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ORIGINAL RESEARCH

Robust optimization of a mathematical model to design a dynamic cell formation problem considering labor utilization

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Abstract Cell formation (CF) problem is one of the most important decision problems in designing a cellular manufacturing system includes grouping machines into machine cells and parts into part families. Several factors should be considered in a cell formation problem. In this work, robust optimization of a mathematical model of a dynamic cell formation problem integrating CF, production planning and worker assignment is implemented with uncertain scenario-based data. The robust approach is used to reduce the effects of fluctuations of the uncertain parameters with regards to all possible future scenarios. In this research, miscellaneous cost parameters of the cell formation and demand fluctuations are subject to uncertainty and a mixed-integer nonlinear programming model is developed to formulate the related robust dynamic cell formation problem. The objective function seeks to minimize total costs including machine constant, machine procurement, machine relocation, machine operation, inter-cell and intra-cell movement, overtime, shifting labors between cells and inventory holding. Finally, a case study is carried out to display the robustness and effectiveness of the proposed model. The tradeoff between solution robustness and model robustness is also analyzed in the obtained results.

Keywords Dynamic cell formation problem · Scenario-based robust optimization · Mixed-integer nonlinear model · Worker assignment

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Introduction

Today, global competitive environment has persuaded manufacturing practitioners to deliver low-cost and high-quality products. Some recently applied approaches have been put into practice to cope with the ever growing manufacturing costs, such as location, material handling system, and energy. One of these recent manufacturing approaches is Group Technology (GT). GT is one of the main building blocks to implementing Just-In-Time (JIT) philosophy. This approach is based upon grouping parts and machines together with respect to their similarities in production processes, functionalities, etc. The aspect of GT which associates with the configuration of manufacturing firms is cellular manufacturing system (CMS). The most outstanding benefit of CMS can be noted as reduction in some production factors, such as lot sizes, lead times, work-in-process inventories and setups, while higher level of investment is inevitable to implement this system. Designing of a CMS involves four main steps. The first step associates with cell formation problem which comprises assigning parts to their families and machines to their corresponding machine cells based on some features, such as similar geometric design or processing requirements. Second, intra-cell and inter-cell layouts are defined through Group Layout (GL). This step determines the location of machines and cells in the shop floor. Third, Group Scheduling (GS) is accomplished to schedule parts within part families. Finally, required resources such as labors and material handling devices are assigned to the manufacturing cells.

It has been clarified by Wu et al. (2007) that these four steps are interrelated and in other words, the solution for each step influences the other one. Thus, simultaneously solving these problems has to be applied by the

researchers; that is, the matter not been paid attention enough. Nevertheless, due to the complexity and NP-complete nature of CF, GL, and GS decisions, most researchers have addressed two or three decisions sequentially or independently. However, the benefits gained from CMS implementation are highly affected by how theses stages of the CMS design have been performed in collaboration with each other.

Shorter product life cycles are an increasingly significant issue in CM. As a result, neglecting new products emerging at future imposes subsequent unplanned changes to the CMS design and causes production disruptions and unexpected costs. Hence, those changes should be incorporated in the design process. To come up with a solution to handle those changes, the dynamic cellular manufacturing system (DCMS) was introduced in which it is assumed that the product mix or volume changes of demands can be predicted in a multi-period planning horizon (Rheault et al. 1995).

Most DCMS models assume that the input parameters are deterministic and certain. However, in practical situations many parameters are uncertain and imprecise. DCMS design has to be implemented in many environments based on some parameters with uncertain values. However, there are few studies on designing cellular manufacturing systems under dynamic and uncertain conditions. These studies can be divided into four classes as fuzzy programming approach, stochastic programming approach, scenario-based programming approach, and robust optimization approach in terms of uncertainty expression type in the problem. Different robust optimization approaches have been introduced in the recent years to deal with the uncertainty of the data. In this study, a scenario-based robust optimization approach is used to cope with uncertainty and to find a solution that is robust with regard to data uncertainties in part demand, inter-cell and intra-cell movement cost, machine purchase cost, selling machine revenue, machine fixed/variable cost, machine relocation cost, inter-cell movement labor cost, process variable cost and inventory holding cost. It is the first time that this vast coverage of input parameters in a DCMS are considered uncertain to be handled by a robust optimization approach.

The aims of this study are twofold. The first one is to formulate a new mathematical model with an extensive coverage of important manufacturing features including batch intra-cell/inter-cell movement, production planning strategies (i.e., internal production, inventory holding, and lost sale as under-fulfilled demand), selling/purchase machine, labor movement, labor assignment, labor capacity, machine relocation, regular/overtime machine capacity, cell size limit, flexible operation sequence, machine/labor processing time, and uncertain scenario-based parameters (i.e., part demand and miscellaneous costs). The second aim is to develop a robust model based on the deterministic proposed model using scenario-based robust optimization approach. The important concern of the employed robust methodology is to obtain an optimal CM design that is robust with regard to data uncertainties in part demand and miscellaneous costs. The objective function of the integrated model is to minimize the total costs of machine constant, machine procurement, machine relocation, machine operation, inter-cell and intra-cell movement, overtime, shifting labors between cells and inventory holding. The main constraints are operator-machine-cell assignment, machine capacity, machine number equilibrium, labor capacity, cell size limit, and balancing inventory.

Recently, Kia et al. (2012) have formulated a mathematical model integrating the CF and GL decisions in a dynamic environment by considering some advantages including: (1) considering flexible configuration of cells, (2) calculating relocation cost based on the locations assigned to machines, (3) distance-based calculation of intra- and inter-cell material handling costs and (4) considering multi-rows layout of equal sized facilities. One disadvantage in their work was ignoring the assignment of operators to machines located in different cells. In another study, Bagheri and Bashiri (2014) investigated the simultaneous consideration of the cell formation problem with inter-cell layout and operator assignment problems in a dynamic environment by formulating a mathematical model with the objectives of minimization of inter–intra cell part trips, machine relocation cost and operator-related issues. A main drawback in both mentioned studies was that all parameters were considered deterministic despite the fact some of them should be predicted for the future periods in a dynamic environment with high level of uncertainty.

Generally, the presented study is an extension of the previous studies Kia et al. (2012), Bagheri and Bashiri (2014) by integrating the CF, production planning (PP) and worker assignment in a mathematical model with data uncertainties in most parameters of model including part demand and miscellaneous costs which is solved by a scenario-based robust optimization approach. The robust approach is used to reduce the effects of fluctuations of the uncertain parameters with regards to all possible future scenarios.

To investigate the effect of turbulence in the values of uncertain data on the model performance and obtained solutions, a robust model is developed. Then, a case study is carried out to demonstrate the validity of the employed robust approach and verify the integrated DCMS model. The obtained results of implementing the case study also illustrate the applicability of the proposed model in real industrial cases.

The remainder of this paper is organized as follows. In "Literature review" section, the literature review is carried out. The background of the robust optimization approach employed in this study is described in "Robust optimization" section. A mathematical model is formulated integrating CF, PP and worker assignment decisions in "Mathematical model and model description" section followed using some linearization procedures. In addition, a robust model is developed in this section. "A case study" section illustrates the case study that is implemented to investigate the features of the proposed model and assess the performance of the developed robust model. Finally, conclusion is given in "Conclusion" section.

Literature review

One of the most important issues which have received less attention in the literature body of DCMS is consideration of human-related issues. The first mathematical model developed for human-related aspects of DCMS was presented by Aryanezhad et al. (2009). They developed a new mathematical model to deal with DCMS and worker assignment problems, simultaneously. The objective function of this model contains system costs including machine purchase, operating, inter-cell material handling, machine relocation, worker hiring, training, salary and firing costs. Balakrishnan and Cheng (2005) presented a flexible framework for modeling cellular manufacturing when product demand changes during the planning horizon.

