



Dairy herd improvement (DHI) programs and national genetic evaluation at the individual and/or herds levels rely on adjusted 305 d lactation performance predicted by lactation curve functions. These functions are approximations of real curves. To find the best function, multi-step assessment of predicted lactation curves is required. The purpose of this study was to investigate four-step examination of two-parameter Pollott mechanistic function and compare it with two empirical functions (Wood and Wilmink) to choose the one that best suited individual lactation curves in Jersey and Simmental cattle populations, independently. Wilmink had the lowest BIC for both breeds, while the Pollott had the lowest AICc value (although the difference with other functions is negligible) and produced the most typical curves, so could be the best fit. Moreover, the correlations between curve parameters in the Pollott function were the lowest for both breeds; demonstrating the independence of the evaluated parameters and the strength of that. In the best function (Pollott), the mean and annual trends for the estimated total lactation milk yield were 6082.1 kg and 48.01 kg for Simmental, and 6747.9 kg and 148.33 kg for Jersey cows, respectively. Overall, our results confirm that the Pollott's mechanistic function outperforms the other two functions for fitting individual lactation curves. It is more robust in terms of: (1) maximum number of standard curves, (2) lowest AICc, (3) independent curve parameters, and (4) biological interpretation of typical curves. Therefore, it could be recommended for practical implications of fitting and standardization of test-day yield of these two breeds.

KEY WORDS dairy breed, empirical, mechanistic function, milk production.

INTRODUCTION

Milk production plays a great role in the dairy farm economy. Dairy Herd Improvement (DHI) programs and national genetic evaluation at the individual and/or herds levels (such as annual trend reports) are based on milk production. Different lengths of lactation result in fluctuations in milk yield and therefore, corrections should be made toward a305-daystandard record to improve the accuracy of the evaluations. In this regard, mathematical functions can be applied to model and predict changes like peak and persistency in a cow lactation curve (Macciotta *et al.* 2011). Different empirical and mechanistic (biological) mathematical functions have been proposed to illustrate the lactation curve of dairy cows in recent years (Macciotta *et al.* 2011). The empirical functions such as Wood (1967) and Wilmink (1987), rely on statistical analyses without considering lactation biology (e.g. Wood, 1967; Rook *et al.* 1993). On the contrary, mechanistic functions explain lactation curves based on the biology of the lactation process (e.g. Dijkstra *et al.* 1997; Pollott, 2000). Pollott's mechanistic function is based on two exponential parts: the first part represents the maximum milk secretion capacity from all parenchymal cells, and the second part is the relative decrease in the number of these cells over time (Pollott, 2000).

Lactation curve functions (models) are well-studied in Holstein cows in many countries, including America (Dematawewa et al. 2007), England (Dijkstra et al. 2010), China (Kong et al. 2018), as well as Iran (Hossein-Zadeh, 2016; Torshizi and Farhangfar, 2020). However, there is limited research available on the lactation curves of Jersey and Simmental cows. These breeds have low milk production compared to Holstein but are highly valued for their high milk content, reproductive efficiency, and longevity, making them favored for purebred and crossbred programs (Knob et al. 2019). Cankaya et al. (2011) suggested that, Wood function best fits the lactation curves of pasturedependent Jersey cows in Turkey. It was also superior in predicting milk yield of Simmental cows in the Czech Republic (Kopec et al. 2013; Kopec et al. 2021) and Slovenia (Jeretina et al. 2013). Nevertheless, comparisons among several empirical functions including Wood and Wilmink, neither of these two functions was proposed as the superior function in the first to third calving in Indian Jersey cows (Mohanty et al. 2019). Considering the importance of determining the most appropriate function to describe the lactation curve and the need to predict the adjusted 305-day milk production during lactation, and the lack of studies fitting milk production curve functions for Simmental (dual purpose) and Jersey cows in Iran, it seems it is necessary to calculate and compare different functions and choose the most suitable.

