Original Research .

Measures of Repairable Series-Parallel Computer Network with Fault Tolerant Device via Copula

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Received: 04 Jan 2023 / Accepted: 31 Aug 2024 / Published online: 07 Sep 2024

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Abstract
This study introduces a comprehensive stochastic model for analyzing the behavior of a complex computer network, consisting of a cloud server, two load balancers, a fog node, and multiple clients. The primary aim is to evaluate the reliability measures of a series-parallel computer network incorporating fault-tolerant devices. The system is exposed to two partial and complete or total failures. The research delves into two corrective maintenance techniques: general and copula repair for partial and total failures, respectively. All failure rates in the model are assumed to follow an exponential distribution. A system of first-order partial differential equations (PDEs) is derived and solved using supplementary variables and Laplace transformation to obtain explicit expressions for availability, reliability, mean time to failure (MTTF), MTTF sensitivity, and cost function. The results from numerical experiments are presented in tables and graphs, offering valuable insights for system designers, engineers, maintenance managers, and reliability engineers. These findings provide practical guidance for optimizing system performance and maintenance strategies.

Keywords- Reliability; Computer network; Cloud server; MTTF; Sensitivity

INTRODUCTION

In many scenarios, computer systems utilized number of distributed networks to provide available and optimal network to the clients. The study of computer network system present its economic and technical feasibility as the better choice to multipurpose network. With the advancement in technology, availability of computer network is a object of research and discussion. Meeting optimal level of availability is of paramount important needed in information, communication, military and institutional sector. Where reliability could not attend its maximum level, the computer network will be very poor. High computer system reliability is vital to industrial growth due to the fact that revenue mobilization is proportional to system performance. Due to its importance in industrial, domestic, institutional and manufacturing sector, literature study on dependability, reliability, maintainability and availability modelling of different computer network, developed models are used to address the computer network performance subject to system failure. To this end, literatures have studied dependability, reliability, maintainability and availability problems of different computer network. The technique of redundancy is thoroughly used to enhance reliability, dependability and availability of the system. In some computer network, the availability and dependability rely on the design of the system and strength of the units. To retain availability and dependability of complex computer network to an optimal level, the structure of the system and its components of optimal availability are required. Generally, system designers can develop technologies in a serial network to improve network availability, dependability and reliability. [1] Explore on article on performance analysis on computer network system that comprises of centralized database server, load balancer and distributed database server, [2] discuss on reliability metrics of network communication system having receiver, relay and transmitter. [3] Conducts a comprehensive exploration of the reliability and dependability assessment of a complex system composed of two subsystems. In this system, Subsystem 1 operates under a k-out-of-n configuration, utilizing the G policy, while Subsystem 2 consists of four identical units functioning in active parallel. The study delves into the intricacies of how these configurations contribute to the overall system reliability.[4] Undertakes an in-depth study of the reliability of computer networks through the application of genetic algorithms. The research not only examines the effectiveness of these algorithms in assessing reliability but also develops optimization techniques specifically designed to enhance the reliability of the network. The study provides valuable insights into how genetic algorithms can be utilized for the robust optimization of network reliability. However, [5] presented the stochastic performance of computer based test having four subsystems arranged in series namely load balancer, clients, centralized server and database server.

[6] Conducts a reliability analysis of a computer network composed of three critical subsystems: a router, a workstation, and a hub. [7] Examines the performance metrics of a network utilizing a transparent bridge, focusing on specific configurations such as 1-out-of-2: G, 2-out-of-3: F, a bridge unit, and D 3-out-of-5: G schemes. [19, 21] Analyzes a distributed system consisting of five standby subsystems arranged in a series-parallel configuration: A (clients), B (two load balancers), C (two distributed database servers), D (two mirrored distributed database servers), and E (a centralized database server [8] Focuses on enhancing reliability within an intuitionistic fuzzy space. [9] Discusses heterogeneity in clients using remote procedure calls (RPC). [10] Develops models for analyzing the strength and performance of computer networks under various maintenance scenarios.[11] Examines the reliability of database clusters, virtual router redundancy protocols, and load balancers, analyzing availability by comparing the reliability when a load balancer, virtual router redundancy protocol, and high-availability proxy are implemented. [12] Investigates how system structure impacts the reliability measures of software agents and client-server architectures. [13] Considers online transaction processing (OLTP) applications with incremental repartitioning of shared-nothing distributed databases. [14] Examines the implications of load balancing in distributed systems. [15] Provides a comprehensive analysis of the current TCAs (Traffic Control Algorithms) in FANETs (Flying Ad Hoc Networks), introducing a new taxonomy based on FANET topology architectures and underlying mathematical models. [16] Addresses repairable systems with reboot delay, focusing on a single repair policy and imperfect coverage. [17, 18]

Investigates a parallel system, identifying three types of failures: human failure, unit failure, and major failure. The literature above focus on reliability and performance analysis of distributed systems using different techniques, claiming an improved the performance of the distributed systems. Researchers have long used the copula technique to evaluate the dependability traits of many complex repairable communication systems, among other things. Similar to other methods, researchers made various system designs and parameter assumptions to assessed systems' dependability components. The copula policy has been in use by numerous scholars worldwide. To mention a few, [22, 28] analyzes the impact of unit failure and sensitivity on reliability and performance of a serial system through copula. [5] focus on measuring the performance indices of a complex system consisting *n*-identical units under a k-out-of-n: G; configuration through the use of copula. [3] delves into evaluation of reliability characteristics of a complex system working under k-out-of-n: G configuration. [23] using copula approach delves into performance enhancement of a distributed system with data replica, [20,29] delves into copula performance modelling and reliability estimation of serial photovoltaic system attended by human operator, [27] focus on performance evaluation of feeding unit in paper plant using copula linguistic.

From the literature above, it is clear that there is a lack of comprehensive stochastic models specifically tailored to analyze the behavior of complex computer networks that include a cloud server, load balancers, a fog node, and multiple clients and the need to evaluate reliability measures for series-parallel computer networks incorporating fault-tolerant devices has not been thoroughly addressed in existing research. With this in mind, this paper focuses on the performance analysis of a computer network system that includes a cloud server, two load balancers, a fog node, and multiple clients, utilizing a copula approach. The study is driven by the need to improve the performance of distributed systems by carefully managing critical metrics such as mean time to failure (MTTF), cost, availability, and reliability. These metrics significantly impact the development and operation of IT and process sectors. By employing a copula approach, the study aims to model and evaluate these performance metrics, establish explicit expressions, and statistically validate them. The objectives of study are three folds. First is to develop models that accurately represent both total and partial failure situations within the system. The second is to assess the reliability and performance of a complex tree-topology computer networking system. This involves examining how effectively the system functions under normal conditions and its ability to withstand failures and to analyze the specific configuration of these subsystems, including their roles (e.g., a cloud server, two load balancers, a fog node, and multiple clients) and how they are interconnected in a series-parallel arrangement. This involves utilizing techniques such as the Copula family for total failures and general repair distributions for partial failures. The third is to optimize system operation, minimize failure rates, and guide managerial decisions regarding resource allocation and service pricing to maximize profit while maintaining system performance.

This research work further improved the work of previous investigations were five subsystems are considered subsystem A consist of 2-clients, subsystem B comprises of a load balancer I, 2-fog node are in subsystem C, subsystem D comprises of load balancer II and lastly, subsystem E consist of 2-cloud server. However, analysis of the model in terms of fault tolerant, general repair and copula was thoroughly investigated. Reliability analysis measures such as availability, cost and MTTF, sensitivity analysis were carried out for different scenarios to check optimality of the entire system with respect both failure and repair rate. Moreover, some practical applications were considered. This work is organized into several key sections. Section 2 provides a comprehensive description of the system under investigation and introduces the corresponding notations used throughout the study. Section 3 discusses in detail the formulation of the models and their presentation, laying the groundwork for the analytical approaches employed. In Section 4, the numerical experiments conducted as part of the study are presented, showcasing the practical application and validation of the proposed models. Finally, Section 5 offers a conclusive summary of the findings, highlighting the implications of the results and potential directions for future research.

DESCRIPTION AND NOTATION OF THE SYSTEM

I. Description of the System

The computer network examined in this study is structured with five subsystems configured in a series-parallel arrangement. Subsystem A includes two clients operating in active parallel, while subsystem B features a first-tier load balancer. Subsystem C contains two fog nodes functioning in active parallel, subsystem D incorporates two second-tier load balancers, and subsystem E consists of two cloud servers running in parallel. The computer network in this study comprises five subsystems arranged in a series-parallel configuration, designed to ensure both reliability and efficiency.

- **Subsystem A**: This subsystem includes two clients operating in active parallel. These clients represent the end-users or devices that initiate requests and interact with the network, ensuring continuous service even if one client fails.
- **Subsystem B**: This subsystem consists of a single first-tier load balancer. The load balancer is responsible for distributing incoming traffic from the clients to the subsequent network components, optimizing resource utilization, and preventing any single node from becoming a bottleneck.
- **Subsystem C**: This subsystem houses two fog nodes working in active parallel. Fog nodes provide intermediate processing and storage capabilities, bringing computation closer to the clients to reduce latency and improve response times. Their parallel arrangement enhances reliability, as the system can continue functioning if one fog node fails.
- **Subsystem D**: This subsystem includes two second-tier load balancers. Similar to the first-tier load balancer, these second-tier load balancers further distribute the traffic received from the fog nodes to the final stage of the network, ensuring balanced load distribution and efficient processing.

- **Subsystem E**: This subsystem consists of two cloud servers in parallel. The cloud servers provide robust processing power and storage capabilities, handling the bulk of data processing and management. Their parallel configuration ensures that the system remains operational even if one server encounters issues, thereby enhancing the overall reliability of the network.

By arranging these subsystems in a series-parallel configuration, the network is designed to maintain high availability and reliability, with each component contributing to the system's seamless operation and fault tolerance.

