



ABSTRACT

Anaerobic digestion (AD) of biowastes is one of the most common ways to produce methane-rich biogas, which has considerable potential to replace the fossil fuel used in multiple applications, such as vehicular transportation, internal combustion engines, cogeneration of heat and power systems and many other systems. Many companies are involved in the design and construction of anaerobic digestion systems. Empirical methods have been used to improve AD facilities, but these have needed time-consuming studies and construction of expensive prototype systems. On the other hand, design and optimization of AD processes for biogas production can be enhanced via validated mathematical models. In this paper a dynamic mathematical model has been developed to a pilot anaerobic reactor fed dairy cow / cattle manure. The model is based upon material balances and comprises four state variables, namely biodegradable volatile solids, acid generating microbes (acidogens), methane generating microbes (methanogens) and volatile fatty acids. The model predicts the methane gas flow produced in the reactor. At the end, a sensitivity analysis is done to show how the gas flow rate, maximum reaction rate of acidogens, maximum reaction rate of methanogens, reactor temperature and also reactor volume.

KEY WORDS anaerobic digestion, biogas, livestock waste, mathematical model.

INTRODUCTION

In recent years, the expanding use of biomass as an energy source forms a major part of the global energy system. By increasing its use as a feedstock, biomass contributes in reducing carbon dioxide emissions and other pollutants that cause the global warming (Sawatdeenarunat *et al.* 2015). Bioenergy can play a central role in promoting renewable alternatives. In fact, bioenergy is estimated to be the fourth largest energy resource in the world (Chunlan *et al.* 2015). Long-term economic and environmental concerns that offers the prospect of replacing fossil fuels in the transporta-

tion sector and limiting the net greenhouse gas emissions implicated and bioenergy to replace fossil fuels in the past decades (Magnusson and Alvfors, 2012). Ruminant animals (cattle, buffalo, sheep, goat and camel) produce significant amounts of methane as part of their normal digestive process, a large proportion of methane produced by the manure of these animals can be captured. Livestock manure keeps releasing methane due to the anaerobic decomposition of organic material contained in the manure by bacteria exited along with the manure from the animal manure deposited on fields and pastures also produces significant amounts of methane. Manure lagoons and holding tanks, which are commonly used at larger dairy and swine operations, also release significant quantities of methane (Tauseef *et al.* 2013). Biogas, which is generally referring to gas from anaerobic digestion units, is a promising means of addressing global energy needs and providing multiple environmental benefits, as shown in Table 1 (Tambone *et al.* 2010; Rehl and Müller, 2011; Jiang *et al.* 2011).

 Table 1
 Biogas environmental benefits analysis (Tambone *et al.* 2010;

 Rehl and Müller, 2011; Jiang *et al.* 2011)

Biogas	Corresponding contents
Green energy	Electricity
Production	Heat Vehicle fuel Tri-generation
	Agricultural residues
Organic waste dis-	Industrial wastes
posal	Household wastes
-	Organic waste mixtures
Environmental	Pathogen reduction through sanitation
	Less nuisance from insect flies
	Air and water pollution reduction
Protection	Eutrophication and acidification reduction
	Forest vegetation conservation Replacing inorganic fertilizer
Greenhouse gas (GHG) emission reduction	Substituting conventional energy sources

Methane (CH₄) is an atmospheric greenhouse gas (GHG) produced by methanogenic Archaea (methanogens) in diverse anaerobic environments, such as waterlogged soil and the digestive tract of animals (Elsgaard *et al.* 2016). Anaerobic digestion (AD) is becoming an important industrial process in all societies. Anaerobic digestion (AD) is a set of processes by which microorganisms break down biodegradable material in the absence of oxygen including the generation of methane-rich biogas via the biological degradation of regionally available biomass like agricultural and municipal solid wastes and waste waters.