Nevertheless, only functions that are not only easy to use and well-fit, but also capable of estimating 305-day milk production with high accuracy based on available data and features are desirable and practical (Torshizi et al. 2011). Evaluation of the sufficiency of mathematical functions or models is possible only through a combination of various statistical analyses and a precise definition of the purposes for which the mathematical function was originally designed and developed. Therefore, incorrect use of the methods and techniques provided to compare and rank the mentioned functions may lead to unfavorable results for given scenarios (Tedeschi, 2006). In this regard, according to Burnham and Anderson (2002), the information criteria (AIC, BIC, AICc) as of goodness-of-fit measures are relative rather than absolute values to select one or more appropriate mathematical functions. For example, suppose AICc is used to rank functions; the absolute value of AICc of each function is unimportant, but the differences can be directly related to the information. These differences denoted as "delta" (in Burnham and Anderson, 2002; Anderson, 2008) and are the distance between the AICc value of the best (selected) function, which has the smallest AICc value, and the AICc value of ith function. In fact, such a difference defines the best model as always having delta =0. By this method, the AICc values are standardized by the AICc value of the best model. These differences apply when using AIC, BIC and AICc. This approach is based on information theoretic (I-T) framework which considers relative importance of the models versus the best model and gives us a better view and will reduce comparison bias. This study was conducted to examine and compare the two-parameter Pollott mechanistic function performance with two well-known empirical equations (Wood and Wilmink), based on individual lactation curves of Jersey and Simmental cows (each breed separately), considering 4 main steps: (1) the information criteria (especially AICc and the relative difference between functions), (2) lower rate of atypical elimination, (3) small number of parameters and independencies between estimated curve parameters, and (4) lactation curve biological interpretation.

MATERIALS AND METHODS

Data

Test-day (TD) milk yield records of primiparous Jersey and Simmental cows, calving between 2003 to 2020 were obtained from the National Animal Breeding Center of Iran (INABC). In general, cows with 2 or more and 5 or more TD records were kept during the first half and the entire lactation period, respectively. Outlier data were excluded for records from < 5 day and > 305 DIM, and the first TD record after 60 DIM. The final edited data included 6692 TD records belonging to 776 Jersey cows and 7659 TD records for 977 Simmental cows.

Lactation curve functions

Three mathematical functions were used to fit and estimate the parameters of lactation curves, based on each cow's TD milk production. A function with the following characteristics would be desirable: easy to apply, more likely to fit based on different biological patterns (biologically meaningful), less constrained by atypical lactation curves, and predictable for projected records and short lactations (Pollott, 2000; Macciotta *et al.* 2011). The equations describing the lactation curve and their properties are presented as follows:

(1) The Wood function (Wood, 1967), is computed as follows:

$$y_t = at^b e^{-ct}$$

Where:

yt: milk yield on day t of lactation (kg).

a, *b* and *c*: parameters that determine the shape and scale of the curve.

a: scaling factor to represent yield at the beginning of lactation.

b: inclining slope up to peak yield.

c: declining slope after peak (Macciotta *et al.* 2011; Abbasi *et al.* 2021).

(2) The quasi-linear Wilmink function (WIL; Wilmink, 1987), is as follows:

 $y_t = a + ct + b(e^{(-kt)})$

Where:

yt: milk yield on day t of lactation (kg).

a: associated with the level of production.

b and *c*: associated with the inclining and declining slope of the curves.

k: peak time and usually takes a fixed value (k=0.05; Wilmink, 1987; Torshizi *et al.* 2011). This implies that the function has only three parameters to be estimated.

(3) Pollott mechanistic function: Although the empirical models described above generates parameters by fitting equations, ultimately milk production is a biological process based on the number of secretory cells and the rate of secretion per cell during lactation. The two-parameter Pollott mechanistic (biological) function (Albarran-Portilo and Pollott, 2013; Abbasi *et al.* 2021) is computed as follows:

$$y_t = (b/1 + z \times e^{[-0.1 \times (t-150)]}) \times (2 - e^{[c \times (t-150)]})$$

Where:

yt: milk yield on day t of lactation (kg).

b: maximum secretion potential of the lactation (inclining slope).

c: relative decline in cell numbers as the lactation progressed (declining slope).

z=[(1-0.9999999)/0.9999999]

The function consisted of 2 logistic curves; the first curve accounted for the maximum value of the trait involved (e.g., milk) as a function of time (t). The second part determined the proportional reduction in cell numbers as lactation proceeded, which is the result of the relative death rate of cells as a function of time (Pollott, 2000; Albarran-Portilo and Pollott, 2013).

