A profit Malmquist productivity index

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Abstract: In some situations the producers desire to maximize total profit of Decision Making Units (DMUs) while the inputs and outputs prices of DMUs change from one time period to another. In this paper, the researchers develop productivity index when producers are going to maximize total profit when the price of inputs and outputs are known. The proposed method uses all price information about inputs and outputs for determining productivity index while previous methods use only cost of inputs to determine index of productivity. Therefore, the proposed Mulmquist productivity index in this paper is more precise than the other Mulmquist productivity index. Here, productivity change is decomposed into profit efficiency and profit technical change. Furthermore, profit efficiency change is decomposed into technical and allocative efficiency change and profit technical change into a part capturing shifts of input and output quantities and shifts of input and output prices. These decompositions provide a clearer picture of the root sources of productivity change. Finally, the proposed fractional programming problems are converted to the linear programming problems. By an illustrative example, we explain the proposed profit Malmquist productivity index.

1. Introduction

Data envelopment analysis (DEA) has been recognized as an excellent method for analyzing performance and modeling organization and operational process. DEA can be applied to panel data to measure the productivity changes between two periods of activities fulfilled by a specific set of DMUs.

In early work, productivity change was explained in term of technical change but recently it has become widely accepted that efficiency change can also contribute to it. In this framework, the Malmquist index was first introduced in productivity literature by Cave *et al.* (1982). Nischimizu and Page (1982) used a parametric programming approach to compute the index for the first time in the empirical context. Fare *et al.* (1989) decomposed productivity change into a part attributable to change of technical efficiency and technical change and used non-parametric mathematical programming models for its computation. The Malmquist index has seen many applications and extensions (Chen, 2003; Pastor *et al.,* 2005; Shestalova 2003). However, it does not capture an important form of efficiency, namely allocative. Thus, the Malmquist index may not give a full picture of the sources of productivity change such as those resulting from a unit aligning its input and output mix better with the prevailing input and output prices.

Bauer (1990) and Balk (1997) have attempted to decompose productivity change so that the contribution of allocative efficiency change (AEC) is identified. Baure (1990) used an econometric framework and Balk (1997) used index numbers. Maniadakis and Thanassoulis (2004) proposed an approach that provides similar decompositions of productivity change. They developed a dual Malmquist index, which is defined in terms of cost rather than input distance function. The index was applicable when producers can be assumed to be cost minimizers and input-output quantity and input price data are available. The proposed method by Maniadakis and Thanassoulis (2004) doesn't use all price information about inputs and outputs for determining productivity index.

In some situations, the producers want to determine Malmquist Productivity index, they would like to maximize the total profit, and the price of inputs and outputs are available. Therefore, here an alternative approach that provides similar decompositions of productivity change is proposed such that the proposed method uses all price information about inputs and outputs for determining productivity index. In other words the index developed here is defined in terms of inputs cost and outputs profit rather than input cost. In

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particular, a dual Malmquist index, which is defined in term of profit rather than input cost, is developed and computed using non-parametric linear programming models, known as DEA.

The rest of paper is organized as follows: Section 2 introduces the Malmquist productivity index based on profit. In Section 3 we decompose the profit Malmquist productivity index. The component of profit Malmquist productivity index are computed in Section 4. An illustrative example is then presented in Section 5. Conclusion is made at the last section.

2. Malmquist productivity index based on profit

Consider that in time period *t*, producers are using inputs $x^t \in \mathrm{R}_+^n$, to produce outputs $y^t \in \mathrm{R}_+^m$. We define now the production technology of period *t*, which is:

$$P^t(x^t, y^t) = \{(x^t, y^t) : x^t \text{ can produce } y^t\}. \quad (1)$$

We assume that $P^t(x^t, y^t)$ is non-empty. $P^t(x^t, y^t)$ has upper bound as:

$$\Lambda = \{(x^t, y^t) : (x^t, y^t) \in P^t(x^t, y^t), \quad (2)$$
$$(\lambda x^t, \phi y^t) \notin P^t(x^t, y^t), \lambda < 1, \phi > 1\},$$

that defines a boundary (frontier) to the $P^t(x^t, y^t)$ in the sense that any radial increase of output vectors and decrease of input vectors that lie on the frontier is not possible within $P^t(x^t, y^t)$.