II. Notations of the System

t := Variable of time

FT := Fault tolerant

CP := Copula

GR := General repair

s := Variable of laplace transform

φ_1 := Revenue generated

φ_2 := Service cost per unit

f_1 := to mean rate of failure of clients

f_2 := to mean rate of failure of load balancer I

f_3 := to mean rate of failure of Fog node

f_4 := Rate of failure due to load balancer II

f_5 := Rate of failure due to cloud server

$k(r_1)$:= Rate of repair of clients

$k(r_2)$:= to mean rate of repair of load balancer I

$k(r_3)$:= Rate of repair of Fog node

$k(r_4)$:= Rate of repair of load balancer II

$k(r_5)$:= Rate of repair of cloud server

$q_0(r_1)$:= Copula repair of clients at complete failed state

$q_0(r_2)$:= Copula repair of load balancer I at complete failed state

$q_0(r_3)$:= Copula repair of Fog node at complete failed state

$q_0(r_4)$:= Copula repair of load balancer II at complete failed state

$q_0(r_5)$:= Copula repair of cloud server at complete failed state

$g_i(t)$:= System probability at S_i for $i \in [0,14]$

$\overline{G}$:= to mean Laplace transformation of the transition probability g(t)

$G_i(r,t)$:= to mean probability that a system in the System S_i, is under repair with repair variable r a

$\Delta_1 = gf_1 + gf_2 + 2gf_3 + gf_4 + 2gf_5 + s + 1 \qquad \Delta_2 = 2gf_1 + gf_2 + 2gf_3 + gf_4 + gf_5 + s + 1$

$$\Delta_3 = 2gf_1 + gf_2 + gf_3 + gf_4 + 2gf_5 + s + 1 \qquad \Delta_4 = 4g^3 f_1^2 f_3 + 4g^3 f_1^2 f_5$$
$$\Delta_5 = 4g^3 f_5^2 f_1 + 4g^3 f_5^2 f_3 \qquad \Delta_6 = 4g^3 f_3^2 f_1 + 4g^3 f_3^2 f_5 + 2g^3 f_3^2 + 2gf_3$$
$$\Delta_7 = 2g^2 f_1 f_2 + 2g^2 f_1 f_3 + 2g^2 f_1 f_5 + gf_2 \qquad \Delta_8 = 2g^2 f_1 f_4 + 2g^2 f_4 f_3 + 2g^2 f_4 f_5 + gf_4$$
$$\Delta_9 = 2gf_1 + gf_2 + 2gf_3 + gf_4 + 2gf_5 + s \qquad \Delta_{10} = gf_1 + gf_2 + 2gf_3 + gf_4 + 2gf_5 + s$$

$$\Delta_{11} = 2gf_1 + gf_2 + 2gf_3 + gf_4 + gf_5 + s \qquad \Delta_{12} = 2gf_1 + gf_2 + gf_3 + gf_4 + 2gf_5 + s$$
$$\Delta_{13} = 2gf_1 + gf_2 + 2gf_3 + gf_4 + 2gf_5 \qquad \Delta_{14} = gf_1 + gf_2 + 2gf_3 + gf_4 + 2gf_5$$
$$\Delta_{15} = 2gf_1 + gf_2 + 2gf_3 + gf_4 + gf_5 \qquad \Delta_{16} = 2gf_1 + gf_2 + gf_3 + gf_4 + 2gf_5$$

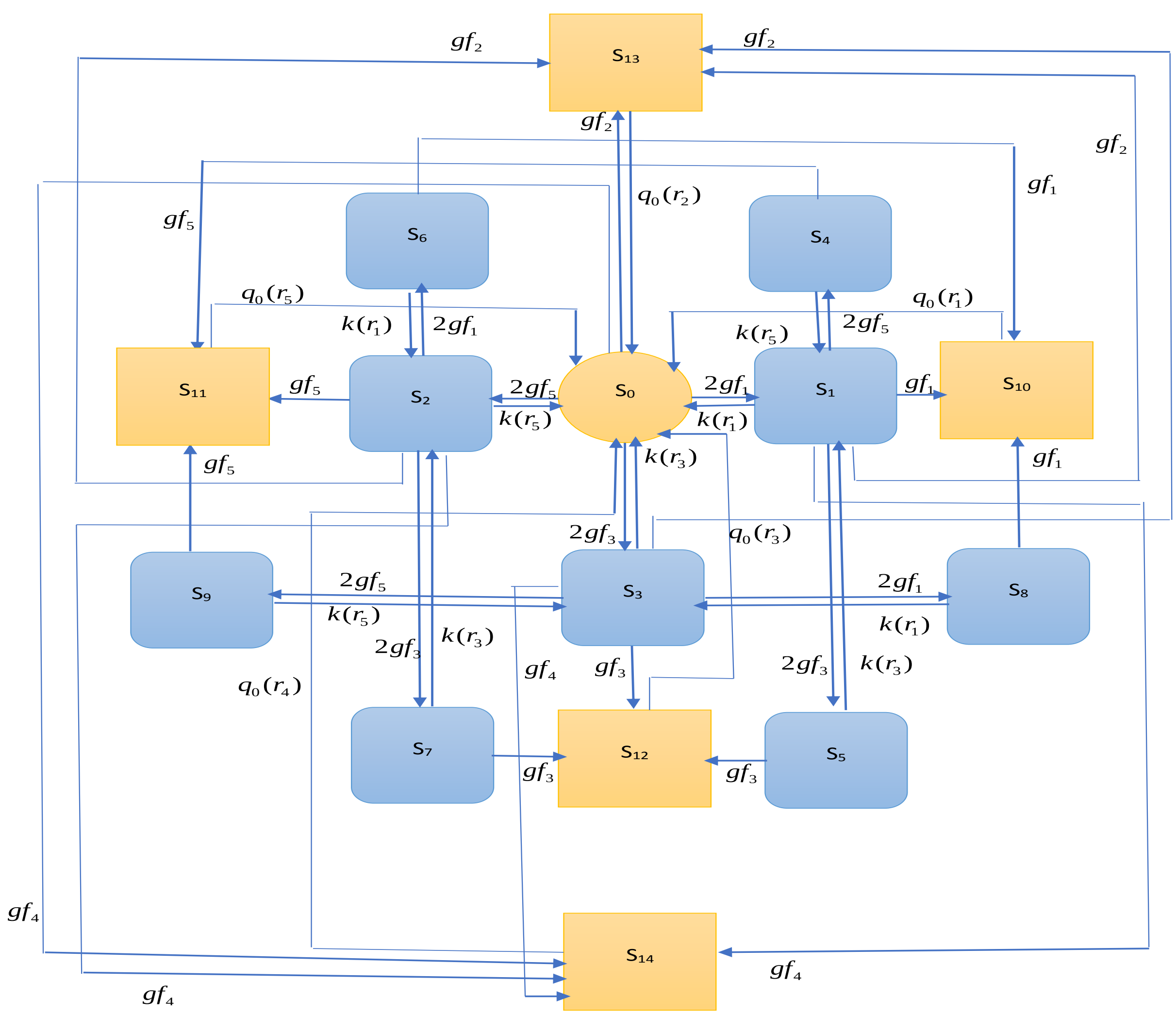

FIGURE 1
TRANSITION DIAGRAM OF THE SYSTEM

MODEL DEVELOPMENT

The supplementary variable technique uses an additional variable to represent the system's state at a specific moment. This method simplifies the partial differential equations (PDEs) that describe the system, making them easier to solve. Laplace transforms are then used to convert these PDEs into algebraic equations, which are much simpler to handle. These probabilities are the foundation for creating reliability models, which assess the system's performance and identify potential failure modes. According to studies by [3,4, 11, 22], developing reliability and performance models involves deriving the partial differential equations, applying Laplace transforms, solving the resulting algebraic equations, and using the solutions to calculate state probabilities. In this study, the procedure outlined above is meticulously followed, utilizing Figure 1 as a foundational reference to derive the corresponding system of linear partial differential equations. These equations are subsequently solved through the application of the supplementary variable technique and Laplace transformation. This approach enables the explicit determination of key system metrics, including availability, reliability, meantime to failure (MTTF), cost analysis, and the sensitivity of MTTF with respect to various system parameters.

$$\left(\frac{\delta}{\delta t}+2gf_1+2gf_2+2gf_3+gf_4+2gf_5\right)G_0(t)=\int_0^{\infty}\sum_{i,j=1,2,3,10,11,12,13,14} e^{[r^{\theta}+(\log q_0(r_i))^{\theta}]^{\frac{1}{\theta}}}dr_i \tag{1}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_1}+\left(gf_1+gf_2+2gf_3+gf_4+2gf_5+k(r_1)\right)\right)G_1(t)=0 \tag{2}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_5}+\left(2gf_1+gf_2+2gf_3+gf_4+gf_5+k(r_5)\right)\right)G_2(t)=0 \tag{3}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_3}+\left(2gf_1+gf_2+gf_3+gf_4+2gf_5+k(r_3)\right)\right)G_3(t)=0 \tag{4}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_3}+\left(gf_3+k(r_3)\right)\right)G_4(t)=0 \tag{5}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_3}+\left(gf_3+k(r_3)\right)\right)G_5(t)=0 \tag{6}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_1}+\left(gf_1+k(r_1)\right)\right)G_6(t)=0 \tag{7}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_3}+\left(gf_3+k(r_3)\right)\right)G_7(t)=0 \tag{8}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_1}+\left(gf_1+k(r_1)\right)\right)G_8(t)=0 \tag{9}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_5}+\left(gf_5+k(r_5)\right)\right)G_9(t)=0 \tag{10}$$

$$\sum_{i,j=1,2,3,4,5}\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_i}+\left(q_0(r_i)\right)\right)G_j(t)=0 \tag{11}$$

III. Boundary Conditions

$$G_1(0,t)=2gf_1G_0(t) \tag{12}$$
$$G_2(0,t)=2gf_5G_0(t) \tag{13}$$
$$G_3(0,t)=2gf_3G_0(t) \tag{14}$$
$$G_4(0,t)=2g^2f_1f_5G_0(t) \tag{15}$$
$$G_5(0,t)=4g^2f_3f_1G_0(t) \tag{16}$$
$$G_6(0,t)=2g^2f_1f_5G_0(t) \tag{17}$$
$$G_7(0,t)=2g^2f_5f_3G_0(t) \tag{18}$$
$$G_8(0,t)=2g^2f_1f_3G_0(t) \tag{19}$$
$$G_9(0,t)=4g^2f_3f_5G_0(t) \tag{20}$$
$$G_{10}(0,t)=\left(4g^3f_1^2f_5+4g^3f_1^2f_3\right)G_0(t) \tag{21}$$
$$G_{11}(0,t)=\left(4g^3f_5^2f_1+4g^3f_5^2f_3\right)G_0(t) \tag{22}$$
$$G_{12}(0,t)=\left(2gf_3+2g^2f_3^2+4g^3f_1^2f_5+4g^3f_1^2f_3\right)G_0(t) \tag{23}$$
$$G_{13}(0,t)=\left(gf_2+2g^2f_1f_2+2g^2f_2f_5+2g^3f_2f_3\right)G_0(t) \tag{24}$$
$$G_{14}(0,t)=\left(gf_4+2g^2f_1f_4+2g^2f_4f_5+4g^2f_4f_3\right)G_0(t) \tag{25}$$

