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The Precision Approach of the Lactation Curve in Sirohi Goats Using Non-Linear Models

Research Article

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ABSTRACT

Lactation knowledge enables total milk yield prediction from single and multiple lactation test days. The objective of this study was to compare different non-linear lactation curve models and to select the best fit model for evaluation of the Sirohi goat's milk production curve. Data retrieved fortnightly test day milk yield (FTDMY) in the various days (15, 30, 45, 60, 75, 90, 105, 120, 135 and 150) at 22.630 fortnightly test day milk yield of 2,263 Sirohi does in different lactations at All India Coordinated Research project area period from 2004 to 2016. Gamma, inverse quadratic polynomial, exponential, mixed log, and polynomial regression were evaluated to describe the lactation curve. The mean FTDMY increased from 0.811 ± 0.004 kg on Td_1 (15^{th} day of lactation) to 1.025 ± 0.005 kg on Td_3 (45^{th} day of lactation) and then decreased to 0.379 ± 0.001 kg on Td_{10} (150^{th} day of lactation), with a coefficient of variation ranging from 20.40% to 28.68%. The polynomial regression function had the best adjusted R^2 value of 99.4% and the smallest root mean square error of 0.003 kg., with expected peak yield, persistency, and total milk yield were 1.03 kg, 60.8%, and 115.73%, respectively. Out of the five lactation curve models examined, the polynomial regression function produced an outstanding model for predicting fortnightly test day milk output in Sirohi goats, with a relatively strong R^2 and a low root mean square error (RMSE).

KEY WORDS exponential, gamma, inverse quadratic polynomial, mixed log, polynomial regression.

INTRODUCTION

Goats play an important role in livelihood and food security of small and marginal farmers of Rajasthan and are considered as "moving ATMs" for goat-keepers. These are considered as assurance for livelihood and survivability of "Ghumantu (migratory) and land-less livestock-keepers, especially shepherds". The National Bureau of Animal Genetic Resources of India has thirty four recognized goat breeds (NBAGR, 2020). With 148.88 million goats, India ranks second within the world in terms of goat population, accounting for 27.80 percent of the country's total livestock population (FAOSTAT, 2019). Rajasthan with its 56.8 Million livestock population ranks second in the country and shares more than 10% livestock population of India. Rajasthan ranks first in country with 20.84 million populations of goats and 14% share of goat population from country (Rajasthan AHD, 2020) and the state has high genetic diversity amongst India's goat population. The goat, a poor man's cow, provides milk and meat, which are important sources of animal protein. Goats produce about 3% of total milk in India, while they contribute 14.25

percent to meat production. (DADF Annual Report, 2017).

Sirohi goats are a dual-purpose goat breed found primarily in the southern region of Rajasthan. They are well-known for their milk and meat production. According to Kapadiya *et al.* (2016) goat milk is considered as superior to cow and buffalo milk. It has a number of health benefits over cow's milk, including improved digestion, increased alkalinity, and a lower s1-casein content, making it less allergenic. Goat milk is alkaline, unlike cow milk, which is acidic, making it ideal for those who suffer from acid reflux (Nazli, 2017).

The lactation curve is outlined as a graph that shows the connection between milk yields and also the length of your time since biological process (Brody, 1964). Milk production generally peaks within the early stages of lactation then bit by bit declines (Leon-Velarde *et al.* 1995). It's a valuable tool for genetic analysis and management selections involving time (Macciotta *et al.* 2011). Lactation knowledge allows for the prediction of total milk yield from single and multiple test days of lactation. The evaluation of lactation curve models is useful for monitoring individual yields for diet planning, early disease detection, and selecting superior animals to be parents in the next generation (Gipson and Grossman, 1989). A variety of empirical models have been developed to explain the lactation curve (Nelder, 1966; Wood, 1967; Wilmink 1987; Guo and Swalve, 1995; White *et al.* 1999).

MATERIALS AND METHODS

Data

The data were collected from All India Co-ordinated Research Project (AICRP) on Sirohi goat improvement, Livestock Research Station, Vallabhnagar, Udaipur, India. Data retrieved fortnightly test day milk yield (FTDMY) in the various days (15, 30, 45, 60, 75, 90, 105, 120, 135 and 150) at 22.630 FTDMY of 2.263 Sirohi does in different lactations during period from 2004 to 2016. The project area is located in Southern Rajasthan state and situated at 582 m above sea level on 24.67° N 74.00° E. which characterized by semi-arid climate and having annual normal rainfall of the State is 594.9 mm. out of which 75 to 95% of the rainfall mostly precipitates in the monsoon period i.e. from 1st June to 30th September. Similarly, the average temperature ranges from 10^C to 35^C in project area. Records pertaining to culling in the middle of lactation, abortion, still birth, or any other pathological causes affecting the lactation yield of the animals were considered abnormal, and thus such records were not included in the current study.