Statistical analyses

The iterative nonlinear curve fitting module of minpack.lm package (R software, Version 4.0.0; Elzhov et al. 2016) was used for function fitting. This module applies the Levenberg-Marquardt algorithm with a convergence criterion of 10⁻⁶. Estimates of the function parameters were obtained for each individual lactation curve. Strong correlation between the estimated curve parameters resulted in higher sensitivity to data distribution and imprecise estimates of curve parameters and predicted milk yield at different stages of lactation (Macciotta et al. 2011). Moreover, the independence of the estimated parameters from each other seems to be an important issue when fitting different equations. Variance inflation factor (VIF) is a measure to evaluate the multicollinearity and independence of response variable and the function parameters. The values above 5 imply dependencies and relationships between the parameters and weaknesses in the fitted function (Montgomery et al. 2021; Bang et al. 2022). The VIF of each parameter and correlation between parameters calculated with car and olsrr packages (R software, version 4.0.0; Hebbali and Hebbali, 2017).

Combination of 'b' and 'c' parameters used to select a typical (standard) curve shape. Parameters 'b' and 'c' greater than zero for Wood function (Macciotta *et al.* 2005; Abbasi *et al.* 2021), and parameters 'b' and 'c' smaller than zero for Wilmink function (Macciotta *et al.* 2005) were considered as a combination of parameters for selecting a typical curve. The results for Wood function parameter combinations and number of standard curves are as reported by Abbasi *et al.* (2021). A range of 0.1 to 85 for parameter 'b' and a range of 0.0001 to 0.1 for parameter 'c' were considered as a combination of parameters for selecting a typical curve with the Pollott function (Albarran-Portilo and Pollott, 2013).

In comparison, Abbasi *et al.* (2021) used a wider range of parameters suggested by Albarran-Portilo and Pollott (2013). To assess function strength, we compared predicted and actual curves by plotting mean weekly observed milk yield based on actual (or real) data against predicted lactation curves for fitted functions.

Information criteria including AIC, AICc (the corrected AIC) and BIC were calculated for each individual lactation curve and used as measures of goodness of fit for the analyzed function (model) to compare functions and determine which one could be recommended for fitting lactations. AICc is more accurate due to the limited number of TD records per cow and different number of parameters in the functions (Burnham and Anderson, 2002). AIC, AICc and BIC were calculated as:

AIC= N * ln (MSE)+2k AICc= AIC + [(2k(k+1)) / (n-k-1)]BIC= AIC - 2k + k × ln (N)

Where:

N: number of data points.

k: number of independent parameters in the function (Burnham and Anderson, 2002; Dematawewa *et al.* 2007).

This approach is based on I-T frame work which consider relative importance of the models versus the best model (Burnham et al. 2011). That is, we want the model from within the model set that loses the least information about full reality, hence, the model that is closest to full reality in the current model set. Generally, one computes AICc for each of the R models and selects the model with the smallest AICc value as "best" and obtains a ranking of the rest. Then the differences between AICc of each model (AICc_i) and the best (AICcbest) should be compared. For a better comparison and a guideline, when ranking the criteria, Burnham et al. (2011) proposed, if the difference between the information criterion for each function (or model) and the best-fitting function is 2 or less, it can be concluded that this difference is negligible; and if the difference is in range of 2 to 7, there is some support or still credibility and should rarely be dismissed (Burnham and Anderson, 2002). If the results are similar for different functions, the simpler(est) is best and preferred (SAS, 2004). For more details about I-T refer to Anderson (2008) and Burnham et al. (2011).

Phenotypic trend for typical curves of the preferred function was reported. Regression coefficient for estimated total lactation milk yield (305-d TMY) on animals' birth year used as phenotypic trend.