Most CMS models assume that the input parameters are deterministic and certain. However, in practical situations, many parameters such as parts demands, processing times and machines capacities are uncertain. Robust optimization as a strong technique was used to deal with uncertainty in the systems. Robust optimization can be very efficient and useful because of generation of the good and robust solutions for any possible occurrences of uncertain parameters (Mulvey et al. 1995). The concept of robust optimization in operation research was presented by Mulvey et al. (1995). They extended a robust counterpart approach with a nonlinear function that penalizes the constraint violations and addresses uncertainties via a set of discrete scenarios. Bai et al. (1997) demonstrated that the traditional stochastic linear program fails to determine a robust solution despite the presence of a cheap robust point. They evaluated properties of risk-averse utility functions in robust optimization. They discussed that a concave utility function should be incorporated in a model whenever the decision maker is risk averse. Ben-Tal and Nemirovski (1998) proposed a robust optimization approach to formulate continuous uncertain parameters. Ben-Tal and Nemirovski (1998), Ben-Tal and Nemirovski (2002) and Ben-Tal et al. (2002) developed robust theory of linear, quadratic and conic quadratic problems. Bertsimas and Sim (2002) and Bertsimas and Thiele (2003) proposed robust optimization methods for discrete optimization in continuous spaces.

Mirzapour Al-E-Hashem et al. (2011) studied multi-site aggregate production planning problems under uncertainty by defining multi-objective robust optimization models.

Mahdavi et al. (2010) proposed a mathematical model for solving dynamic cellular manufacturing problem considering two areas of cell configuration and assigning the operators to the machines. In the proposed model, some factors have been considered including machine capacity, multi-period planning horizon and the worker idleness time. Rafiei and Ghodsi (2013) designed a two-objective mathematical model for solving the operator assignment and cell configuration simultaneously. Minimizing total costs of machines purchase, machine relocation and overhead, parts intra-cell and inter-cell movements and the operator inter-cell movements were considered in the first objective function. The second objective function increased the utilization level of the operators.

In similar studies, Kia et al. (2013), Shirazi et al. (2014) presented multi-objective mixed-integer nonlinear programming models to combine the problems of dynamic cell formation and group layout. They utilized the multi-row layout for locating machines inside the cells with flexible size regarding the lot splitting feature and several other features (i.e., operation sequence, processing time, machine duplicates, and machine capacity).

Bashiri and Bagheri (2013) proposed a two-phase heuristic method for cell formation and operator assigning, where in the first phase, clustering technique and in the second phase, a mathematical model is used. Kia et al. (2011) presented a mathematical model for a multi-period CM system layout with fuzzy parameters. By taking the linear intra-cell machines layout, operation sequence, processing times and the machines capacity into account, the model intended to minimize the intra/inter-cell movements costs, the machines overhead costs and machines relocation costs.

Ghezavati et al. (2011) proposed a robust model for cell formation and group scheduling with supply chain approach. In this model, the uncertainty resulted from demand and parts processing time were expressed by stochastic scenarios with given probabilities. They formulated the problem with the objective to minimize delaying costs for parts delivery due time, the parts outsourcing costs to suppliers and the underutilization cost of machines and solved it by a hybrid meta-heuristic algorithm. Paydar et al. (2013) presented a mathematical model for integration of cell formation, machine layout and production planning. They considered customer demand and machine

capacity uncertain and proposed a robust model. Forghani et al. (2012) suggested a robust model to determine cell formation and group layout where the parts demand is uncertain.

Sakhaii et al. (2015) developed a robust optimization approach for a new integrated MILP model to solve a DCMS with unreliable machines and a production planning problem simultaneously. They adopted a robust optimization approach immunized against even worst-case to cope with the parts processing time uncertainty. Hassannezhad et al. (2014) performed sensitivity analysis of modified self-adaptive differential evolution (MSDE) algorithm for basic parameters of cell formation problem. First, they presented a DCMS model. Then, two basic test CF problems were introduced to assess the performance of MSDE algorithm by diverse problems sizes.

Regarding this section, it could be concluded that no study has been done on simultaneous integrating of three problems as cell configuration, production planning and operator assigning so far with uncertainty considered in the most model parameters including part demands and cost parameters.

Robust optimization

Mulvey et al. (1995) presented a framework for robust optimization that involves two types of robustness: ''solution robustness'' (the solution is nearly optimal in all scenarios) and ''model robustness'' (the solution is nearly feasible in all scenarios). The robust optimization method extended by Mulvey et al. (1995), in fact, develops stochastic programming through replacing traditional expected cost minimization objective by one that explicitly addresses cost variability. The framework of robust optimization is briefly demonstrated by Feng and Rakesh (2010). The form of the robust optimization model is as follows:

$$\text{Min } c^T x + d^T y \tag{1}$$

$$Ax = b \tag{2}$$

$$Bx + Cy = e \tag{3}$$

$$x, y \geq 0 \tag{4}$$

where x defines the vector of decision variables that should be determined under the uncertainty of model parameters. B, C and e demonstrate random technological coefficient matrix and right- hand side vector, respectively. Assume a finite set of scenarios $\Omega = \{1, 2,\ldots,s\}$ to model the uncertain parameters; with each scenario $s \in \Omega$, we associate the subset $\{d_s; B_s; C_s; e_s\}$ and the probability of the scenario $p_s(\sum_{s=1}^{s} p_s = 1)$.

Note that a scenario is a series of data realizations over the planning horizon. In addition, control variable y, can be denoted as y_s for scenario s. δ_s represents the infeasibility of the model under scenario s, because of parameter uncertainty the model may be infeasible for some scenarios. If the model is feasible, δ_s will be equal to 0, otherwise; δ_s will receive a positive value according to Eq. (7). A robust optimization model is formulated as follows:

$$\text{Min } \sigma(x, y_1, \ldots, y_s) + \omega\rho(\delta_1, \delta_2, \ldots, \delta_s) \tag{5}$$

$$Ax = b \tag{6}$$

$$B_s x + C_s y_s + \delta_s = e_s \quad \text{for all} \quad s \in \Omega \tag{7}$$

$$x \geq 0, \quad y_s \geq 0 \quad \text{for all} \quad s \in \Omega \tag{8}$$

The first term presents solution robustness, a single choice for an aggregate objective in (1). The second term demonstrates model robustness, feasibility penalty function, which is used to penalize violation of the control constraint under some of the scenarios. Mulvey et al. (1995) used Eq. (9) to indicate solution robustness as follows:

$$\sigma(0) = \sum_{s\in\Omega} \psi_s p_s + \lambda \sum_{s\in\Omega} p_s \left(\psi_s - \sum_{s'\in\Omega} p_{s'} \psi_{s'} \right)^2 \tag{9}$$

As can be seen, there is a quadratic term in Eq. (9). Yu and Li (2000) proposed an absolute deviation instead of the quadratic term, because the computational effort required due to the quadratic term is less, shown as follows:

$$\sigma(0) = \sum_{s\in\Omega} \psi_s p_s + \lambda \sum_{s\in\Omega} p_s \left| \psi_s - \sum_{s'\in\Omega} p_{s'} \psi_{s'} \right| \tag{10}$$

Mathematical model and model description

In this section, a new mixed-integer nonlinear programming model of a DCMS integrating CF, PP and worker assignment is presented to minimize total costs including machine constant, machine procurement, machine relocation, machine operation, inter-cell and intra-cell movement, overtime, shifting labors between cells and inventory holding respecting to the following assumptions.

Assumptions

1. Each part type has several operations which must be processed according to their sequence data.
2. Process time and manual workload time required for performing operations of a part type on various machine types are known and deterministic.
3. Part demands in each period are uncertain and defined in scenarios.