IV. Model Solution

Taking Laplace transformation of equations (1) – (25) to obtain the following:

$$\left(s+2gf_1+2gf_2+2gf_3+gf_4+2gf_5\right)\overline{G}_0(s)=1+\int_0^\infty \sum_{i,j=1,2,3,10,11,12,13,14} e^{[r^\theta+(\log q_0(r_i))^\theta]^{\frac{1}{\theta}}}dr_i \tag{26}$$

$$\left(s+\frac{\delta}{\delta r_1}+gf_1+gf_2+2gf_3+gf_4+2gf_5+k(r_1)\right)\overline{G}_1(s)=0 \tag{27}$$

$$\left(s+\frac{\delta}{\delta r_5}+2gf_1+gf_2+2gf_3+gf_4+gf_5+k(r_5)\right)\overline{G}_2(s)=0 \tag{28}$$

$$\left(s+\frac{\delta}{\delta r_3}+2gf_1+gf_2+gf_3+gf_4+2gf_5+k(r_3)\right)\overline{G}_3(s)=0 \tag{29}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_5}+\left(gf_5+k(r_5)\right)\right)G_4(s)=0 \tag{30}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_3}+\left(gf_3+k(r_3)\right)\right)\overline{G}_5(s)=0 \tag{31}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_1}+\left(gf_1+k(r_1)\right)\right)\overline{G}_6(s)=0 \tag{32}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_3}+\left(gf_3+k(r_3)\right)\right)\overline{G}_7(s)=0 \tag{33}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_1}+\left(gf_1+k(r_1)\right)\right)\overline{G}_8(s)=0 \tag{34}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_5}+\left(gf_5+k(r_5)\right)\right)\overline{G}_9(s)=0 \tag{35}$$

$$\sum_{i,j=1,2,3,4,5}\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta r_i}+\left(q_0(r_i)\right)\right)G_j(s)=0 \tag{36}$$

$$\overline{G}_1(0,s)=2gf_1\overline{G}_0(s) \tag{37}$$
$$\overline{G}_2(0,s)=2gf_5\overline{G}_0(s) \tag{38}$$
$$\overline{G}_3(0,s)=2gf_3\overline{G}_0(s) \tag{39}$$
$$\overline{G}_4(0,s)=4g^2 f_1 f_5\overline{G}_0(s) \tag{40}$$
$$\overline{G}_5(0,s)=4g^2 f_1 f_3\overline{G}_0(s) \tag{41}$$
$$\overline{G}_6(0,s)=4g^2 f_1 f_5\overline{G}_0(s) \tag{42}$$
$$\overline{G}_7(0,s)=4g^2 f_3 f_5\overline{G}_0(s) \tag{43}$$
$$\overline{G}_8(0,s)=4g^2 f_1 f_3\overline{G}_0(s) \tag{44}$$
$$\overline{G}_9(0,s)=4g^2 f_3 f_5\overline{G}_0(s) \tag{45}$$

$$\overline{G}_{10}(0,s)=\left(4g^3 f_1^2 f_5+4g^3 f_1^2 f_3\right)\overline{G}_0(s) \quad (46)$$

$$\overline{G}_{11}(0,s)=\left(4g^3 f_5^2 f_1+4g^3 f_3^2 f_5\right)\overline{G}_0(s) \quad (47)$$

$$\overline{G}_{12}(0,s)=\left(2gf_3+2g^2 f_3^2+4g^3 f_3^2 f_1+4g^3 f_3^2 f_5\right)\overline{G}_0(s) \quad (48)$$

$$\overline{G}_{13}(0,s)=\left(gf_2+2g^2 f_1 f_2+2g^2 f_5 f_2+2g^2 f_3 f_2\right)\overline{G}_0(s) \quad (49)$$

$$\overline{G}_{14}(0,s)=\left(gf_4+2g^2 f_1 f_4+2g^2 f_5 f_4+2g^2 f_3 f_4\right)\overline{G}_0(s) \quad (50)$$

$$\overline{G}_1(s)=2gf_1\left\{\frac{1-\overline{S}_k\left(s+gf_1+gf_2+2gf_3+gf_4+2gf_5\right)}{s+gf_1+gf_2+2gf_3+gf_4+2gf_5}\right\}\overline{G_0}(s) \quad (51)$$

$$\overline{G}_2(s)=2gf_5\left\{\frac{1-\overline{S}_k\left(s+2gf_1+gf_2+2gf_3+gf_4+gf_5\right)}{s+2gf_1+gf_2+2gf_3+gf_4+gf_5}\right\}\overline{G_0}(s) \quad (52)$$

$$\overline{G}_3(s)=2gf_3\left\{\frac{1-\overline{S}_k\left(s+2gf_1+gf_2+gf_3+gf_4+2gf_5\right)}{s+2gf_1+gf_2+gf_3+gf_4+2gf_5}\right\}\overline{G_0}(s) \quad (53)$$

$$\overline{G}_4(s)=4g^2 f_1 f_5\left\{\frac{1-\overline{S}_k\left(s+gf_5\right)}{s+gf_5}\right\}\overline{G_0}(s) \quad (54)$$

$$\overline{G}_5(s)=4g^2 f_1 f_3\left\{\frac{1-\overline{S}_k\left(s+gf_3\right)}{s+gf_3}\right\}\overline{G_0}(s) \quad (55)$$

$$\overline{G}_6(s)=4g^2 f_1 f_5\left\{\frac{1-\overline{S}_k\left(s+gf_1\right)}{s+gf_1}\right\}\overline{G_0}(s) \quad (56)$$

$$\overline{G}_7(s)=4g^2 f_3 f_5\left\{\frac{1-\overline{S}_k\left(s+gf_3\right)}{s+gf_3}\right\}\overline{G_0}(s) \quad (57)$$

$$\overline{G}_8(s)=4g^2 f_1 f_3\left\{\frac{1-\overline{S}_k\left(s+gf_1\right)}{s+gf_1}\right\}\overline{G_0}(s) \quad (58)$$

$$\overline{G}_9(s)=4g^2 f_3 f_5\left\{\frac{1-\overline{S}_k\left(s+gf_5\right)}{s+gf_5}\right\}\overline{G_0}(s) \quad (59)$$

$$\overline{G}_k(s)=\sum_{i,j=1,2,3,4,5}\left\{\frac{1-\overline{S}_{q_0}\left(s\right)}{s}\right\}\overline{G_j}(s) \quad (60)$$

for k=10,11,12,13,14 and $\overline{G_0}=\dfrac{1}{L(s)}$

$$L(s) = s + 2gf_1 + 2gf_2 + 2gf_3 + gf_4 + 2gf_5 - \left(\begin{aligned} &2gf_1\left\{\bar{S}_k\left(s+gf_1+gf_2+2gf_3+gf_4+2gf_5\right)\right\}+ \\ &\Bigg(2gf_5\left\{\bar{S}_k\left(s+2gf_1+gf_2+2gf_3+gf_4+2gf_5\right)\right\}+ \\ &\Bigg(2gf_5\left\{\bar{S}_k\left(s+2gf_1+gf_2+2gf_3+gf_4+2gf_5\right)\right\}+ \\ &\Bigg(2gf_3\left\{\bar{S}_k\left(s+2gf_1+gf_2+gf_3+gf_4+2gf_5\right)\right\}+ \\ &\left(4g^3f_5f_1^2+4g^3f_3f_1^2\right)\left\{\bar{S}_{q_0}(s)\right\}+\left(4g^3f_1f_5^2+4g^3f_3f_5^2\right)\left\{\bar{S}_{q_0}(s)\right\}+ \\ &\left(2gf_3+2g^2f_3^2+4g^3f_1f_3^2+4g^3f_5f_3^2\right)\left\{\bar{S}_{q_0}(s)\right\}+ \\ &\left(gf_2+2g^2f_1f_2+2g^2f_2f_5+2g^2f_2f_3\right)\left\{\bar{S}_{q_0}(s)\right\}+ \\ &\left(gf_4+2g^2f_1f_4+2g^2f_4f_5+2g^2f_4f_3\right)\left\{\bar{S}_{q_0}(s)\right\}\Bigg)\Bigg)\Bigg)\end{aligned}\right) \quad (61)$$

$$\bar{G}_{UP}(s) = \bar{G}_0(s) + \bar{G}_1(s) + \bar{G}_2(s) + \bar{G}_3(s) + \bar{G}_4(s) + \bar{G}_5(s) + \bar{G}_6(s) + \bar{G}_7(s) + \bar{G}_8(s) + \bar{G}_9(s) \quad (62)$$

$$\bar{G}_{down}(s) = 1 - \bar{G}_{UP}(s) \quad (63)$$

NUMERICAL ANALYSIS OF THE MODEL FOR PARTICULAR CASES

I. Availability Analysis

- Availability analysis of the model with fault tolerant (without Copula and General repair)

Setting all the repairs $k(r_1)$, $k(r_2)$, $k(r_3)$, $k(r_4)$, $k(r_5)$, $q_0(r_1)$, $q_0(r_2)$, $q_0(r_3)$, $q_0(r_4)$ and $q_0(r_5)$ to 1. While the failure rates are set as $f_4 = 0.02$, $f_1 = 0.05$, $f_3 = 0.03$, $f_2 = 0.04$, and $f_5 = 0.01$, and setting the fault tolerant g = 0.002 table 1 and figure 2 below were obtained, by varying t=0 through 0.09.

$$A_{FT} = \frac{1 + \frac{2gf_1}{\Delta_1} + \frac{2gf_5}{\Delta_2} + \frac{2gf_3}{\Delta_3} + \frac{4g^2f_1f_5}{gf_5+s+1} + \frac{4g^2f_1f_5}{gf_1+s+1} + \frac{4g^2f_1f_3}{gf_3+s+1} + \frac{4g^2f_3f_5}{gf_3+s+1} + \frac{4g^2f_1f_3}{gf_1+s+1} + \frac{4g^2f_3f_5}{gf_5+s+1}}{\left(s+2gf_1+gf_2+2gf_3+gf_4+2gf_5\right) - \frac{2gf_1}{\Delta_1} - \frac{2gf_5}{\Delta_2} - \frac{2gf_3}{\Delta_3} - \frac{q_0}{s+q_0}\left(\Delta_4+\Delta_5+\Delta_6+\Delta_7+\Delta_8\right)} \quad (64)$$

TABLE 1
AVAILABILITY ANALYSIS OF THE MODEL WITH FAULT TOLERANT

Time	Availability
0.000000	1.000000
0.010000	0.999998
0.020000	0.999997
0.030000	0.999996
0.040000	0.999995
0.050000	0.999994
0.060000	0.999992
0.070000	0.999991
0.080000	0.999990
0.090000	0.999989

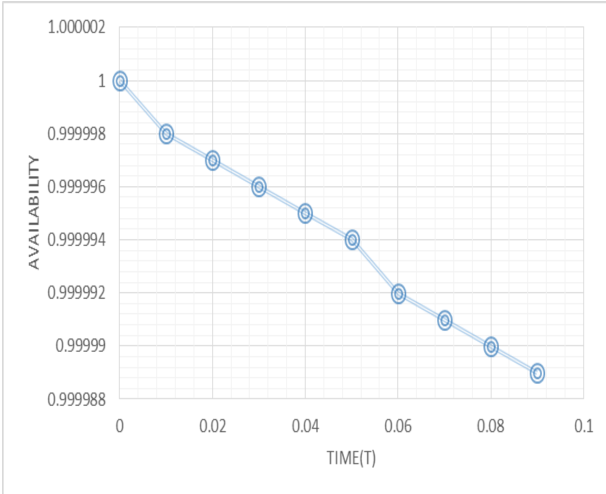

FIGURE 2
AVAILABILITY OF THE MODEL WITH FAULT TOLERANT

- Availability analysis of the model with Copula (without Fault tolerant and general repair)

Setting all the repairs $k(r_1)$, $k(r_2)$, $k(r_3)$, $k(r_4)$ and $k(r_5)$ to 1 with $q_0 = 2.7183$ and substituting $f_4 = 0.02$, $f_1 = 0.05,\ f_3 = 0.03,\ f_2 = 0.04,$ and $f_5 = 0.01$, and setting fault tolerant g = 1.0 table 2 and figure 3 below were obtained, by varying t=0 through 0.09.

