Original Research

Improving discrimination power based on reducing dispersion of weights in data envelopment analysis

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Abstract

The main drawbacks that arise from data envelopment analysis (DEA) are lack of discrimination power amongst efficient decision making units (DMUs) and dispersion of input-output weights. In the DEA, sometimes the mismatch of the input or output weights in the decision-making units (DMUs) under consideration leads to assigning higher weight to variables with the less significant variables and/or the lower or zero weight to the variables with high significance. Accordingly, most DEA models introduce more than one efficient DMU in evaluating the relative efficiency of decision-making units. The present paper is conducted to overcome these problems. In this regard, we present a novel DEA model based on minimizing the average of absolute deviations of all input-output weights from each other. The proposed model is to enhance the discrimination power and adjusts the balanced dispersion of input-output weights. Finally, well-known numerical experiments are considered to demonstrate the efficiency and validation of the suggested model.

***Keywords*:** Data Envelopment Analysis; Discrimination power; Dispersion of weights; Scale transformation

INTRODUCTION

DEA was originally introduced by Charnes et al. (1978) as a procedure for assessing the comparative efficiency of decision making units (DMUs) with multiple inputs and outputs. Subsequently, Banker et al. (1984) developed the basic DEA models under variable returns to scale assumption. In DEA, we occasionally encounter two problems, the first problem associates with the inability to discriminate among extremely efficient DMUs, and the second one concerns the unbalanced input-output weights of DMUs. Basic DEA models sometimes assign zero to input and/or output weights of DMUs under evaluation. The problem of lack of discrimination power emerges when the total number of inputs-outputs is large compared to the number of DMUs under evaluation. In such cases, basic DEA models identify more than one DMUS as efficient after evaluating the comparative efficiency of DMUs. Consequently, improving the discrimination power in DEA is of great significance. To improve the discrimination power of DEA, some DEA models have been presented based on techniques of super efficiency, multi criteria and cross-efficiency approaches. Super efficiency models are presented to improve the discrimination power by removing a given DMU from the reference set and then compute its super efficiency score (Andersen and Petersen, 1993, Lee, Chu, Zhu, 2011, Chen, Du, Huo, 2013). However, it has been proved that super efficiency DEA models may encounter the infeasibility problems in some cases (Seiford and Zhu, 1999). Chen (2005) discussed the relation between super efficiency and the infeasibility of super efficiency DEA model and showed that not only input-oriented but also output-oriented super efficiency models should be needed for the purpose of characterizing the super-efficiency model.

Moreover, the cross efficiency model was explored by Sexton et al. for ranking DMUs using the cross evaluation scores related to all DMUs in order to achieve the best comparative efficiency. Although this technique is widely used, it has some drawbacks resulting from the classical DEA. One of the shortcomings of cross efficiency method is the non-uniqueness solution to optimal weights. Liu and Peng proposed a method based on the common set of weights in order to determine the rank of efficient DMUs. The multi criteria DEA (MCDEA) model was initially proposed by Li and Reeves (1999) to increase the discrimination power and to adjust the dispersion of weights. The model considers three objective functions that each of them should be minimized, where the first objective is the inefficiency of the DMU under evaluation, the second one is minimax of inefficiency within the set of assessed DMUs and the third objective is the sum of the inefficiencies of each DMUs. The application of MCDEA model is limited due the generation of the non-dominated solutions and complexity of this model. Bal et al. (2010) attempted to solve the weighted multi objective linear programming model of Li & Reeves (1999) using a goal programming DEA (GPDEA) approach under CRS and VRS assumptions in order to increase the discrimination power of DEA as well as to obtain more realistic input and output weights. However,

Ghasemi et al. (2014) illustrated that the GPDEA models suffer from some mathematical and conceptual flaws. In addition, Ghasemi et al. (2014) proposed a bi-objective MCDEA (BiO-MCDEA) model involving only two criteria of the model of Li & Reeves (1999) to improve discriminating power and achieve better weight dispersion. HatamiMarbini and Toloo (2016) mentioned some drawbacks of the model of Ghasemi et al. (2014) and extended three models involving a weighted MCDEA model to three evaluation criteria based on the maximum lower bound for input and output weights, a super-efficiency model for efficient units under the proposed MCDEA model and an epsilon based minisum BCC-DEA model in order to proceed our research objectives under variable returns to scale (VRS). Amin (2007) proposed two-phase algorithm to detect efficient DMUs based on inverse optimization. Razavian and Tohidi used integrated DEA models to rank all extreme and non-extreme efficient DMUs and then applied integrated DEA ranking method as a criterion to modify Genetic Algorithm for finding Pareto optimal solutions of a Multi-Objective programming problem. Ziari and Reissi (2016) presented a super efficiency model to rank the extreme efficient units based on minimizing distance. Also, Ziari (2016) provided a transformation to linearize proposed model by Jahanshahloo et al (2004) for ranking efficient DMUs using l1-norm.

