Original Research

Using a new warranty policy to define optimum values for burn-in and warranty periods

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Abstract

In recent years, offering warranty services has become a regular practice in the selling process. Manufacturers try to offer suitable warranty services on their products as an efficient tool to attract new customers and satisfy their current customers. The failure rate curve of many systems shows a bathtub shape with three distinguishable phases. In the first phase, it shows a decreasing rate. During this phase, manufacturers use burn-in methods to eliminate products with early failures. In the second phase, where the curve shows approximately a flat rate, manufacturers offer warranty services. These services assure consumers about the manufacturer's support for the purchased products in case they face failure during the warranty period. In this study, a non-repairable component with a bathtub-shaped failure rate function is considered. In the first phase, if a component fails during the burn-in period, the manufacturer replaces it with a new component. In the second phase, the manufacturer offers a new pro-rata warranty service if a component fails. Considering these assumptions, a cost model is constructed and optimal burn-in time and warranty period are obtained. A numerical illustration is presented to evaluate the impact of the proposed model by calculating the optimum values of burn-in and warranty periods.

Keywords*:* Bathtub-shaped failure rate; Burn-in method; Linear rebate function; Optimization; Pro-rata warranty

INTRODUCTION

For many electrical components, their failure rate curves form a bathtub shape. There are three distinct regions in this curve. The first region, where the failure rate decreases rapidly during a short period of time, is known as infant mortality region. In the second region, the failure rate curve shows approximately a flat form. During this period which is generally longer than the first period length, the failure rate is approximately constant and known as the useful life period. The main reason of failure during this region is usage and operation. Usually, manufacturers offer different types warranty policy during this period. After these two regions, the failure rate starts to increase monotonically. This region is known as wear-out or post warranty region. The main reason of failure during this region is the effect of aging.

Based on bathtub curve theory and Markov decision process, Chen *et.al.* [1] developed a dynamic product failure rate forecasting model to enable third-party repair service providers to effectively predict the demand for service parts. Zheng and Su [16] in their paper, proposed a warranty policy for repairable products with bathtub-shape failure rate. They employed a two-fold Weibull competing risk model to describe the bathtub-shape cure. In their study, they presented a numerical optimization model to achieve the optimal preventive maintenance strategy.

A typical bathtub shaped failure rate curve with three regions, is shown in Figure 1.