4. Time-capacity in regular time and overtime for each machine type are known and deterministic over the planning horizon.
5. Purchasing price and revenue from selling of each machine type are uncertain.
6. Constant cost of each machine type is uncertain. It covers overall service and maintenance cost. It is burdened for each machine even when a machine is idle.
7. Variable cost of each machine type in regular time and overtime is uncertain. It covers operating cost depending on the workload allocated to the machine.
8. Holding inventory is allowed and its related cost is uncertain.
9. In each period, the number of cells and the maximum cell size is known.
10. All machine types are multipurpose. Therefore, each operation of each part can be processed by more than one machine which brings flexibility for processing routes. However, each operation is allowed to be assigned to only one machine. In addition, there is no changeover cost for performing different operations by a machine.
11. Total number of labors is constant for all periods. Firing and hiring are not allowed.
12. Relocation cost of each machine between cells and shifting cost of operators between cells during successive periods are uncertain.
13. Batch sizes are fixed for moving parts between and within cells during planning horizon. However, inter-cell and intra-cell batches have different sizes. It is supposed that inter-cell and intra-cell transferring of batches has uncertain costs.

Indices

c Index for cells ($c = 1,\ldots,C$).
m Index for machine types ($m = 1,\ldots,M$).
p Index for part types ($p = 1,\ldots,P$).
h Index for time periods ($h = 1,\ldots,H$).
j Index for operations of part p ($j = 1,\ldots,Op$).
s Index for scenarios ($s = 1,\ldots,S$).

Input parameters

L Total number of labors.
D_{phs} Demand for part p in period h under scenario s.
ϑ_{phs} 1 if part p is planned to be produced in period h under scenario s; 0 otherwise.
B_p^{inter} Batch size for inter-cell movements of part p.
B_p^{intra} Batch size for intra-cell movements of part p.
γ_s^{inter} Inter-cell movement cost per batch under scenario s.
γ_s^{intra} Intra-cell movement cost per batch under scenario s. For justification of CMS, it is assumed that $(\gamma_s^{\text{intra}}/B_p^{\text{intra}}) < (\gamma_s^{\text{inter}}/B_p^{\text{inter}})$.
φ_{ms} Marginal cost to purchase machine type m under scenario s.
h_{phs} Inventory cost for holding part p at the end of period h under scenario s.
W_{ms} Marginal revenue from selling machine type m under scenario s.
α_{ms} Constant cost of machine type m in each period under scenario s.
ρ_{hs} Constant cost of inter-cell labor movement in period h under scenario s.
β_{ms} Variable cost of machine type m for each unit time in regular time under scenario s.
δ_{ms} Relocation cost of machine type m under scenario s.
T_{mh} Time-capacity of machine type m in period h in regular time.
T'_{mh} Time-capacity of machine type m in period h in overtime.
θ_{mhs} Variable cost of processing on machine type m per hour in overtime in period h under scenario s.
UB Maximal cell size.
t_{jpm} Processing time required to perform operation j of part type p by machine type m.
t'_{jpm} Manual workload time required to perform operation j of part type p by machine type m.
a_{jpm} 1 if operation j of part p can be processed by machine type m; 0 otherwise.
p_s Occurrence probability of scenario s.
WT Available time capacity per worker.

Decision variables

N_{mch} Number of machine type m allocated to cell c in period h.
k_{mch}^{+} Number of machine type m added in cell c in period h.
k_{mch}^{-} Number of machine type m removed from cell c in period h.
I_{mh}^{+} Number of machine type m purchased in period h.
I_{mh}^{-} Number of machine type m sold in period h.
X_{jpmchs} 1 if operation j of part type p is processed by machine type m in cell c in period h under scenario s; 0 otherwise.
L_{ch} Number of labors assigned to cell c in period h.
T'_{mch} Extra time needed for machine type m allocated to cell c in period h.
δ_{phs} the under-fulfillment of demand of part type p in period h under scenario s.

I_{phs} The inventory level of part p at the end of time period h under scenario s.

Q_{phs} Number of demand of part type p produced in period h under scenarios s.

Problem formulation

The objective function consists of nine components, given in Eqs. (1.1)–(1.9), seeks to minimize the sum of miscel-

$$\text{Min } Z = \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C} N_{mch}\cdot\alpha_{ms} \tag{1.1}$$

$$+\sum_{h=1}^{H}\sum_{m=1}^{M} I_{mh}^{+}\cdot\varphi_{ms} - \sum_{h=1}^{H}\sum_{m=1}^{M} I_{mh}^{-}\cdot w_{ms} \tag{1.2}$$

$$+\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{O_p}\sum_{m=1}^{M}\beta_{ms}\cdot Q_{phs}\cdot t_{jpm}\cdot X_{jpmchs} \tag{1.3}$$

$$+1/2\sum_{h=1}^{H}\sum_{p=1}^{P}\gamma_{s}^{\text{inter}}\cdot\left[\frac{Q_{phs}}{B_{p}^{\text{inter}}}\right]\sum_{j=1}^{O_p-1}\sum_{c=1}^{C}\left|\sum_{m=1}^{M}X_{(j+1)pmchs}-\sum_{m=1}^{M}X_{jpmchs}\right| \tag{1.4}$$

$$+1/2\sum_{h=1}^{H}\sum_{p=1}^{P}\gamma_{s}^{\text{intra}}\cdot\left[\frac{Q_{phs}}{B_{p}^{\text{intra}}}\right]\sum_{j=1}^{O_p-1}\sum_{c=1}^{C}\left(\sum_{m=1}^{M}\left|X_{(j+1)pmchs}-X_{jpmchs}\right|-\left|\sum_{m=1}^{M}X_{(j+1)pmchs}-\sum_{m=1}^{M}X_{jpmchs}\right|\right) \tag{1.5}$$

$$+\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}T_{mch}^{'}\cdot\theta_{mhs} \tag{1.6}$$

$$+1/2\sum_{h=1}^{H}\sum_{c=1}^{C}\rho_{hs}\cdot\left(\left|L_{c(h+1)}-L_{ch}\right|\right) \tag{1.7}$$

$$+1/2\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\delta_{ms}\cdot\left(k_{mch}^{+}+k_{mch}^{-}\right) \tag{1.8}$$

$$+\sum_{h=1}^{H}\sum_{p=1}^{P}h_{phs}\cdot I_{phs} \tag{1.9}$$

s.t:

$$\sum_{c=1}^{C}\sum_{m=1}^{M}X_{jpmchs}\cdot a_{jpm}=\vartheta_{phs} \qquad \forall j,p,h,s \tag{2}$$

$$X_{jpmchs}\le a_{jpm} \qquad \forall j,p,m,c,h,s \tag{3}$$

$$\sum_{p=1}^{P}\sum_{j=1}^{O_p}X_{jpmchs}\cdot Q_{phs}\cdot t_{jpm}\le T_{mh}\cdot N_{mch}+T'_{mch} \qquad \forall m,c,h,s \tag{4}$$

$$\sum_{c=1}^{C}N_{mch}-\sum_{c=1}^{C}N_{mc(h-1)}=I_{mh}^{+}-I_{mh}^{-} \qquad \forall m,h \tag{5}$$

$$N_{mc(h-1)}+k_{mch}^{+}-k_{mch}^{-}=N_{mch} \qquad \forall m,c,h \tag{6}$$

$$\sum_{c=1}^{C}T'_{mch}\le T'_{mh} \qquad \forall m,h \tag{7}$$

$$\sum_{c=1}^{C}L_{ch}\le L \qquad \forall h \tag{8}$$

$$\sum_{m=1}^{M}N_{mch}\le UB \qquad \forall c,h \tag{9}$$

$$\sum_{j=1}^{O_p}\sum_{p=1}^{P}\sum_{m=1}^{M}X_{jpmchs}\cdot Q_{phs}\cdot t'_{jpm}\le WT\cdot L_{ch} \qquad \forall c,h,s \tag{10}$$

$$D_{phs}=Q_{phs}-I_{phs}+I_{p(h-1)s} \qquad \forall p,h,s \tag{11}$$

$$Q_{phs}\le M\vartheta_{phs} \qquad \forall p,h,s \tag{12}$$

X_{jpmchs} in binary $\forall j,p,m,c,h,s$ (13)