Saeidi et al. (2013) used data envelopment analysis for ranking woven fabric defects (WFDs) observed in textile manufacturing companies. Saeidi et al. (2015) proposed a methodology for ranking defects based on the ordered weighted averaging (OWA) operator. Rodder et al (2018) developed a formula for returns to scale (RTS) and the proposed formula applied for interior points of technology. Didehkhani et al (2019) attempted to extend basic models for benchmarking of efficient units under practical condition by constructing the practical production possibility set (PPPS) using the concept of artificial decision-making units. Bal et al. (2008) provided a CVDEA model including coefficient of variation (CV) for input-output weights to rank efficient DMUs and adjusts more homogeneous dispersion of input-output weights. Wang et al. (2009) revealed serious drawbacks of the CVDEA model. The significant drawbacks of the CVDEA model aforementioned by Wang et al. (2009) are illustrated in the followings. The first drawback is concerned with the averaging process of the input-output weights with different dimensions and units. The second problem is related to the changes in CVDEA efficiency by scale transformation on the set of input and output data. The third shortcoming is dealt with the producing multiple local optimal solutions arising from nonlinearity of the CVDEA model. It is noted that the analysis of performance evaluation and comparison based on local optimal solutions are not valid. The forth shortcoming is related to the aggregation of inputs and outputs with equal weights. To remove this drawback, more weights should be imposed on the DEA efficiency in the objective function. The CVDEA model was further modified by Jahanshahloo and Firoozi Shahmirzadi (2013), who presented some approaches with the aim of ranking efficient DMUs based on L1-norm by considering the average of input-output weights. However, the model suggested by them had the same problems as CVDEA model. In this trends, we present a novel approach to rank the efficient DMUs which overcomes some drawbacks in the previous models. The present study is structured as follows: Section 2 reviews the concept of DEA framework. The CVDEA model suggested by Bal et al., (2008) is discussed in Section 3. Section 4 explains the new procedure for ranking efficient DMUs based on minimizing the average of absolute deviations of0 all input-output weights from each other. Some numerical experiments are elaborated in Section 5. Finally, conclusions are drawn in the last section of the paper.

DATA ENVELOPMENT ANALYSIS

DEA is a powerful tool to assess the comparative performance of $DMUs$ consisting multiple inputs and outputs.

Considering there are n $DMUs$ and each DMU_j $(j = 1, \dots, n)$ a column vector of inputs (X_j) is consumed to produce a vector of outputs (Y_j), where X_j=$(x_{1j}, x_{2j}, \dots, x_{mj})$ and Yj=$(y_{1j}, y_{2j}, \dots, y_{sj})$. Moreover, it is supposed that X_j≥0, Y_j≥0, $X_j \neq 0$, and $Y_j \neq 0$ for every

$j = 1, \dots, n.$

The relative performance of the kth DMU (X_k, Y_k) is measured by solving the nonlinear fractional programming problem below:

$$\max z = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \qquad (Model\ 1) \quad (1)$$

$$s.t. \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, \dots, n \qquad (2)$$

$$u_r \geq 0, \qquad r = 1, \dots, s$$

$$v_i \geq 0, \qquad i = 1, \dots, m$$

where $v = (v_1, \dots, v_m)^T$and $u = (u_1, \dots, u_s)^T$ are column vectors of unknown variables used for weights of the input and output vectors.

z^* stands for the efficiency score of DMU_kin (1), where the asterick (*) signifies optimality.

DMU_k relatively is efficient if and only if in the case of optimality, the objective value of (1) equals to one. The above mentioned fractional programming problem can be reformulated a linear form where the objective value on optimality represents the relative efficiency of DMU_k. The linear form of the above nonlinear programming problem is as follows, which is also reputed as the CCR model:

$$\max \quad z = \sum_{r=1}^{s} u_r y_{rk} \qquad (Model\ 2) \quad (3)$$

$$s.t. \quad \sum_{i=1}^{m} v_i x_{ik} = 1, \qquad (4)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \qquad (5)$$

$$u_r, v_i \geq 0, \quad r = 1, \dots, s, \quad i = 1, \dots, m$$

The solution to the above model allocates the value one foe efficient $DMUs$. The super efficiency concept is presented for fully rank efficient DMU_S. One suggested super efficiency approach developed to rank efficient DMU_Sin DEA was offered by Andersen and Petersen (1993), which is also reputed as AP model.The AP model assigns a value greater than one to the extreme efficient unit k by erasing the kth constraint in the envelopment linear programming. This approach is as follows:

$$\max \quad z = \sum_{r=1}^{s} u_r y_{rk} \qquad (Model\ 3) \quad (6)$$

$$s.t. \quad \sum_{i=1}^{m} v_i x_{ik} = 1, \qquad (7)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad j \neq k \qquad (8)$$

$$u_r, \quad v_i \geq 0, \quad r = 1, \dots, s, \quad i = 1, \dots, m$$

THE CVDEA MODEL

In the current section, the model that was originally proposed by Bal et al. (2008) is described. The Coefficient of Variation (CV), the proportion of sample standard deviation to the sample mean, presents the variability of the weights relative to their mean. It compares the relative dispersion of two sample of data with different types. Let u_r for $r = 1,2, \dots, s$ stand for the weight on output r and let $\bar{u}$ stand the average of the u_r for $r = 1,2, \dots, s$. Then the CV for the weights $u_r$can be described in the following way:

$$CV_1 = \frac{\sqrt{\sum_{r=1}^{s}(u_r-\bar{u})^2/(s-1)}}{\bar{u}}. \qquad (9)$$

In similar manner, the CV for the weights v_i for $i = 1,2, \dots, m$, can be calculated as follows:

$$CV_2 = \frac{\sqrt{\sum_{i=1}^{m}(v_i-\bar{v})^2/(m-1)}}{\bar{v}} \qquad (10)$$

Bal et al. suggested the following model integrating the CV for input-output weights to the model (2), which is labeled CVDEA model which is abbreviation form of Coefficient of Variation DEA model:

$$\max \quad z = \sum_{r=1}^{s} u_r y_{rk} - CV_1 - CV_2 \qquad (Model\ 4) \quad (11)$$

$$s.t. \quad \sum_{i=1}^{m} v_i x_{ik} = 1, \qquad (12)$$

$$\sum_{r=1}^{s} u_r y_{rk} - \sum_{i=1}^{m} v_i x_{ik} \leq 0, \quad j = 1, \dots, n, \qquad (13)$$

$$u_r\ ,\ v_i \geq 0, \quad r = 1, \dots, s, \quad i = 1, \dots, m \qquad (14)$$

The Krash-Kuhn-Tuker algorithm can easily be applied to solve this nonlinear optimization model. The CVEDA can be performed for increasing the discriminatory power of DEA occurring more than one efficient DMUs.

THE PROPOSED MODEL BASED ON DISPERSION OF WEIGHTS

As the coefficient of variation (CV) is free from scale of measurement, it can be applied for comparing the dispersion of two or more samples of data from different type of variables or the same type of variables when the means are significantly different. Therefore, employing CV of input or output weights would not be of interest for method in DEA. Meanwhile, it seems that suggested model by Bal et al. (2008) has a problem in applying CV inputs and outputs weights. Now, we introduce the following model based on minimizing the average of absolute deviations of all input-output weights ($u_r$andv_i) from each other in order to rank all efficient DMUs. The proposed model can simply be transformed into a linear optimization model. It should be noted that the objective function of the proposed method is as following multi-objective:

$$\max \quad z_1 = \sum_{r=1}^{s} u_r\, y_{rk}\ , \qquad (15)$$

$$\min \quad z_2 = \frac{1}{s^2} \sum_{r=1}^{s-1} \sum_{p=r+1}^{s} \left| u_r - u_p \right|, \qquad (16)$$

$$\min \quad z_3 = \frac{1}{m^2}\sum_{i=1}^{m-1}\sum_{q=i+1}^{m}\left|v_i - v_q\right|. \quad (17)$$

We can incorporate the above aforementioned multi-objective functions into the one function.
Hence, the suggested model is considered as below:

$$\max\ z = \sum_{r=1}^{s} u_r\, y_{rk} - \frac{1}{s^2}\sum_{r=1}^{s-1}\sum_{p=r+1}^{s}\left|u_r - u_p\right| \ - \frac{1}{m^2}\sum_{i=1}^{m-1}\sum_{q=i+1}^{m}\left|v_i - v_q\right| \quad (18)$$

$$s.t. \quad \sum_{i=1}^{m} v_i x_{ik} = 1, \qquad (Model\ 5) \quad (19)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1, \dots, n, \qquad (20)$$

$$u_r,\ v_i \ge 0, \quad r = 1, \dots, s, \quad i = 1, \dots, m,$$

where $v = (v_1, \dots, v_m)^T$ and $u = (u_1, \dots, u_s)^T$ are column vectors of unknown variables used for weights of the input and output vectors. The mathematical software such as GAMS can be used for finding the solution of above-mentioned model. It should be noted that, the required data should be normalized before performing each of the models are presented in this section.