Anaerobic digestion (AD) technology has been extensively adopted by Germany and Denmark, which have implemented rigorous waste disposal legislation. Since 2000, annual electricity generation from AD projects in the USA has increased almost 25-fold from 14 million kilowatthours (kWh) to an estimated 331 million kWh per year (Environmental Protection Agency U.S., 2010).

Anaerobic digestion of different organic wastes by microbes, involving the members of the bacteria, appears to be the effective biotechnological method to convert bio wastes into bioenergy. Various agricultural and industrial waste materials can be anaerobically converted to energy-rich methane by complex microbial consortia. Most of the existing facilities for agricultural biogas digestion of chicken, cow, and pig manure are completed with the cooperation of media to increase the content of organic functional materials and supplies (Appels *et al.* 2008). The co-substrates are typically harvest residues (e.g. sugar beet leaves and tops), agricultural organic wastes (e.g. energy crops), urban food waste and organic waste collected from restaurants and families (Appels *et al.* 2008; Cakir and Stenstrom, 2005).

Digestibility of substrates and biogas production are affected by substrate salinity, loading rate, mineral and volatile fatty acid composition, carbon-to-nitrogen ratio, and pH, as well as reactor temperature and hydraulic retention time (HRT) (Krzystek *et al.* 2001; Novak *et al.* 2003; Sanchez *et al.* 2006). Understanding the mechanisms and kinetics of anaerobic rumen environment for the design and comparison with solid reactor, where operating conditions, methane (CH₄) production, stability and effluent quality requirements can be predicted or identified.

As mentioned before, biogas plants systems use a bacteriological process called anaerobic digestion to convert organic waste into biogas. Biogas is a clean energy source that may be converted to heat, electricity or bio-fuel for automotive applications. Figure 1 shows the vast majority of this waste comes from the agricultural sector.



Figure 1 Different waste comes from the agricultural sector

Biogas that is composed of methane (CH₄), carbon dioxide (CO₂) and various other gases, can also be refined into bio methane and injected into the existing natural gas network. Unlike natural gas, bio methane is a clean and renewable carbon-neutral fuel. The typical composition of anaerobic digestion raw biogas is shown in Table 2.

Nowadays, a significant fraction of the non-agricultural organic waste produced in the world is land filled due to a lack of alternative disposal outlets. It was estimated that only 85% (2.8 M tones/year) of this organic waste would be readily available for anaerobic digestion. Moreover, biogas plants are capable of efficiently converting energy crops

into biogas energy. Figure 2 shows energy potential of available material (Electrigaz Technologies Inc, 2007).

Table 2 Typical	acomposition	of anarohio	digastion	row biogos
Table 2 Typical	composition	of anaerobic	urgestion	raw biogas

Gas features	Composites/formula	Percentage in biogas
Methane	CH_4	50%-80%
Carbon dioxide	CO_2	20%-50%
Ammonia	NH_3	0-300 PPM
Hydrogen Sul- phide	H_2S	50-5000 PPM
Nitrogen	N_2^*	1-4%
Oxygen	O_2^*	< 1%
Water vapor	H_2O	Saturated 2-5% (mass)

* Only present if air is injected into the digester for H₂S reduction.



Figure 2 Energy potential of available material

Various models have been proposed to provide more indepth understanding of the biological and chemical mechanisms affecting the AD process. Since the primary digester dynamic mathematical models of the late 1960 s (Andrews, 1968; Graef and Andrews, 1974), additional and more sophisticated models have been developed to account for the interaction of biological and inhibition (Stamatelatou *et al.* 2009; Batstone *et al.* 2002; Vavilin *et al.* 2007).