$$A_{CP} = \frac{1+\frac{2gf_1}{\Delta_1}+\frac{2gf_5}{\Delta_2}+\frac{2gf_3}{\Delta_3}+\frac{4g^2 f_1 f_5}{gf_5+s+1}+\frac{4g^2 f_1 f_5}{gf_1+s+1}+\frac{4g^2 f_1 f_3}{gf_3+s+1}+\frac{4g^2 f_3 f_5}{gf_3+s+1}+\frac{4g^2 f_1 f_3}{gf_1+s+1}+\frac{4g^2 f_3 f_5}{gf_5+s+1}}{\left(s+2gf_1+gf_2+2gf_3+gf_4+2gf_5\right)-\frac{2gf_1}{\Delta_1}-\frac{2gf_5}{\Delta_2}-\frac{2gf_3}{\Delta_3}-\frac{2.7183}{s+2.7183}\left(\Delta_4+\Delta_5+\Delta_6+\Delta_7+\Delta_8\right)} \tag{65}$$

TABLE 2
AVAILABILITY ANALYSIS OF THE MODEL WITH COPULA

Time	Availability
0.000000	1.000000
0.010000	0.999599
0.020000	0.999230
0.030000	0.998891
0.040000	0.998580
0.050000	0.998298
0.060000	0.998043
0.070000	0.997814
0.080000	0.997610
0.090000	0.997431

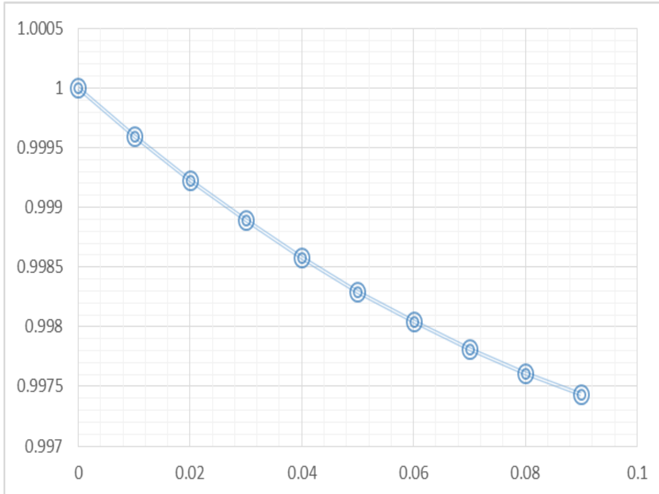

FIGURE 3
AVAILABILITY OF THE MODEL WITH COPULA

- Availability analysis of the model with General repair (without Fault tolerant and Copula)

Setting all the repairs $k(r_1)$, $k(r_2)$, $k(r_3)$, $k(r_4)$ and $k(r_5)$ to 1 with $q_0 = 1$ and substituting $f_4 = 0.02$, $f_1 = 0.05$, $f_3 = 0.03$, $f_2 = 0.04$, and $f_5 = 0.01$, and setting fault tolerant g = 1.0 table 3 and figure 4 below were obtained, by varying t=0 through 0.09.

$$A_{GR} = \frac{1 + \frac{2gf_1}{\Delta_1} + \frac{2gf_5}{\Delta_2} + \frac{2gf_3}{\Delta_3} + \frac{4g^2 f_1 f_5}{gf_5 + s + 1} + \frac{4g^2 f_1 f_5}{gf_1 + s + 1} + \frac{4g^2 f_1 f_3}{gf_3 + s + 1} + \frac{4g^2 f_3 f_5}{gf_3 + s + 1} + \frac{4g^2 f_1 f_3}{gf_1 + s + 1} + \frac{4g^2 f_3 f_5}{gf_5 + s + 1}}{\left(s + 2gf_1 + gf_2 + 2gf_3 + gf_4 + 2gf_5\right) - \frac{2gf_1}{\Delta_1} - \frac{2gf_5}{\Delta_2} - \frac{2gf_3}{\Delta_3} - \frac{1}{s+1}\left(\Delta_4 + \Delta_5 + \Delta_6 + \Delta_7 + \Delta_8\right)} \quad (66)$$

TABLE 3
AVAILABILITY ANALYSIS OF THE MODEL WITH GENERAL REPAIR

Time	Availability
0.000000	0.999999
0.010000	0.999588
0.020000	0.999185
0.030000	0.998791
0.040000	0.998405
0.050000	0.998028
0.060000	0.997660
0.070000	0.997299
0.080000	0.996947
0.090000	0.996603

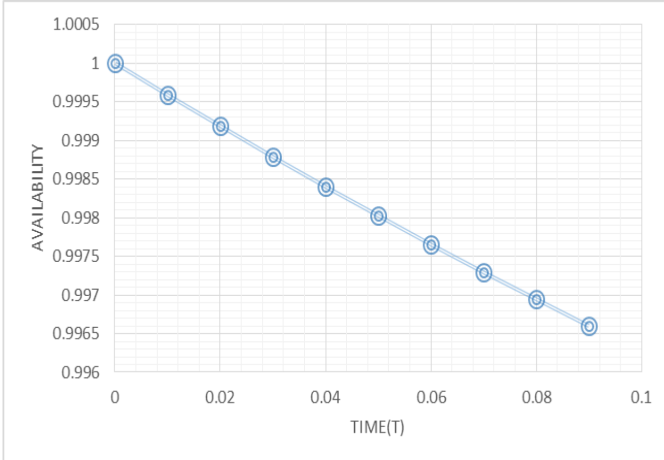

FIGURE 4
AVAILABILITY OF THE MODEL WITH GENERAL REPAIR

- Availability analysis of the model with General repair, Fault tolerant and Copula

Setting all the repairs $k(r_1)$, $k(r_2)$, $k(r_3)$, $k(r_4)$ and $k(r_5)$ to 1 with $q_0 = 2.7183$ and substituting $f_4 = 0.02$, $f_1 = 0.05$, $f_3 = 0.03$, $f_2 = 0.04$, and $f_5 = 0.01$, and setting fault tolerant g = 0.02 table 4 and figure 5 below were obtained, by varying t=0 through 0.09.

$$A_{All} = \frac{1+\frac{2gf_1}{\Delta_1}+\frac{2gf_5}{\Delta_2}+\frac{2gf_3}{\Delta_3}+\frac{4g^2 f_1 f_5}{gf_5+s+1}+\frac{4g^2 f_1 f_5}{gf_1+s+1}+\frac{4g^2 f_1 f_3}{gf_3+s+1}+\frac{4g^2 f_3 f_5}{gf_3+s+1}+\frac{4g^2 f_1 f_3}{gf_1+s+1}+\frac{4g^2 f_3 f_5}{gf_5+s+1}}{\left(s+2gf_1+gf_2+2gf_3+gf_4+2gf_5\right)-\frac{2gf_1}{\Delta_1}-\frac{2gf_5}{\Delta_2}-\frac{2gf_3}{\Delta_3}-\frac{2.7183}{s+2.713}\left(\Delta_4+\Delta_5+\Delta_6+\Delta_7+\Delta_8\right)} \quad (67)$$

TABLE 4
AVAILABILITY ANALYSIS OF THE MODEL WITH FAULT TOLERANT, COPULA AND GENERAL REPAIR

Time	Availability
0.000000	0.999999
0.010000	0.999998
0.020000	0.999997
0.030000	0.999996
0.040000	0.999995
0.050000	0.999994
0.060000	0.999993
0.070000	0.999993
0.080000	0.999992
0.090000	0.999991

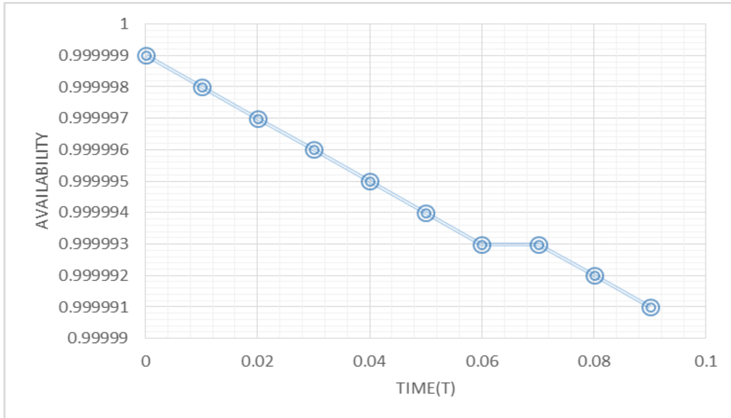

FIGURE 5
AVAILABILITY OF THE MODEL WITH FAULT TOLERANT, COPULA AND GENERAL REPAIR

II. Cost Analysis

If the system is operational and expected revenue during the interval [0,t) then the cost of the system will be given by the equation below, however table 5, 6, 7, 8 through figure 6, 7, 8, and 9 shows the result of cost analysis with different scenarios as can be seen below.

$$\Delta_p(t) = \varphi_1 \int_0^t G_{up}(t)dt - \varphi_2 t \tag{68}$$

- Cost Analysis of the model with fault tolerant

$$\Delta_{P_{FT}}(t) = \varphi_1 \begin{bmatrix} 1.454388891*10^{-8} * e^{-1.0001*t} + 6.577447813*10^{-9} * e^{-1.0002*t} + 1.394532296*10^{-7} * e^{-1.0007*t} \\ +1.872570045*10^{-10} * e^{-1.0004*t} + 1.814126611*10^{-10} * e^{-1.00040*t} - 8326.289744 * e^{-1.12010-3*t} \\ +1.399391880*10^{-8} * e^{-1.00006*t} \end{bmatrix} - \varphi_2 t \tag{69}$$

TABLE 5
COST ANALYSIS OF THE MODEL WITH FAULT TOLERANT

Time(t)	$\Delta_P(t)$ $\varphi_2 = 0.1$	$E_P(t)$ $\varphi_2 = 0.2$	$E_P(t)$ $\varphi_2 = 0.3$	$E_P(t)$ $\varphi_2 = 0.4$	$E_P(t)$ $\varphi_2 = 0.5$	$E_P(t)$ $\varphi_2 = 0.6$
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.89994	0.79994	0.69994	0.59994	0.49994	0.39994
2	1.79976	1.59976	1.39976	1.19976	0.99976	0.79976
3	2.69946	2.39946	2.09946	1.79946	1.49946	1.19946
4	3.59904	3.19904	2.79904	2.39904	1.99904	1.59904
5	4.49850	3.99850	3.49850	2.99850	2.49850	1.99850
6	5.39783	4.79783	4.19783	3.59783	2.99783	2.39783
7	6.29705	5.59705	4.89705	4.19705	3.49705	2.79705
8	7.19615	6.39615	5.59615	4.79615	3.99615	3.19615
9	8.09513	7.19513	6.29513	5.39513	4.49513	3.59513