$L_{ch},N_{mch},k_{mch}^{+},k_{mch}^{-},I_{mh}^{+},I_{mh}^{-}$ are positive and integer $\forall m,c,h$ (14)

$Q_{phs},I_{phs},T_{mch}^{'}\ge 0$ are positive and continuous $\forall p,m,c,h,s$ (15)

laneous costs. Term (1.1) demonstrates sum of constant cost of all machines which have been used over the planning horizon for entire cells. Term (1.2) shows the total purchase cost minus selling income for entire machines during all periods. Term (1.3) indicates the variable cost of processing operations by different machines in whole cells and periods. Terms (1.4) and (1.5) calculate inter-cell and intra-cell movement costs, respectively. Term (1.6) represents the total costs for overtime working of machines which is required to produce the partial fraction of demand. Term (1.7) demonstrates the total costs of shifting labors between cells over the planning horizon. Various parameters such as labors training, wage rate of skilled labors and labors transference among the cells affect this expenditure. Term (1.8) indicates the cost of machines relocations. Finally, the last term of the objective function considers inventory holding costs. It is worth mentioning that all components (1.1)–(9) in the objective function are calculated under scenario s.

The first constraint introduced in Eq. (2) ensures that each operation of part p is allocated to only one machine capable of processing that part operation and one cell in period h on condition that part p is planned to be produced in that period. Equation (3) guarantees that an operation of a part is assigned to a machine provided that the machine is capable of processing that part operation. Equation (4) guarantees that machine capacity is not exceeded. Equation (5) calculates the number of each machine type bought or sold during each period. Equation (6) shows that the number of machines type m in cell c at the current period h equals to the number of that machines moved into cell c, plus the number of the same machine type present in the previous period and minus the number of machines removed from that cell. Equation (7) shows that summation of the extra time dedicated to all cells per machine type m cannot exceed the total capacity of machine type m in period h in overtime. Equation (8) ensures the number of labors allocated to all cells in each period is equal to the total number of available labors. Equation (9) determines the number of machines assigned to a cell in each period is less than the upper cell size limit. Equation (10) guarantees that available time capacity per worker is not exceeded. Equation (11) shows the balancing inventory constraint between periods for each part type at each period. It means that the inventory level of each part at the end of each period is equal to the quantity of production plus the inventory level of the part at the end of the previous period minus the part demand volume in the current period. Equation (12), complementary to Eq. (2), ensures that a portion of the part demand can be produced at the given period if its operations are assigned in the constraint given in Eq. (2). Logical binary, non-negativity integer or continuous necessities for the decision variables are determined in Eqs. (13), (14) and (15).

Linearization of the proposed model

The proposed model is a mixed-integer nonlinear programming model because of absolute terms in Eqs. (1.4), (1.5) and (1.7) and the product of decision variables in Eqs. (1.3), (4) and (10).

The linearization process for absolute terms (1.4), (1.5) and (1.7) is accomplished by transforming the absolute terms into the linear form as follows:

To linearize term (1.4), non- negative variables Z^1_{jpchs} and Z^2_{jpchs} are introduced and term (1.4) is rewritten as follows:

$$1/2\gamma_s^{\text{inter}} \cdot \sum_{h=1}^{H}\sum_{p=1}^{P}\left[\frac{Q_{phs}}{B_p^{\text{inter}}}\right]\sum_{j=1}^{O_p-1}\sum_{c=1}^{C}\left(Z^1_{jpchs}+Z^2_{jpchs}\right) \tag{11}$$

where the following constraint must be added to the original model.

$$Z^1_{jpchs}-Z^2_{jpchs}=\sum_{m=1}^{M}X_{(j+1)pmchs}-\sum_{m=1}^{M}X_{jpmchs} \quad \forall j,p,c,h,s \tag{12}$$

Likewise, to transform the term (1.5) to the linear form, non- negative variables Y^1_{jpmchs} and Y^2_{jpmchs} are introduced and this term is rewritten as follows:

$$1/2\gamma_s^{\text{intra}}\sum_{h=1}^{H}\sum_{p=1}^{P}\left[\frac{Q_{phs}}{B_p^{\text{intra}}}\right]\sum_{j=1}^{O_p-1}\sum_{c=1}^{C} \times\left(\sum_{m=1}^{M}\left(Y^1_{jpmchs}+Y^2_{jpmchs}\right)-\left(Z^1_{jpchs}+Z^2_{jpchs}\right)\right) \tag{13}$$

where the following constraint must be added to the original model.

$$Y^1_{jpmchs}-Y^2_{jpmchs}=X_{(j+1)pmchs}-X_{jpmchs} \quad \forall j,p,m,c,h,s \tag{14}$$

Equation (11) is still nonlinear term. In the next step, to transform Eq. (11) to the linear form, non-negative variable φ^1_{jpchs} is introduced, and this equation is rewritten as follows:

$$1/2\gamma_s^{\text{inter}}\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O_p-1}\sum_{c=1}^{C}\left[\frac{\varphi^1_{jpchs}}{B_p^{\text{inter}}}\right] \tag{15}$$

where the following constraints set must be added to the original model.

$$\varphi^1_{jpchs}\geq Q_{phs}-M\left(1-Z^1_{jpchs}-Z^2_{jpchs}\right) \quad \forall j,p,c,h,s \tag{16}$$

$$\varphi^1_{jpchs} \le Q_{phs} + M\left(1 - Z^1_{jpchs} - Z^2_{jpchs}\right) \quad \forall j,p,c,h,s \tag{17}$$

Likewise, to transform Eq. (13) to the linear form, non-negative variable φ^2_{jpchs} is introduced, and this equation is rewritten as follows:

$$1/2\gamma^{\text{intra}}_s \sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O_p-1}\sum_{c=1}^{C}\left[\frac{\varphi^2_{jpchs}}{B^{\text{intra}}_p}\right] \tag{18}$$

where the following constraints must be added to the original model.

$$\varphi^2_{jpchs} \ge Q_{phs} - M\left\{1 - \sum_{m=1}^{M}\left(Y^1_{jpmchs} + Y^2_{jpmchs}\right) + \left(Z^1_{jpchs} + Z^2_{jpchs}\right)\right\} \quad \forall j,p,c,h,s \tag{19}$$

$$\varphi^2_{jpchs} \le Q_{phs} + M\left\{1 - \sum_{m=1}^{M}\left(Y^1_{jpmchs} + Y^2_{jpmchs}\right) + \left(Z^1_{jpchs} + Z^2_{jpchs}\right)\right\} \quad \forall j,p,c,h,s \tag{20}$$

To transform product terms in Eqs. (1.3), (4) and (10) to the linear forms, non-negative variable φ_{jpmchs} is introduced and replaced by $X_{jpmchs} \times Q_{phs}$ in the aforementioned terms. Then, the following constraints must be added to the original model.

$$\varphi_{jpmchs} \ge Q_{phs} - M\left(1 - X_{jpmchs}\right) \quad \forall j,p,m,c,h,s \tag{21}$$

$$\varphi_{jpmchs} \le Q_{phs} + M\left(1 - X_{jpmchs}\right) \quad \forall j,p,m,c,h,s \tag{22}$$

