Original Research

EPQ model with inspection of batches and disposal of defective items

Eveline Libânio Zidan[1]. Ludmylla da Silva Moreira[1] ▪ Fabíola Negreiros de Oliveira[1] ▪ Adriana Leiras[1*]

Received: 2021-01-12/ Accepted: 2021-05-13/ Published online: 2021-05-23

* Corresponding Author, adrianaleiras@puc-rio.br
1- Department of Industrial Engineering, Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.

Abstract
Manufacturing processes can produce imperfect items, and inventory disruptions may occur in the process, causing shortages. This paper presents a production inventory model that considers the inspection of the produced batches and disposal of defective items in an Economic Production Quantity (EPQ) model with partial backorders and a discount on batches that are imperfect but not defective. Furthermore, the proposed model also considers the holding cost of these imperfect items while they are not sold. A step-by-step is conducted to find the optimal solution and a numerical example is provided to perform sensitivity analysis. We conclude that the setup cost and the holding cost are the ones with the most significant impacts on the total cost function. Although the inspection cost has been added to the model, this cost has little effect on the total cost and does not increase it significantly.

Keywords - Backorder; disposal; EPQ; imperfect quality; inspection

INTRODUCTION

The Economic Order Quantity (EOQ) model, proposed by Harris [1], aimed to determine the order quantity based on economic considerations, including holding and setup costs. From the EOQ model, Taft [2] developed the classic Economic Production Quantity (EPQ) model, recognised by researchers as one of the most applicable models in production and inventory control management [3].

The EOQ/EPQ inventory control models intend to minimise the total inventory-related costs, usually the holding cost and ordering costs, and because of their simplicity and effectiveness, the models are still a common approach for practical inventory management [4]-[6]. Nevertheless, several EOQ/EPQ models consider unrealistic assumptions and conditions regarding model input parameters that do not match real-world situations [7]-[9]. To increase the EOQ/EPQ models' applicability, a significant number of researchers have expanded these initial models by considering more realistic propositions [10]-[23].

One of the very restrictive assumptions of the EPQ model is that the manufacturing process must necessarily result in good-quality products. However, the manufacturing process may produce imperfect items for many reasons, such as imperfect raw materials, the skill level of the workforce, machine capability, maintenance policies, or malfunction during the manufacturing process [24]. These imperfect-quality items may not necessarily be defective [25]; they can also be non-defective if their problem does not affect their primary function. For example, in the textile industry, the manufacturing process may produce imperfect clothes out of strict quality standards. Some customers may consider the possibility of buying items, even with imperfections, at a lower price. On the other hand, if a product is unable to perform its primary function, it is classified as a defective item, and it will therefore be disposed of.

To evaluate the quality of batches and classify them as perfect or imperfect, and if imperfect, as defective or non-defective, an inspection needs to be performed. Many scholars have developed studies considering the inspection cost in production [26]-[29]. Salameh and Jaber [30] evaluated the possibility of selling imperfect items at a discount in a single lot by the end of the inspection. When such items are classified as defective, this is, does not fulfill their main function, some customers will not receive their orders. If such customers are willing to wait and receive their orders in the following period, the orders will turn into backorders; otherwise, the orders become lost sales. Thereby, customers' behaviours in the shortage condition have led to another assumption in formulating the different models. Montgomery, Bazaraa, and Keswani [31] extended the basic EOQ pattern through partial backordering (EOQ-PBO). Thereafter, a wide range of researchers has explored the EOQ/EPQ-PBO problem [32]-[38]. Based on EPQ-PBO and imperfect items discussion, Cunha et al. [25] proposed an EPQ model that investigates how the partial backordering (EPQ- PBO) and the discount on imperfect quality of items affect the EPQ model when demand is not fully met.

In light of the above, this work intends to extend the EPQ model of Cunha et al. [25], adding more realistic dimensions to the problem, by considering: (i) the inspection cost of all batches in the manufacturing process; (ii) the disposal cost of defective items; and (iii) the holding cost of non-defective items. This paper differs from other works and contributes to academia and practitioners in several ways. Firstly, through a review of the literature, and to the best of the authors' knowledge, no previous work considering an EPQ model with partial backordering, lost sales, inspection, disposal of defective items, and holding cost of imperfect (non-defective) items sold at a discount was found. Thus, this study endorses the literature discussion on inventory management by developing a new cost function. Secondly, EPQ models considering imperfect non-defective items are particularly relevant to industries that need to sell the products at a lower price. Besides, we consider that the holding cost of non-defective items is relevant as non-defective products may not be sold immediately, needing, therefore, to be managed and stocked until they are sold at a discount. Thus, through our EPQ model, managers in charge of inventory decisions can determine the best-integrated options for a manufacturing system.

The remainder of the paper is organised as follows. Section 2 presents the literature review. Section 3 states the problem definition. Section 4 describes the mathematical modelling problem and model optimisation, including the analytic solution procedure for finding the optimal solution to the EPQ inventory model, proving its optimality. Section 5 presents the numerical example and sensitivity analysis, whereas Section 6 discusses the managerial implications. Finally, Section 7 summarises the concluding remarks, the limitations of this study, and recommendations for future research.

Background

EOQ is one of the initial inventory models and is still used in production and inventory control environments. Studies directed after Harris [1] focused on certain critical features, such as developing new cost and revenue functions for particular applications [4]. For EPQ and EOQ models, the cost function can be optimised in numerous ways by combining multi-decision variables and various inventory approaches.

In the literature, one specific focus of study is to consider the imperfect process in determining the EPQ/EOQ value [39]-[41] due to several factors such as human error, machine, and equipment breakdowns [42]. Various works have studied the problem of handling imperfect items in EPQ inventory models [43]-[52], and to evaluate the quality of batches, many studies have included inspection and screening procedures to identify these imperfect products. Lee and Rosenblatt [53] presented an inspection policy for the EOQ model with a known fraction of defective items. A year later, Porteus [54] introduced the concept of defective products into the manufacturing process for inventory by assuming a fixed probability of a malfunction. Chan, Ibrahim, and Lochert [55] modified the classical EPQ model, integrating 100% inspection of goods, lower pricing, and rework situations. The authors assumed that imperfect-quality items, not necessarily defective, could be used in a different production process or sold to a specific customer at a lower price.

In his EPQ model, Chiu [56] suggested that not all defective items can be repairable; but some would need to be discarded. The mathematical model proposed by Chiu [56] included repair and holding costs per reworked item and disposal cost per scrapped item. Later, Chiu and Wang [10] considered a procedure to determine the optimal runtime and production quantity for an EPQ model with repairable defective items, scrap, and stochastic machine breakdowns. Tsou, Hejazi, and Barzoki [57] proposed a production inventory model assuming that perfect items were maintained in stock until sold; imperfect items were sold at a discounted price, and defective items could be reworked or rejected. Jaggi, Khanna, and Kishore [58] developed an EPQ model by combining the effects of imperfect-quality items, a defective inspection process, an imperfect rework process, and disposal cost. Selvaraju and Ghuru [59] presented an imperfect production system with rework and scrap at a single-stage manufacturing system, integrating cost reduction delivery policy. More recently, Nobil, Sedigh, and Afshar-Nadjafi [60] studied a flawed production system with inspection, rework, shortage and scrap.

Considering the production of defective items, several researchers had proposed distinguishing between holding costs of defective and perfect items [61]-[63]. Paknejad, Nasri, and Affisco [64] considered the extra cost of holding the defective items in the batch during the period before items were returned to the supplier. More recently, Miranda, Vilela, and Leiras [65] presented an EPQ model considering items with a range of imperfection levels, with the selling price varying accordingly to it.

Another classical assumption of EPQ models is that shortages are not allowed. According to Roy, Sana, and Chaudhuri [66], shortages are natural events in any inventory. Yu, Wee, and Chen [67] affirmed that when some customers are willing to wait for stock replenishment, shortages are partially backordered; otherwise, shortages may result in lost sales. It is possible to backorder the partial or full shortage amount. Montgomery, Bazaraa, and Keswani [31] were some of the initial authors to developed a model for the basic EOQ with partial backordering (EOQ-PBO). After that, many authors have focused their efforts on studying EOO/EPQ models, including imperfect-quality items combined with backorder-allowed cases [68]-[69].