Alternatively, with reference to the $P^t(x^t, y^t)$, we define the technology of production in terms of the distance function as:

$$D^t(y^t, x^t) = \max\{\frac{\theta}{\eta} : (\frac{x}{\theta}, \frac{y}{\eta}) \quad (3)$$
$$\in P^t(x^t, y^t), \theta, \eta > 0\}.$$

$D^t(y^t, x^t)$ characterizes the technology of production completely in the sense that $D^t(y^t, x^t) \geq 1$ is sufficient for:

$$(x^t, y^t) \in P^t(x^t, y^t),$$

and

$$D^t(y^t, x^t) = 1,$$

if and only if (x^t, y^t) lie on the frontier. $D^t(y^t, x^t)$ is the reciprocal to measure of technical efficiency, which is as follows:

$$TE(y^t, x^t) = \min\{\frac{\alpha}{\beta} : (\alpha x, \beta y) \in P^t(x^t, y^t)\}. \quad (4)$$

When output prices, $p^t \in R_+^m$ and input prices, $c^t \in R_+^m$, are available one may define technology in terms of the profit function (Cooper *et al.*, 2006), which is:

$$PR^t(x^t, y^t, c^t, p^t) = \max\{\frac{p^t y^t}{c^t x^t} : (x^t, y^t) \quad (5)$$
$$\in P^t(x^t, y^t), c^t, p^t > 0\}, PR^t(x^t, y^t, c^t, p^t)$$

defines the maximum profit when the output prices is p^t and input prices is c^t.

The set of units (x^t, y^t) which correspond to the scalar $PR^t(x^t, y^t, c^t, p^t)$ lie on a frontier which defines a price boundary, as:

$$\Gamma = \{(x^t, y^t) : \frac{p^t y^t}{c^t x^t} = PR^t(x^t, y^t, c^t, p^t)\}. \quad (6)$$

By using (3) and (5) we have the following inequality:

$$PR^t(x^t, y^t, c^t, p^t) \geq D^t(y^t, x^t) \frac{p^t y^t}{c^t x^t}. \quad (7)$$

The profit efficiency for (x^t, y^t) under output prices p^t and input prices c^t is as follows (Cooper *et al.*, 2006):

$$E_p(x^t, y^t, c^t, p^t) = \frac{p^t y^t / c^t x^t}{PR^t(x^t, y^t, c^t, p^t)}. \quad (8)$$

This measure compares the maximum feasible production profit $PR^t(x^t, y^t, c^t, p^t)$ to that observed. (8) is equal or less than one. If profit efficiency is less than one it will be either because production is based on shortfall output and excessive input usage or because it takes place at the wrong mix in the light of output and input prices, or both. The first factor is captured by the

technical efficiency measure in (4) and the second by the allocative efficiency. By (4) and (5), we define allocative efficiency as follows:

$$AE^t(x^t,y^t,c^t,p^t) = \frac{p^t(\dfrac{y^t}{\eta})/c^t(\dfrac{x^t}{\theta})}{PR^t(x^t,y^t,c^t,p^t)}$$

$$= \frac{(\dfrac{\theta}{\eta})(\dfrac{p^t y^t}{c^t x^t})}{PR^t(x^t,y^t,c^t,p^t)} = E_p D^t(y^t,x^t). \qquad (9)$$

2.1. Theorem

$AE = 1$ if and only if:

$$E_P = \frac{1}{D^t(y^t,x^t)} = TE(y^t,x^t).$$

Proof: By (9) the proof is evident.