The absolute term Eq. (1.7) is transformed into the linear form as follows:

$$1/2\sum_{h=1}^{H}\sum_{c=1}^{C}\rho_{hs}\left(W^1_{ch} + W^2_{ch}\right) \tag{23}$$

where the following constraint must be added to the original model:

$$W^1_{ch} - W^2_{ch} = L_{c(h+1)} - L_{ch} \quad \forall c,h \tag{24}$$

The final linear model is written as follows:

$$\begin{aligned}\text{Min } Z = {} & \sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{O_p}\sum_{m=1}^{M}\beta_{ms}\cdot t_{jpm}\cdot\varphi_{jpmchs} \\ & + \text{Eq. (1.1)} + \text{Eq. (1.2)} + \text{Eq. (1.6)} + \text{Eq. (1.8)} \\ & + \text{Eq. (1.9)} + \text{Eq. (15)} + \text{Eq. (18)} + \text{Eq. (23)}\end{aligned}$$

s.t:

Equations (2) and (3)

$$\sum_{p=1}^{P}\sum_{j=1}^{O_p}\varphi_{jpmchs}t_{jpm} \le T_{mh}N_{mch} + T'_{mch} \quad \forall m,c,h,s \tag{25}$$

Equations (5)–(9)

$$\sum_{j=1}^{O_p}\sum_{p=1}^{P}\sum_{m=1}^{M}\varphi_{jpmchs}t'_{jpm} \le WTL_{ch} \quad \forall c,h,s \tag{26}$$

Equations (11)–(15), (12), (14), (16), (17), (19–22) and (24)

$$\varphi_{jpmchs}, Z^1_{jpchs}, Z^2_{jpchs}, \varphi^1_{jpchs}, Y^1_{jpmchs}, Y^2_{jpmchs}, \varphi^2_{jpchs}, W^1_{ch}, W^2_{ch} \ge 0$$

Robust optimization formulation

In this paper, a robust optimization approach based on Mulvey's model is employed in which uncertainty is represented by a set of discrete scenarios. The extended robust optimization model for the mentioned problem can be stated as follows:

$$\begin{aligned}TC_s = {} & \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}N_{mch}\alpha_{ms} + \sum_{h=1}^{H}\sum_{m=1}^{M}I^+_{mh}\varphi_{ms} - \sum_{h=1}^{H}\sum_{m=1}^{M}I^-_{mh}w_{ms} \\ & + \sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{O_p}\sum_{m=1}^{M}\beta_{ms}\varphi_{jpmchs}t_{jpm} \\ & + 1/2\gamma^{\text{inter}}_s\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O_p-1}\sum_{c=1}^{C}\left[\frac{\varphi^1_{jpchs}}{B^{\text{inter}}_p}\right] \\ & + 1/2\gamma^{\text{intra}}_s\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O_p-1}\sum_{c=1}^{C}\left[\frac{\varphi^2_{jpchs}}{B^{\text{intra}}_p}\right] \\ & + \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}T'_{mch}\theta_{mhs} + 1/2\sum_{h=1}^{H}\sum_{c=1}^{C}\rho_{hs}\left(W^1_{ch} + W^2_{ch}\right) \\ & + 1/2\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\delta_{ms}\left(k^+_{mch} + k^-_{mch}\right) + \sum_{h=1}^{H}\sum_{p=1}^{P}h_{phs}I_{phs}\end{aligned} \tag{27}$$

$$\begin{aligned}\text{Min } Z = {} & \sum_{s=1}^{S}p_sTC_s + \lambda_1\sum_{s=1}^{S}p_s\left|TC_s - \sum_{s'}^{S}p_{s'}TC_{s'}\right| \\ & + \omega\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{h=1}^{H}p_s\delta_{phs}\end{aligned} \tag{28}$$

s.t

$$D_{phs} = \delta_{phs} + Q_{phs} - I_{phs} + I_{p(h-1)s} \quad \forall p,h,s \tag{29}$$

Equations (2), (3), (5)–(9), (12)–(15), (12), (14), (16), (17), (19)–(22), (24), (25), (26).

The first and second terms in the objective function (28) are the expected value and variance of the objective function (27), respectively, and they measure solution robustness. The third term in (28) measures the model robustness with regards to infeasibility associated with control constraints (29) under scenario s. Equation (29) is a control constraint that is used to specify the level of inventory and the under-fulfillment of part demand via

violation level δ_{phs} under scenario s. It is noted that if the total quantity of products produced in period h plus previous inventory at period h-1 is greater than market demand D_{phs}, then the inventory at period h will be equal to $I_{phs} = I_{p(h-1)s} + Q_{phs} - D_{phs}$ and under minimization, the violation level $\delta_{phs} = 0$; whereas if $I_{p(h-1)s} + Q_{phs}$ is less than market demand D_{phs}, then $I_{phs} = 0$, and $\delta_{phs} = D_{phs} - Q_{phs} - I_{p(h-1)s}$, demonstrating under-fulfillment of part demand, thus an infeasible solution is obtained.

Although Eq. (28) is a nonlinear function, the absolute term is transformed into the linear form as follows:

$$\min \quad = \sum_{s=1}^{S} p_s TC_s + \lambda_1 \sum_{s=1}^{S} p_s (p_s + q_s) + \omega \sum_{s=1}^{S} \sum_{m=1}^{M} \sum_{h=1}^{H} p_s \delta_{phs}. \tag{30}$$

s.t:

$$p_s - q_s = TC_s - \sum_{s'}^{S} p_{s'} TC_{s'} \qquad \forall s \tag{31}$$

$\delta_{phs} \geq 0$, Eqs. (2), (3), (5)–(9), (12)–(15), (12), (14), (16), (17), (19)–(22), (24), (25), (26), (29).

A case study

Case data description

A case study is conducted for a typical equipment manufacturer located in the Mazandaran province in the north of Iran. Badeleh Machinery Company was pioneered in 1988 with a factory for producing different kinds of tanked and trailed sprayers. Parallel with an increment in production rate, there came a variety of other types of machines, thus an increase in the factory's area, as far as 15,000 meters for production section with another 15,000 meters of area left for future developments, in which 70 people consisting of workers and specialists work seven days a week. Regarding the customized demand in such case study, different scenarios in different season could be defined. Eight part types (farm equipment) consisting of (1) sprinkler, (2) Rot cultivator, (3) Stalk-Shredder, (4) chipper, (5) Roller Chisel, (6) Borers with hydraulic inverter, (7) Borers with hydraulic inverter, and (8) Rear Hydraulic Crane Arm are produced in the company. To validate the proposed model and investigate the credibility of the employed robust optimization approach, the case study is solved using GAMS 22.0 software (solver CPLEX). First, the input data are described. Next, the obtained results are analyzed. This case study suggested in an uncertain environment includes 8 parts ($p1,\ldots,p8$), six types of machines ($m1,\ldots,m6$), three time periods ($h1$, $h2$, $h3$) and three types of cells ($c1$, $c2$, $c3$). For each part, three operations ($j1$, $j2$, $j3$) have to be processed sequentially considering processing times. The maximum available time for each worker in a time period is 40 h and the number of workers is 70. Besides, it has been assumed that the future economic scenarios will fit four probable scenarios that, respectively, are boom, good, fair and poor with the related probabilities 0.45, 0.25, 0.2, and 0.15.