RESULTS AND DISCUSSION

Lactation curve functions are well known to provide useful information for DHI and genetic selection programs, especially in primiparous cows. The number or percent of individual typical curves or in other words, animals with typical curves is a crucial factor in comparing and choosing the best function (Macciotta *et al.* 2011; Abbasi *et al.* 2021). The highest percentage of fitted curves in Simmental cows produced by the Pollott function and was 82%. This means, the mentioned function is able to fit 82% of the individual curves accurately. This is about 16% more than Wilmink, that produced 66% typical curves and 11% more than Wood (71% typical curves; based on Abbasi *et al.* 2021) function. Abbasi *et al.* (2021) taking into account the wider range for Pollott function parameters (Albarran-Portillo and Pollott, 2008), reported about 85.2% of the curves as typical, which is approximately 3.2% more than the results of the present study. In order to confirm the results of the present study, the difference of 3.2% was also checked visually. The incidence of atypical lactation patterns in individual cows, which may partly account for poor function fits, can be up to 30% (Macciotta *et al.* 2011). The Pollott function seems less susceptible to the problem of fewer test-days at the onset of lactation and before the peak, as is the case with many commercial dairy farms (Albarran-Portillo and Pollott, 2008), so it could be a reason to generate more typical curves relative to the other two studied functions.

Table 3 shows the VIF of each parameter and correlation between parameters for the fitted functions on the typical curves. The VIF was the lowest for both breeds, close to 1, for the Pollott function; 1.2 and 1.07 for Simmental and Jersey cows, respectively. This may indicate the strength of this fitted function and the independence of the estimated parameters. Analogous to our results for parameters correlation, Albarran-Portillo and Pollott (2008) reported that the 'b' and 'c' parameters are not correlated in the Pollott function, and stated that this issue is important for two reasons. Firstly, it demonstrates that the two logistic curves used to construct the biological function were estimated independently of each other. This feature is deemed to be important in lactation curve functions but does not happen with the Wood function with the VIF value of 5.26 and 6.67 for Simmental and Jersey cows, respectively, for parameter 'b' (an evidence of dependency among the parameters). Secondly, using these independent parameters could lead to more accurate evaluations in milk production improvement, because it would allow independent selection for different phases and aspects of the lactation curve.

The information criteria (AIC, BIC, AICc) as measures of goodness-of-fit based on typical lactation curves for the studied functions in Simmental cows are presented in Table 1.

As mentioned earlier, these ranking criteria are relative rather than absolute (Burnham and Anderson, 2002). Hence, it is important to select one or more appropriate mathematical functions that can effectively describe the amount of milk production during lactation. As shown in Table 1, the Wilmink function is the best in terms of AIC and BIC for Simmental cows, but the differences with Wood (second) and Pollott (third) functions were 0.20 and 1.14 for AIC, and 0.19 and 1.11 for BIC, respectively. As a result, these differences are negligible (Burnham *et al.* 2011).

Function	Breed	Goodness-of-fit items						
		Ν	R^2	RMSE	AIC	BIC	AICc	
Wood	SM	620	0.62	1.84	20.89	21.02	30.43	
	JE	548	0.55	2.57	16.34	16.93	23.95	
Wilmink	SM	568	0.63	1.76	20.69	20.83	30.22	
	JE	508	0.57	2.54	15.61	16.20	23.20	
Pollott	SM	814	0.47	2.16	21.83	21.94	26.32	
	JE	638	0.40	3.06	17 59	18.03	21.37	

 Table 1
 Evaluation criteria for comparison of lactation fitted function in primiparous Simmental and Jersey cows (calving between 2003 to 2020)

 obtained from the National Animal Breeding Center of Iran (INABC)

N: number of typical individual lactation curves; R²: coefficient of determination; RMSE: square root of mean square error; AIC: Akaike information criterion; BIC: bayesian information criterion; AICc: corrected AIC; SM: Simmental and JE: Jersey.

 Table 2
 Means and standard deviation (SD) of estimated function parameters (a, b, and c) and Means with standard deviation (SD) of total milk yield (305-d TMY) for primiparous Simmental (SM) and Jersey (JE) cows (calving between 2003 to 2020) obtained from the INABC. Values adopted from the typical curves¹

Breed	Lactation	N		205 J TMV (SD)		
	function	19	a (SD) b (SD)		c (SD)	505-u-1M1 (SD)
SM	Wood	620	11.65 (7.63)	0.37 (0.52)	0.0051 (0.0055)	6082.0 (1504.0)
	Wilmink	568	29.3 (6.55)	-23.56 (25.84)	-0.051 (0.0343)	6052.2 (1599.8)
	Pollott	817	-	20.94 (4.59)	0.0018 (0.0015)	6082.1 (1459.1)
JE	Wood	548	14.44 (7.33)	0.23 (0.21)	0.003 (0.0021)	6744.3 (1391.0)
	Wilmink	508	28.84 (5.99)	-17.67 (17.51)	-0.036 (0.0224)	6727.3 (1437.67)
	Pollott	638	-	22.93 (4.55)	0.0013 (0.0009)	6747.9 (1380.4)

¹N: number of typical individual lactation curves and bold: the preferred lactation function.