Remark 1. Suppose that DMU_p and DMU_q be arbitrary efficient DMUs. After the performing Model (5), the rank of DMU_p is better than DMU_q, when $z_p^* > z_q^*$.

Theorem 1. The model (9) is feasible and bounded.

Proof. For proving of feasibility of the model, it is sufficient to make a feasible solution for the model.
Suppose, $x_{ik} = \max_{1\le j\le n}\{x_{ij}\}$, for $i = 1, \dots, m.$. Let $v_i = \frac{1}{\sum_{i=1}^{m} v_i x_{ik}}$, for $i = 1, \dots, m$. We select the output weghts as:

$$u_r = \min_{1\le j\le n}\left\{\frac{\sum_{i=1}^{m} x_{ij}}{(\sum_{i=1}^{m} x_{ik})(\sum_{r=1}^{s} y_{rj})}\right\}, \quad r = 1, \dots, s. \qquad (21)$$

It is easy to check that (v_i, u_r), $i = 1, \dots, m, r = 1, \dots, s$, is a feasible solution for model (5).

Also, the value of objective funtion at the feasible solution is $z_0 = \sum_{r=1}^{s} \min_{1\le j\le n}\left\{\frac{\sum_{i=1}^{m} x_{ij}}{(\sum_{i=1}^{m} x_{ik})(\sum_{r=1}^{s} y_{rj})}\right\} y_{rj}.$

According to the objective function of Model (5), it is maximization form, therefore $z^* \ge z_0$.
On the other hand, if a DMU under evaluation based on Model (2) is efficient the its score of efficiency is one, and since from the objective function of Model (5) the positive terms was subtracted, it is conclude that conclude that $z^* \le 1$. So, the proof is complete. To encounter more than one efficient DMUs, we can apply the super-efficiency model for ranking all efficient DMUs:

$$\max\ z = \sum_{r=1}^{s} u_r y_{rk} - \sum_{r=1}^{s-1}\sum_{p=r+1}^{s}\left|u_r - u_p\right| \ - \sum_{i=1}^{m-1}\sum_{q=i+1}^{m}\left|v_i - v_q\right| \qquad (22)$$

$$s.t. \quad \sum_{i=1}^{m} v_i x_{ik} = 1, \qquad (Model\ 6) \qquad (23)$$

$$\sum_{r=1}^{s} u_r y_{ij} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1, \dots, n, \qquad j \ne k \qquad (24)$$

$$u_r, v_i \ge 0, \quad r = 1, \dots, s, \quad i = 1, \dots, m.$$

To convert the model (5) into the linear programming form, we let $\acute{a}_r = \frac{1}{2}\left(\left|u_r - u_p\right| + u_r - u_p\right)$ and $a_p^{"} = \frac{1}{2}\left(\left|u_r - u_p\right| - (u_r - u_p)\right)$ and also we take $b_i' = \frac{1}{2}\left(\left|v_i - v_q\right| + v_i - v_q\right)$ and $b_q^{"} = \frac{1}{2}\left(\left|v_i - v_q\right| - (v_i - v_q)\right)$.

Hence, model (6) can be rewritten as the following linear form:

$$\max z = \sum_{r=1}^{s} u_r y_{rk} - \sum_{r=1}^{s-1} \sum_{p=r+1}^{s} \left(a_r' + a_p^{"}\right) - \sum_{i=1}^{m-1} \sum_{q=i+1}^{m} \left(b_i' + b_q^{"}\right) \qquad (25)$$

$$s.t. \quad \sum_{i=1}^{m} v_i x_{ik} = 1, \qquad (Model\ 7) \quad (26)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad k = 1, \dots, n, \qquad (27)$$

$$a_r' - a_p^{"} = u_r - u_p,$$

$$r = 1, \dots, s-1, \;\; r+1 \leq p \leq s$$

$$a_r^{"} - a_p^{"} = v_i - v_q,$$

$$i = 1, \dots, m-1, \;\; i+1 \leq q \leq m$$

$$u_r, v_i \geq 0, \;\; r = 1, \dots, s, \;\; i = 1, \dots, m,$$

$$a_r', a_p^{"} \geq 0, \quad r = 1, \dots, s-1, \qquad r+1 \leq p \leq s,$$

$$b_i', b_q^{"} \geq 0, \quad i = 1, \dots, m-1, \qquad i+1 \leq q \leq m.$$

Note that if there are more than one efficient DMUs, then we perform the supper efficiency model correspond to the above model for ranking all efficient DMUs.