Demand for part type p in period h under scenario s is shown Table 1. Batch size for inter and intra-cell movement of part p are shown Table 2. Inter-cell and intra-cell movement costs per batch under scenario s are shown Table 3. Purchase cost of machine type m under scenario s is shown Table 4. Marginal revenue from selling machine type m under scenario s is shown Table 5. Constant cost of machine type m in each period under scenario s is shown Table 6. Variable cost of machine type m for each unit time in regular time is shown Table 7. Relocation cost of machine type m under scenario s is shown Table 8. Fixed cost of inter-cell labor moving in period h under scenario s is shown Table 9. Time-capacity of machine type m in regular and overtime are shown Table 10. Variable cost of processing on machine type m in overtime in period h under scenario s is shown Table 11. Processing time required

Table 1 Demand for eight part types in two periods under four scenarios

Dphs	Scenario	*P*1	*P*2	*P*3	*P*4	*P*5	*P*6	*P*7	*P*8
*h*1	Boom	550	800	0	500	0	450	0	800
	Good	0	0	250	300	0	200	300	0
	Fair	350	500	0	0	200	0	250	250
	Poor	0	0	100	100	100	100	100	100
*h*2	Boom	700	800	0	500	0	800	0	950
	Good	0	0	500	300	0	500	300	0
	Fair	500	400	0	0	300	0	200	350
	Poor	0	0	200	100	100	100	100	100
*h*3	Boom	400	650	0	500	0	700	0	750
	Good	0	0	300	300	0	300	400	0
	Fair	200	400	0	0	250	0	300	200
	Poor	0	0	100	100	100	200	200	100

Table 2 Batch size for inter-cell and intra-cell movement of four part types

	*P*1	*P*2	*P*3	*P*4	*P*5	*P*6	*P*7	*P*8
B^{inter}	35	25	20	40	45	30	35	40
B^{intra}	7	5	4	8	9	5	7	8

Table 3 Inter-cell and intra-cell movement cost per batch under four scenarios

	Boom	Good	Fair	Poor
γ_s^{inter}	50	40	30	20
γ_s^{intra}	8	7	6	5

Table 4 Purchase cost of six machine types under four scenarios

φ_{ms}	Boom	Good	Fair	Poor
*M*1	14,000	12,000	11,000	10,000
*M*2	14,000	12,000	11,000	10,000
*M*3	15,000	13,000	12,000	11,000
*M*4	14,000	13,000	12,000	11,000
*M*5	15,000	13,000	12,000	11,000
*M*6	16,000	13,000	12,000	11,000

Table 5 Marginal revenue from selling six machine types under four scenarios

ω_{ms}	Boom	Good	Fair	Poor
*M*1	9800	8100	7700	7000
*M*2	9800	8100	7700	7000
*M*3	10,500	8700	8400	7700
*M*4	9800	8100	7700	7000
*M*5	9100	8000	7700	7000
*M*6	11,200	9000	8400	7700

Table 6 Constant cost of six machine types in each period under 4 scenarios

α_{ms}	Boom	Good	Fair	or
*M*1	1400	1200	1100	1000
*M*2	1400	1200	1100	1000
*M*3	1500	1400	1200	1100
*M*4	1400	1300	1200	1100
*M*5	1300	1200	1100	1000
*M*6	1600	1300	1200	1100

Table 7 Variable cost of six machine types for each unit time in regular time

β_{ms}	Boom	Good	Fair	Poor
*M*1	9	8	7	5
*M*2	9	8	7	6
*M*3	8	7	6	5
*M*4	8	7	6	5
*M*5	9	8	7	6
*M*6	8	6	5	4

Table 8 Relocation cost of six machine types under four scenarios

δ_{ms}	Boom	Good	Fair	Poor
*M*1	650	600	550	500
*M*2	700	650	600	550
*M*3	750	700	650	600
*M*4	700	650	600	550
*M*5	650	600	550	500
*M*6	800	750	700	650

Table 9 Fixed cost of inter-cell moving of a labor in three periods under four scenarios

ρ_{hs}	Boom	Good	Fair	Poor
*H*1	200	150	100	70
*H*2	200	150	100	70
*H*3	200	150	100	70

Table 10 Time-capacity of six machine types in regular and over time

	T_{mh}	T'_{mh}
*M*1	500	200
*M*2	500	200
*M*3	500	200
*M*4	500	200
*M*5	500	200
*M*6	500	200

Table 11 Variable cost of processing on six machine types in overtime in three periods under four scenarios

θ_{mhs}	Scenario	*M*1	*M*2	*M*3	*M*4	*M*5	*M*6
h1	Boom	15	11	17	12	10	20
	Good	14	10	16	11	9	19
	Fair	13	9	15	10	8	18
	Poor	10	8	10	9	7	10
h2	Boom	15	11	17	12	10	20
	Good	14	10	15	11	9	19
	Fair	13	9	13	10	8	18
	Poor	10	8	10	9	7	10
h3	Boom	15	11	17	12	10	20
	Good	13	10	12	11	9	19
	Fair	12	9	11	10	8	18
	Poor	10	8	10	9	7	10

to perform operation j of part type p on machine type m is shown Table 12. Manual workload time required to perform operation j of part type p on machine type m is shown Table 13. Inventory holding cost for part type p in period h under scenario s is shown Table 14.

Table 12 Processing time required to perform the operations of eight part types on six machine types

t^{jpm}	*P*1			*P*2			*P*3			*P*4			*P*5			*P*6			*P*7			*P*8		
	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3
*M*1	0	0	0	0.76	0	0.39	0	0	0	0	0.83	0	0	0	0	0	0.57	0	0	0	0	0	0	0.54
*M*2	0	0	0	0	0	0	0.99	0	0	0	0	0.74	0	0	0	0.72	0	0.47	0.44	0	0.28	0	0.17	0
*M*3	0.73	0.93	0	0	0	0	0	0	0	0.45	0	0	0	0	0	0	0	0	0	0.97	0	0	0	0.15
*M*4	0	0	0.46	0	0.81	0	0	0	0	0	0	0	0	0.26	0	0.55	0	0	0	0.47	0	0.84	0.86	0.78
*M*5	0	0	0	0	0	0	0	0.48	0	0	0	0	0.12	0	0.75	0	0	0.12	0.76	0	0.86	0.2	0	0
*M*6	0	0	0	0	0	0	0.36	0	0.00	0	0.78	0	0	0.45	0.59	0.81	0.48	0	0	0	0	0	0	0

Table 13 Manual workload time required to perform the operations of eight part types on six machine types

t^{jpm}	*P*1			*P*2			*P*3			*P*4			*P*5			*P*6			*P*7			*P*8		
	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3	*J*1	*J*2	*J*3
*M*1	0	0	0	0.076	0	0.039	0	0	0	0	0.083	0	0	0	0	0	0.057	0	0	0	0	0	0	0.054
*M*2	0	0	0	0	0	0	0.099	0	0	0	0	0.074	0	0	0	0.072	0	0.047	0.044	0	0.028	0	0.017	0
*M*3	0.073	0.093	0	0	0	0	0	0	0	0.045	0	0	0	0	0	0	0	0	0	0.097	0	0	0	0.015
*M*4	0	0	0.046	0	0.081	0	0	0	0	0	0	0	0	0.026	0	0.055	0	0	0	0.047	0	0.084	0.086	0.078
*M*5	0	0	0	0	0	0	0	0.048	0	0	0	0	0.012	0	0.075	0	0	0.012	0.076	0	0.086	0.2	0	0
*M*6	0	0	0	0	0	0	0.036	0	0	0	0.078	0	0	0.045	0.059	0.081	0.048	0	0	0	0	0	0	0

Table 14 Cost of inventory holding for eight part types in three periods under four scenarios

H_{phs}	Scenario	$P1$	$P2$	$P3$	$P4$	$P5$	$P7$	$P8$
$h1$	Boom	17	17	25	20	19	33	25
	Good	15	15	21	18	14	25	22
	Fair	13	13	17	15	13	22	19
	Poor	10	10	10	10	10	10	10
$h2$	Boom	18	22	22	22	17	32	22
	Good	15	15	18	19	12	27	19
	Fair	14	13	15	14	11	22	13
	Poor	11	10	12	11	10	12	11
$h3$	Boom	17	18	20	17	20	17	29
	Good	13	13	15	15	15	15	22
	Fair	12	12	11	12	13	13	17
	Poor	10	10	10	10	11	10	10

Results analysis

As stated in "Robust optimization" section, robustness means that the model output should not be highly sensitive to the exact values of the model input parameters, and if the model remains feasible for each certain scenario, the model is robust. According to the objective function Z [Eq. (28)], the model robustness is calculated through third term in objective function. Because of the uncertainty of the demand parameter and the cost parameters related to the cell formation, the model might be infeasible for some various scenarios. Thus, third term of objective function (28) that is the penalty function for infeasibility penalizes the violation of the control constraint (29). The violation of the control constraint means an infeasible solution is obtained under some scenarios. In fact, δ_{phs} is the violation vector showing the infeasibility level in control constraint (29) under a given scenario. If the under-fulfilled demand (δ_{phs}) equals zero, the model is feasible, otherwise, δ_{phs} will be positive. Table 15 presents sensitivity analysis for the robustness of Model Z with different values for parameter ω.