Ohuchi *et al.* (2015) studied the thermophilic anaerobic codigestion of sugar beet tops silage (SBT) with dairy manure (DM) at four SBT silage proportions. The highest methane yield of 422 mL/g volatile solids (VS) and VS reduction of 57%, were obtained when the mixture contained the lowest SBT proportion (40%) while the system failure was observed for the highest SBT proportion (Ohuchi *et al.* 2015). Cisneros *et al.* (2015) proposes an extremum seeking control approach based on sliding mode to achieve the dynamic optimization of methane outflow rate in anaerobic digestion processes. Open-loop analysis

for a two population models have shown that the system becomes unstable due at the accumulation of volatile fatty acids (VFA). Then the controller is designed to achieve the regulation of VFA concentration close to the optimal setpoint while maximizing the methane production (Cisneros *et al.* 2015).

Increasingly sophisticated models for metabolic reactions have been developed with the recognition of various microbial groups and substrates in AD system (Angelidaki et al. 1999; Batstone et al. 2002). Nevertheless, the difficulty in identifying the composition of complex, undefined substrates led to previous attempts to simulate co-digestion of various wastes that for readily were assumed to be defined by general compositions. For instance, water waste was considered as carbohydrates, proteins, fats, and others have been formed (Jeyaseelan, 1997). More advanced course of AD and stoichiometric matrix based on these assumptions (Tomei et al. 2009) were presented in the following (Angelidaki et al. 1999) model and Anaerobic Digestion Model No.1 (ADM1) model (Batstone et al. 2002). Various theoretical and practical aspects of AD processes are described (Tchobanoglous et al. 2003; Deublein and Steinhauser, 2010; Husain, 2012). Foss Biolab, (Haugen et al. 2013a), is a pilot biological plant at Foss dairy farm in Skien, Norway, for nutrient and energy recovery from animal waste. The aims of this paper are to adapt a dynamic mathematical model of the AD processes of the reactor that able to predict the methane gas flow produced in the reactor and to adapt a dynamic model able to predict the reactor temperature (Haugen et al. 2013a).

MATERIALS AND METHODS

Throughout this paper the modified version of Hill's AD model developed by Haugen et al. (2013a) is used to predict the biogas produced from cow manure in an agricultural farm in Iran-Zanjan, with a total of 3000 dairy cows. Data required for running the model are obtained from laboratory analysis and online-data from sensors. Samples for laboratory analysis have been taken regularly from the reactor since August 2014. A number of different variables characterizing the reactor influent and effluent are analyzed. Among these, concentration of volatile solids (VS) and concentration of total volatile fatty acids (VFA) are used for model adaptation in the present study. Online-data include feed flow (load rate), reactor temperature, ambient (air) and feed temperature (assumed to be the same and therefore measured with one sensor), biogas flow, and methane gas concentration.

Mathematical model of AD

Modified Hill's AD model (Haugen *et al.* 2013a) is chosen for prediction of biogas flow as it satisfies all the criteria and because it is simpler than the comparable models. Hill's model (Haugen *et al.* 2013a) includes equations 1 to 12 below. Equations of the model in the Hill's model defining that portion of the raw waste which can serve as substrate:

$$S_{bvs \text{ in}} = B_0 S_{vs \text{ in}} \quad (1)$$

Where:

 $S_{bvs in}$: concentration of BVS in influent [g BVS / L]. B_0 : biodegradability constant [(g BVS/L) / (g VS/L)]. $S_{vs in}$: concentration of volatile solids in influent [g VS / L].

Defining that portion of the biodegradable material that is initially in the acid form:

 $S_{vfa \text{ in}} = A_f S_{bvs \text{ in}}$ (2) Where:

 $S_{vfa \text{ in}}$: concentration of VFA in biodegradable part of influent [g VFA / L].

 A_f : acidity constant [(g VFA/L) / (g BVS/L)]. $S_{bvs in}$: concentration of BVS in influent [g BVS / L].

Mass balance of biodegradable volatile solids is defined as:

$$S_{bvs} = (S_{bvs \text{ in}} - S_{bvs}) \times (F_{feed}/V) - (\mu k_1 X_{acid}) \quad (3)$$

Where:

 S_{bvs} : concentration of BVS in reactor [g BVS / L]. $S_{bvs in}$: concentration of BVS in influent [g BVS / L]. F_{feed} : influent or feed flow or load rate, assumed equal to effluent flow (constant volume) [L / d]. V: effective reactor volume [L].