ILLUSTRATIVE EXAMPLES

In order to illustrate the enhancement of the dispersion of input-output weights and the ability of the proposed model to improve the discrimination power, we have applied two examples incorporating the data of six DMUs with two inputs and two outputs in Sixton et al. (1986) and the data seven DMUs with three inputs and three outputs in Wong & Beasley (1990).

Example 1. Efficiency assessment of six nursing homes (Sexton, Silkman, & Hogan, 1986).

Six nursing homes consist of two inputs and two output variables of staff hours per day, including nurses, physicians etc. (x1); supplies per day, assessed in thousands of dollars (x2); total Medicare-plus Medicaid-reimbursed patient days (y1); and total privately paid patient days (y2), respectively. The input and output data relating the six nursing homes are given in Table 1. Using the data in Table 1, each of the above-mentioned models will be applied to these data in order to attain the efficiencies and levels of input-output weights. The results obtained with the DEA-CCR and its corresponding supper efficiency models for data set of Table 1 are summarized in Table 2. In Table 3, we presented the objective value of CVDEA model for each DMU and the values of input-output weights. Furthermore, the proposed model has been employed for data set and the results are presented in Table 4, which include the objective values related to each DMU and values of input-output weights. According to objective function values obtained by different models, the results of ranking DMUs are presented in Table 5. By considering Table 4, we find the input-output weights obtained with the proposed model are scattered more homogeneous compared to the DEA-CCR model and the proposed model discriminates all efficient DMUs.

TABLE 1
DATASET RELATED TO SIX NURSING HOMES

DMU	y1	y2	x1	x2
1	1.40	0.35	1.50	0.2
2	1.40	2.10	4.00	0.7
3	4.20	1.05	3.20	1.2
4	2.80	4.20	5.20	2.0
5	1.90	2.50	3.50	1.2
6	1.40	1.50	3.20	0.7

TABLE2
THE CCR-EFFICIENCIES AND SUPER EFIICIENCY VALUES OF DMUS

DMU	Efficiency	Super efficiency	u1	u2	v1	v2
1	1	2	0.714	0	0	5.000
2	1	1.395	0	0.476	0	1.429
3	1	1.412	0.238	0	8.172	0.374
4	1	1.131	0	0.238	0.069	0.321
5	0.977	0.977	0.115	0.304	0.110	0.513
6	0.867	0.867	0.162	0.427	0.155	0.722

TABLE 3
THE CVDEA-CCR RESULTS BASED ON DATA

DMU	Efficiency	u1	u2	v1	v2
1	1	0.571	0.571	0.517	1.120
2	0.863	0.176	0.293	0.181	0.392
3	0.991	0.189	0.189	0.227	0.227
4	0.983	0.103	0.165	0.138	0.138
5	0.948	0.158	0.259	0.212	0.212
6	0.735	0.190	0.312	0.256	0.256

TABLE 4
THE PROPOSED MODEL RESULTS BASED ON DATA

DMU	Efficiency	u1	u2	v1	v2
1	1	2.400	2.400	2.574	2.574
2	0.744	0.713	1.311	0.893	0.893
3	0.963	0.800	0.800	0.897	0.747
4	0.916	0.399	0.734	0.500	0.500
5	0.839	0.627	1.153	0.785	0.785
6	0.667	0.966	0.966	1.036	1.036

TABLE 5
RANKING DMUS FOR THE DATA BY VARIOUS MODELS

DMU	Super efficiency (AP model)	CVDEA	Proposed model
1	1	1	1
2	3	5	4
3	2	2	2
4	4	3	3
5	5	4	—
6	6	6	—

Example2. Evaluation of efficiency related to seven departments in a university (Wong & Beasley, 1990).

The evaluation of efficiency related to seven university departments (DMUs) consists of three inputs and three outputs. The variables for this problem are described below and the relevant data are represented in Table 6:

x1: number of academic staff number
x2: academic staff salaries in thousands of pounds
x3: support staff salaries in thousands of pounds
y1: number of undergraduate students
y2: number of postgraduate students
y3: number of research papers

The DEA-efficiency and its super efficiency related to each department has been assessed using Models (2), (3) and the efficiency results are summarized in Table 7. Also, in Table 7, it can be noticed the DEA model assigns the efficiency score one to all DMUs except DMU4. Additionally, Table 8 consists of the efficiency and super efficiency values of DMUs and the input-output weights applying the CVDEA model. In Table 8, it can be observed that the CVDEA determines DMU 1, DMU 5 and DMU 7 as efficiency. Hence, to rank the remaining efficiency DMUs (DMU 1, DMU 5 and DMU 7), the super efficiency CVDEA model has been performed on data set of Example 2. Wang and Luo (2009) mentioned that the CVDEA model gives the multiple locally optimal solutions due to its nonlinearity. Hence, the results of efficiency evaluation by CVDEA model are not reliable to compare the other approaches. Furthermore, the proposed model has been applied for data set in Table 7 and results are included in Table 9, which contain the objective function values of DMUs, and the input output weight values. The results of Table 9 show that our model decreases the number of efficiency DMUs. Ranking results for the seven departments are presented in Table 10 which contains the objective function values of DMUs obtained by the various models.