It is seen from Table 15 that the objective function Z is sensitive in return for various values of ω, (δ_{phs}) obtains a positive value and the objective function Z is positive under some scenarios. At the point $\omega = 0$, the part under-fulfilled demand (δ_{phs}) obtains the maximum value since no production occurs and this way, it acquires a positive value in a descending manner until at the point $\omega = 800$, the part under-fulfilled demand (δ_{phs}) equals zero and the model becomes feasible. Figure 1 depicts sensitivity analysis for the model robustness and objective function value Z. As Fig. 1 illustrates, the value of Z increases as ω increases and the objective function value Z goes up. In fact, Fig. 1 shows that model Z has penalized the violation of control constraint (29) under some scenarios and as ω increases, the objective function value gets higher because the infeasibility penalty function acquires a positive value.

Table 15 Sensitivity analysis for model Z

ω	Z
0	2850
100	264,416.1
200	281,446.5
300	292,060.5
400	298,182.5
500	301,576.6
600	301,941
700	308,054.6
800	308,636.1

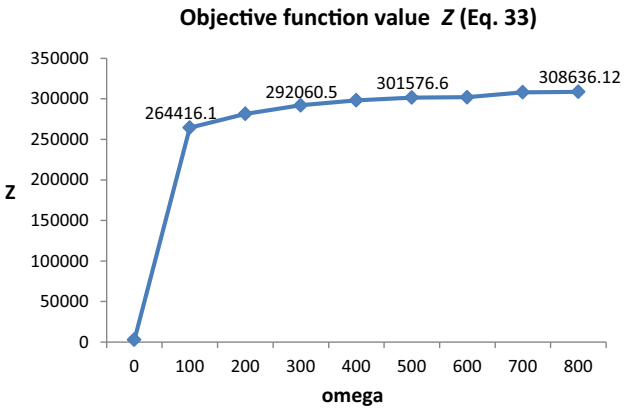

Fig. 1 Sensitivity analysis for the model robustness and the objective function value Z

Here, the model solution is analyzed considering $\omega = 300$. The computational results are given in Tables 16 and 17. Table 16 depicts the under-fulfilled demand of part type p in the period h under scenario s. As can be seen, the under-fulfilled demand of parts 1 and 2 obtain positive values in periods 1 and 2 for a boom scenario. Since the infeasibility penalty function (28) obtains positive value, it penalizes control constraint violation under some scenarios. While the demand for part 2 in period 1 under the boom scenario is 800, the optimal production value is 783 and the under-fulfilled demand is 17. Similarly, the demand for part 1 in period 2 is 700, the optimal production value is 625 and the under-fulfillment demand is 75 under the boom scenario. The demand for part 2 in period 2 is 800, the optimal production is 683 and the under-fulfilled demand is seven under the boom scenario. That is, violation of the control constraint (29) for the boom scenario in parts 1 and 2 in periods 1 and 2 happened at $\omega = 300$.

Table 17 illustrates the total costs based on Eq. (27) including costs of machine constant, machine variable,

Table 16 The under-fulfilled demand of eight part types in three periods under four scenarios

δ_{phs}	Scenario	$P1$	$P2$	$P3$	$P4$	$P5$	$P6$	$P7$	$P8$
$h1$	Boom	0	17	0	0	0	0	0	0
	Good	0	0	0	0	0	0	0	0
	Fair	0	0	0	0	0	0	0	0
	Poor	0	0	0	0	0	0	0	0
$h2$	Boom	75	117	0	0	0	0	0	0
	Good	0	0	0	0	0	0	0	0
	Fair	0	0	0	0	0	0	0	0
	Poor	0	0	0	0	0	0	0	0
$h3$	Boom	0	0	0	0	0	0	0	0
	Good	0	0	0	0	0	0	0	0
	Fair	0	0	0	0	0	0	0	0
	Poor	0	0	0	0	0	0	0	0

machine purchase, intra and inter-cell movement, part inventory holding and overtime under various scenarios.

According to Table 1, it is clear that the part demands and the scenario-based cell formation cost parameters are incremental from poor scenario to the boom one. As can be seen in Table 17, all cost components have increased from the poor scenario to the boom one, except the inventory holding cost. Since the part under-fulfilled demand has obtained positive value for parts 1 and 2 according to Table 16 in the boom scenario, violation occurred and according to the control constraint (29), the part inventory amount and its related cost is zero in the boom scenario. However, from the boom scenario to the poor one the part inventory has increased and similarly, the part inventory cost has increased as well. Since in the good, fair and poor scenarios, δ_{phs} equals zero, the part inventory level gets a positive level and the inventory holding cost gets a positive level as well.

Figure 2 depicts the cells configuration in three periods for the main model of the DCMS under boom scenario. The part operation assignments to machines and the machines assignments to cells are also shown in Fig. 2. For example, in the first period, 2 units of machines types 5 and 3 have been assigned to the cells 1 and 3, respectively.

In period 1, operations 1 and 2 of part 1 are processed inside cell 1 by machines 5 and 4, respectively, and operation 3 inside cell 2 by machine 6. Then, there is need for an intra-cell movement for operations 1 and 2 and an inter-cell movement for operations 2 and 3. In period 1, eight inter-cell movements and two intra-cell movement are performed for the parts processing. In period 2, seven inter-cell movements and three intra-cell movement are performed for the parts processing.

Figure 3 shows the cells configuration in periods 1, 2 and 3 for DCMS model solved by the robust optimization approach respect to 4 scenarios. Here, compared with the cell configurations obtained for the main model under boom scenario, there are some similarities and some differences. For example, in period 1, operations 1 and 2 of part 1 are processed inside cell 1 by machines 5 and 2, and operation 3 inside cell 2 by machine 6 as shown in Fig. 3. In period 2, according to Fig. 3, three inter-cell movements and seven intra-cell movement are performed for the parts processing. In period 3, five inter-cell movements and five intra-cell movement are performed for the parts processing. Totally, the number of inter-cell movements and the number of machines decrease; as a result, the relocation cost, machine constant cost and inter-cell movement cost become lower.