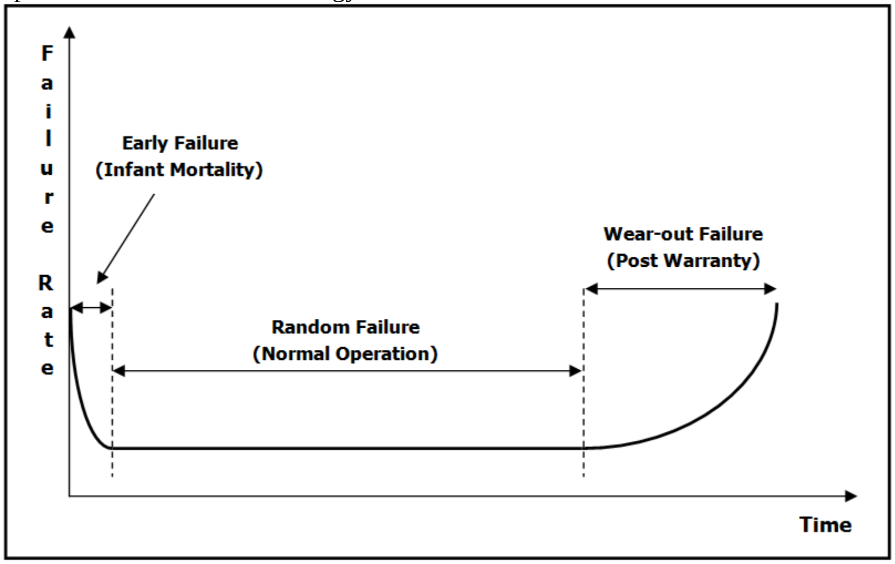

FIGURE 1
A TYPICAL BATHTUB FAILURE RATE WITH THREE DISTINCT REGIONS

Warranties play a crucial role in the new market. Sellers try to encourage their customers by offering suitable warranty services for their products. Nowadays warranty service has become an important factor in customer decision making. Although better warranty service can increase sales and improve market share, it contains some additional costs. Generally, depending on the industry (manufacturer) and product, warranty costs constitute 2 to 10 percent of the selling price [7]. By offering a warranty service, the manufacturer wants to assure their customer that the product can serve its purpose successfully during the warranty period. It is worth mentioning that among products with the similar conditions, customers will select the one with the best warranty offer [13]. Warranty specifications consist of the length of the warranty period, method of performing warranty service, and conditions that violate warranty terms. By increasing the warranty period, warranty costs increase as well. Researchers classify warranty policies from different points of view. One of them can be based on the costs paid by a consumer during the warranty period if the product fails. From this point of view, there are pro-rata warranty and free replacement/free repair warranty (FRW) policies. Another class is non-renewing and renewable warranties. Some researchers divide warranty policies into two main groups; one-dimensional and two-dimensional warranties. Most of the researchers consider a one-dimensional warranty in their studies to avoid dealing with complexities in the computation of two-dimensional warranty models.

Jain and Maheshwari [4] prepared a model based on renewable pro-rata warranty. In their model, they supposed that components failure rate, preventive maintenance cost, and replacement cost are fixed. Determining optimal number and length of preventive maintenance after warranty period is the capability of their model. Wuand Huang[14] developed a cost model to determine the optimum burn-in and warranty periods by considering non-repairable products and fully renewing contribution free replacement warranty (FRW) and pro-rata warranty (PRW) policies. In their model, if the product fails up to time W', the seller has to replace it with a new product at no cost to the buyer. However, any failure after W' and before W (length of the warranty period), results in a pro-rata replacement. Shafiee[11] considered a product with a bathtub-shaped failure rate and obtained optimal burn-in time and imperfect preventive maintenance strategies. In another research, Shafiee *et.al.* [12] derived an expected cost function to determine the optimum value of the burn-in period, degree of preventive maintenance and interval of preventive maintenance. MoghimiHadji [5] in his recent research investigated the total cost incurred during the burn-in and warranty periods from manufacturer's perspective by considering different types of repair service to calculate optimal length of warranty and burn-in periods. He obtained the expected total cost in each phase. Yeh and Fang [15] integrated the PRW policy with the pricing and the production strategy using a Bayesian decision-making model. Moreover, they provided a heuristic algorithm to deal with the computation complexity of their model. Park*et.al.* [9] developed a model to determine optimal warranty period in which they considered both repair and replacement costs simultaneously from manufacturer's perspective. Nasrollahi and Asgharizadeh [8] considered a non-repairable product and defined a new pro-rata warranty policy. They obtained the expected cost during the warranty period from the seller and buyer's perspective. Chen*et.al.* [2] assessed warranty costs by considering the influence of different failure states, different phases of product reliability and warranty policies. They proposed a warranty cost model considering burn-in, free replacement warranty

(FRW), and pro-rata warranty (PRW) for repairable products with minor and major (catastrophic) failures involving minimal repair and replacement. Their cost function was the combination of these three phases. Finally by minimizing the expected warranty cost, they obtained the failure rate distribution and optimal warranty length. Based on a two-stage repair-or-full-refund maintenance strategy, Park *et. al.*[10] considered an optimal policy to determine an optimum length of warranty period from the dealer's perspective. They assumed only minimal repairs for failed products.

In this study, a non-repairable component with a burn-in period and a new pro-rata warranty is considered. If this component fails during the burn-in process, it will be replaced with a new component. If a component passes the burn-in test successfully, it will reach the market with a new pro-rata warranty. In traditional pro-rata warranty, the manufacturer is responsible for the failed products in all failures. However, in the new pro-rata warranty, only *p* percent of failures are covered by warranty service. Generally, some types of failures are not acceptable to the manufacturer. For example, if a component fails because of a physical impact, the consumer has to pay for replacement or repair (if this component is repairable), or if an electrical component fails because of pouring water on it, again, the consumer is responsible for replacement costs. Thus, we assume that on average *p* percent of failures are under warranty coverage.

The rest of the paper is organized as follows. The notations are listed in the nest section. In Section 3, the model is given. A numerical example is presented in Section 4. Finally, concluding remarks are given in the last section.