 μ : reaction (growth) rate of acidogens [d⁻¹]. k1: a yield constant [g BVS / (gacidogens/L)].

 X_{acid} : concentration of acidogens [gacidogens / L].

Mass balance of total VFA is defined as:

$$S_{vfa} = (S_{vfa \text{ in}} - S_{vfa}) \times (F_{feed}/V) - (\mu k 2X_{acid}) - (\mu c k 3X_{meth})$$
(4)

Where:

 S_{vfa} = concentration of VFA acids in reactor [g VFA / L]. $S_{vfa \text{ in}}$: concentration of VFA in biodegradable part of influent [g VFA / L].

 F_{feed} : influent or feed flow or load rate, assumed equal to effluent flow (constant volume) [L / d].

V: effective reactor volume [L].

 μ : reaction (growth) rate of acidogens [d⁻¹].

K2: a yield constant [g VFA / (g acidogens/L)].

 X_{acid} : concentration of acidogens [gacidogens / L].

 μc : reaction (growth) rate of methanogens [d⁻¹].

K3: a yield constant [g VFA / (g methanogens / L)].

 X_{meth} : concentration of methanogens [g methanogens / L]. Mass balance of acidogens is shown in equation 5:

Where:

 $\begin{aligned} X_{acid} &= (\mu - K_d - (F_{feed}/b)/V)) \times X_{acid} \quad (5) \\ X_{acid} &: \text{ concentration of acidogens [gacidogens / L].} \\ \mu : \text{ reaction (growth) rate of acidogens [d⁻¹].} \\ K_d &: \text{ specific death rate of acidogens [d⁻¹].} \\ F_{feed} &: \text{ influent or feed flow or load rate, assumed equal to effluent flow (constant volume) [L / d].} \\ b &: \text{ retention time ratio [d / d].} \\ V &: \text{ effective reactor volume [L].} \end{aligned}$

Equations 6 and 7 indicate the mass balance of methanogens and Methane gas flow rate (gas production) respectively.

 $X_{meth} = (\mu c - K_{dc} - (F_{feed}/b)/V)) \times X_{meth} \quad (6)$

Where:

 μc : reaction (growth) rate of methanogens [d⁻¹]. K_{dc} : specific death rate of methanogens [d⁻¹]. F_{feed} : influent or feed flow or load rate, assumed equal to effluent flow (constant volume) [L / d]. b: retention time ratio [d / d]. V: effective reactor volume [L].

 X_{meth} : concentration of methanogens [g methanogens / L].

$$F_{meth} = V \mu c K 5 X_{meth} \quad (7)$$

Where: *V*: effective reactor volume [L].

 μc : reaction (growth) rate of methanogens [d⁻¹].

K5: a yield constant [L / g methanogens].

 X_{meth} : concentration of methanogens [g methanogens / L].

Where the reaction rates, with Monod kinetics, are defined as follows:

$$\mu = \mu m (S_{bvs} / (Ks + S_{bvs})) \quad (8)$$

Where:

 μ : reaction (growth) rate of acidogens [d⁻¹]. μ *m*: maximum reaction rate for acidogens [d⁻¹]. S_{bvs} : concentration of BVS in reactor [g BVS / L]. *Ks*: monod half-velocity constant for acidogens [g BVS / L].

$$\mu c = \mu m c (S_{vfa} / (Ksc + S_{vfa})) \quad (9)$$

Where:

 μc = reaction (growth) rate of methanogens [d⁻¹]. μmc : maximum reaction rate for methanogens [d⁻¹]. S_{vfa} : concentration of VFA acids in reactor [g VFA / L]. *Ksc*: monod half-velocity constant for methanogens [g VFA / L].