Tradeoff between solution robustness and model robustness

Tradeoff between solution robustness (expected total costs) and model robustness (expected under-fulfillment) can be found using different values of ω in the objective function (28). Robust optimization approach allows for infeasibility in the control constraints by means of penalties. When ω is considered equal to zero, δ_{phs} in constraint (29) is equal to D_{phs} due to the minimization of objective function (28). In fact, the total under-fulfillment obtains its highest value, and obviously this decision cannot be upheld. Therefore, it is necessary to evaluate the proposed robust optimization model with various values of ω. Tradeoff between feasibility and costs is illustrated in Fig. 4. As the value of ω increases, the expected total costs representing solution robustness increases exponentially, and the expected under-fulfilled demand representing model robustness drops. This means that for larger value of ω, the obtained solution is approaching 'almost' feasible for any realization

Table 17 Cost components of total costs [Eq. (27)] in four scenarios

	Total costs	Machine constant	Machine variable	Purchasing machine	Inter-cell movement	Intra-cell movement	Inventory holding	Overtime
Boom	275,040.3	30,000	87,218	100,000	12,571.7	1862.1	0	28,813.4
Good	189,211.5	27,000	28,888.1	88,000	4422.7	1689.9	388.5	25,826. 1
Fair	179,160.1	24,300	25,414.8	81,000	3313	1226.5	634.9	22,807.2
Poor	134,714.6	22,000	9528	74,000	1266.6	1250	1569.6	19,819.9

Period h1			C1		C2		C3	
			P1	P2	P4		P6	P7
C1	2	M5	1					
	1	M4	2					
	1	M2		2	3		1	
C2	1	M6	3		2			3
	3	M1		1			3	
C3	2	M3		3				
	1	M4						
	1	M6			1			2
	1	M5					2	1

Period h2	units		C1		C2		C3	
			P1	P2	P4		P6	P7
C1	2	M1	1				2	
	1	M2			3			
	1	M3		2			1	
C2	1	M4			1			
	3	M2	2				3	
	2	M1		1				
	1	M6		3				1
C3	2	M2	3					3
	1	M4						2
	1	M6			2			

Period h3	units		C1		C2		C3	
			P1	P2	P4		P6	P7
C1	2	M5	1				2	
	1	M4	2		3			
	1	M2		2				
C2	1	M6	3		1			
	3	M1		1			3	
C3	2	M3		3				3
	1	M4					1	
	1	M6			2			1
		M5						2

Fig. 2 Cell configurations for the main DCMS model under boom scenario

Period h1			C1		C2		C3	
			P1	P2	P4		P6	P7
C1	2	M5	1	1				
	1	M2	2	2,3	3			
C2	1	M6	3		2			
	3	M1			1			1
C3	2	M3					3	
	1	M4					1	3
	1	M6					2	2

Period h2			C1		C2		C3	
			P1	P2	P4		P6	P7
C1	2	M1	1					
	1	M2	2	3	3			
	1	M3	3	2				
C2	1	M4			1			
	3	M2			2		3	
	2	M1		1				
C3	2	M2					1	3
	1	M4					2	1,2

Period h3			C1		C3		
			P1	P2	P4	P6	P7
C1	2	M5	1			2	
	1	M4	2	3			
	1	M2		2	3		
C3	2	M3	3				
	1	M4		1	1	1	3
	1	M6				3	1
	2	M5			2		2

Fig. 3 Cell configurations for the DCMS model by the proposed robust optimization approach

of scenario s through the payment of more total costs. In addition, e expected under-fulfillment will eventually drop to zero with an increase in value of ω to 800.

Comparing the effectiveness of robust model and mean-value based model

To illustrate the robust dynamic cell formation that could be obtained by the proposed MIP model, expected values of uncertain parameters are used in the primary mixed-integer linear programming model presented in "Linearization of

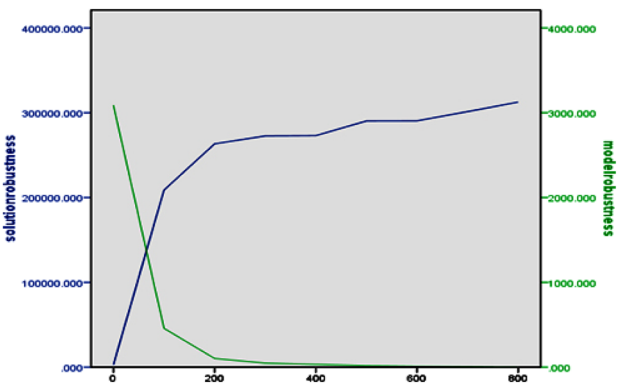

Fig. 4 Trade-off between expected total costs and expected under-fulfillment

Table 18 Total objective function values obtained by the robust and mean-value based models

Problem number	1	2	3	4	5	6	7	8	9	10
Robust model	257,361.8	286,914.8	294,127.3	285,993.8	271,391.6	301,591.6	298,255.4	308,646.9	279,743.5	309,443
Mean-value based model	316,642.7	394,204.2	389,787.9	421,225.4	410,668.2	432,788.6	371,073.8	439,419.8	406,668.8	434,360.9

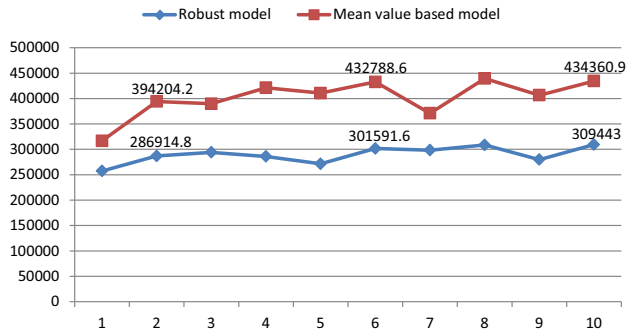

Fig. 5 Comparison of total objective function between robust and mean-value based models

the proposed model" section as certain value parameters, hereafter called mean-value based model. The results of these two models (i.e., robust model and mean-value based model) are compared with each other at the following. Robust optimization is used to attain a robust solution against the fluctuation of uncertain parameters in the future. Note that at the inception of planning horizon, some parameters are uncertain, and only in the execution time of the plan, the real values of uncertain parameters will be realized. For this purpose, we simulate some real and conceivable scenarios that may occur after executing the cell formation in the future. We consider 10 random occurrences for the uncertain parameters and compute the objective function Z of each instance for the dynamic cell formation problem obtained by the robust and mean-value based models.

The objective function values for the scenarios with probabilities 0.15, 0.2, 0.25 and 0.45 are shown in the Table 18 and Fig. 5. As shown in Fig. 5, the objective function values of dynamic cell formation problem obtained by the proposed MIP model are robust against the amount of uncertain parameters in the future and yield a series of solutions that are less sensitive to realizations of the uncertain data. In other words, the violation of results attained by the robust optimization model is less than that by mean-value based model.

In fact, the values of the objective function Z for different scenarios are closer to each other than these values for the mean-value based model. The curve of values in the proposed method follows a more robust incline, but the fluctuation in the curve of values for the classical approach is very high. This achievement indicates that the proposed approach is efficient for any systems that the robustness of solution is important in addition to objective function value Z of production for their managers. Indeed, for such systems having a solution with minimum total objective is not adequate, but the fluctuation in real scenarios in future should be handled. Therefore, numerical results show the robustness and effectiveness of the proposed model.

Conclusion

In this study, a mathematical model based on a robust optimization approach has been presented in dynamic cell formation problem with uncertain data to integrate CF, PP and worker assignment. The robust optimization approach reduces the effect of the fluctuations of uncertain parameters under certain scenarios. In this study, the majority of cell formation parameters including cost parameters and part demand fluctuation were considered uncertain.

Next, sensitivity analysis has been presented for solution robustness and model robustness. Since the objective function has been influenced by ω, the relationship between the model robustness and solution robustness has been analyzed only for the objective function value.

The computational experiments obtained from a set of real-world data for an Iranian farm tanked and trailed sprayers manufacturer illustrated that the proposed robust model is more practical for handling uncertain parameters in the production environments. The tradeoff between optimality and infeasibility was used for obtaining robust solution based on the opinion of decision-makers. The results showed the robustness and effectiveness of the model in real-world cell formation problem.

In addition, the results obtained by the robust MIP model indicated the advantages of robust optimization in generating more robust cell configurations with less cost over the considering expected value of uncertain parameters in a deterministic mean-value based model. In fact, in such systems designed here as the mean-value based model, having only solution with the minimum value of the objective function and lower costs is not sufficient rather the fluctuations in the related scenarios have to be lowered in future.

The future studies in the following of the present study can be pursued in multi-objective DCMS modeling,

employing the other robust optimization methods, taking into account the setup time, defining the processing times and time-capacity of machines as uncertain, consideration of machine layout, allowing partial or total subcontracting, workload balancing among the cells, and using meta-heuristics to tackle large-sized problems.

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