Zipkin [32] explored backorders and lost sales. Eroglu and Ozdemir [70] modified the model of Salameh and Jaber [30] by allowing shortages to be fully backordered and considered the scenario where defective items were scrapped, and imperfect-quality items could be sold at a discount as a single lot. Cárdenas-Barrón [24] adjusted the approach of Jamal, Sarker, and Mondal [71] by including the planned backorders into an EPQ inventory model. Pentico and Drake [33] formulated a deterministic EOQ model with partial backordering, developing a new solution for cost function optimisation. Later, Pentico, Drake, and Toews [34] extended this model to EPQ-PBO.

Still considering the several modifications on EOQ/ EPQ-PBO models, Taleizadeh, Sadjadi, and Niaki [72] studied two joint production systems with and without rework, where shortage was allowed and backordered. Hsu and Hsu [73] developed an integrated inventory model with an imperfect production process, inspection errors, and fully backordered shortage. Taleizadeh and Pentico [15] formulated an EOQ-PBO model with discounts for all units and a solution algorithm. Taleizadeh, Cárdenas-Barrón, and Mohammadi [74] elaborated an EPQ model with interruptions in the process, scrapping and rework, for multiple products and a single machine, resulting in limited production capacity and shortages. Salehi, Taleizadeh, and Tavakkoli-Moghaddam [38] presented an EOQ model in which some customers expected the orders to be delivered in the subsequent period when a shortage occurred due to quality issues; hence, in this model, the shortage was allowed by the unplanned interruption and partial backorders. Cheng, Wang, and Wei [75] studied an inventory model in which demand and backorder rates were, respectively, dependent on stock and backorder levels. More recently, Keshavarzfard et al. [28] presented an economic production system with backorder, inspection, and holding costs of both perfect and imperfect items. The major findings of the relevant literature are summarised in Table 1, presenting the features, model objective functions, and stochastic conditions of the models.

TABLE 1
LITERATURE FINDINGS

Features	**Reference**
Imperfect items	[9], [10], [13], [20], [25], [30], [39], [38], [40], [51], [56], [57], [58], [61], [62], [63], [68], [73], [74], [76], [77],[78] [79], [80]
Inspection Process	[9], [10], [13], [20], [30], [39], [40], [51], [56], [58], [61], [62], [63], [68], [73], [74], [77], [78], [80]
Discounts	[20], [25], [30], [57], [61], [63], [73], [77]
Disposal / scrapping	[9], [10], [20], [39], [40], [51], [56], [57], [58], [62], [74], [76], [80], [81]
Backorder / PBO	[13], [25], [38], [56], [58], [68], [72], [73], [74] [76], [78], [79], [80], [81]
Lost Sales	[25], [38], [62], [76],[81]
Model Objective	**Reference**
Maximise total profit	[20], [30], [51], [57], [58], [61], [73], [77]
Minimise total inventory costs	[9], [10], [13], [20], [25], [38], [39], [40], [56], [62], [63], [68], [74], [76], [78], [79], [80],[81]
Minimise total warehouse space	[79]
Uncertainty Conditions	**Reference**
Percentage of defective items	[13], [30], [38], [51], [58], [61], [73], [74], [78], [79]
Percentage of imperfect items	[56], [57], [68], [77]
Percentage of reworked items	[39], [57], [58], [61]
Percentage of scrapped items	[39]
Number of machine breakdowns	[10], [40]
Proportion of errors Type I/ II	[51], [58], [61], [73]
Preventive maintenance time	[62]
Frequency of orders	[9]
Number of consecutive imperfect batches	[25]
Storage area of product	[76]
Repair time/ breakdown time	[78]
Deterioration rate	[80]
Demand	[81]

Based on the wide-ranging literature on the EPQ models, it is noted that there is still a strong growing interest in the topic of imperfect production systems, which is a real-world manufacturing segment concern. On this, the present paper investigates the imperfect production systems, distinguishing non-defective items from defective ones and addressing the holding cost of these non-defective items - until they are sold at a discount - and the disposal cost of defective items. It is also possible to highlight that most of the papers do not address the lost sales costs together with backorder conditions. Thus, this study considers partial backordering and lost sales besides inspection costs and the other aforementioned costs.

PROBLEM DEFINITION

This paper addresses an EPQ problem in a production system. We assume that once a product has been manufactured, all produced batches are inspected through quality control. The batches are identified as perfect or imperfect; there are no batches of intermediate quality. Imperfect batches can be classified as defective products (do not perform their primary function) or non-defective products (still perform their primary function but are not considered perfect, e.g., due to a scratch, a crack, or a stain). In many cases, the producer can sell the imperfect-not defective items at a discount. In the case of defective items, they can become a backorder, if the customer accepts to receive the order in the following period, or lost sale if the customer does not accept (Figure 1).

If the demand is not met in the production system because of insufficient stock, the inventory may be negative. Still, if the demand is met, the inventory may be non-negative. When production exceeds demand and all batches are perfect, these batches will be sold at full price. Suppose the demand exceeds production, and all batches are perfect. In that case, the producer will sell all batches, and the shortages can become backorders if the customers are willing to wait for delivery until the next period. Alternatively, the shortages can also result in lost sales if the customer refuses to receive the product in the next period. When production exceeds demand and imperfect batches are produced, these items may be sold at a discount, and the remaining orders can become backorders or lost sales. Lastly, suppose the demand exceeds production and imperfect items are produced. In that case, non-defective batches can be sold at a discount, and the remaining orders may become either backorders or lost sales.

In this work, we consider six different types of costs. Setup cost is incurred for all batches since it is the cost to prepare the equipment to process a distinct batch. Once production has been finished, the batches undergo a quality inspection, resulting in inspection costs. As previously explained, batches can be classified as perfect or imperfect, and the imperfect ones are divided into defective and non-defective. All perfect and non-defective batches are stored, incurring holding costs, and all defective batches are rejected, incurring disposal costs. Backordering costs occur when the customer agrees to receive the order in the next period, and the lost sales cost is incurred when the customer does not accept the order in the next period. The lost sales cost includes the lost profit on the respective unit and any goodwill loss. Figure 1 represents all the costs involved in this manufacturing system.

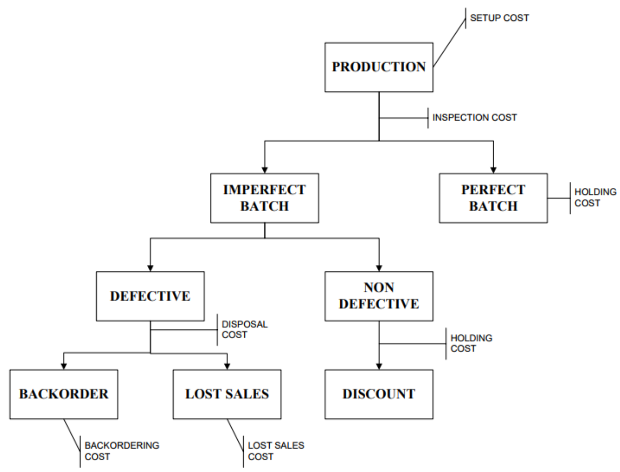

FIGURE 1
ROADMAP FOR THE COSTS IN THE STUDIED PRODUCTION SYSTEM

The production schedule is fixed, and the batches are manufactured at equally-spaced intervals T. Production has a periodic review policy. After having been produced, each batch is subject to quality control that classifies it as perfect or imperfect. X is the number of consecutive imperfect batches. As the production of an imperfect batch occurs independently, it is presumed that the probability distribution followed is a geometric distribution [38, 82]. Therefore, X is a geometric random variable with parameter θ equal to the probability of an imperfect batch occurrence. There will thus be cycles where imperfect batches are produced (XT) and cycles where all batches are perfect (T). As such, the length of any inventory cycle is defined by $T'=(X+1)T$.