Assume two time periods t and $t+1$ respectively and define in each one of them technology and production as in the previous section. Taking time period t as the reference period, the Malmquist index, M, is defined as follows (Fare *et al.*, 1989):

$$M^t = [\frac{D^t(y^{t+1},x^{t+1})}{D^t(y^t,x^t)}], \qquad (10)$$

where, M^t compares (x^{t+1},y^{t+1}) and (x^t,y^t) by measuring their respective distances from the constant returns to scale production boundary of the reference period t. In a similar fashion, with reference to period $t+1$, the following index is defined (Fare *et al.*, 1989):

$$M^{t+1} = \left[\frac{D^{t+1}(y^{t+1},x^{t+1})}{D^{t+1}(y^t,x^t)}\right]. \qquad (11)$$

We use the geometric mean of the M^t and M^{t+1} so that the M (Fare *et al.*, 1989) is:

$$M = \left[\frac{D^t(y^{t+1},x^{t+1})}{D^t(y^t,x^t)}\frac{D^{t+1}(y^{t+1},x^{t+1})}{D^{t+1}(y^t,x^t)}\right]^{\frac{1}{2}}. \qquad (12)$$

Clearly, given the definition of the distance function (3), when $M>1$ on average (x^{t+1},y^{t+1})

further from the efficient boundary than is (x^t,y^t) and so we have a deterioration in productivity between t and $t+1$. Productivity remains unchanged if $M=1$ and improve if $M<1$.

By the above discussion we have the following definition.

2.2. Definition

In the spirit of the indices in (10)-(12) the profit Malmquist (PM) productivity index of periods $t,t+1$ and their geometric mean is defined as follows, respectively:

$$PM^t = \left[\frac{\dfrac{PR^t(x^{t+1},y^{t+1},c^t,p^t)}{\dfrac{p^t y^{t+1}}{c^t x^{t+1}}}}{\dfrac{PR^t(x^t,y^t,c^t,p^t)}{\dfrac{p^t y^t}{c^t x^t}}}\right],$$

$$PM^{t+1} = \left[\frac{\dfrac{PR^{t+1}(x^{t+1},y^{t+1},c^{t+1},p^{t+1})}{\dfrac{p^{t+1} y^{t+1}}{c^{t+1} x^{t+1}}}}{\dfrac{PR^{t+1}(x^t,y^t,c^{t+1},p^{t+1})}{\dfrac{p^{t+1} y^t}{c^{t+1} x^t}}}\right]$$

and

$$PM = \left[\frac{\dfrac{PR^t(x^{t+1},y^{t+1},c^t,p^t)}{\dfrac{p^t y^{t+1}}{c^t x^{t+1}}}}{\dfrac{PR^t(x^t,y^t,c^t,p^t)}{\dfrac{p^t y^t}{c^t x^t}}}\right.$$

$$\times\left.\frac{\dfrac{PR^{t+1}(x^{t+1},y^{t+1},c^{t+1},p^{t+1})}{\dfrac{p^{t+1} y^{t+1}}{c^{t+1} x^{t+1}}}}{\dfrac{PR^{t+1}(x^t,y^t,c^{t+1},p^{t+1})}{\dfrac{p^{t+1} y^t}{c^{t+1} x^t}}}\right]^{\frac{1}{2}}.$$

3. Decomposition of the profit malmquist productivity index

We can decompose the *PM* into Profit Efficiency Change (PEC) and Profit Technical Change (PTC). Moreover, both of these components can be further decomposed into quantity and price components.

3.1. First stage decomposition of the PM index

We can decompose PM index into PEC and PTC components, as follows:

$$PM = \dfrac{\dfrac{PR^{t+1}(x^{t+1},y^{t+1},c^{t+1},p^{t+1})}{\dfrac{p^{t+1}y^{t+1}}{c^{t+1}x^{t+1}}}}{\dfrac{PR^{t}(x^{t},y^{t},c^{t},p^{t})}{\dfrac{p^{t}y^{t}}{c^{t}x^{t}}}}$$

$$\times \left[\dfrac{\dfrac{PR^{t}(x^{t+1},y^{t+1},c^{t},p^{t})}{\dfrac{p^{t}y^{t+1}}{c^{t}x^{t+1}}}}{\dfrac{PR^{t+1}(x^{t+1},y^{t+1},c^{t+1},p^{t+1})}{\dfrac{p^{t+1}y^{t+1}}{c^{t+1}x^{t+1}}}} \right.$$