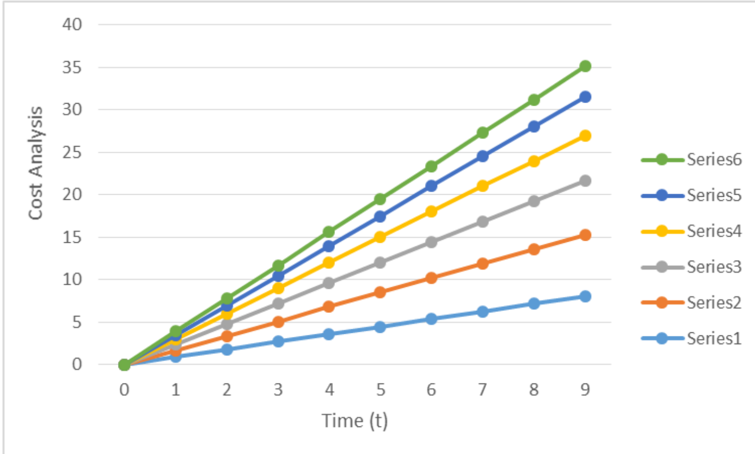

FIGURE 6
COST ANALYSIS OF THE SYSTEM WITH FAULT TOLERANT

- Cost Analysis of the model with Copula

$$\Delta_{P_{CP}}(t)=\varphi_1\begin{bmatrix}-0.1671439783*10^{-1}*e^{-2.8631*t}+0.271785673*10^{-1}*e^{-1.3351*t}+0.1588950944*10^{-3}*e^{-1.0007*t}\\+1.872570045*10^{-10}*e^{-1.0004*t}+1.814126611*10^{-10}*e^{-1.00040*t}-8326.289744*e^{-1.2661*t}\\+0.1626234200*10^{-3}*e^{-1.2013*t}+26.561675*e^{0.3755*t}+0.3386504056*e^{-1.0300*t}\\+0.1641814010*10^{-2}*e^{-1.0100*t}+0.3418433812*10^{-2}*e^{-1.0500*t}\end{bmatrix}-\varphi_2 t \quad (70)$$

TABLE 6
COST ANALYSIS OF THE MODEL WITH COPULA

Time(t)	$\Delta_P(t)$ $\varphi_2=0.1$	$\Delta_P(t)$ $\varphi_2=0.2$	$\Delta_P(t)$ $\varphi_2=0.3$	$\Delta_P(t)$ $\varphi_2=0.4$	$\Delta_P(t)$ $\varphi_2=0.5$	$\Delta_P(t)$ $\varphi_2=0.6$
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.90658	0.80658	0.70658	0.60658	0.50658	0.40658
2	1.85564	1.65564	1.45564	1.25564	1.05564	0.85564
3	2.84942	2.54942	2.24942	1.94942	1.64942	1.34942
4	3.88656	3.48656	3.08656	2.68656	2.28656	1.88656
5	4.96767	4.46767	3.96767	3.46767	2.96767	2.46767
6	6.09413	5.49413	4.89413	4.29413	3.69413	3.09413
7	7.26756	6.56756	5.86756	5.16756	4.46756	3.76756
8	8.48973	7.68973	6.88973	6.08973	5.28973	4.48973
9	9.76251	8.86251	7.96251	7.06251	6.16251	5.26251

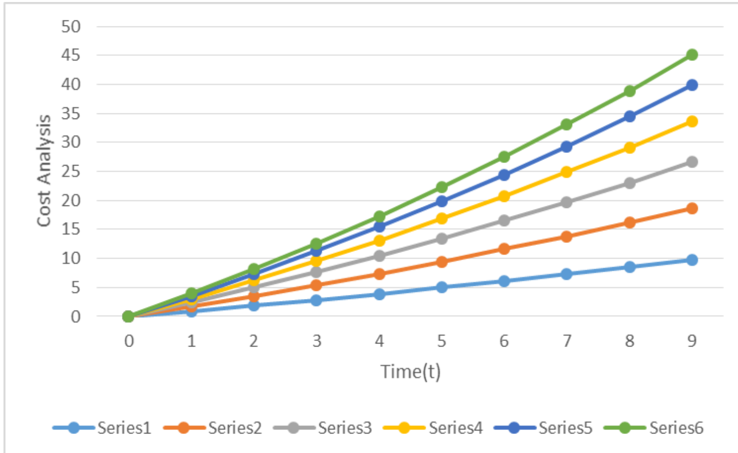

FIGURE 7
COST ANALYSIS OF THE SYSTEM WITH COPULA

- Cost Analysis of the model with General repair

$$\Delta_{P_{GR}}(t) = \varphi_1 \begin{bmatrix} -0.2668117991*10^{-2}*e^{-1.0300*t} - 0.2710665579*10^{-3}*e^{-1.0100*t} - 0.1233622754*10^{-1}*e^{-1.4138*t} \\ -0.2010907834*10^{-3}*e^{-1.2266*t} - 0.2980458159*e^{-1.0627*t} + 26.63758441*e^{0.3512*t} \\ -0.1136216230*10^{-1}*e^{-1.0500*t} \end{bmatrix} - \varphi_2 t \quad (71)$$

TABLE 7
COST ANALYSIS OF THE MODEL WITH GENERAL REPAIR

Time(t)	$\Delta_P(t)$ $\varphi_2 = 0.1$	$\Delta_P(t)$ $\varphi_2 = 0.2$	$\Delta_P(t)$ $\varphi_2 = 0.3$	$\Delta_P(t)$ $\varphi_2 = 0.4$	$\Delta_P(t)$ $\varphi_2 = 0.5$	$\Delta_P(t)$ $\varphi_2 = 0.6$
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.89050	0.79050	0.69050	0.59050	0.49050	0.39050
2	1.78903	1.58903	1.38903	1.18903	0.98903	0.78903
3	2.71452	2.41452	2.11452	1.81452	1.51452	1.21452
4	3.67384	3.27384	2.87384	2.47384	2.07384	1.67384
5	4.67007	4.17007	3.67007	3.17007	2.67007	2.17007
6	5.70518	5.10518	4.50518	3.90518	3.30518	2.70518
7	6.78076	6.08076	5.38076	4.68076	3.98076	3.28076
8	7.89832	7.09832	6.29832	5.49832	4.69832	3.89832
9	9.05939	8.15939	7.25939	6.35939	5.45939	4.55939

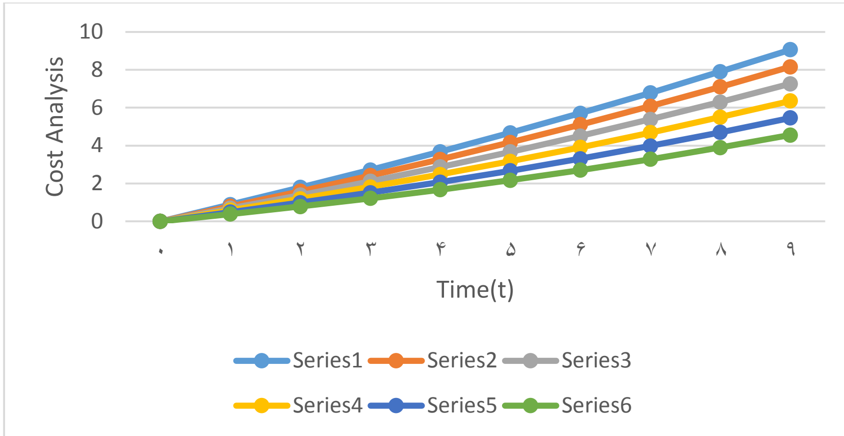

FIGURE 8
COST ANALYSIS OF THE SYSTEM WITH GENERAL REPAIR

- Cost Analysis of the model with Fault tolerant, Copula and General repair

$$\Delta_{P_{All}}(t)=\varphi_1\begin{bmatrix} 6.576164477*10^{-9}*e^{-1.00002*t}+1.399133682*10^{-8}*e^{-1.00006*t}-0.3248548604*10^{-4}*e^{-2.71854*t} \\ +2.748450113*10^{-7}*e^{-1.00076*t}+6.144116983*e^{-1.00045*t}+6.529942975*e^{-1.00040*t} \\ +8342.668772*10^{-3}*e^{0.11985*t}+1.454137869*10^{-8}*e^{-1.00010*t} \end{bmatrix}-\varphi_2 t \quad (72)$$

TABLE 8
COST ANALYSIS OF THE MODEL WITH FAULT COVERAGE, COPULA AND GENERAL REPAIR

Time(t)	$\Delta_P(t)$ $\varphi_2 = 0.1$	$\Delta_P(t)$ $\varphi_2 = 0.2$	$\Delta_P(t)$ $\varphi_2 = 0.3$	$\Delta_P(t)$ $\varphi_2 = 0.4$	$\Delta_P(t)$ $\varphi_2 = 0.5$	$\Delta_P(t)$ $\varphi_2 = 0.6$
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.89999	0.79999	0.69999	0.59999	0.49999	0.39999
2	1.80009	1.60009	1.40009	1.20009	1.00009	0.80009
3	2.70030	2.40030	2.10030	1.80030	1.50030	1.20030
4	3.60064	3.20064	2.80064	2.40064	2.00064	1.60064
5	4.50108	4.00108	3.50108	3.00108	2.50108	2.00108
6	5.40166	4.80166	4.20166	3.60166	3.00166	2.40166
7	6.30235	5.60235	4.90235	4.20235	3.50235	2.80235
8	7.20316	6.40316	5.60316	4.80316	4.00316	3.20316
9	8.10409	7.20409	6.30409	5.40409	4.50409	3.60409

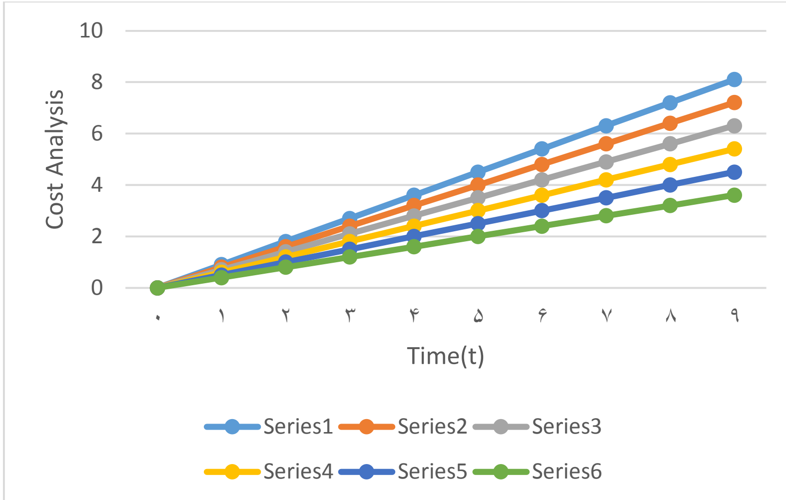

FIGURE 9
COST ANALYSIS OF THE SYSTEM WITH FAULT TOLERANT, COPULA AND GENERAL REPAIR

III. Reliability Analysis

- Reliability analysis of the model with Fault tolerant (Without Copula and General repair)

Setting all the repairs $k(r_1)$, $k(r_2)$, $k(r_3)$, $k(r_4)$, $k(r_5)$, $q_0(r_1)$, $q_0(r_2)$, $q_0(r_3)$, $q_0(r_4)$ and $q_0(r_5)$ to 0 and substituting $f_1 = 0.05,\ f_2 = 0.04,\ f_3 = 0.03,\ f_4 = 0.02$, $f_5 = 0.01$ and setting the fault-tolerant g = 0.002 table 9 and figure 10 below were obtained, by varying t = 0 through 0.09

$$R_{FT} = \frac{1}{\Delta_9}\left(1 + \frac{2gf_1}{\Delta_{10}} + \frac{2gf_5}{\Delta_{11}} + \frac{2gf_3}{\Delta_{12}} + \frac{4g^2 f_1 f_5}{gf_5 + s} + \frac{4g^2 f_1 f_3}{gf_3 + s} + \frac{4g^2 f_1 f_5}{gf_1 + s} + \frac{4g^2 f_3 f_5}{gf_3 + s} + \frac{4g^2 f_1 f_3}{gf_1 + s} + \frac{4g^2 f_3 f_5}{gf_5 + s}\right) \quad (73)$$