Figure 2 illustrates three consecutive cycles, where the first and second batches produced are imperfect $(X=1)$, and the third produced batch is perfect. As a result, the first inventory cycle is $T' = 2T$, and then $T' = T$.

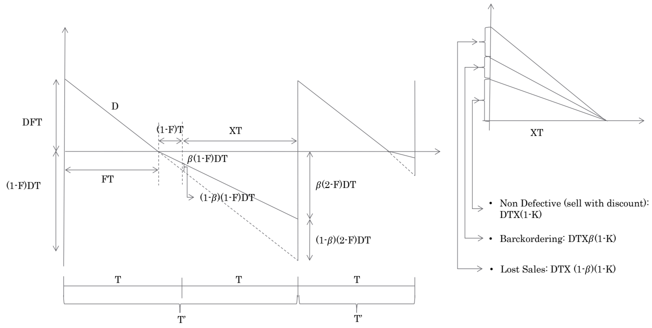

FIGURE 2
THREE CONSECUTIVE CYCLES IN THE EPQ MODEL WITH IMPERFECT AND PERFECT BATCHES

FUNDAMENTAL ASSUMPTIONS AND NOTATIONS

This section presents the model notations, assumptions, the Total Cost Function, and the model optimisation. Additionally, we provide a step-by-step to prove the solution's optimality.

I. Notations

The following notations are used in the model:

Parameters:

C_0: Setup cost per unit time
C_i: Inspection cost per unit time
C_h: Cost of holding a unit in inventory per unit time
C_d: Disposal cost per unit time
C_b: Cost of keeping a backordered unit per unit time
C_l: Lost sales cost (lost profit and goodwill loss) per unit
D: Demand rate
P: Production rate
β: Fraction of backordered shortage ($0 < \beta < 1$)
K: Fraction of imperfect batches sold at a discount ($0 < K < 1$)
X: Number of consecutive imperfect batches manufactured per unit time
θ: Probability of producing an imperfect batch

Decision variables:

T: Time interval between two successive production runs
T': Time interval between two successive perfect production runs ($T' = (X + 1)T$)
F: Fraction of time intervals with positive inventory level $0 < F < 1$

II. Assumptions

The following assumptions are adopted to develop the proposed inventory model:

- The planning horizon is infinite [51, 52, 69];
- The demand rate is known and constant [52, 58];
- Shortages are permitted and partially backordered [25, 38, 67];
- The production schedule is fixed, in which batches are manufactured at equally-spaced intervals T [25, 38];
- Each produced batch is inspected, and imperfect batches occur independently from each other [25, 38];
- Defective items are not reworked [25];
- The total quantity of any defective batch or shortage being backordered is routinely added to the amount of scheduled production [25, 38];
- An inspection of 100% of goods is performed [52, 80];
- In the inspection of the batches, the whole batch is considered either perfect or imperfect; there are no half-imperfect batches [38, 52];
- The inspection process is perfect [60]; and
- The defective batches are disposed of immediately without warehousing [20, 60].

Salehi, Taleizadeh, and Tavakkoli-Moghaddam [38] developed an EOQ model assuming that the total quantity of any rejected batch is routinely added to the batch quantity of the next planned delivery. Using the same reasoning of the authors, we assume that the total quantity of any defective batch or shortage being backordered is routinely added to the amount of scheduled production. Besides that, to render the model more appropriate for real-world situations, we considered that imperfect non-defective items may not be sold immediately and therefore have a holding cost.

III. Total Cost Function

This study includes three new costs in the total cost function of the EPQ model presented by Cunha et al. [25]. We added: (i) an inspection cost of all batches, (ii) a disposal cost of defective items, and (iii) the cost of holding the non-defective items, assuming that these items are not immediately sold. Therefore, the total cost function of the production system is expressed in terms of the following costs: setup cost, inspection cost, holding cost, disposal cost, backordering cost, and lost sales cost, as demonstrated in (1) with respect to F, T, and X.

$$\boldsymbol{C(F,T,X)} = \overbrace{C_0(X+1)}^{\text{Setup Cost}} + \overbrace{C_i(X+1)}^{\text{Inspection Cost}} + \overbrace{\frac{C_h D T^2 F^2}{2}\left(1-\frac{D}{P}\right)}^{\text{Holding Cost of Perfect Items}} + \overbrace{\frac{C_h D T^2 X^2 K}{2}\left(1-\frac{D}{P}\right)}^{\text{Holding Cost of Non-Defective Items}} \tag{1}$$

$$\underbrace{+\frac{C_d D T^2 X^2 (1-K)}{2}}_{\text{Disposal Cost}}$$

$$\underbrace{+ C_l DT(1-\beta)(X(1-K)+1-F)}_{\text{Lost Sale Cost}}$$

$$\underbrace{+\frac{C_b \beta D T^2 (1-F)^2}{2}\left(1-\frac{D}{P}\beta\right)}_{\text{Backordering Cost when inventory < 0}}$$

$$\underbrace{+\frac{C_b \beta D T^2 X^2 (1-K)}{2}\left(1-\frac{D}{P}\beta\right)}_{\text{Backordering Cost when there are imperfect batches}}$$

$$\underbrace{+ C_b \beta D T^2 X (1-K)(1-F)}_{\text{Backordering Cost when inventory < 0 and there are imperfect batches}}$$

Since X is the number of consecutive imperfect batches that occur independently, it is a geometric random variable with parameter θ equal to the probability of an imperfect batch. The probability mass function is $P(X = x) = \theta^x(1-\theta), x \geq 0$. The expected value and the second moment of X are given by $E(X) = \frac{\theta}{1-\theta}$ and $E(X^2) = \frac{\theta}{(1-\theta)^2} + \frac{\theta^2}{(1-\theta)^2} = \frac{\theta(1+\theta)}{(1-\theta)^2}$, respectively. Thus, the expected cost of any cycle in terms of F and T is given by:

$$\begin{aligned}
&\boldsymbol{CC(F,T)} \\
&= C_0\left(\left(\frac{\theta}{1-\theta}\right)+1\right) + C_i\left(\left(\frac{\theta}{1-\theta}\right)+1\right) \\
&+ \frac{C_h D T^2 F^2}{2}\left(1-\frac{D}{P}\right) \\
&+ \frac{C_h D T^2 K}{2}\left(1-\frac{D}{P}\right)\left(\frac{\theta(1+\theta)}{(1-\theta)^2}\right) \\
&+ \frac{C_d D T^2 (1-K)}{2}\left(\frac{\theta(1+\theta)}{(1-\theta)^2}\right) \\
&+ C_l DT(1-\beta)\left(\left(\frac{\theta}{1-\theta}\right)(1-K)+1-F\right) \\
&+ \frac{C_b \beta D T^2 (1-F)^2}{2}\left(1-\frac{D}{p}\beta\right) \\
&+ + \frac{C_b \beta D T^2 (1-K)}{2}\left(1-\frac{D}{P}\beta\right)\left(\frac{\theta(1+\theta)}{(1-\theta)^2}\right) \\
&+ C_b \beta D T^2 (1-K)(1-F)\left(\frac{\theta}{1-\theta}\right)
\end{aligned} \tag{2}$$

As aforementioned, the length of any cycle is given by $T' = (X+1)T$. Thus, the expected length of each inventory cycle is $T' = (E(X)+1)\,T = \frac{T}{1-\theta}$. Accordingly, the total cost function is:

$$\begin{aligned}
\boldsymbol{TC(F,T)} &= \frac{CC(F,T)}{E(T')} \\
&= \frac{C_0}{T} + \frac{C_i}{T} + \frac{C_h D T F^2\left(1-\frac{D}{P}\right)(1-\theta)}{2} \\
&+ \frac{C_h D T K\left(1-\frac{D}{P}\right)\theta(1+\theta)}{2(1-\theta)} \\
&+ \frac{C_d D T (1-K)\theta(1+\theta)}{2(1-\theta)} \\
&+ C_l D(1-\beta)\big(-\theta K + 1 - F(1-\theta)\big) \\
&+ \frac{C_b \beta D T (1-F)^2\left(1-\frac{D}{P}\beta\right)(1-\theta)}{2} \\
&+ \frac{C_b \beta D T\left(1-\frac{D}{P}\beta\right)\theta(1+\theta)(1-K)}{2(1-\theta)} \\
&+ C_b \beta D T \theta (1-F)(1-K)
\end{aligned} \tag{3}$$