$$\left. \times \dfrac{\dfrac{PR^{t}(x^{t},y^{t},c^{t},p^{t})}{\dfrac{p^{t}y^{t}}{c^{t}x^{t}}}}{\dfrac{PR^{t+1}(x^{t},y^{t},c^{t+1},p^{t+1})}{\dfrac{p^{t+1}y^{t}}{c^{t+1}x^{t}}}} \right]^{\frac{1}{2}}. \tag{13}$$

The component outside the square brackets in (13) captures PEC between periods t and $t+1$. The term inside the square brackets in (13) will be referred to as PTC. It measures the shift of the profit boundary evaluated at the mixes (x^{t},y^{t}) and (x^{t+1},y^{t+1}). The technical change component compares the maximum profit of securing certain input and output in one period relative to that in another period.

3.2. Second stage decomposition of the PM index

The terms obtained in the first stage decomposition of the PM index can themselves be decomposed as follows:

The decomposition of PEC: The PEC component in (13) can be decomposed into technical (TEC) and allocative efficiency change (AEC) terms. Therefore, we have the following decomposition:

$$E_p^t = \dfrac{AE^t}{D^t(y^t,x^t)} = \dfrac{\dfrac{(\dfrac{\vartheta}{\eta})\left(\dfrac{p^t y^t}{c^t x^t}\right)}{PR^t(x^t,y^t,c^t,p^t)}}{D^t(y^t,x^t)}$$

$$= \dfrac{D^t(y^t,x^t)\left(\dfrac{p^t y^t}{c^t x^t}\right)}{PR^t(x^t,y^t,c^t,p^t)} \times \dfrac{1}{D^t(y^t,x^t)},$$

$$E_p^{t+1} = \dfrac{D^{t+1}(y^{t+1},x^{t+1})\left(\dfrac{p^{t+1}y^{t+1}}{c^{t+1}x^{t+1}}\right)}{PR^{t+1}(x^{t+1},y^{t+1},c^{t+1},p^{t+1})}$$

$$\times \dfrac{1}{D^{t+1}(y^{t+1},x^{t+1})}.$$

Therefore, we have:

$$PEC = \dfrac{E_P^t}{E_P^{t+1}} = \dfrac{D^{t+1}(y^{t+1},x^{t+1})}{D^t(y^t,x^t)}$$

$$\times \dfrac{\dfrac{PR^{t+1}(x^{t+1},y^{t+1},c^{t+1},p^{t+1})}{D^{t+1}(y^{t+1},x^{t+1})\left(\dfrac{p^{t+1}y^{t+1}}{c^{t+1}x^{t+1}}\right)}}{\dfrac{PR^{t}(x^{t},y^{t},c^{t},p^{t})}{D^{t}(y^{t},x^{t})\left(\dfrac{p^{t}y^{t}}{c^{t}x^{t}}\right)}}. \tag{14}$$

The decomposition of PTC: Shift in the profit boundary may be caused either by shifts of the production boundary and /or by relative input and output price shifts. Thus, the PTC factor, can be further decomposed. So, we have the following decomposition for the first term of PTC in (13).

$$\frac{PR^t(x^{t+1}, y^{t+1}, c^t, p^t)}{D^t(y^{t+1}, x^{t+1})\left(\dfrac{p^t y^{t+1}}{c^t x^{t+1}}\right)}$$

$$\frac{PR^{t+1}(x^{t+1}, y^{t+1}, c^{t+1}, p^{t+1})}{D^{t+1}(y^{t+1}, x^{t+1})\left(\dfrac{p^{t+1} y^{t+1}}{c^{t+1} x^{t+1}}\right)}$$

$$\times \frac{D^t(y^{t+1}, x^{t+1})}{D^{t+1}(y^{t+1}, x^{t+1})}, \tag{15}$$

and from the second term of PTC in (13) we have:

$$\frac{\dfrac{PR^t(x^t, y^t, c^t, p^t)}{D^t(y^t, x^t)\left(\dfrac{p^t y^t}{c^t x^t}\right)}}{\dfrac{PR^{t+1}(x^t, y^t, c^{t+1}, p^{t+1})}{D^{t+1}(y^t, x^t)\left(\dfrac{p^{t+1} y^t}{c^{t+1} x^t}\right)}} \times \frac{D^t(y^t, x^t)}{D^{t+1}(y^t, x^t)}, \tag{16}$$

by using (15) and (16), we can write PTC as follows:

$$PTC = \left[\frac{D^t(y^{t+1}, x^{t+1})}{D^{t+1}(y^{t+1}, x^{t+1})} \times \frac{D^t(y^t, x^t)}{D^{t+1}(y^t, x^t)}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\dfrac{PR^t(x^{t+1}, y^{t+1}, c^t, p^t)}{D^t(y^{t+1}, x^{t+1})\left(\dfrac{p^t y^{t+1}}{c^t x^{t+1}}\right)}}{\dfrac{PR^{t+1}(x^{t+1}, y^{t+1}, c^{t+1}, p^{t+1})}{D^{t+1}(y^{t+1}, x^{t+1})\left(\dfrac{p^{t+1} y^{t+1}}{c^{t+1} x^{t+1}}\right)}}\right.$$

$$\left.\times \frac{\dfrac{PR^t(x^t, y^t, c^t, p^t)}{D^t(y^t, x^t)\left(\dfrac{p^t y^t}{c^t x^t}\right)}}{\dfrac{PR^{t+1}(x^t, y^t, c^{t+1}, p^{t+1})}{D^{t+1}(y^t, x^t)\left(\dfrac{p^{t+1} y^t}{c^{t+1} x^t}\right)}}\right]^{\frac{1}{2}}. \tag{17}$$

The first term in the right-hand side of (17) is the technical change (TC). It reflects the shift of the production boundary between periods t and $t+1$,

evaluated at (x^t, y^t) and (x^{t+1}, y^{t+1}). The term in the second square brackets in (17) capture the residual impact of relative output and input price changes on the shift of the profit boundary. This term will be referred as Price Effect (PE).

4. Computation of the index and its components

Let us have in each time period, $j= 1,...,J$ production units. In period t, the kth unit employs amount x_{kn}^t of input n(n=1,...,N), with price c_{kn}^t to produce amount y_{km}^t of output $m\,(m=1,...,M)$, with price p_{km}^t. To compute the term $PR^t(x^t, y^t, c^t, p^t)$ for unit k, the researchers propose the following model:

$$PR^t(x^t, y^t, c^t, p^t) =$$

$$\frac{p^t y^{t*}}{c^t x^{t*}} = \max \sum_{m=1}^{M} p_{km}^t y_m \Big/ \sum_{n=1}^{N} c_{kn}^t x_n \tag{18}$$

Subject to:

$$y_m = \sum_{j=1}^{J} \lambda_j y_{jm}^t \geq y_{km}^t,$$

$$x_n = \sum_{j=1}^{J} \lambda_j x_{jn}^t \leq x_{kn}^t,$$

$$\lambda_j \geq 0, j = 1,...,J.$$

For computing the term $PR^t(x^{t+1}, y^{t+1}, c^t, p^t)$, the researchers propose the following:

$$PR^t(x^{t+1}, y^{t+1}, c^t, p^t) =$$

$$\frac{p^t y^{t*}}{c^t x^{t*}} = \max \sum_{m=1}^{M} p_{km}^t y_m \Big/ \sum_{n=1}^{N} c_{kn}^t x_n \tag{19}$$