NOTATION

b: Length of the burn-in period
C_O: Operating cost during the burn-in period
C_S: Cost of supplying, installing, and setting up a component during the burn-in period
N(t): Number of failure during (0, t)
f(t): Lifetime density function
F(t): Lifetime distribution function
$\bar{F}(t)$: Survival function of a component which is given by $\bar{F}(t) = \int_t^{\infty} f(x)\,dx$
h(t): failure rate function which is given by $h(t) = \frac{f(t)}{\bar{F}(t)}$
W: Warranty period length
C_M: Repair cost during the warranty period
P: the probability of manufacturer taking responsibility to repair/replace a failed component
x: failure time of the component

THE MODEL

In this study, a component with a bathtub-shaped failure rate function is considered. The manufacturer puts the component in the burn-in test with length *b*. If during this period a component fails, it will be replaced with a new one and burn-in test restarts. Components that pass the burn-in test, can reach the market with a non-renewing pro-rata warranty service with length *W*.

A. Costs during the burn-in period, (0, b]

The incurred costs during this period are operating cost C_O per unit time per each component and supplying, installation, and setting up cost C_S of each component. Thus, the total cost during the burn-in period, *(0, b]* is given by

$$TC_b = C_O\left(\sum_{i=1}^{N(b)-1} t_i + b\right) + C_S\big(N(b)\big) \qquad (1)$$

where t_i is the lifetime of the i^{th} failed component. *N(b)-1* is the number of failed items during this period, *(0, b]* until for the first time one component can pass this period successfully. It is worth mentioning that between *N(b)* items which put in the burn-in test, the first *N(b)-1* items fail during this test and only the $N(b)^{th}$ item can pass this burn-in test successfully. Hence, there are totally *N(b)* burn-in test and only the last one (*the* $N(b)^{th}$ item) can pass this test successfully. Thus, by considering the definition of the burn-in procedure, it is clear that *N(b)* has a geometric distribution given by

$$P_r[N(b) = n] = \bar{F}(b).F(b)^{n-1} \qquad ,n = 1,2,3,\ldots \qquad (2)$$

Thus, based on the geometric distribution properties, the expected value of *N(b)* can be written as $E[N(b)] = \frac{1}{\bar{F}(b)}$.

Based on Wald's identity, we have

$$\sum_{i=1}^{N(b)-1} t_i = E[N(b)].E(t_i) - E\big[t_{N(b)}\big] = \frac{\int_0^b \bar{F}(t)dt}{\bar{F}(b)} - b \quad (3)$$

Thus, the expected value of the total cost during the burn-in period can be expressed by

$$E(TC_b) = C_O \frac{\int_0^b \bar{F}(t)dt}{\bar{F}(b)} + C_S \frac{1}{\bar{F}(b)} \qquad (4)$$

B. Costs during the warranty period, (b, b+W]

When a component passes the burn-in test successfully, it inters this period at age *b*. During this period, the seller offers a non-renewing pro-rata warranty for the product. Generally, a rebate function, say *q(x)* may be employed to characterize the pro-rata warranty [6]. In this case, suppose that *p* percent of failures are warranted by the manufacturer, who uses a linear form of pro-rata warranty given by

$$q(x) = \begin{cases} \left(1 - \frac{x}{b+W}\right) C_M b < x \leq b + W \\ 0 \qquad\qquad\qquad x > b + W \end{cases} \qquad (5)$$

where*q(x)* is a rebate function at time *x*. It is well-known, when *h(t)* is a failure rate function, the number of failure in a non-renewable warranty policy during *(b, b+W]* can be obtained by $\int_b^{b+W} h(t)\,dt$. Reasoning similar to the Nasrollahi and Asgharizadeh [8], the expected cost for the manufacturer for the above rebate function during the warranty period is

$$E(TC_W) = \int_b^{b+W} q(t)\,.\,f(t)dt\,.\,p.\int_b^{b+W} h\,(t)dt \qquad (6)$$

Thus, the expected total cost during the warranty period, *(b, b+W]* using this linear rebate function can be written as

$$E(TC_W) = C_M\left[\int_b^{b+W} f(t)\,dt - \frac{\int_b^{b+W} t.f(t)dt}{b+W}\right].p.\int_b^{b+W} h\,(t)dt \quad (7)$$