IV. Procedure for determining the optimal values

To perform model optimisation, the function $TC(F,T)$ needs to be minimised subject to the constraint $0 \leq F \leq 1$. To minimise the function $TC(F,T)$, the total cost function has to be derived for F and T. To simplify (3) and to derive the total cost function, artificial variables were used, as proposed by Cunha et al. [25]. The total cost function can be rewritten as:

$$\boldsymbol{TC(F,T)} = \frac{\alpha_1}{T} + T(\alpha_2 F^2 + \alpha_3 - 2\alpha_3 F + \alpha_4 - \alpha_4 F + \alpha_5) - \alpha_6 F + \alpha_7 \quad (4)$$

Where:

$$W = \left(1 - \frac{D}{P}\right) \quad (5)$$

$$Y = \left(1 - \frac{D}{P}\beta\right) \quad (6)$$

$$\alpha_1 = C_0 + C_i \quad (7)$$

$$\alpha_2 = \frac{D(1-\theta)(C_h W + C_b \beta Y)}{2} \quad (8)$$

$$\alpha_3 = \frac{C_b \beta D Y(1-\theta)}{2} \quad (9)$$

$$\alpha_4 = C_b \beta D \theta (1-K) \quad (10)$$

$$\alpha_5 = \frac{D\theta(1+\theta)(C_h K W + C_d(1-K) + C_b \beta Y(1-K)}{2(1-\theta)} \quad (11)$$

$$\alpha_6 = C_l D(1-\beta)(1+\theta) \quad (12)$$

$$\alpha_7 = C_l D(1-\beta)(1-\theta K) \quad (13)$$

Again, (4) can be rewritten as:

$$\boldsymbol{TC(F,T)} = \frac{\alpha_1}{T} + Tr(F) + q(F) \quad (14)$$

Where:

$$r(F) = \alpha_2 F^2 + \alpha_3 - 2\alpha_3 F + \alpha_4 - \alpha_4 F + \alpha_5 \quad (15)$$

$$q(F) = -\alpha_6 F + \alpha_7 \quad (16)$$

Aiming to minimise the total cost, the partial derivatives of (14) were computed with respect to the decision variables T (cycle length) and F (fraction of time interval with the positive level of inventory), and both derivatives were set equal to zero. The optimal value of cycle length is:

$$T^*(F) = \sqrt{\frac{\alpha_1}{r(F)}} = \sqrt{\frac{\alpha_1}{\alpha_2 F^2 + \alpha_3 - 2\alpha_3 F + \alpha_4 - \alpha_4 F + \alpha_5}} \quad (17)$$

The optimal value of the fraction of time interval with the positive inventory level is:

$$F^*(T) = \frac{2\alpha_3 T + \alpha_4 T + \alpha_6}{2\alpha_2 T} \quad (18)$$

By substituting $F^*(T)$ given by (18) into (17) and after algebraic transformations, we finally have:

$$T^* = \sqrt{\frac{4\alpha_1\alpha_2 - \alpha_6^2}{\alpha_4 + 4\alpha_2\alpha_3 + 4\alpha_2\alpha_4 - 4\alpha_3\alpha_4 - 2\alpha_4^2 + 4\alpha_2\alpha_5}} \quad (19)$$

V. Proof of the solution's optimality

As discussed in the previous section, (17) shows the optimal value of the cycle length. The discriminant of $r(F) = \alpha_2 F^2 + \alpha_3 - 2\alpha_3 F + \alpha_4 - \alpha_4 F + \alpha_5$ is as follows:

$$\Delta = (2\alpha_3 + \alpha_4)^2 - 4\alpha_2(\alpha_3 + \alpha_5) \quad (20)$$

Once the discriminant shown in (20) is always negative and there are no roots, the unique optimal value of the cycle length that minimises the cost function is given by (17). The latter provides the optimal value for each period of the cycle when the inventory is positive. Because the discriminant presented in (20) has no roots, $r(F)$ is either positive for all F or negative for all F. Therefore, for each F, (17) gives a unique root $T^* = T^*(F)$ that minimises the cost function given by (14). By substituting the expression for $T^*(F)$ in (17) into (14), it is established that:

$$\widehat{TC}(F) = \frac{\alpha_1}{\sqrt{\frac{\alpha_1}{r(F)}}} + \sqrt{\frac{\alpha_1}{r(F)}}\, r(F) + q(F) \qquad (21)$$

$$= 2\sqrt{\alpha_1\, r(F)} + q(F)$$

This equation provides the minimum cost for each value of F, and because $\widehat{TC}(F)$ is a continuous function of F, it has one or more local minimum points within the interval (0, 1). The smallest value minimum is the global minimum of the cost function. So, to discover the global minimum, the first and second derivatives of (21) with respect to F are computed as follows:

$$\widehat{TC}'(F) = \sqrt{\alpha_1}\left(\frac{r'(F)}{\sqrt{r(F)}}\right) + q'(F)$$

$$= \sqrt{\alpha_1}\left(\frac{2F\alpha_2 - 2\alpha_3 - \alpha_4}{\sqrt{F^2\alpha_2 + \alpha_3 - 2F\alpha_3 + \alpha_4 - F\alpha_4 + \alpha}}\right) - \alpha_6 \qquad (22)$$

$$\widehat{TC}''(F) = \sqrt{\alpha_1}\left[\frac{2\, r''(F) r(F) - (r'(F)^2}{2r(F)^{3/2}}\right] \qquad (23)$$

$$= \sqrt{\alpha_1}\left(\frac{8\alpha_2\alpha_3 - 8F\alpha_2\alpha_3 + 4\alpha_2\alpha_4 - 2F\alpha_2\alpha_4 + 4\alpha_2\alpha_5 + 4\alpha_3^2 + \alpha_4^2 + 2\alpha_3\alpha_4}{2\sqrt{(F^2\alpha_2 + \alpha_3 - 2F\alpha_3 + \alpha_4 - F\alpha_4 + \alpha_5)^3}}\right)$$

Assuming the inventory level to be positive, with *F=1*, if (22) is $\widehat{TC}'(1) \geq 0$, then (21) has a unique minimum in the open interval (0, 1). Otherwise, if $\widehat{TC}'(1) < 0$, then (21) reaches the minimum at the boundary point ($F = 1$). The model is solved using the algorithm presented in Cunha et al. [25] and Salehi, Taleizadeh, and Tavakkoli-Moghaddam [38]. Hence, to determine the optimal values to *F*, *T* and *TC*, we follow the next steps.

(1) Calculate $\widehat{TC}'(1)$, presuming the inventory to be positive (*F*=1), using (22);
(2) If $\widehat{TC}'(1) < 0$, then go to stage 3. Otherwise, go to stage 4;
(3) To calculate the cycle length T^* that minimises the total cost, assume that $F = 1$ and replace it in (17). Then, compute the total cost using (3) and compare that to the cost of losing all demand $(C_l D)$. If the total cost is greater than the cost of losing all demand, then the producer would allow the inventory level to always be negative during the cycle length that should be infinite. The optimal value of decision variables is $F^* = 0$ and $T^* = \infty$. Otherwise, if the producer does not allow the inventory to be negative ($F^* = 1$) in any cycle length, T^* is calculated using (17);
(4) If $\widehat{TC}'(1)$ is non-negative, determine T^* using (19);
(5) Replace the optimal cycle length value (T^*) into (18) to find the fraction of cycle length in which the inventory level is positive (F^*); and
(6) Determine the cost function by (4).