 Table 3
 Variance inflation factor (VIF) of each parameter (a, b, and c), on the diagonal, and correlations between parameters, off diagonal, based on typical curves of the fitted functions for primiparous Simmental (SM) and Jersey (JE) cows(calving between 2003 to 2020) obtained from the INABC

	Parameter	Functions							
Breed		Wood			Wilmink			Pollott	
		а	b	с	а	b	с	а	b
	а	1.47	-0.57	-0.50	1.75	-0.65	-0.31	1.21	-0.43
SM	b	-0.57	5.26	0.89	-0.65	1.75	-0.29	-0.43	1.22
	с	-0.50	0.89	4.76	-0.31	-0.29	1.23	-	-
JE	а	2.94	-0.79	-0.56	1.59	-0.60	-0.22	1.07	-0.25
	b	-0.79	6.67	0.83	-0.60	1.57	0.24	-0.25	1.06
	с	-0.56	0.83	3.57	-0.22	0.24	1.08	-	-

On the other hand, as already mentioned, the AICc is more appropriate for comparing more complex functions with different numbers of parameters, and fewer data points or fewer individual TD records (Burnham and Anderson, 2002). Based on the results presented in Table 1, the Pollott function has the lowest AICc value and is the best, and its difference with the other two functions is 3.9 (Wilmink) and 4.11 (Wood), respectively, which could be some support and should rarely be ignored (Anderson, 2008). Consequently, based on: (1) the information criteria; (2) considering the biological importance of interpreting lactation curves (Albarran-Portillo and Pollott, 2013; Angeles-Hernandez et al. 2018); (3) the lower rate of atypical elimination, i.e. producing the higher number of typical curves (Silvestre et al. 2009), and (4) small number of parameters and independency between estimated curve parameters; the Pollott's mechanistic function is thought to be the most suitable for Simmental breed.

In accordance with our results, Rekaya *et al.* (2000) proposed that functions with a minimal number of parameters and the capacity for biological interpretation are most desirable.

The highest number of typical curve indicated that the highest value belonged to the Pollott function for Jersey cows, equal to 84%; which is about 20% and 26% larger than Wood (based on Abbasi *et al.* 2021) and Wilmink (58%) functions, respectively (Table 1). Regarding the AIC and BIC values, the results (Table 1) showed the lowest values for the Wilmink function, but the differences with the subsequent functions (i.e. Wood and Pollott) were very small and negligible. In contrast, the Pollott function offered the best fit according to AICc (the smallest value), with negligible differences were observed in this breed with respect to Akaike or Bayesian information criteria, but, similar with the Simmental breed, the Pollott function

outperformed given the biological significance of lactation curve interpretation (Albarran-Portillo and Pollott, 2013; Angeles-Hernandez *et al.* 2018), more typical curves and independencies between estimated curve parameters.

In the studies of Albarran-Portillo and Pollott (2008) and Albarran-Portillo and Pollott (2013), the two-parameter Pollott function used to fit the lactation curves for UK commercial dairy herds. They declare that the Pollott function performed better than Wood function in both dairy cattle and dairy sheep.

The mean values of the estimated parameters related to typical curves of the Pollott (the best function) and the other two functions for the studied breeds are shown in Table 2.The Wood and Wilmink parameter estimates for Jersey cattle are in good agreement with Torshizi *et al.* (2011) for Iranian Holstein cows; with their estimates for the 'a', 'b', and 'c' parameters of the Wood function were 14.98, 0.21, and 0.002, respectively, and were 35.23, -20.22, and -0.3636 for the Wilmink function (with 'K' equal to 0.05), respectively. In a study on first parity Jersey cows in Turkey, the estimated parameters of Wood function ('a', 'b', and 'c') were 13.13, 0.12, and 0.06, respectively, and were 27.11, -14.18, and -0.94 for Wilmink function, respectively (Cankaya *et al.* 2011).