The aim of this study is to find the optimum values of the burn-in period, b^* and warranty period, W^*. Thus, one should minimize the average total cost (ATC) during these two periods. Hence, considering this fact that the value of b is very smaller than the value of W, the objective function can be written as

$$MinTC = \frac{E(TC_b)+E(TC_W)}{W} \quad (8)$$

NUMERICAL EXAMPLE

In this example, we consider Dhillon [3] five-parameter Bathtub-shaped function as the failure rate function for the component which is given by

$$h(t) = KC\lambda t^{C-1} + (1-K)Bt^{B-1}\theta e^{\theta t^B} \quad (9)$$

where λ and θ are the scale parameters, both equal to *1*, and C and B are the shape parameters, equal to 0.3 and 2.5, respectively. By making changes in C and B, it is possible to obtain different shapes for $h(t)$. K is a number between *0* and *1*, inclusive. Figure 2 depicts the failure rate curve of the above failure rate function using the specified parameters, for different values of B.

In this illustration, the cost parameters are chosen to be $C_O=1$, $C_S=100$ and $C_M=10$. Let p, the probability of manufacturer responsibility in a failure be equal to *0.7*. Table 1 shows the average total cost (ATC) during burn-in and warranty periods(using equation number 8) considering different values for b and W. Using equation (4), the average total cost during burn-in period, $E(TC_b)$ computed and then using equation (7), the average total cost during warranty period, $E(TC_W)$ obtained. Finally using equation (8), the average total cost (ATC) calculated.

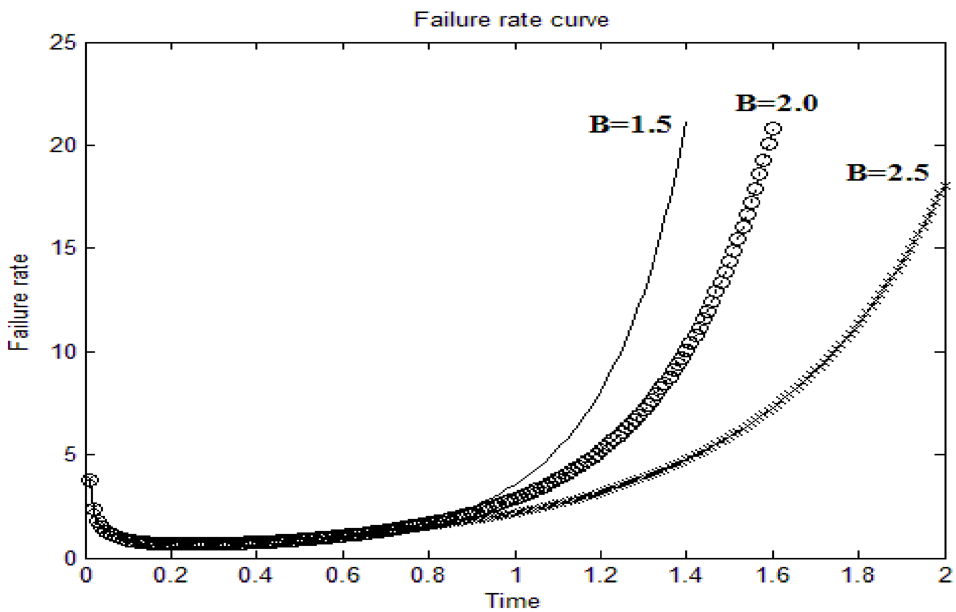

FIGURE 2
DIFFERENT SHAPES OF $h(t)$ USING ABOVE PARAMETERS AND DIFFERENT VALUES OF SHAPE PARAMETER, B.