Empirical Study in National Gas Company of Iran

The product of natural gas are used in industries, power plants, commercial and household consumptions. Thus, performance measurement of the companies related to this product can be useful for improvement of the gas industry (see J.-Sharahii et.al. (2021)). In order to show the applicability of the proposed model with a new real-world data, we utilize the variables for 23 large branches of National Gas Company of Iran in 22 provinces consisting of three input and four output variables in Amirteimoori & ordrostami(2012). The three input variables are Budget (x1), number of staff (x2) and cost (x3) including operational cost, labor cost, maintenance and other services. The four output variables are number of customers (y1), length of gas network (y2)(Km), the delivered volumes (y3)(m3), the sold-out gas (y4). The data set of above mentioned input-output variables are shown in Table 11.

The DEA- efficiency and its super efficiency related to each DMUs has been evaluated by Models (2), (3) and the results are represented in Tables 12 and 13, respectively. Also, in Tables 12 and 13, are included the input-output weights concerned each DMUs. Moreover, the proposed model has been applied for data set in Table 11 and results are represented in Table 14, which contain the objective function values of DMUs, and values of input-output weights values. The results of Table 14, illustrate that our model decreases the number of efficiency DMUs as well. Ranking results are demonstrated in Table 15 which contain the objective function values of DMUs obtained by the various models. The four output variables are number of customers (y1), length of gas network (y2)(Km), the delivered volumes (y3)(m3), the sold-out gas (y4). The data set of above mentioned input-output variables are shown in Table 11. The DEA-efficiency and its super efficiency related to each DMUs has been evaluated by Models (2), (3) and the results are represented in Tables 12 and 13, respectively. Also, in Tables 12 and 13, are included the input-output weights concerned each DMUs. Moreover, the proposed model has been applied for data set in Table 11 and results are represented in Table 14, which contain the objective function values of DMUs, and values of input-output weights values. The results of Table 14, illustrate that our model decreases the number of efficient DMUs as well. Ranking results are demonstrated in Table 15 which contains the objective function values of DMUs obtained by the various models.

Table 6

DATASET RELATED TO SEVEN DEPARTMENTS OF THE UNIVERSITY

DMU	y1	y2	y3	x1	x2	x3
1	60	35	17	12	400	20
2	139	41	40	19	750	70
3	225	68	75	42	1500	70
4	90	12	17	15	600	100
5	253	145	130	45	2000	250
6	132	45	45	19	730	50
7	305	159	97	41	2350	600

TABLE 7
THE CCR-EFFICIENCIES AND SUPER EFFICIENCY VALUES OF DMUS

DMU	Efficiency	Super efficiency	u1	u2	u3	v1	v2	v3
1	1	1.829	0.983	10172	0	0	0.250	0
2	1	1.048	0.719	0	0	0	0.133	0
3	1	1.198	0	0	0.133	0	0.033	0.711
4	0.820	0.819	0.911	0	0	6.415	0.006	0
5	1	1.220	0	0.432	0.288	0	0.05	0
6	1	1.190	0.639	0	0.347	0	0.137	0
7	1	1.266	0.121	0.334	0.105	0.732	0.030	0

TABLE 8
THE CVDEA-CCR RESULTS BASED ON DATA

DMU	Efficiency	Super efficiency	u1	u2	u3	v1	v2	v3
1	1	1.368	0.847	0.971	0.893	0.870	0.193	0.618
2	0.983	0.983	0.462	0.403	0.438	0.106	0.124	0.063
3	0.990	0.990	0.293	0.162	0.293	0.756	0.021	0.514
4	0.820	0.820	0.812	0.420	0.350	1.107	0.109	0.179
5	1	1.311	0.038	0.293	0.366	0.101	0.032	0.125
6	0.980	0.980	0.440	0.440	0.440	0.133	0.133	0
7	1	1.253	0.178	0.178	0.178	0.513	0.025	0.033