NUMERICAL EXAMPLE

To demonstrate the application and usefulness of the model, we present a numerical example, following the parameters based on Jaggi, Khanna, and Kishore [58] and Pentico, Toews, and Drake [83]. Thus, the parameters values are set to D = 1100 units per year; C_o = \$275; C_h = \$2 per unit time; C_b = \$3.2 per unit time; C_l = \$4; β = 0.77; K = 0.4; P = 9200 units per year; θ = 0.10; C_i = \$10 per unit time; C_d = \$8 per unit time. Following the solution algorithm presented in the previous section, the optimal values obtained from this work were F^* = 1, T^* = 0.46 and TC^*= \$1090.53.

The model was solved using MATLAB software. Despite the inclusion of three more costs in the total cost function, the proposed solution algorithm showed a good performance once the optimal values were achieved in few seconds compared to other algorithms in the literature. After calculating the optimal solution with the data above, a sensitivity analysis is performed to verify how the parameter changes affect the optimal F^*, T^* and TC^*. The results of this analysis are presented in Table 2.

TABLE 2
SENSITIVITY ANALYSIS

Parameters	Changes	Optimal Values			% Changes		
		*T**	*F**	*TC**	*T**	*F**	*TC**

D		1100.0	0.462	1.00	1090.53			
	50%	1650.0	0.388	1.00	1258.19	84%	100%	115%
	25%	1375.0	0.419	1.00	1183.12	91%	100%	108%
	-25%	825.0	0.528	1.00	974.20	114%	100%	89%
	-50%	550.0	0.638	1.00	822.10	138%	100%	75%
Co		275.0	0.462	1.00	1090.53			
	50%	412.5	0.563	1.00	1358.61	122%	100%	125%
	25%	343.8	0.515	1.00	1231.13	112%	100%	113%
	-25%	206.3	0.403	1.00	931.67	87%	100%	85%
	-50%	137.5	0.333	1.00	744.78	72%	100%	68%
Ch		2.0	0.462	1.00	1090.53			
	50%	3.0	0.399	1.00	1287.36	86%	100%	118%
	25%	2.5	0.427	1.00	1192.58	92%	100%	109%
	-25%	1.5	0.509	1.00	979.23	110%	100%	90%
	-50%	1.0	0.572	1.00	855.58	124%	100%	78%
Cb		3.2	0.462	1.00	1090.53			
	50%	4.8	0.455	1.00	1111.23	98%	100%	102%
	25%	4.0	0.459	1.00	1100.92	99%	100%	101%
	-25%	2.4	0.467	1.00	1080.05	101%	100%	99%
	-50%	1.6	0.471	1.00	1069.48	102%	100%	98%
Cl		4.0	0.462	1.00	1090.53			
	50%	6.0	0.463	1.00	1019.69	100%	100%	94%
	25%	5.0	0.463	1.00	1055.11	100%	100%	97%
	-25%	3.0	0.463	1.00	1125.95	100%	100%	103%
	-50%	2.0	0.388	0.96	1179.38	84%	96%	108%
Ci		10.0	0.462	1.00	1090.53			
	50%	15.0	0.467	1.00	1101.29	101%	100%	101%
	25%	12.5	0.465	1.00	1095.92	101%	100%	100%
	-25%	7.5	0.461	1.00	1085.11	100%	100%	100%
	-50%	5.0	0.459	1.00	1079.67	99%	100%	99%
Cd		8.0	0.462	1.00	1090.53			
	50%	12.0	0.437	1.00	1163.03	95%	100%	107%
	25%	10.0	0.449	1.00	1127.30	97%	100%	103%
	-25%	6.0	0.477	1.00	1052.63	103%	100%	97%
	-50%	4.0	0.493	1.00	1013.49	107%	100%	93%
θ		0.1	0.462	1.00	1090.53			
	50%	0.2	0.424	1.00	1132.71	92%	100%	104%

	25%	0.1	0.443	1.00	1109.75	96%	100%	102%
	-25%	0.1	0.483	1.00	1075.02	104%	100%	99%
	-50%	0.1	0.503	1.00	1063.22	109%	100%	97%
β		0.8	0.462	1.00	1090.53			
	50%	1.2	0.325	0.22	936.28	70%	22%	86%
	25%	1.0	0.385	0.76	1188.18	83%	76%	109%
	-25%	0.6	0.466	1.00	962.27	101%	100%	88%
	-50%	0.4	0.470	1.00	833.40	102%	100%	76%
K		0.4	0.462	1.00	1090.53			
	50%	0.6	0.484	1.00	1016.40	105%	100%	93%
	25%	0.5	0.473	1.00	1053.76	102%	100%	97%
	-25%	0.3	0.453	1.00	1126.73	98%	100%	103%

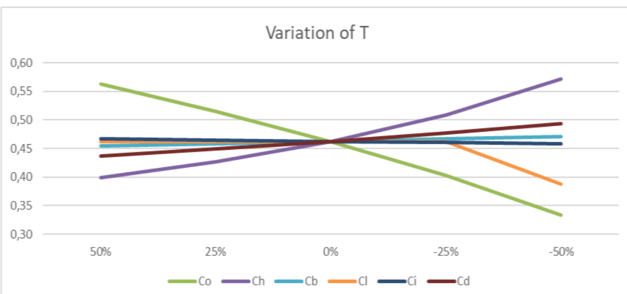

FIGURE 3
SENSITIVITY ANALYSIS OF T^*

As to the others parameters, such as the fraction of the time interval with the positive inventory level (F^*), this parameter is only affected by two costs. F^* decreases when C_l decreases by 50% and when the fraction of backordered batches (β) increases by 25% and 50%. Finally, the total cost (TC^*) is influenced positively by the setup cost (C_0), the holding cost (C_h), the cost of keeping a backorder (C_b), the inspection cost (C_i), the disposal cost (C_d), and the demand (D). As these parameters increase, TC^* also increases, and if they decrease, TC^* also does. The only cost that negatively affects TC^* is the lost sales cost (C_l). Regarding demand (D) decrease, T^* varies in the opposite direction and its value increase.

The sensitivity analysis also shows that when the probability of producing an imperfect batch (θ) increases, T^* decreases. Thus, this analysis makes it evident that with the increase in imperfect batches' probability, it will be necessary to produce more batches, as many batches will be detected as imperfect. Hence, the duration of time between two production runs (T^*), decreases. Similarly, if demand (D) increases, T^* varies in the opposite direction to speed up production and meet demand. Considering F^*, it is unaffected by the increase or decrease of θ. TC^* is directly affected by θ; if θ increases, TC^* will also increase, and if θ decreases, TC^* also decreases.

Considering the fraction of backordered batches (β), the increase of β will make T^* decrease, and the decrease of β will make T^* increase. Additionally, if β increases, F^* will be affected negatively. On the other hand, if β decreases, F^* will not be affected. Analysing TC^*, we can conclude that TC^* is positively affected by β. If β increases, TC^* will also increase, and if β decreases, TC^* also will. As to the fraction of imperfect batches sold at a discount (K), TC^* will decrease if K increases and the opposite also happens. K also does not affect F*. If K increases, T^* also increases, and if K decreases, T^* also does.

DISCUSSION AND MANAGERIAL IMPLICATIONS

This study takes into account the behavioural factors of the real-world scenarios, considering a flawed manufacturing system with permissible shortages, which can be partially backordered or lead to lost sales. The imperfect products can be distinguished as non-defectives and defectives through an inspection process. Then, they are sold at a discount (in case of non-defectiveness) or disposed of after the production process. Moreover, non-defective items need to be stocked while they are not sold at a discount because it depends on the customer's willingness to buy them. Thus, this model indicates the best-integrated decisions for a manufacturing system, allowing the managers to better use the available resources. Compared to similar models in the literature, this model provides a good framework for a complex manufacturing system with inspection, partial backordering, discount for imperfect-quality batches, lost sales conditions, warehousing conditions for non-defective items, and disposal of defective items. This makes our model closer to real-life inventory systems and more robust, trying to cover some gaps in the inventory systems literature. Cunha et al. [25], for instance, do not consider the inspection process to identify imperfect items and the holding cost for imperfect-quality batches, therefore not accounting for the fact that these products may not be sold instantly. Gharaei, Hoseini Shekarabi, and Karimi [20] do not take into account shortages in terms of backorders and lost sales in all periods, not considering that shortages are very common in any production system.