The maximum reaction rate μm , μmc which are functions of the reactor temperature are shown in equation 10.

 $\mu m(T_{reac}) = \mu mc(T_{reac}) = 0.013T_{reac} - 0.129 \quad (10)$ $(20 \ ^{\circ}C < T_{reac} < 60 \ ^{\circ}C)$

Where:

 μm : maximum reaction rate for acidogens [d⁻¹]. μmc : maximum reaction rate for methanogens [d⁻¹]. T_{reac} : reactor temperature [°C].

In the original Hill's model the retention time of the biomass (here:acidogens and methanogens) is equal to the hydraulic retention time (HRT) as indicated in equation 11, but since the retention time of the biomass is larger than the hydraulic retention time in up-flow sludge bed reactors such as applied in this study, where biomass is conserved by gravity (Haugen *et al.* 2013a), the retention time ratio b is introduced. The biomass retention time, which is denoted the solids retention time (SRT), is obtained using 12 equation.

$$T_{hr} = (V/F_{feed}) = HRT \quad (11)$$

Where:

 T_{hr} : hydraulic retention time. V: effective reactor volume [L]. F_{feed} : influent or feed flow or load rate, assumed equal to effluent flow (constant volume) [L / d].

HRT: hydraulic retention time.

$$T_{br} = bT_{hr} = (bV/F_{feed}) = (V/(F_{feed}/b) = SRT \quad (12)$$

 T_{br} = biomass retention time. b: retention time ratio [d/d]. T_{hr} : hydraulic retention time. V: effective reactor volume [L]. F_{feed} : influent or feed flow or load rate, assumed equal to effluent flow (constant volume) [L / d]. *SRT*: solids retention time.

Above it is supposed that VFA is total volatile fatty acid consisting mainly of propionate, vale rate, butyrate and acetate. The main component of VFA is acetate that is used in methanogens which is the main methane-generating process. Methane is also produced in hydrogen trophic methanogens. Hydrogen is produced from various components including the VFA components propionate, vale rate and butyrate. In order to include effects of the hydrogen trophic methanogens, S_{vfa} in the model represents total VFA. Figure 3 shows an overall block diagram displaying the parameters and variables of the Hill's model.

Table 3 shows the values of inputs and states in the relevant steady-state operation point which is t = 66 d, that used for model adaption.

 Table 3 Parameters in Hill's model adapted to AD reactor [24]

Assumed data	Quantity	Assumed data	Quantity
K _d	0.02 (L/d)	K _{meth}	0.5
K_{dc}	0.02 (L/d)	\mathbf{K}_{sc}	3
b	2.90	V (reactor design)	250 (m ³)
$\mathbf{X}_{\mathrm{acid}}$	1.32 (g/L)	\mathbf{K}_1	3.89
\mathbf{X}_{meth}	0.36 (g/L)	K ₂	1.76
	0.69		
A_{f}	(gVFA/L) /	K ₃	31.7
	(gBVS/L)		
_	0.25		
\mathbf{B}_{0}	(gVFA/L) /	K_5	15.4
-	(gBVS/L)		
F _{feed}	45(L/d)	Ks	15.5
T_{reac}	35 °C	S_{bvs}	5.21 (g/L)
S_{vfa}	1.0094 (g/L)	${S}_{vs}$ in	30.2 (g/L)



Figure 3 Overall block diagram of the modified Hill's mode (Haugen *et al.* 2013a)

RESULTS AND DISCUSSION

The modified Hill's model which has been adapted to the pilot bioreactor fed dairy manure is a relatively simple model compared with alternative models since the model does contain neither ammonia, alkalinity, nor pH as variables. These variables are more important in reactors fed manure from swine or poultry because their values may have higher impact on the stability of such reactors (Haugen *et al.* 2013b).