TABLE 9
RELIABILITY ANALYSIS OF THE MODEL WITH FAULT TOLERANT

Time	Reliability
0.000000	1.000000
0.010000	0.999880
0.020000	0.999759
0.030000	0.999639
0.040000	0.999519
0.050000	0.999399
0.060000	0.999278
0.070000	0.999158
0.080000	0.999037
0.090000	0.998917

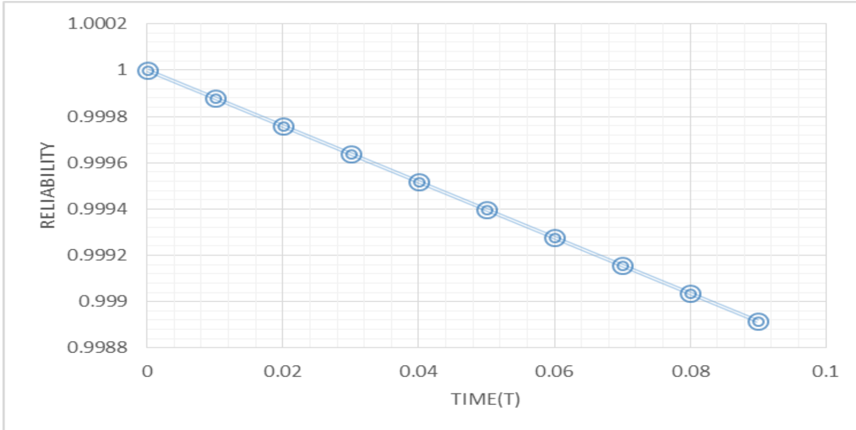

FIGURE 10
RELIABILITY OF THE MODEL WITH FAULT TOLERANT

- Reliability analysis of the model with Copula (Without Fault tolerant and general repair)

Setting all the repairs $k(r_1)$, $k(r_2)$, $k(r_3)$, $k(r_4)$, $k(r_5)$, $q_0(r_1)$, $q_0(r_2)$, $q_0(r_3)$, $q_0(r_4)$ and $q_0(r_5)$ to 0 and substituting $f_1 = 0.05, f_2 = 0.04, f_3 = 0.03, f_4 = 0.02$, $f_5 = 0.01$ and setting the fault tolerant g = 1 table 10 and figure 11 below were obtained, by varying t = 0 through 0.09

$$R_{CP} = \frac{1}{\Delta_9}\left(1+\frac{2gf_1}{\Delta_{10}}+\frac{2gf_5}{\Delta_{11}}+\frac{2gf_3}{\Delta_{12}}+\frac{4g^2 f_1 f_5}{gf_5+s}+\frac{4g^2 f_1 f_3}{gf_3+s}+\frac{4g^2 f_1 f_5}{gf_1+s}+\frac{4g^2 f_3 f_5}{gf_3+s}+\frac{4g^2 f_1 f_3}{gf_1+s}+\frac{4g^2 f_3 f_5}{gf_5+s}\right) \quad (74)$$

TABLE 10
RELIABILITY ANALYSIS OF THE MODEL WITH COPULA

Time	Reliability
0.000000	0.999999
0.010000	0.947077
0.020000	0.878613
0.030000	0.802746
0.040000	0.724999
0.050000	0.648997
0.060000	0.577001
0.070000	0.510315
0.080000	0.449570
0.090000	0.394942

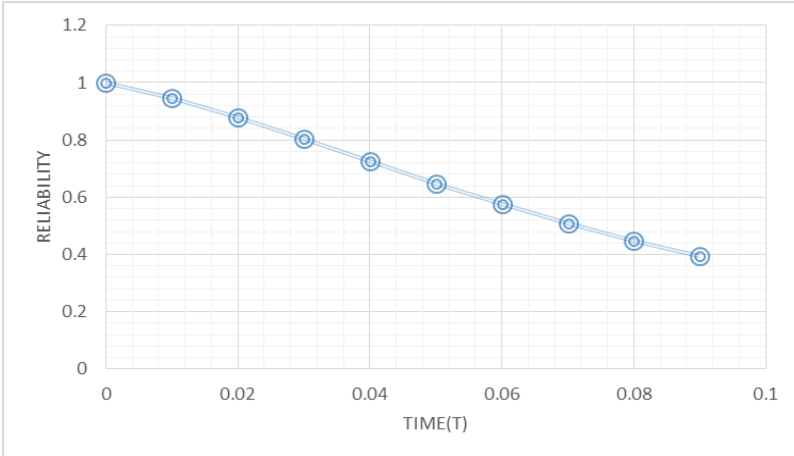

FIGURE 11
RELIABILITY OF THE MODEL WITH COPULA

- Reliability analysis of the model with General repair (Without Fault tolerant and Copula)

Setting all the repairs $k(r_1)$, $k(r_2)$, $k(r_3)$, $k(r_4)$, $k(r_5)$, $q_0(r_1)$, $q_0(r_2)$, $q_0(r_3)$, $q_0(r_4)$ and $q_0(r_5)$ to 0 and substituting $f_1 = 0.05, f_2 = 0.04, f_3 = 0.03, f_4 = 0.02$, $f_5 = 0.01$ and setting the fault tolerant g = 1 table 11 and figure 12 below were obtained, by varying t = 0 through 0.09

$$R_{GR} = \frac{1}{\Delta_9}\left(1+\frac{2gf_1}{\Delta_{10}}+\frac{2gf_5}{\Delta_{11}}+\frac{2gf_3}{\Delta_{12}}+\frac{4g^2f_1f_5}{gf_5+s}+\frac{4g^2f_1f_3}{gf_3+s}+\frac{4g^2f_1f_5}{gf_1+s}+\frac{4g^2f_3f_5}{gf_3+s}+\frac{4g^2f_1f_3}{gf_1+s}+\frac{4g^2f_3f_5}{gf_5+s}\right) \quad (75)$$

TABLE 11
RELIABILITY ANALYSIS OF THE MODEL WITH GENERAL REPAIR

Time	Reliability
0.000000	0.999999
0.010000	0.947077
0.020000	0.878613
0.030000	0.802746
0.040000	0.724999
0.050000	0.648997
0.060000	0.577001
0.070000	0.510315
0.080000	0.449570
0.090000	0.394942

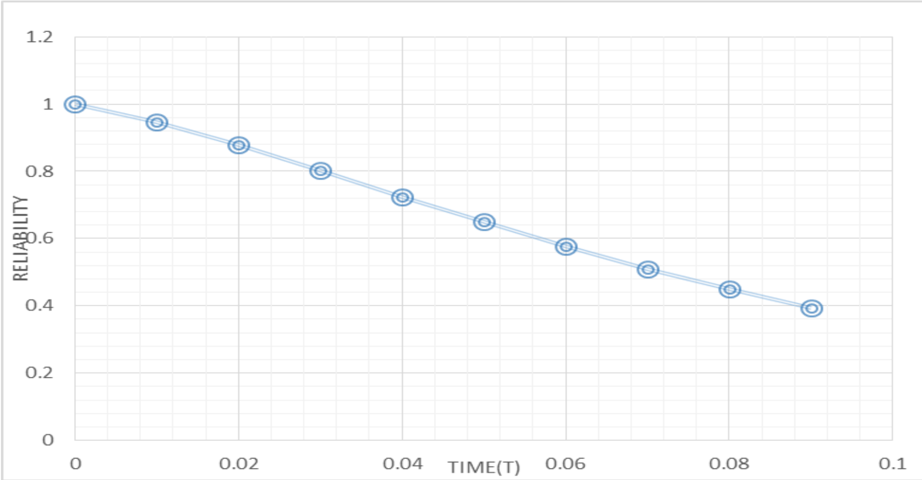

FIGURE 12
RELIABILITY OF THE MODEL WITH GENERAL REPAIR

- Reliability analysis of the model with General repair, Fault tolerant and Copula

The objective of this section is to express numerical experiment so as to see effect of the parameters on the performance. However, all the repairs $k(r_1)$, $k(r_2)$, $k(r_3)$, $k(r_4)$, $k(r_5)$, $q_0(r_1)$, $q_0(r_2)$, $q_0(r_3)$, $q_0(r_4)$ and $q_0(r_5)$ are set to be 0 and substituting $f_1 = 0.05, f_2 = 0.04, f_3 = 0.03, f_4 = 0.02$, $f_5 = 0.01$ and setting the fault tolerant g = 1 table 12 and figure 13 below were obtained, by varying t = 0 through 0.09

$$R_{All} = \frac{1}{\Delta_9}\left(1+\frac{2gf_1}{\Delta_{10}}+\frac{2gf_5}{\Delta_{11}}+\frac{2gf_3}{\Delta_{12}}+\frac{4g^2 f_1 f_5}{gf_5+s}+\frac{4g^2 f_1 f_3}{gf_3+s}+\frac{4g^2 f_1 f_5}{gf_1+s}+\frac{4g^2 f_3 f_5}{gf_3+s}+\frac{4g^2 f_1 f_3}{gf_1+s}+\frac{4g^2 f_3 f_5}{gf_5+s}\right) \quad (76)$$

TABLE 12
RELIABILITY ANALYSIS OF THE MODEL WITH GENERAL REPAIR, FAULT TOLERANT AND COPULA

Time	Reliability
0.000000	1.000000
0.010000	0.999880
0.020000	0.999759
0.030000	0.999639
0.040000	0.999519
0.050000	0.999399
0.060000	0.999278
0.070000	0.999158
0.080000	0.999037
0.090000	0.998917

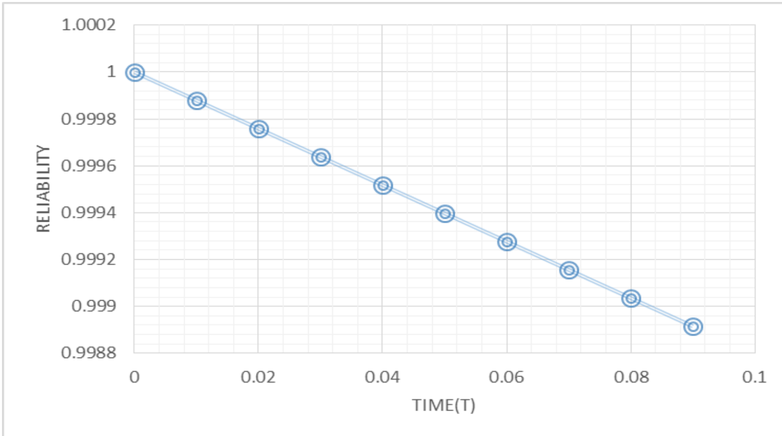

FIGURE 13
RELIABILITY OF THE MODEL WITH GENERAL REPAIR, FAULT TOLERANT AND COPULA

IV. MTTF of the model with Fault-tolerant

Taking all repairs to zero to obtain (77) and setting g=0.002, g=0.004, and g=0.006, the formula for MTTF can be written as:

$$MTTF=\frac{1}{\Delta_{13}}\left(1+\frac{2gf_1}{\Delta_{14}}+\frac{2gf_5}{\Delta_{15}}+\frac{2gf_3}{\Delta_{16}}+\frac{4g^2 f_1 f_5}{gf_5}+\frac{4g^2 f_1 f_3}{gf_3}+\frac{4g^2 f_1 f_5}{gf_1}+\frac{4g^2 f_3 f_5}{gf_3}+\frac{4g^2 f_1 f_3}{gf_1}+\frac{4g^2 f_3 f_5}{gf_5}\right) \quad (77)$$