Table 1
AVERAGE TOTAL COSTS FOR DIFFERENT VALUES OF BURN-IN AND WARRANTY PERIODS

Burn-in	Warranty	ATC	Burn-in	Warranty	ATC	Burn-in	Warranty	ATC
0.005	0.1	1108.33	0.010	0.1	1134.53	0.015	0.1	1153.02
0.005	0.2	554.62	0.010	0.2	567.64	0.015	0.2	576.84
0.005	0.3	370.07	0.010	0.3	378.71	0.015	0.3	384.81
0.005	0.4	277.84	0.010	0.4	284.29	0.015	0.4	288.85
0.005	0.5	222.56	0.010	0.5	227.70	0.015	0.5	231.33
0.005	0.6	185.79	0.010	0.6	190.05	0.015	0.6	193.06
0.005	0.7	159.63	0.010	0.7	163.26	0.015	0.7	165.83
0.005	0.8	140.17	0.010	0.8	143.32	0.015	0.8	145.55
0.005	0.9	125.24	0.010	0.9	128.02	0.015	0.9	129.99
0.005	1.0	113.61	0.010	1.0	116.08	0.015	1.0	117.83
0.005	1.1	104.56	0.010	1.1	106.77	0.015	1.1	108.35
0.005	1.2	97.73	0.010	1.2	99.71	0.015	1.2	101.14
0.005	1.3	93.07	0.010	1.3	94.85	0.015	1.3	96.16
0.005	1.4	90.90	0.010	1.4	92.48	0.015	1.4	93.69
0.005	1.5	92.08	0.010	1.5	93.47	0.015	1.5	94.61
0.005	1.6	98.55	0.010	1.6	99.75	0.015	1.6	100.86
0.020	0.1	1167.82	0.025	0.1	1180.40	0.030	0.1	1191.45
0.020	0.2	584.20	0.025	0.2	590.46	0.030	0.2	595.97
0.020	0.3	389.70	0.025	0.3	393.86	0.030	0.3	397.52
0.020	0.4	292.50	0.025	0.4	295.61	0.030	0.4	298.34
0.020	0.5	234.24	0.025	0.5	236.71	0.030	0.5	238.89
0.020	0.6	195.47	0.025	0.6	197.53	0.030	0.6	199.34
0.020	0.7	167.89	0.025	0.7	169.65	0.030	0.7	171.19
0.020	0.8	147.35	0.025	0.8	148.88	0.030	0.8	150.22
0.020	0.9	131.57	0.025	0.9	132.93	0.030	0.9	134.12
0.020	1.0	119.25	0.025	1.0	120.47	0.030	1.0	121.55
0.020	1.1	109.63	0.025	1.1	110.74	0.030	1.1	111.72
0.020	1.2	102.32	0.025	1.2	103.34	0.030	1.2	104.25
0.020	1.3	97.25	0.025	1.3	98.20	0.030	1.3	99.07
0.020	1.4	94.72	0.025	1.4	95.64	0.030	1.4	96.49
0.020	1.5	95.62	0.025	1.5	96.56	0.030	1.5	97.44
0.020	1.6	101.93	0.025	1.6	102.96	0.030	1.6	103.99

As it can be seen in the above table, using the same value for burn-in period, by increasing the length of warranty period, the average total cost decreases up to 1.4 (in this example) and after that, it starts to increase. By considering the same value for the warranty period, by increasing the length of burn-in period, the average total cost increases slowly. It can be seen from Table 1, the optimum value of average total cost is 90.90 and consequently, optimum burn-in period is equal to 0.005 and optimum warranty period is 1.4. Since the burn-in test is an expensive procedure, the model tries to keep the length of burn-in period at a minimum level. Although one can reach other optimum values by changing the cost parameters or make changes in the failure rate function parameters.

The value of W (length of warranty period), again depends on the failure rate function parameters. As it can be seen in the Figure 2, using the parameters specified in this example, at time 1.4, failure rate function starts to increase rapidly. Thus, up to this point, the average total cost during burn-in and warranty periods decreases slowly and from this point, it starts to increase.

CONCLUSIONS

Nowadays many products are sold with a pro-rata warranty service. In this paper, a new pro-rata warranty policy is considered to find the optimum burn-in and warranty periods from the manufacturer's perspective. In this study, a non-repairable component with a non-renewing pro-rata warranty is considered. However, in our model, only p percent of failures are covered by warranty service and the manufacturer is responsible to repair/replace the failed component. A component with a bathtub-shaped failure rate is considered and the optimum values for the burn-in and warranty periods are obtained based on the specified parameters. In order to extend this research, one can consider other warranty policies or consider a system, which consists of several components to find the optimum length of the burn-in and warranty periods.

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