Subject to:

$$y_m = \sum_{j=1}^{J} \lambda_j y_{jm}^t \geq y_{km}^{t+1},$$

$$x_n = \sum_{j=1}^{J} \lambda_j x_{jn}^t \leq x_{kn}^{t+1},$$

$$\lambda_j \geq 0, j = 1,...,J.$$

The terms $PR^{t+1}(x^{t+1}, y^{t+1}, c^{t+1}, p^{t+1})$ and $PR^{t+1}(x^t, y^t, c^{t+1}, p^{t+1})$ can be computed using models (18) and (19) respectively, after changing round the time periods t and $t+1$. The term $\left[D^t(y^t, x^t)\right]^{-1}$ can be computed using a model. Then, we propose the following models:

$$\left[D^t(y^t, x^t)\right]^{-1} = \min \frac{\alpha}{\beta} \qquad (20)$$

Subject to:

$$\sum_{j=1}^{J} \lambda_j y^t_{jm} \geq \beta y^t_{km},$$

$$\sum_{j=1}^{J} \lambda_j x^t_{jn} \leq \alpha x^t_{kn},$$

$$\lambda_j \geq 0, j = 1, \ldots, J,$$

and

$$\left[D^t(y^{t+1}, x^{t+1})\right]^{-1} = \min \frac{\alpha}{\beta} \qquad (21)$$

Subject to:

$$\sum_{j=1}^{J} \lambda_j y^t_{jm} \geq \beta y^{t+1}_{km},$$

$$\sum_{j=1}^{J} \lambda_j x^t_{jn} \leq \alpha x^{t+1}_{kn},$$

$$\lambda_j \geq 0, j = 1, \ldots, J.$$

$D^{t+1}(y^t, x^t)$ and $D^{t+1}(y^{t+1}, x^{t+1})$ can be computed using models (20) and (21) respectively, after changing round the time periods t and $t+1$.

By $\dfrac{1}{\sum_{n=1}^{N} C^t_{kn} x_n} = z$, $\dfrac{1}{\beta} = z$ and setting:

$$y'_m = z y_m, \quad m = 1, \ldots, M$$

$$x'_n = z x_n, \quad n = 1, \ldots, N$$

$$\lambda'_j = z \lambda_j, \quad j = 1, \ldots, J$$

$$\alpha' = z \alpha,$$

the fractional programming problems (18), (19), (20) and (21) are transformed the linear programming problems (22), (23), (24) and (25) respectively, which are as follows:

$$PR^t(x^t, y^t, c^t, p^t) = \max \sum_{m=1}^{M} p^t_{km} y'_m \qquad (22)$$

Subject to:

$$\sum_{n=1}^{N} c^t_{kn} x'_n = 1,$$

$$z y^t_{km} \leq \sum_{j=1}^{J} \lambda'_j y^t_{jm} = y'_m,$$

$$z x^t_{kn} \geq \sum_{j=1}^{J} \lambda'_j x^t_{jn} = x'_n,$$

$$\lambda'_j, z \geq 0, j = 1, \ldots, J;$$

$$PR^t(x^{t+1}, y^{t+1}, c^t, p^t) = \max \sum_{m=1}^{M} p^t_{km} y'_m \qquad (23)$$

Subject to:

$$\sum_{n=1}^{N} c^t_{kn} x'_n = 1,$$

$$z y^{t+1}_{km} \leq \sum_{j=1}^{J} \lambda'_j y^t_{jm} = y'_m,$$

$$z x^{t+1}_{kn} \geq \sum_{j=1}^{J} \lambda'_j x^t_{jn} = x'_n,$$

$$\lambda'_j, z \geq 0, j = 1, \ldots, J;$$

$$\left[D^t(y^t, x^t)\right]^{-1} = \min \alpha' \qquad (24)$$

Subject to:

$$\sum_{j=1}^{J} \lambda'_j y^t_{jm} - y^t_{km} \geq 0,$$

$$\sum_{j=1}^{J} \lambda'_j x^t_{jn} - \alpha' x^t_{kn} \leq 0,$$

$$\lambda'_j, \alpha' \geq 0, j = 1, \ldots, J;$$

and

$$\left[D^{t}(y^{t+1}, x^{t+1}) \right]^{-1} = \min \ \alpha' \qquad (25)$$

Subject to:

$$\sum_{j=1}^{J} \lambda'_{j} y^{t}_{jm} - y^{t+1}_{km} \geq 0,$$

$$\sum_{j=1}^{J} \lambda'_{j} x^{t}_{jn} - \alpha' x^{t+1}_{kn} \leq 0,$$

$$\lambda'_{j}, \alpha' \geq 0, j = 1, \ldots, J.$$

$(z = 1, \ \lambda'_{k} = 1, \ \lambda'_{j} = 0 (j \neq k))$ is a feasible solution of model (22). Hence the optimal value of objective function at least is $\sum_{m=1}^{M} p^{t}_{km} y^{t}_{km}$ and by (2) is not unbounded.

$(\alpha' = 1, \ \lambda'_{k} = 1, \ \lambda'_{j} = 0 (j \neq k))$ is a feasible solution of model (24). Therefore, the optimal α' is not greater than 1. On the other hand, the constraint $\sum_{j=1}^{J} \lambda'_{j} y^{t}_{jm} - y^{t}_{km} \geq 0$ force λ' to be nonzero because $y^{t}_{km} \geq 0$ and $y^{t}_{km} \neq 0$. Hence, from $\sum_{j=1}^{J} \lambda'_{j} x^{t}_{jn} - \alpha' x^{t}_{kn} \leq 0, \ \alpha'$ must be greater than zero.

If $(x^{t+1}_{kn}, y^{t+1}_{km}) \in p^{t}(x^{t}, y^{t})$, (25) is similar to (24), otherwise the constraints of (25) require the activity $(\alpha' x^{t+1}_{kn}, y^{t+1}_{km})$ to belong to $p^{t}(x^{t}, y^{t})$, while the objective seeks the minimum α' that increases the input vector x^{t+1}_{kn} radially to $\alpha' x^{t+1}_{kn}$ and guarantees at least the output level y^{t+1}_{km} in all components. Hence, $\alpha' \geq 1$ and objective function of (25) is not unbounded.

5. Numerical example

Units 1-5 in Table 1 use one input to produce a single output. Assume now that in period 1 all units increase their input level by 10% output level by 20% and the price of their input reduces by 5%.

The data in Table 1 can now be used to compute the components of the PM and M indices in respect of unit 2 as detailed below. The results appear in Table 2.

[a] M=TC×TEC, [b] PM=TC×TEC×AEC×PE

In this example two indices show opposite trends in productivity at unit level. Unit 2 shows

productivity growth by the M index (M=0.9167). But, when input and output prices are taken into consideration the PM index indicates productivity regress (PM=1.004). Here TEC=1 shows that the technical efficiency of unit 2 remains unchanged. The TC component is defined as the square root of the distance between the efficient frontiers at the input and output mixes of unit 2 in periods 0 and 1. The value of AEC being below 1 reflects the improvement in the allocative efficiency of unit 2 between period 0 and 1.

6. Conclusion

Because between time periods t and $t+1$ the price some inputs and outputs are different and this changes the productivity of DMUs. The contribution of this paper is that it addressed the above subject and proposed a new Mulmquist productivity index, so called profit Mulmquist productivity index, by price information of all inputs and outputs when the producers desire to maximize total profit and the price of inputs and outputs are available, while the previous methods do not use price information of all inputs and outputs. The proposed Mulmquist productivity index in this paper is more precise than the other Mulmquist productivity index. Because, it uses all price information of data.

Then, we decomposed the defined index into the profit efficiency and profit technical change. Finally, the proposed fractional programming problems are converted to the linear programming problems. Therefore, to computa- tion of the defined Malmquist productivity index the researchers used the linear programming problems.

Table 1: Numerical example data for period 0.

Unit	Input level	Input price	Output level	Output price
1	3	3	1	6
2	5	1	2	5
3	6	2	3	5
4	5.5	2	2	4
5	7	1	4	3

Table 2: Results.

M[a]	TC	TEC	PM[b]
0.9167	0.9167	1	1.004
TEC	AEC	TC	PE
1	0.9999	0.9167	1.0954

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