TABLE 13
MTTF OF THE MODEL WITH FAULT TOLERANT

t	$g=0.002$					$g=0.004$					$g=0.006$				
	f_1	f_2	f_3	f_4	f_5	f_1	f_2	f_3	f_4	f_5	f_1	f_2	f_3	f_4	f_5
0.01	5403	4234	4695	4904	5403	2702	2118	2349	2453	2702	1802	1413	1567	1637	1802
0.02	4912	3959	4283	4547	4912	2457	1981	2143	2275	2457	1639	1321	1429	1517	1639
0.03	4530	3715	3959	4234	4530	2266	1858	1981	2118	2266	1512	1240	1321	1413	1512
0.04	4219	3497	3695	3959	4219	2111	1749	1849	1981	2111	1408	1167	1233	1321	1408
0.05	3959	3301	3473	3715	3959	1981	1652	1738	1858	1981	1321	1102	1160	1240	1321
0.06	3735	3125	3284	3497	3735	1869	1564	1644	1749	1869	1247	1043	1097	1167	1247
0.07	3540	2966	3120	3301	3540	1771	1484	1561	1652	1771	1182	990.3	1042	1102	1182
0.08	3367	2821	2975	3125	3367	1685	1411	1489	1564	1685	1124	941.9	993.8	1043	1124
0.09	3212	2688	2845	2966	3212	1607	1345	1424	1484	1607	1073	897.7	950.7	990.3	1073

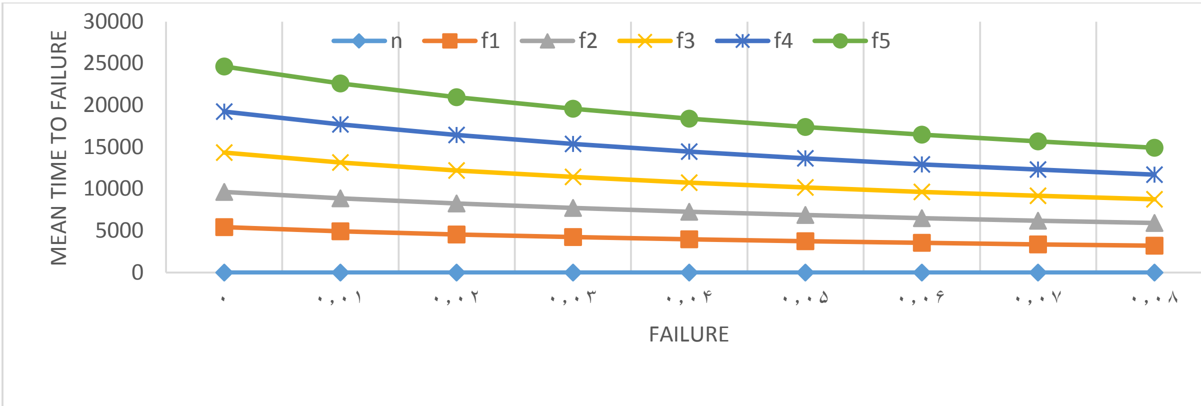

FIGURE 14

MEAN TIME TO FAILURE AGAINST THE DIFFERENT FAILURE RATES WHEN $g = 0.002$

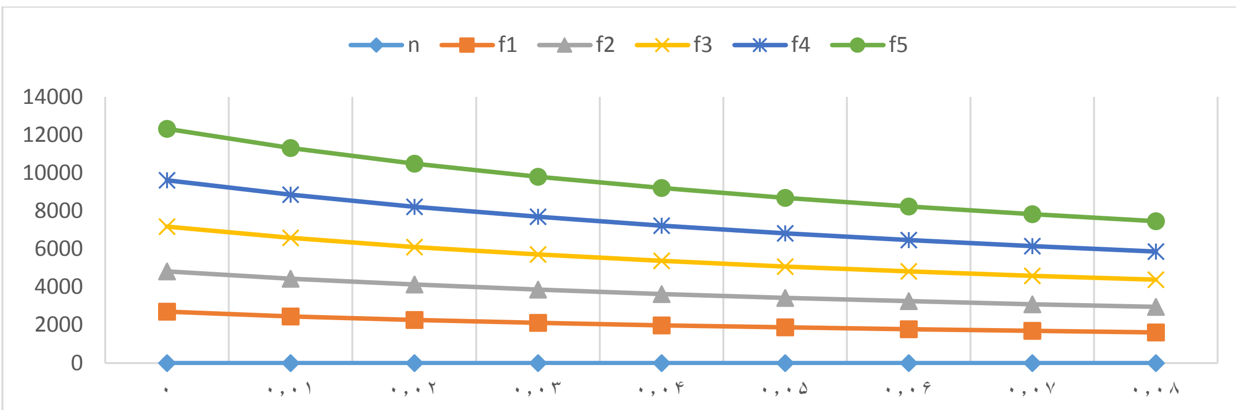

FIGURE 15

MEAN TIME TO FAILURE AGAINST THE DIFFERENT FAILURE RATES WHEN $g = 0.004$

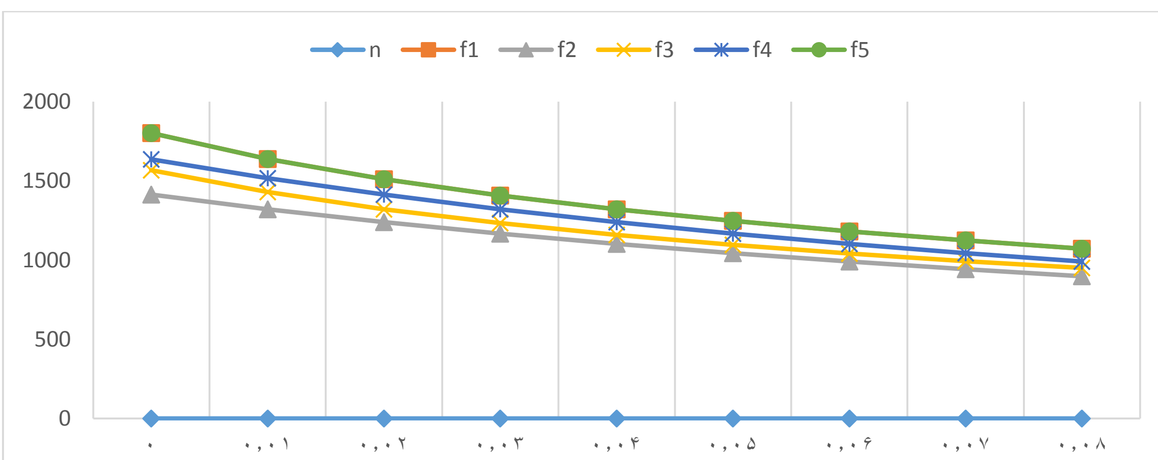

FIGURE 16

MEAN TIME TO FAILURE AGAINST THE DIFFERENT FAILURE RATES WHEN $g = 0.006$

Figures 14: (a), (b) and (c): Comparison of the numerical solutions of the sensitivity analysis for different values of $f_4 = 0.02$, $f_1 = 0.05$, $f_3 = 0.03$, $f_2 = 0.04$, and $f_5 = 0.01$, $q_0(r_1)$, $q_0(r_2)$, $q_0(r_3)$, $q_0(r_4)$ and $q_0(r_5)$

V Sensitivity Analysis of the model with Fault tolerant

Sensitivity of the system with Fault tolerant, by setting the fault tolerant (g) as g = 0.3, g = 0.5 and g = 0.7 in table 14, one can observe that how a sensitive a model is to change in parameter of the model and the analysis shows that as the fault tolerant g increases the sensitivity also increases.

TABLE 14
SENSITIVITY OF THE MODEL WITH FAULT TOLERANT

t	$g = 0.3$					$g = 0.5$					$g = 0.7$				
	f_1	f_2	f_3	f_4	f_5	f_1	f_2	f_3	f_4	f_5	f_1	f_2	f_3	f_4	f_5
0.01	-357	-208	-301	-271	-357	-206	-130	-175	-169	-206	-142	-97	-122	-125	-142
0.02	-270	-185	-231	-237	-270	-156	-116	-135	-148	-156	-107	-86.4	-93.6	-110	-107
0.03	-216	-165	-185	-208	-216	-124	-103	-107	-130	-124	-85.7	-77.2	-74.7	-97.2	-85.7
0.04	-178	-147	-153	-185	-178	-103	-93.0	-89	-116	-103	-70.9	-69.4	-61.6	-86.4	-70.9
0.05	-152	-133	-129	-165	-152	-87.8	-83.9	-75	-103	-87.8	-60.3	-62.7	-52.0	-77.2	-60.3
0.06	-131	-120	-111	-147	-131	-76.2	-76.1	-65	-93.0	-76.2	-52.4	-56.9	-44.9	-69.4	-52.4
0.07	-116	-109	-98.1	-133	-116	-67.2	-69.2	-56	-83.9	-67.2	-46.2	-51.9	-39.3	-62.7	-46.2
0.08	-103	-100	-87.0	-120	-103	-59.9	-63.3	-50	-76.1	-59.9	-41.2	-47.5	-34.9	-56.9	-41.2
0.09	-93.1	-91.7	-78.1	-109	-93.1	-53.9	-58.0	-45	-69.2	-53.9	-37.2	-43.6	-31.3	-51.9	-37.2