The modified Hill's AD model is supposed to be sufficiently accurate as a basis for optimal reactor design and operation, control and state-estimation for a reactor fed dairy manure where the main output is methane gas flow. In applications requiring a prediction of hydrogen or carbon dioxide gas production alternative models must be used (Haugen *et al.* 2013b).

The parameters B_0 (biodegradability constant) and A_f (acidity constant) are estimated from data from one experiment only. Ideally, more experimental data should have been used (Haugen *et al.* 2013b).

The steady-state data used for the model adaptation are reactor temperature, methane gas flow, feed flow (loading rate) and laboratory analysis values of in fluent and effluent VFA and VS concentrations at one specific steady-state operating point.

It should be noted that, data used for modeling are online-data from laboratory analysis and sensors.

Using modified Hill's AD model and data obtained from the animal farm, it is determined that the optimum methane gas flow produced in the reactor is 376.53 (L CH₄/d) when the volume of reactor is 250 cubic meters and the reactor temperature is 35 °C (other parameters required for modeling are shown in Table 3). The results of the estimation of gas flow using modified Hill's model are shown in Table 4.

Methane production in cattle manure showed a typical mesophilic temperature response with an optimum around 35 °C after 17 h of incubation. The shorter 3 h incubation resulted in a slightly higher optimum temperature (41 °C); this was interpreted as a transient metabolic response of mesophiles to temperatures above their normal optimum, a phenomenon observed also for other metabolic types of microorganisms.

Methane production had a thermophilic temperature response with an optimum temperature exceeding the highest temperature employed (52 °C), which was also the operating temperature of the digester, and thus, a thermophilic methanogenic community clearly predominated at this time. Yet, there was also CH₄ production at around 15 °C even though a typical span between minimum and optimum temperatures for growth of most microorganisms, including methanogens, is 20-30 °C.

While it has been documented that microorganisms may show activity at temperatures somewhat below the minimum temperature for growth, the response of CH_4 productio could also reflect the presence of both mesophilic and thermophilic methanogenic populations. In accordance with this interpretation, CH4 production showed an optimum temperature at 35-47 °C and a steep rate increase with temperature in the range from 30 to 40 °C, suggesting that successional changes took place during post-digestion storage favouring mesophilic populations of methanogens (Elsgaard *et al.* 2016).

The results shown in Table 4 indicate that the amount of methane flow rate will increase with the increase of the reactor temperature. At low temperature (20-40 °C) methane flow rate greatly increased with a small increase in temperature. On the other hand, methane production increasing rate is lower at higher temperatures (40-80 °C). Table 4 also reveals that maximum reaction rate of acidogens, maximum reaction rate for methanogens, reaction (growth) rate of acidogens and reaction (growth) rate of methanogens are increase with increasing the reactor temperature.

As the reactor may be operated at different temperatures, the present model should be able to represent the temperature dependency of the dynamics of the AD process. This relation is shown in Figure 4. Figure 4 shows the Change of methane flow rate with change of reactor temperature. The changes of some other parameters with respect to the reactor temperature are also indicated in Figure 5.

Table 4	Results of	estimation c	f gas t	flow rate and	other 1	parameters	with c	lifferent	reactor tem	perature

Reactor tem- perature (*C)	Methane flow rate (L CH ₄ /d)	Maximum reaction rate of acidogens [d ⁻¹]	Reaction (growth) rate of acidogens [d ⁻¹]	Reaction (growth) rate of methanogens [d ⁻¹]	Maximum reaction rate for methanogens [d ⁻¹]
20	151.31	0.131	0.0357	0.1091	0.131
25	226.38	0.196	0.0534	0.1633	0.196
30	301.45	0.261	0.07116	0.2175	0.261
35	376.53	0.326	0.088	0.2716	0.326
40	451.61	0.391	0.1066	0.3258	0.391
45	526.68	0.456	0.1243	0.38	0.456
50	601.76	0.521	0.142	0.434	0.521
55	676.83	0.586	0.1597	0.4883	0.586
60	751.9	0.651	0.1774	0.5425	0.651
65	826.98	0.716	0.1952	0.5966	0.716
70	902.05	0.781	0.2129	0.65083	0.781
75	977.13	0.846	0.23066	0.705	0.846
80	1052.2	0.911	0.2483	0.7591	0.911