The sensitivity analysis allows us to analyse how the changes in the variables affect the total cost function, which provides a useful tool for strategic decision-making. Among all costs, C_0 is the cost that most affects $TC*$. The setup cost (C_0) followed by the holding cost (C_h) are the variables with the highest inclination once they are the most elastic variables. In other words, the variation of these variables cause the most significant change in the total cost (TC). Thus, since savings in holding costs result in a total cost reduction, it is better to sell imperfect items at a discount as quickly as possible rather than stock them. Here, managers need to design strategies to make imperfect items more attractive to the customers, for instance, by increasing the discount over the imperfect items.

On the other hand, inspection cost (C_i) together with backordering cost (C_b) are the costs that least affect $TC*$. These costs have the least variation in the total cost (TC). This finding is important when a decision must be made and where trade-offs exist. Additionally, we have concluded that the lost sales cost (C_l) is the only cost that negatively affects the total cost (TC), i.e., when this variable decreases, the total cost increases. This finding means that there must be a lost sales cost small enough to not warrant production. In this scenario, it would be more efficient to keep the negative inventory for a longer period. Moreover, as C_i affects the total cost less than C_l, it is worth investing in screening and inspection procedures to ensure higher process quality and reduce the risk of defective products and, consequently, the cost of lost sales. Figure 4 presents the sensitivity analyses of variable costs.

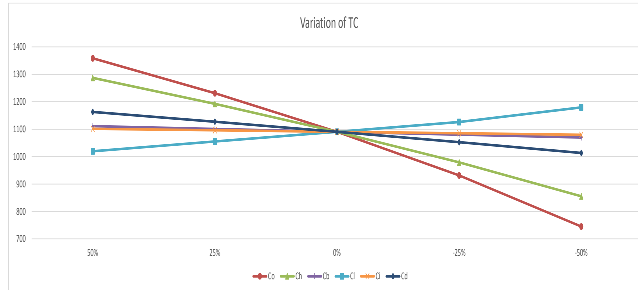

FIGURE 4
SENSITIVITY ANALYSIS OF VARIABLE COSTS

CONCLUSIONS AND SUGGESTIONS

This study develops an EPQ model including imperfect items, distinguishing them into defective and non-defective where defective products are disposed of, and non-defective products can be sold at a lower price. Additionally, this study incorporates partial backordering, lost sales conditions, and inspection to evaluate the quality of the batches. Through a numerical example, the behaviour of decision variables was explored, primarily by examining the optimal total cost, when the values of the parameters of the model were varied. This sensitivity analysis was carried out to demonstrate how decision-making can be performed strategically to reduce the total cost when certain variables change. The parameter that most affects the optimal total cost is the setup cost. In contrast, the inspection cost is the parameter that least affects the optimal total cost. It is interesting to note that although the inspection cost has been added to the model, it does not significantly increase the total cost. This shows a positive finding to the companies since the inspection is an important procedure to identify and separate perfect, imperfect, and defective items.

Moreover, the literature review presented here provides an overview of the work on inventory systems and it can be useful for academics interested in this field of research. According to the literature findings, quality management influences consumer satisfaction that translates into reputation and revenue – the company's ultimate goal. In agreement with the literature and this work's findings, manufacturers should explore sources of imperfections, identifying their causes to minimise the proportion of defective items, once the costs of disposal and holding the non-defective items increase the total cost.

Although this study investigates many real-life costs and considerations, our model still suffers from some limitations. This research does not consider the rework of imperfect items. Rework plays an important role in reverse logistics since second-hand products are reworked to reduce waste, environmental damages, and overall production/inventory costs.

Another limitation of this research is that the model considers inspection as a perfect process. However, the human inspection process is imperfect, often involving errors. We also consider 100% of inspection, which is not always possible, especially when inventory levels are large. Future studies may extend the present model in a few directions: it can extend this model to consider inspection errors, addressing both Type I and Type II inspection errors, and considering sampling inspection rather than perform 100% screening. Future studies may also apply different costs, e.g., assuming that defective items are not disposed of immediately and need to be stored, while having different holding costs. Another suggestion is to address other forms of demand, such as stochastic demand, as research gaps also exist in the integration of strategical with operational decisions [84] - [85]. Finally, an interesting avenue of research can address sustainability issues such as inventory emissions costs (cost of inventory holding emissions and cost of production emissions), as environmental and sustainability issues are receiving enormous attention in business environments and academic researches.

Acknowledgment

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

References

[1] Harris F W. (1913). How Many Parts to Make at Once. *Factory, the Magazine of Management* 10 (2): 135 – 136, 152.
[2] Taft E W. (1918). The Most Economical Production Lot. *Iron Age*, 101: 1410–1412.
[3] Pasandideh, S. H. R., Niaki, S. T. A., & Yeganeh, J. A. (2010). A parameter-tuned genetic algorithm for multi-product economic production quantity model with space constraint, discrete delivery orders and shortages. *Advances in Engineering Software*, *41*(2), 306-314.
[4] Andriolo, A., Battini, D., Grubbström, R. W., Persona, A., & Sgarbossa, F. (2014). A century of evolution from Harris′ s basic lot size model: Survey and research agenda. *International Journal of Production Economics*, *155*, 16-38.
[5] Chiarini, A. (2017). An adaptation of the EOQ formula for JIT quasi-pull system production. *Production Planning & Control*, *28*(2), 123-130.
[6] Khan, M., Hussain, M., & Cárdenas-Barrón, L. E. (2017). Learning and screening errors in an EPQ inventory model for supply chains with stochastic lead time demands. *International Journal of Production Research*, *55*(16), 4816-4832.
[7] Huang, Y. F. (2006). An inventory model under two levels of trade credit and limited storage space derived without derivatives. *Applied Mathematical Modelling*, *30*(5), 418-436.
[8] Pasandideh, S. H. R., & Niaki, S. T. A. (2008). A genetic algorithm approach to optimize a multi-products EPQ model with discrete delivery orders and constrained space. *Applied Mathematics and Computation*, *195*(2), 506-514.
[9] Kangi, F., Javanmard, S., & Pasandideh, S. H. R. (2017). Economic production quantity model with imperfect products and random order frequency under due date and limited storage capacity. *Journal of Industrial and Production Engineering*, *34*(5), 344-361.
[10] Chiu, S. W., Wang, S. L., & Chiu, Y. S. P. (2007). Determining the optimal run time for EPQ model with scrap, rework, and stochastic breakdowns. *European Journal of Operational Research*, *180*(2), 664-676.
[11] Li, J., Wang, S., & Cheng, T. E. (2008). Analysis of postponement strategy by EPQ-based models with planned backorders. *Omega*, *36*(5), 777-788.
[12] Zhang, R. Q. (2009). A note on the deterministic EPQ with partial backordering. *Omega*, *37*(5), 1036-1038.
[13] Sarkar, B., & Moon, I. (2011). An EPQ model with inflation in an imperfect production system. *Applied Mathematics and Computation*, *217*(13), 6159-6167.
[14] Chiu, S. W., Chou, C. L., & Wu, W. K. (2013). Optimizing replenishment policy in an EPQ-based inventory model with nonconforming items and breakdown. *Economic Modelling*, *35*, 330-337.
[15] Taleizadeh, A. A., & Pentico, D. W. (2014). An economic order quantity model with partial backordering and all-units discount. *International Journal of Production Economics*, *155*, 172-184.
[16] Nobil, A. H., Sedigh, A. H. A., & Cárdenas-Barrón, L. E. (2016). A multi-machine multi-product EPQ problem for an imperfect manufacturing system considering utilization and allocation decisions. *Expert Systems with Applications*, *56*, 310-319.
[17] Mokhtari, H., Naimi-Sadigh, A., & Salmasnia, A. (2017). A computational approach to economic production quantity model for perishable products with backordering shortage and stock-dependent demand. *Scientia Iranica*, *24*(4), 2138-2151.
[18] Manna, A. K., Das, B., Dey, J. K., & Mondal, S. K. (2018). An EPQ model with promotional demand in random planning horizon: population varying genetic algorithm approach. *Journal of Intelligent Manufacturing*, *29*(7), 1515-1531.
[19] Chang, C. T., Ouyang, L. Y., Teng, J. T., Lai, K. K., & Cárdenas-Barrón, L. E. (2019). Manufacturer's pricing and lot-sizing decisions for perishable goods under various payment terms by a discounted cash flow analysis. *International Journal of Production Economics*, *218*, 83-95.
[20] Gharaei, A., Hoseini Shekarabi, S. A., & Karimi, M. (2020). Modelling and optimal lot-sizing of the replenishments in constrained, multi-product and bi-objective EPQ models with defective products: Generalised cross decomposition. *International Journal of Systems Science: Operations & Logistics*, *7*(3), 262-274.
[21] Salas-Navarro, K., Acevedo-Chedid, J., Árquez, G. M., Florez, W. F., Ospina-Mateus, H., Sana, S. S., & Cárdenas-Barrón, L. E. (2019). An EPQ inventory model considering an imperfect production system with probabilistic demand and collaborative approach. *Journal of Advances in Management Research*.
[22] Condeixa, L. D., Silva, P., Moah, D., Farias, B., & Leiras, A. (2020). Evaluating cost impacts on reverse logistics using an Economic Order Quantity (EOQ) model with environmental and social considerations. *Central European Journal of Operations Research*, 1-20.
[23] Londe, M. A., Martinez, G. C., & Leiras, A. (2020, July). Economic Production Quantity for Products with Deterioration and Shortage Under Advanced-Cash-Credit Payment Scheme. In *International Joint conference on Industrial Engineering and Operations Management* (pp. 585-597). Springer, Cham.