In our study, regarding the Pollott function, the estimated value of the 'b' parameter (increasing slope) representing the maximum secretion potential or number of milksecreting cells during lactation, was equal to 20.94 and 22.93 for the Simmental and Jersey cows and the estimated value of the 'c' parameter (decreasing slope) indicating the relative reduction in the number of these cells and thus a decrease in milk secretion due to cell death, was equal to 0.0018 and 0.0013 in the Simmental and Jersey cows, respectively (Table 2). This latter parameter is also a feature related to the lactation persistency, with the lower the value, the greater the persistency of production. The estimated of 'b' and 'c' parameters in a study on first parity British Holstein cows were 30.1 and 0.001, respectively (Albarran-Portillo and Pollott, 2008). The parameters of individual lactation curves can be used as criteria for selecting animals with optimal curves (Albarran-Portillo and Pollott, 2008), and can also be evaluated as a separate genetic trait in genetic evaluations.

The basic variables that affect a part or the whole of a curve shape are defined as follows: rate of increase of yield from parturition to the peak, days or time at peak, peak yield and rate of decrease of milk production after the peak. From this point of view and to assess function strength, the lactation curve plotted based on the average actual TD data showed sensible differences with the curve plotted by the Wood function's predicted parameters and to some extent with the Wilmink function's plot (Figures 1 and 2 for Sim-

mental and jersey cows, respectively). On the other hand, it was in good agreement with the Pollott function (Figures 1 and 2), as expected according to the biological basis and the number of typical curves generated Figures 1 and 2 for Simmental and jersey cows, respectively).



Figure 1 Mean weekly observed milk yield (actual or real data) and predicted lactation curves based on Pollott (better fit), Wilmink (average fit) and Wood (poor fit) functions, for primiparous Simmental cows (calving between 2003 to 2020) obtained from the INABC. The mean shape parameters of the fitted functions are according to Table 2



Figure 2 Mean weekly observed milk yield (actual or real data) and predicted lactation curves based on Pollott (better fit), Wilmink (average fit) and Wood (poor fit) functions, for primiparous Jersey cows (calving between 2003 to 2020) obtained from the INABC. The mean shape parameters of the fitted functions are according to Table 2

The annual trends of 305-d-TMY for both breeds, regarding the Pollott as the best function, are shown in Figure 3. The annual trend was 25.01 kg for Simmental (born in 2005 to 2017) and 148.33 kg for Jersey (born in 2009 to 2017) cows. These trends could be used to monitor current and predict future rate of production. A clear positive trend was observed in Jersey cows, as the average 305-d-TMY gradually increases from 5797 kg to 6825 kg. According to the latest report of the CDCB, the average estimated milk production for Jersey cows born in 2019 was about 9656 kg (21288 lbs). Based on Figure 3, the average 305-d-TMY for Simmental cows increased until 2010 from 5168 kg to 7408 kg and then gradually decreased till 2016 and reached 5897, after that increased slightly to 6037 in 2017.



Figure 3 Estimated phenotypic trend of 305-d-TMY for primiparous Simmental and Jersey cows (calving between 2003 to 2020) obtained from the INABC

Kopec *et al.* (2021) fitted the Wood function for Czech Simmental cows and estimated the average 305-d-TMY of 6058 kg. In a study of Bulgarian Simmental cows, during the 17 years (2001-2017), the authors reported an average of 5016 kg for 305-d-TMY and found no clear trend (Karamfilov and Nikolov, 2019).

In this study, we compared the Pollott function with two empirical functions (Wood and Wilmink) and found that the Pollott function was superior in all aspects (four-step assessment). Further and Complementary research to predict genetic parameters and trends based on test day records and standardized whole lactation production using the Pollott function, is suggested.

CONCLUSION

Applying lactation curve functions provides valuable information for genetic selection and management practices such as DHI programs. Wilmink produces better results compared to Wood function in our study. Overall, our results confirm that the Pollott's mechanistic function outperforms the other two functions for fitting individual lactation curves. It is more robust in terms of: (1) maximum number of standard curves, (2) lowest AICc, (3) independent curve parameters, and (4) biological interpretation of typical curves. Moreover, the Pollott function allows for independent selection of different aspects of the lactation curve, improving genetic evaluations and production performance. Therefore, we recommend employing this function for fitting test day records and standardizing (national) milk production for Simmental and Jersey breeds.

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