In Figure 5, the simulated maximum reaction rate for acidogens (or methanogens), reaction (growth) rate of acidogens and reaction (growth) rate of methanogens based on the estimated model is plotted together with respect to different reactor temperature values. It seems that the change of reactor temperature has a direct effect on these parameters.

As a result, when the temperature raises the rate of these parameters will also increase. Comparing between these three component indicate that at the fixed temperature the maximum amounts of reaction (growth) rate of methanogens is higher than other parameters.



Figure 4 Change of methane flow rate with change of reactor temperature



Figure 5 Reaction rate of methanogens and acidogens changing with change of reactor temperature

Among many items that effect on the biogas production the volume of reactor is an important factor in predicting methane produced. In order to determine the effect of this parameter, it is essential to perform an analysis based on changes in reactor volume. So in this part, the volume of reactor changes from 100 to 1000 and methane flow rate changes are calculated. Table 5 has shown the intuitive results that methane flow rate as well as biomass retention time is quite sensitive to the volume of reactor. From Table 5 it can be observed that given a fixed reactor temperature (35 °C), methane flow rate and biomass retention time would begin to rise when the reactor volume is increased.

Figure 6 shows the change of methane flow rate with respect to the change of reactor temperature and volume, respectively.

 Table 5
 Change of methane flow rate and solids retention time with change of reactor volume_____

Volume of reactor	Methane flow rate	Solids retention time (SRT)	
100	150.61	6.44	
200	301.22	12.88	
300	451.84	19.33	
400	602.45	25.77	
500	753.06	32.22	
600	903.67	38.66	
700	1054.3	45.11	
800	1204.9	51.55	
900	1355.5	58	
1000	1506.1	64.44	

Changes of methane flow rate with change of reactor temperature



Figure 6 Change of methane flow rate with change of reactor temperature and volume

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From this figure it can be observed that, increasing reactor temperature and volume have the same result on increasing the methane gas flow rate, but, increasing the methane flow rate with the increase of reactor volume is higher than the increase of methane flow rate with the increase of reactor temperature. So it can be deduced that the volume of reactor has higher effect on producing methane flow rate.

CONCLUSION

In this paper a dynamic mathematical model has been developed to a pilot anaerobic reactor fed diary manure. Modified Hill's AD model is used to predict the produced biogas as it satisfies all the criteria and because it is simpler than comparable models. Data required for running the model are obtained from a cow farm in Iran, Zanjan. This model is based upon material balances, and comprises four state variables, namely biodegradable volatile solids, volatile fatty acids, acidogens and methanogens. Simulations indicate that the optimum methane gas flow produced in the reactor is 376.53 (L CH₄/d) when the volume of reactor is 250 cubic meters and the reactor temperature is 35 °C.

Moreover, using a sensitivity analysis the following conclusions can be deduced:

1) The amount of methane flow rate will increase with the increase of the reactor temperature. At low temperature methane flow rate greatly increased with a small increase in temperature. On the other hand, methane production increasing rate is lower at higher temperatures.

2) The maximum reaction rate of acidogens, maximum reaction rate for methanogens, reaction rate of acidogens and reaction rate of methanogens are increase with increasing the reactor temperature.

3) At the fixed temperature the maximum amounts of reaction (growth) rate of methanogens is higher than other parameters.

4) Given a fixed reactor temperature (35 °C), methane flow rate and biomass retention time would begin to rise when the reactor volume is increased.

Increasing the methane flow rate with the increase of reactor volume is higher than the increase of methane flow rate with the increase of reactor temperature.

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