[24] Cárdenas-Barrón, L. E. (2009). Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. *Computers & Industrial Engineering*, *57*(3), 1105-1113.
[25] Cunha, L. R. A., Delfino, A. P. S., dos Reis, K. A., & Leiras, A. (2018). Economic production quantity (EPQ) model with partial backordering and a discount for imperfect quality batches. *International Journal of Production Research*, *56*(18), 6279-6293.
[26] Khan, M., Ahmad, A. R., & Hussain, M. (2019). Integrated decision models for a vendor–buyer supply chain with inspection errors and purchase and repair options. *The International Journal of Advanced Manufacturing Technology*, *104*(9), 3221-3228.
[27] Tsai, W. C., Shih, N. H., & Wang, C. H. (2018). The effects of inspection delay and restoration cost on the optimal inspection and production policy. *Opsearch*, *55*(1), 187-195.
[28] Keshavarzfard, R., Makui, A., Tavakkoli-Moghaddam, R., & Taleizadeh, A. A. (2019). Optimization of imperfect economic manufacturing models with a power demand rate dependent production rate. *Sādhanā*, *44*(9), 1-19.
[29] Rout, C., Kumar, R. S., Chakraborty, D., & Goswami, A. (2019). An EPQ model for deteriorating items with imperfect production, inspection errors, rework and shortages: a type-2 fuzzy approach. *Opsearch*, *56*(3), 657-688.
[30] Salameh, M. K., & Jaber, M. Y. (2000). Economic production quantity model for items with imperfect quality. *International journal of production economics*, *64*(1-3), 59-64.
[31] Montgomery, D. C., Bazaraa, M. S., & Keswani, A. K. (1973). Inventory models with a mixture of backorders and lost sales. *Naval Research Logistics Quarterly*, *20*(2), 255-263.
[32] Zipkin P.H (2000). Foundations of Inventory Management. *New York: McGrawHill*
[33] Pentico, D. W., & Drake, M. J. (2009). The deterministic EOQ with partial backordering: a new approach. *European Journal of Operational Research*, *194*(1), 102-113.
[34] Pentico, D. W., Drake, M. J., & Toews, C. (2009). The deterministic EPQ with partial backordering: a new approach. *Omega*, *37*(3), 624-636.
[35] Hsieh, T. P., & Dye, C. Y. (2012). A note on "The EPQ with partial backordering and phase-dependent backordering rate". *Omega*, *40*(1), 131-133.
[36] Taleizadeh, A. A., Pentico, D. W., Aryanezhad, M., & Ghoreyshi, S. M. (2012). An economic order quantity model with partial backordering and a special sale price. *European Journal of Operational Research*, *221*(3), 571-583.
[37] Karimi-Nasab, M., & Wee, H. M. (2015). An inventory model with truncated exponential replenishment intervals and special sale offer. *Journal of Manufacturing Systems*, *35*, 246-250.
[38] Salehi, H., Taleizadeh, A. A., & Tavakkoli-Moghaddam, R. (2016). An EOQ model with random disruption and partial backordering. *International Journal of Production Research*, *54*(9), 2600-2609.
[39] Duffuaa, S. O., & Khan, M. (2005). Impact of inspection errors on the performance measures of a general repeat inspection plan. *International Journal of Production Research*, *43*(23), 4945-4967.
[40] Chiu, S. W. (2010). Robust planning in optimization for production system subject to random machine breakdown and failure in rework. *Computers & Operations Research*, *37*(5), 899-908.
[41] Al-Salamah, M. (2016). Economic production quantity in batch manufacturing with imperfect quality, imperfect inspection, and destructive and non-destructive acceptance sampling in a two-tier market. *Computers & Industrial Engineering*, *93*, 275-285.
[42] Taleizadeh, A. A., Niaki, S. T. A., & Najafi, A. A. (2010). Multiproduct single-machine production system with stochastic scrapped production rate, partial backordering and service level constraint. *Journal of Computational and Applied Mathematics*, *233*(8), 1834-1849.
[43] Sarker, B.R., Jamal, A.M.M., & Mondal, S. (2008). Optimal batch sizing in a process at multi-stage production system with network consideration. *European Journal of Operational research*, *184*, 915-929.
[44] Chung, K. J., Her, C. C., & Lin, S. D. (2009). A two-warehouse inventory model with imperfect quality production processes. *Computers & Industrial Engineering*, *56*(1), 193-197.
[45] Hu, J. S., Zheng, H., Guo, C. Y., & Ji, Y. P. (2010). Optimal production run length with imperfect production processes and backorder in fuzzy random environment. *Computers & Industrial Engineering*, *59*(1), 9-15.
[46] Khan, M., Jaber, M. Y., Guiffrida, A. L., & Zolfaghari, S. (2011). A review of the extensions of a modified EOQ model for imperfect quality items. *International Journal of Production Economics*, *132*(1), 1-12.
[47] Sadjadi, S. J., Yazdian, S. A., & Shahanaghi, K. (2012). Optimal pricing, lot-sizing and marketing planning in a capacitated and imperfect production system. *Computers & Industrial Engineering*, *62*(1), 349-358.
[48] Cárdenas-Barrón, L. E., Taleizadeh, A. A., & Treviño-Garza, G. (2012). An improved solution to replenishment lot size problem with discontinuous issuing policy and rework, and the multi-delivery policy into economic production lot size problem with partial rework. *Expert Systems with Applications*, *39*(18), 13540-13546.
[49] Wee, H. M., Wang, W. T., & Cárdenas-Barrón, L. E. (2013). An alternative analysis and solution procedure for the EPQ model with rework process at a single-stage manufacturing system with planned backorders. *Computers & Industrial Engineering*, *64*(2), 748-755.
[50] Khan, M., Jaber, M. Y., & Ahmad, A. R. (2014). An integrated supply chain model with errors in quality inspection and learning in production. *Omega*, *42*(1), 16-24.
[51] Khanna, A., Kishore, A., & Jaggi, C. (2017). Strategic production modeling for defective items with imperfect inspection process, rework, and sales return under two-level trade credit. *International Journal of Industrial Engineering Computations*, *8*(1), 85-118.
[52] Al-Salamah, M. (2019). Economic production quantity in an imperfect manufacturing process with synchronous and asynchronous flexible rework rates. *Operations research perspectives*, *6*, 100103.
[53] Lee, H. L., & Rosenblatt, M. J. (1985). Optimal inspection and ordering policies for products with imperfect quality. *IIE transactions*, *17*(3), 284-289.
[54] Porteus, E. L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Operations research*, *34*(1), 137-144.
[55] Chan, W. M., Ibrahim, R. N., & Lochert, P. B. (2003). A new EPQ model: integrating lower pricing, rework and reject situations. *Production Planning & Control*, *14*(7), 588-595.
[56] Chiu, Y. P. (2003). Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. *Engineering optimization*, *35*(4), 427-437.
[57] Tsou, J. C., Hejazi, S. R., & Barzoki, M. R. (2012). Economic production quantity model for items with continuous quality characteristic, rework and reject. *International Journal of Systems Science*, *43*(12), 2261-2267.
[58] Jaggi, C. K., Khanna, A., & Kishore, A. (2016, March). Production inventory policies for defective items with inspection errors, sales return, imperfect rework process and backorders. In *AIP Conference Proceedings* (Vol. 1715, No. 1, p. 020062). AIP Publishing LLC.
[59] Selvaraju, P., & Ghuru, S. K. (2018). Imperfect production system with rework and scrap at a single stage manufacturing system and integrates cost reduction delivery policy. *International Journal of Operational Research*, *32*(4), 472-494.
[60] Nobil, A. H., Afshar Sedigh, A. H., & Afshar-Nadjafi, B. (2020). Lot-sizing problem for a defective processing system with categorized items, backordering and pricing policy. *Journal of Revenue and Pricing Management*, *19*(4), 255-265.