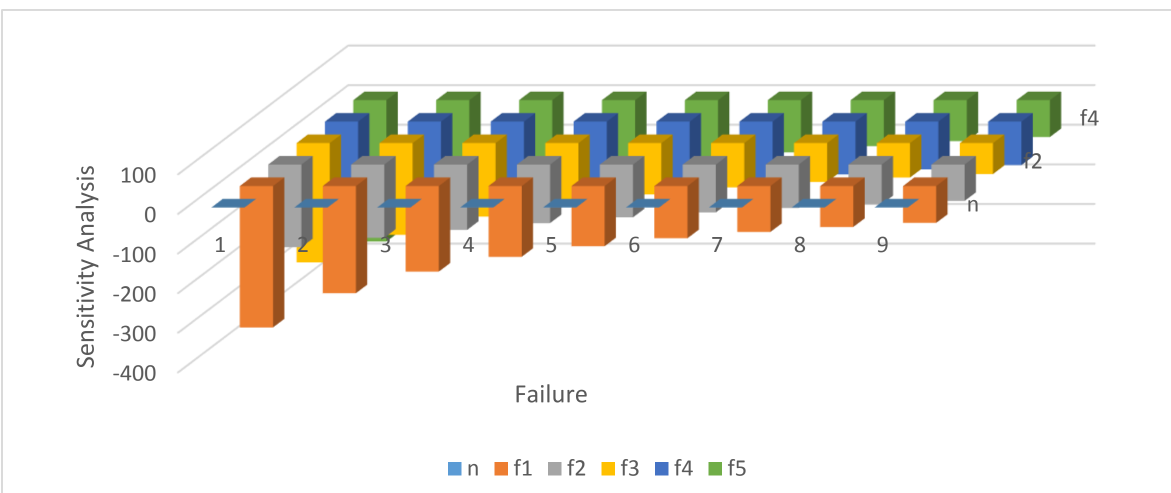

FIGURE 17
MTTF SENSITIVITY AGAINST THE DIFFERENT FAILURE RATES WHEN $g = 0.3$

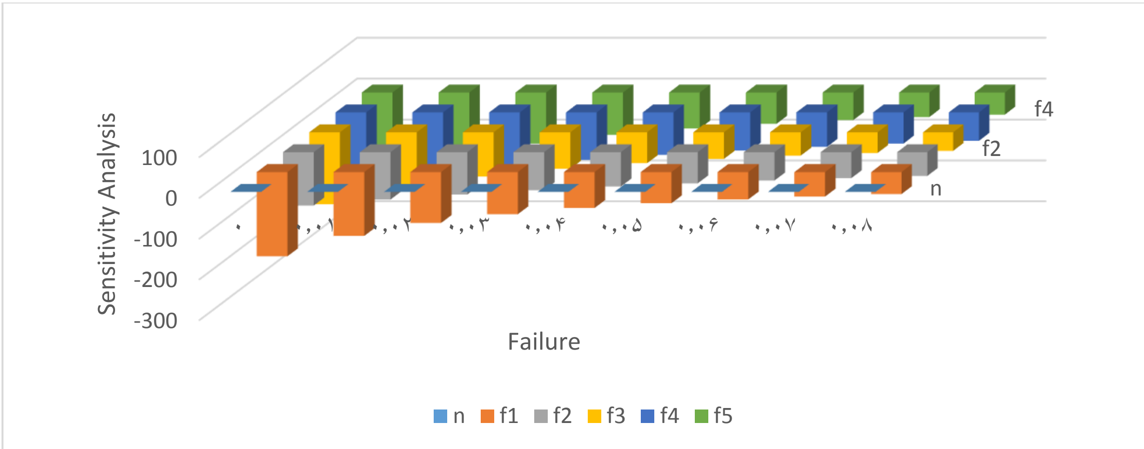

FIGURE 18
MTTF SENSITIVITY AGAINST THE DIFFERENT FAILURE RATES WHEN $g = 0.5$

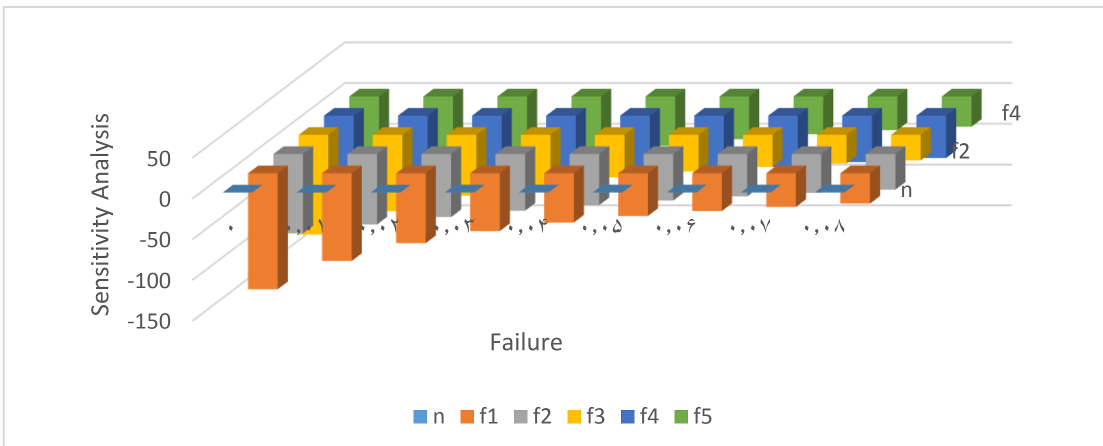

FIGURE 17
MTTF SENSITIVITY AGAINST THE DIFFERENT FAILURE RATES WHEN $g = 0.7$

Figures 15: (d), (e) and (f): Comparison of the numerical solutions of the sensitivity analysis for different values of $f_4 = 0.02$, $f_1 = 0.05$, $f_3 = 0.03$, $f_2 = 0.04$, and $f_5 = 0.01$, $q_0(r_1)$, $q_0(r_2)$, $q_0(r_3)$, $q_0(r_4)$ and $q_0(r_5)$

RESULTS AND DISCUSSION

The study delves into the intricate dynamics of availability, cost, and reliability within four distinct scenarios. Each scenario represents a different approach to system maintenance and fault tolerance, offering valuable insights into their impact on network performance. The results presented in Figure 2 and Table 1, Figure 3 and Table, Figure 4 and Table 3, and Figure 5 and Table 4 provide insight into how the availability of the network evolves with the integration of fault tolerance mechanisms, copula repair corrective maintenance, general repair corrective maintenance, and fault tolerance, copula, and general repair corrective maintenance. It is apparent from both the tables and the figures that the system's availability experiences a slight decline within the time interval from 0 to 0.09 units. This observation underscores the dynamic nature of fault-tolerant, copula repair corrective maintenance, general repair corrective maintenance, and fault tolerance, copula, and general repair corrective maintenance systems, which despite their robust design, are still susceptible to gradual decreases in availability over time. Scenario 1 introduces fault tolerance, a proactive measure aimed at mitigating system failures. This approach yields a remarkable availability rate, as evidenced by the data presented in Table 1 and Figure 2. Despite a slight decrease over time, the system achieves an impressive minimum availability of 0.999989. In contrast, Scenario 2 adopts copula repair for corrective maintenance upon total failure. While still effective, this approach yields slightly lower availability rates compared to Scenario 1. Nevertheless, it demonstrates a commendable minimum availability of 0.997431 from Table 2 and Figure 3. Scenario 3 explores general repair for corrective maintenance at partial failure. Though effective in maintaining system functionality, this approach results in slightly reduced availability rates compared to the previous scenarios. The minimum availability achieved is 0.996603, as illustrated in Table 3 and Figure 4. Scenario 4 integrates both fault tolerance, copula and general repair strategies, providing a comprehensive maintenance framework for partial or complete failures. This combined approach yields the highest availability rates among the scenarios, with a minimum availability of 0.999991 from Table 4 and Figure 5. Through meticulous analysis, it becomes evident that Scenario 4 emerges as the optimal choice for system maintenance and fault tolerance. This finding holds significant implications for system designers, engineers, and maintenance personnel, offering valuable guidance in enhancing network performance and reliability. By leveraging a combination of fault tolerance and general repair strategies, stakeholders can effectively mitigate downtime and optimize system availability, ensuring seamless operation and minimizing disruptions.

Tables 5 through 8 and Figures 6 through 9 provide insight into how the profit of the network evolves with the integration of fault tolerance mechanisms, copula repair corrective maintenance, general repair corrective maintenance, and fault tolerance, copula, and general repair corrective maintenance for different service cost G2 values ranging from 0.1 to 0.6. As depicted, the profit tends to increase with time for each service cost considered. Notably, however, the profit is notably higher when the service cost is set to G2=0.1 for each scenario, particularly when fault tolerance mechanisms are integrated into the network architecture. This observation underscores the significance of optimizing service costs in conjunction with implementing fault tolerance strategies to maximize profitability. The higher profit observed at G2=0.1 suggests that, at this service cost level, the benefits of fault tolerance outweigh the associated expenses, resulting in a more favorable economic outcome for the network operator. This combined approach yields the highest profit of 9.76251 from scenario 2. Through meticulous analysis, it becomes evident that Scenario 2 emerges as the optimal choice for system maintenance and fault tolerance.

The results presented in Tables 9 through 12 and Figures 10 through 13, provide insight into how the Reliability of the network evolves with the integration of fault tolerance mechanisms, copula repair corrective maintenance, general repair corrective maintenance, and fault tolerance, copula, and general repair corrective maintenance. It is apparent from both the tabular and the graphs that the system's availability experiences a slight decline within the time interval from 0 to 0.09 units. Despite a slight decrease over time, in scenario 1 the system achieves an impressive minimum reliability of 0.99817. In contrast, Scenario 2 and scenario3 demonstrate a commendable minimum reliability of 0.394942. Scenario 4 produced a reliability of 0.998917. This meticulous analysis, it becomes evident that Scenario 4 emerges as the optimal choice for system maintenance and fault tolerance. Table 13, along with Figures 14, 15, and 16, illustrate the impact of varying failure rates f1, f2,f3, f4, and f5 on the system's MTTF across different fault tolerance factors g, specifically 0.002, 0.004, and 0.006. The data and visualizations clearly show a trend: as the failure rates increase, the MTTF consistently decreases. This relationship underscores the inverse correlation between failure rate and system longevity, highlighting how higher rates of failure lead to a shorter operational lifespan of the system, regardless of the fault tolerance factor applied. From the table and figures, it is clear that the mean time to failure of the system is higher when g=0.002. On the other hand, Table 14 and Figures 17, 18, and 19 demonstrate the effect of failure rates f1, f2, f3, f4, and f5 on MTTF sensitivity of the system for different fault tolerance factors g ranging from 0.3, 0.5, and 0.7. The table and figures clearly show that MTTF sensitivity decreases as the failure rate increases. Additionally, they demonstrate that the system's MTTF sensitivity is higher when g=0.7g = 0.7g=0.7.

Conclusion

This paper models and evaluates the performance of a computer network system by incorporating fault tolerance, copula methods, general repair, and their combined approaches. Explicit expressions for system MTTF, cost function, reliability, sensitivity, and availability are derived and statistically validated. So, to get the maximum operation of the systems, the availability, MTTF, cost, and reliability must be meticulously maintained to lower the failure rate. The availability, mean time to failure (MTTF), cost, and reliability are critical requirements that significantly influence the development and operational efficiency of any process sector. These factors not only determine the overall performance and sustainability of the systems involved but also play a crucial role in minimizing downtime, optimizing resource allocation, and ensuring the long-term viability of the sector. The interplay between these parameters is essential for achieving high productivity, reducing operational risks, and maintaining competitive advantage in an increasingly demanding market environment. The study contributes by employing a fault tolerance, copula, general repair, and combination of fault tolerance, copula, and general repair approach to model and evaluate the performance of computer network systems. This approach allows for establishing and statistically validating explicit expressions for system mean time to failure (MTTF), availability, cost function, sensitivity, and reliability. The research suggests that a fault tolerance, repair policy based on copula and general distribution can improve system performance. The study advances theoretical understanding by providing insights into key performance metrics such as MTTF, availability, sensitivity, reliability, and cost. It establishes explicit expressions for these metrics, which can aid in theoretical developments within the field of distributed systems. The findings suggest that maintaining key metrics such as MTTF, cost, availability, and reliability is crucial for maximizing system operation. Organizations can use this information to optimize their operational processes and minimize the rate of failure. The cost function analysis highlights the trade-offs between higher service costs and system profitability. This understanding can guide managerial decisions regarding resource allocation and service pricing to maximize profit while maintaining system performance. While the study provides valuable insights, it acknowledges the potential for further research extensions. Specifically, incorporating warm standby components could enhance the understanding of system performance under different configurations or conditions.

ACKNOWLEDGEMENTS: This study was funded by the National Social Science Foundation of China (Grant No. 20BTJ044) and the Provincial Natural Science Foundation of Hunan (Grant No. 2024JJ5453).

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