailer	# Vehicle	# Warehouse	Exact Objective Function	SA Objective Function	Exact Run Time(s)	SA Run Time(s)	Obj. Function Deviation
1	2	3	5	1600	1600	00:00:01	00:00:02	0%
2	2	3	6	2500	2510	00:00:01	00:00:05	0.4%
3	2	3	7	4600	4600	00:00:01	00:00:05	0%
4	2	4	5	2400	2400	00:00:01	00:00:05	0%
5	2	4	6	5700	5700	00:00:01	00:00:04	0%
6	2	4	7	8900	8900	00:00:01	00:00:06	0%
7	3	5	7	3300	3350	00:00:07	00:00:12	1.5%
8	3	5	9	11100	11140	00:00:04	00:00:10	0.4%
9	3	5	10	8640	8690	00:00:03	00:00:04	0.6%
10	4	6	8	3080	3100	00:00:08	00:00:13	0.6%
11	4	6	9	7200	7300	00:00:25	00:00:31	1.4%
12	4	6	10	9000	9100	00:03:32	00:00:32	1.11%
13	4	6	11	11400	11500	00:04:47	00:00:37	0.88%
14	4	6	12	15850	16000	00:05:07	00:00:49	0.95%
15	4	6	13	19400	19600	00:05:03	00:00:25	1.03%
16	4	6	14	23800	24100	00:05:34	00:00:27	1.26%
17	5	7	11	7400	7500	00:05:43	00:00:51	1.35%
18	5	7	12	14115	14200	00:05:50	00:00:54	0.60%
19	5	7	13	16100	16400	00:06:15	00:01:23	1.86%
20	5	7	14	19700	19860	00:05:01	00:01:37	0.81%
21	5	7	15	22470	22600	00:05:48	00:01:54	0.58%
22	5	7	17	28400	28600	00:09:48	00:01:18	0.70%
23	6	10	12	-	5600	Long Time	00:01:47	-
24	6	10	16	-	14500	Long Time	00:02:27	-
25	8	13	18	-	1600	Long Time	00:02:34	-
26	10	15	20	-	20200	Long Time	00:02:45	-
27	10	17	25	-	34400	Long Time	00:02:54	-
28	11	15	20	-	55700	Long Time	00:02:29	-
29	11	17	20	-	43500	Long Time	00:02:47	-
30	12	18	20	-	44670	Long Time	00:03:23	-
31	13	19	25	-	78120	Long Time	00:03:19	-
32	15	30	45	-	67800	Long Time	00:03:15	-

Table 3. Summary of computational results (none constant parameters).

Case	Problem Size # Retailer	# Vehicle	# Warehouse	Exact Objective Function	SA Objective Function	Exact Run Time(s)	SA Run Time(s)	Obj. Function Deviation
1	2	8	6	2009000	2013500	0:00:01	0:00:08	0.20%
				8700	9000	0:00:09	0:00:05	3.40%
				542450	547100	0:04:20	0:00:18	0.90%
				13500	14000	0:04:31	0:00:19	3.70%
2	2	16	6	5439800	5466150	0:00:01	0:00:10	0.50%
				4290100	4318950	0:00:37	0:00:10	0.70%
				5339800	5363950	0:01:31	0:00:11	0.50%
				4018000	4029100	0:03:32	0:00:11	0.30%
3	3	5	6	6026000	6080000	0:00:01	0:00:10	0.90%
				345000	355000	0:01:07	0:00:08	2.90%
				7856000	7912000	0:02:31	0:00:12	0.70%
				5909000	5958400	0:11:53	0:00:11	0.80%
4	3	6	6	24000	24150	0:00:00	0:00:06	0.60%
				145	145	0:00:01	0:00:05	0.00%
				6130	6280	0:00:01	0:00:11	2.40%
				171120	171160	0:00:01	0:00:10	0.00%
5	4	8	7	19780	19860	0:00:01	0:00:13	0.40%
				360	380	0:00:01	0:00:07	5.60%
				3000	3000	0:08:34	0:00:10	0.00%
				3000	3000	0:29:25	0:00:13	0.00%
6	5	10	15	-	1140900	Long Time	0:00:13	-
				-	924200	Long Time	0:00:33	-
				-	725000	Long Time	0:00:28	-
				-	702200	Long Time	0:00:14	-
7	10	17	20	-	4435500	Long Time	0:00:15	-
				-	2041400	Long Time	0:00:13	-
				-	26700	Long Time	0:00:20	-
				-	1076800	Long Time	0:00:19	-
8	15	25	30	-	101310	Long Time	0:00:33	-
				-	157935	Long Time	0:00:58	-
				-	189849	Long Time	0:00:42	-
				-	82379	Long Time	0:00:43	-

Moreover the *SA* algorithm is able to escape from local optima. In lower temperatures, the searching procedure in inclined to the near optima points and in final points it can be easily seen that the method inclination is to the optimum solution.

It was generated by more than 100 random examples by computer and all of them were solved. The result of 32 instances as samples is shown in Table 2.

After solving the generated instances, in order to investigate the model performance, again it was generated by more than 100 random examples by computer. Then the parameters such as vehicles capacity, demands of customers, maximum supply of warehouses and lost sales cost is changed where as the problem size is considered as constant and the result of the 32 instances as samples is shown in Table 3. The CPU times correspond to an Intel Centrino Duo 2 GHz processors.

After solving, the comparison of computational results of solved instances show that either in the samples which the parameters considered constant to decrease the model complexity (Table2) or in samples which the parameters doesn't consider constant (Table3) by increasing the problem dimension, the run times of SA method were better than the exact run times. It is also show that in all solved instances the objective function deviation by proposed SA method comparing with exact method is adoptable which show the appropriate proposed algorithm performance.

6. Conclusion

In this paper a mixed integer linear programming model is presented to solve the warehouses assignment to the retailers, in order to balancing shipment costs for every warehouse. Determination of optimum shipment quantities for retailers in order to minimize total costs is another issue of the proposed model in this paper. Decreasing the amount of the lost sales is considered as a factor of customer service level in the proposed model. The proposed model can be viewed as the combination choice Knapsack problem with capacitated allocation problem simultaneously. So this problem is known to be NP-Hard.

For relatively small size problem we show that the SA algorithm can usually search the near optimum solution. So we believe this algorithm will be an efficient method to solve this kind of problem in large scale size in supply chain management.

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