[61] Yoo, S. H., Kim, D., & Park, M. S. (2009). Economic production quantity model with imperfect-quality items, two-way imperfect inspection and sales return. *International Journal of Production Economics*, *121*(1), 255-265.

[62] Wee, H. M., & Widyadana, G. A. (2013). A production model for deteriorating items with stochastic preventive maintenance time and rework process with FIFO rule. *Omega*, *41*(6), 941-954.

[63] Zhou, Y. W., Chen, J., Wu, Y., & Zhou, W. (2015). EPQ models for items with imperfect quality and one-time-only discount. *Applied Mathematical Modelling*, *39*(3-4), 1000-1018.

[64] Paknejad, J., Nasri, F., & Affisco, J. F. (2005). Quality improvement in an inventory model with finite-range stochastic lead times. *Journal of Applied Mathematics and Decision Sciences*, *2005*(3), 177-189.

[65] Miranda, S., Vilela, J., & Leiras, A. (2020, July). Economic Production Quantity Model Considering Items with Distinct Levels of Imperfection. In *International Joint conference on Industrial Engineering and Operations Management* (pp. 537-547). Springer, Cham.

[66] Roy, M. D., Sana, S. S., & Chaudhuri, K. (2011). An economic order quantity model of imperfect quality items with partial backlogging. *International journal of systems Science*, *42*(8), 1409-1419.

[67] Yu, J. C., Wee, H. M., & Chen, J. M. (2005). Optimal ordering policy for a deteriorating item with imperfect quality and partial backordering. *Journal of the Chinese institute of industrial engineers*, *22*(6), 509-520.

[68] Hayek, P. A., & Salameh, M. K. (2001). Production lot sizing with the reworking of imperfect quality items produced. *Production planning & control*, *12*(6), 584-590.

[69] Sarkar, B., Cárdenas-Barrón, L. E., Sarkar, M., & Singgih, M. L. (2014). An economic production quantity model with random defective rate, rework process and backorders for a single stage production system. *Journal of Manufacturing Systems*, *33*(3), 423-435.

[70] Eroglu, A., & Ozdemir, G. (2007). An economic order quantity model with defective items and shortages. *International journal of production economics*, *106*(2), 544-549.

[71] Jamal, A. M. M., Sarker, B. R., & Mondal, S. (2004). Optimal manufacturing batch size with rework process at a single-stage production system. *Computers & industrial engineering*, *47*(1), 77-89.

[72] Taleizadeh, A. A., Sadjadi, S. J., & Niaki, S. T. A. (2011). Multiproduct EPQ model with single machine, backordering and immediate rework process. *European Journal of Industrial Engineering*, *5*(4), 388-411.

[73] Hsu, J. T., & Hsu, L. F. (2013). Two EPQ models with imperfect production processes, inspection errors, planned backorders, and sales returns. *Computers & Industrial Engineering*, *64*(1), 389-402.

[74] Taleizadeh, A. A., Cárdenas-Barrón, L. E., & Mohammadi, B. (2014). A deterministic multi product single machine EPQ model with backordering, scraped products, rework and interruption in manufacturing process. *International Journal of Production Economics*, *150*, 9-27.

[75] Cheng, Y. L., Wang, W. T., & Wei, C. C. (2018). Optimal production lot sizing when demand is proportional to stock and backorder levels. *International Journal of Industrial Engineering*, *25*(2).

[76] Najafi, M., Ghodratnama, A., & Pasandideh, H. R. (2018). Solving a deterministic multi product single machine EPQ model withpartial backordering, scrapped products and rework. *International Journal of Supply and Operations Management*, *5*(1), 11-27.

[77] Taleizadeh, A. A., Yadegari, M., & Sana, S. S. (2019). Production models of multiple products using a single machine under quality screening and reworking policies. *Journal of Modelling in Management*, *14*(1): 232-259.

[78] Lai, X., Chen, Z., & Bidanda, B. (2019). Optimal decision of an economic production quantity model for imperfect manufacturing under hybrid maintenance policy with shortages and partial backlogging. *International Journal of Production Research*, *57*(19), 6061-6085.

[79] Khalilpourazari, S., Mirzazadeh, A., Weber, G. W., & Pasandideh, S. H. R. (2019). A robust fuzzy approach for constrained multi-product economic production quantity with imperfect items and rework process. *Optimization*.

[80] Nobil, A. H., Afshar Sedigh, A. H., Tiwari, S., & Wee, H. M. (2019). An imperfect multi-item single machine production system with shortage, rework, and scrapped considering inspection, dissimilar deficiency levels, and non-zero setup times. *Scientia Iranica*, *26*(1), 557-570.

[81] Silva, M. A., de Paula, I., & Leiras, A. (2020, July). EPQ Model with Partial Backordering Considering Environmental Aspects and Stochastic Demand. In *International Joint conference on Industrial Engineering and Operations Management* (pp. 115-126). Springer, Cham.

[82] Taleizadeh, A. A. (2017). Lot-sizing model with advance payment pricing and disruption in supply under planned partial backordering. *International Transactions in Operational Research*, *24*(4), 783-800.

[83] Pentico, D. W., Toews, C., & Drake, M. J. (2015). Approximating the EOQ with partial backordering at an exponential or rational rate by a constant or linearly changing rate. *International Journal of Production Economics*, *162*, 151-159.

[84] Leiras, A., Ribas, G., Hamacher, S., & Elkamel, A. (2013). Tactical and operational planning of multirefinery networks under uncertainty: an iterative integration approach. *Industrial & Engineering Chemistry Research*, *52*(25), 8507-8517.

[85] Ribas, G. P., Leiras, A., & Hamacher, S. (2012). Operational planning of oil refineries under uncertainty Special issue: applied stochastic optimization. *IMA Journal of Management Mathematics*, *23*(4), 397-412.

Author (s) Information

Eveline Libânio Zidan, M.Sc., Department of Industrial Engineering, Pontifical Catholic University of Rio de Janeiro.

Ludmylla da Silva Moreira, M.Sc., Department of Industrial Engineering, Pontifical Catholic University of Rio de Janeiro.

Fabíola Negreiros de Oliveira, M.Sc., Department of Industrial Engineering, Pontifical Catholic University of Rio de Janeiro.

Adriana Leiras, Professor, Department of Industrial Engineering, Pontifical Catholic University of Rio de Janeiro.