TABLE 9
THE PROPOSED MODEL RESULTS BASED ON DATA

DMU	Obj. Func.	u1	u2	u3	v1	V2	v3
1	0.895	1.635	1.635	1.635	2.127	2.127	2.127
2	0.915	0.896	0.896	0.896	1.165	1.165	1.165
3	0.793	0.445	0.445	0.445	0.592	0.592	0.592
4	0.510	1.018	1.018	1.018	1.324	1.324	1.324
5	0.945	0.193	0.493	0439	0.441	0.441	0.441
6	1	0.942	0.942	0.942	1.225	1.225	1.225
7	0.858	0.396	0.396	0.279	0.523	0.523	0

TABLE 10
RANKING DMUS FOR THE DATA BY VARIOUS MODELS

DMU	Super efficiency (AP model)	CVDEA	Proposed model
1	1	1	4
2	6	5	3
3	4	4	6
4	7	7	—
5	3	2	2
6	5	6	1
7	2	3	5

TABLE 11
DATASET RELATED TO 23 NATIONAL GAZ COMPANY OF IRAN

Company	x1	x2	x3	y1	y2	y3	y4
1	665771	256	544757	80172	1295	495919	44040
2	368909	127	417595	6415	558	291437	32492
3	18747	107	177253	18526	243	176033	9274
4	765341	793	1600619	60165	1074	1761550	37228
5	1549715	895	1803747	47607	249	2044866	17875
6	392288	317	1120300	111235	932	867519	73714
7	115054	115	278242	10306	347	133925	7270
8	1143899	455	1107969	70124	986	1131640	36047
9	609959	506	759118	26285	351	815333	24860
10	151572	88	266684	7035	129	133694	4023
11	105413	116	219250	9523	222	171782	3768
12	656420	578	1054984	52785	947	660851	26085
13	172068	103	291136	15538	322	340813	10379
14	124778	103	203816	10312	97	176639	4914
15	184814	81	188664	20741	236	201128	13087
16	589694	152	494136	27284	697	393708	7971
17	373247	96	131205	29805	326	240842	13672
18	67801	104	119324	4156	116	24953	2066
19	175572	251	249043	20118	355	185752	13648
20	394181	388	504215	31075	680	479300	16263
21	177725	108	167911	28116	272	195526	15532
22	458883	376	529316	78188	1279	617592	53832
23	154727	159	349983	21085	357	451890	13164

TABLE 12
THE CCR- EFFICIENCIES VALUES OF DMUS

Company	Efficiency	u1	u2	u3	u4	v1		
1	1	0.037	0.208	3.030	0	1	0	0
2	1	0.312	6.524	0	0.047	2.258	0	0
3	0.771	2.134	3.024	3.870	1.659	0.410	4.860	0
4	0.847	0.169	0	1.033	0	0	0.983	0
5	0.823	0.047	0.472	0.481	0	0	0.823	0
6	1	0.342	1.292	0.734	0.347	0	1.540	0
7	1	7.801	3.275	0	0	3.732	0	0
8	0.804	0	1.119	0.702	0	0	1.452	0
9	0.782	0.337	0	2.061	0	0	1.962	0
10	0.467	0.002	10.169	0	0.966	0	6.213	0
11	0.800	13.592	0	0.621	0	3.581	2.219	0
12	0.548	1.659	0.460	0	0	0.567	0.412	0
13	1	4.413	3.797	0.452	0	2.276	2.604	0
14	0.660	2.328	0	7.191	1.295	0	6.254	0
15	0.957	1.541	3.436	4.830	0	0	6.051	2.040
16	1	0.183	4.010	0.910	0	1.119	1.993	0.129
17	1	0.339	7.419	1.683	0	2.070	3.688	0.239
18	0.591	20.345	0	1.661	0	6.593	0	0
19	0.728	8.694	0	0.109	0.257	2.091	1.195	0
20	0.755	0.660	0	2.977	0	0.310	2.525	0
21	1	2.473	0	7.696	1.760	0	5.806	0
22	1	0.617	0	2.785	0	0.290	2.362	0
23	1	1.026	0	4.626	0	0.482	3.924	0

TABLE 13
THE SUPER EFFICIENCY (AP MODEL) RESULTS BASED ON DATA

Company	Obj. Func.	u1	u2	u3	u4	v1	v2	v3
1	1.190	0.040	2.107	0.948	0.056	1.150	0	0
2	1.462	0	6.603	0.272	1.822	0.953	0	0
3	0.771	2.134	3.024	3.870	1.659	0.410	4.860	0
4	0.847	0.169	0	1.033	0	0	0.983	0
5	0.823	0.047	0.472	0.481	0	0	0.823	0
6	1.663	3.950	0	0	1.581	0	0.192	0
7	1.082	13.469	0	0	0	4.038	0	0
8	0.804	0	1.119	0.702	0	0	1.452	0
9	0.782	0.337	0	2.061	0	0	1.962	0
10	0.467	0.002	10.169	0	0.966	0	6.213	0
11	0.800	13.592	0	0.621	0	3.581	2.219	0
12	0.548	1.659	0.460	0	0	0.567	0.412	0
13	1.183	0	8.689	0	0	0.446	6.393	0.044
14	0.660	2.328	0	7.191	1.295	0	6.254	0
15	0.957	1.541	3.436	4.830	0	0	6.051	2.040
16	1.077	0	5.888	0	0	1.173	2.315	0
17	1.547	0	0	13.748	0.647	0.166	11.308	0
18	0.591	20.345	0	1.661	0	6.593	0	0
19	0.728	8.694	0	0.109	0.257	2.091	1.195	0
20	0.755	0.660	0	2.977	0	0.310	2.525	0
21	1.067	2.398	0	7.788	3.910	0	0.818	0
22	1.377	2.416	0	0.970	0	1.062	0	0.449
23	1.310	7.183	0	1.458	0	1.091	4.569	0

TABLE 14

THE PROPOSED MODEL RESULTS BASED ON DATA

Company	Objective function	u1	u2	u3	u4	v1	v2	v3
1	0.8938	0.349	0.349	0.349	0.349	0.941	0.941	0.941
2	0.9651	0.606	0.606	0.606	0.606	1.635	1.635	1.635
3	0.6196	1.095	1.095	1.095	1.095	2.952	2.952	2.952
4	0.6438	0.096	0.096	0.661	0.096	0.441	0.441	0.441
5	0.5779	0	0	0.711	0	0.333	0.333	0.333
6	0.9767	0.344	0.225	0.347	0.347	0.814	0.814	0.814
7	0.5450	1.039	1.039	1.039	1.039	2.801	2.801	2.801
8	0.5420	0.153	0.153	0.767	0.072	0.537	0.537	0.537
9	0.3924	0.158	0.158	1.086	0.158	0.725	0.725	0.725
10	0.3049	1.078	1.078	1.078	1.078	2.907	2.907	2.907
11	0.4556	1.162	1.162	1.162	1.162	3.133	3.133	3.133
12	0.4221	0.224	0.224	0.224	0.224	0.604	0.604	0.604
13	0.6658	0.957	0.957	0.957	0.957	2.580	2.580	2.580
14	0.3853	1.202	1.202	1.202	1.202	3.240	3.240	3.240
15	0.7604	1.180	1.180	1.180	1.180	3.181	3.181	3.181
16	0.4877	0.450	0.450	0.450	0.450	1.213	1.213	1.213
17	0.7251	0.881	0.881	0.881	0.881	2.376	2.376	2.376
18	0.2742	1.640	1.640	1.640	1.640	4.423	4.423	4.423
19	0.5097	0.697	0.697	0.697	0.697	1.880	1.880	1.880
20	0.4828	0.383	0.383	0.383	0.383	1.034	1.034	1.034
21	0.8683	1.129	1.129	1.129	1.129	3.045	3.045	3.045
22	1.0000	0.367	0.367	0.367	0.367	0.990	0.990	0.990
23	0.6801	0.786	0.786	0.786	0.786	2.121	2.121	2.12

TABLE 15
RANKING DMUS FOR THE DATA BY VARIOUS MODELS

DMU	Super efficiency (AP model)	Proposed model
1	6	4
2	3	3
3	17	—
4	12	—
5	13	—
6	1	2
7	8	9
8	14	—
9	16	—
10	23	—
11	15	—
12	22	—
13	7	8
14	20	—

DMU	Super efficiency (AP model)	Proposed model
15	11	—
16	9	10
17	2	6
18	21	—
19	19	—
20	18	—
21	10	5
22	4	1
23	5	7

CONCLUSION

In the present research, we have proposed a new model based on minimizing the average of absolute deviations of all input-output weights from each other which overcomes some of the drawbacks of CVDEA model offered by Bal et al. (2008). One of the problems of CVDEA model is averaging the input-output weights with different dimensions and scales. The other disadvantages are that the CVDEA model changes the efficiency by normalizing the input and output data and it is possible to find the multiple locally optimal solutions because of nonlinearity of the model. The suggested approach overcomes the previously mentioned problems and then was applied on some numerical examples in the literature. The obtained results revealed that the new model provided a more balanced variability of input-output weights and improved the discrimination power. Also, for future study, this model can be applied for supply chain problems and can be extended for two-stage DEA models.

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