Housing, feeding, and lactation parameters

In the project area, Sirohi goats are raised in a semi-intensive system. Every day, goats grazed for six to eight hours on pasture. Goats are typically housed at night in Kacha floors that are covered by soil that has been coated with cow dung and are located at the farmer's house. Various types of trees, shrubs, and grasses can be found in the project area's pasture land at various times of the year. Available trees, shrubs and grasses different seasons as monsoon (Kair, Dhaman, Dudh, Patharchatta, Motha, Akra and Thur), winter (Neem, Motha, Akra, Keekar and Beri) and summer (Post harvest left over residue of Gram pea, Babul, Kair and Khejri) for Sirohi goat in southern Rajasthan. Vaccination and treatment services are provided to registered goat keepers in the project area by project staff and the Rajasthan animal husbandry department.

Estimation of lactation parameter

The data was used to evaluate the lactation curve parameters pertaining to five lactation curve function. Gamma function (GF) (Wood, 1967) (1).

$$Y_t = at^b e^{-ct}$$

Fitted in the log linear form:

$$\log Y = \log a + b\log t - ct$$

Where:
Y_t: average daily yield in the t^{th} fortnight.
a: initial milk yield, just after kidding.
b: inclining slope parameter up to peak yield.
c: declining slope parameter.
t: fortnightly test day.
e: base of natural logarithm (2.71828).

Inverse quadratic polynomial (IQP), (Nelder, 1966) (2).

$$Y^{-1}_t = a + bt^{-1} + ct$$

Where:
Y_t: average daily yield in the t^{th} fortnight.
a: initial milk yield, just after kidding.
b: inclining slope parameter up to peak yield.
c: declining slope parameter.
t: fortnightly test day exponential function (EF), (Wilmink, 1987) (3).

$$Y_t = a + be^{-0.7t} + ct$$

Where:
Y_t: average daily yield in the t^{th} fortnight of lactation.
a: initial milk yield, just after kidding.
b: inclining slope parameter up to peak yield.
c: declining slope parameter.

t: fortnightly test day.

Mixed log function (MLF), (Guo and Swale, 1995) (4).

$Y_t = a + bt^{½} + c\log t + e_t$

Where:
Y_t: average daily milk yield in the t^{th} test day of lactation.
a: initial milk yield just after kidding.
b: ascending slope parameter up to the peak yield.
c: descending slope parameter.
t: length of time since kidding.
e_t: residual error.

Polynomial regression function (PRF), (Ali and Schaeffer, 1987) (5).

$$Y_t = a + bx + cx^2 + d\log\frac{1}{x} + e\log\left(\frac{1}{x}\right)^2$$

Where:
Y_t: average milk yield in t^{th} fortnight of lactation.
a: associated with peak yield.
b and c: associated with decreasing slope.
d: associated with the increasing slope, x= t/150.
e: base of natural logarithm (2.71828).

Fitting the models to lactation curve

The above-mentioned models were fitted to the Sirohi goat's FTDMY (kg) only after it had completed 150 days of milk yield (10 fortnightly) of lactation. The best model was chosen based on the highest adjusted coefficient of determination (R^2) value and the lowest root mean square error (RMSE). The residuals were graphically plotted to determine the model's accuracy in fitting the lactation curves.

Estimated total milk yield

Total milk yield was estimated for the selected equations by the centering date method (CMD), also known as Fleischmann's method (Ruiz *et al.* 2000). The general expression of the CDM method is:

$$ETMY = P_1 \times D_1 + \sum_{i=1}^{k}\frac{P_i + P_{i-1}}{2} \times D_i + P_{k+1} \times 15$$

Where:
ETMY: estimated total milk yield.
D_1: interval between kidding and first recording.
P_i: yield of the record I.
D_i: interval between the record i and the record (i+1) (i=1,...k).
15: days interval of milk recording in lactation period.

Persistency percentage

Persistency= 100 × TMY_{LH}/TMY_{FH}

Where:
TMY_{LH}: cumulative milk yield of last half of lactation curve.
TMY_{FH}: cumulative milk yield of first half of lactation curve.

Coefficient of determination (R2)

R^2 gives the percentage of variance of fortnightly yield explained by the model:

$$R^2 = \left[1 - \left(\frac{SSE}{SST}\right)\right] \times 100$$

Where:
SSE: error sum of square.
SST: total sum of square.

$$R^2_{Adj} = \frac{1 - (n-1)}{(n-p)(1-R^2)}$$

Where:
N: no. of observations.
P: no. of parameter in the model.

Root mean square error (RMSE)

Root mean square error is a kind of generalized deviation.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n}}$$

Where:
n: number of observations.
y_i: actual values.
ỹ: values predicted by the regression model.

RESULTS AND DISCUSSION

A total of ten FTDMY records (15^{th}, 30^{th}, 45^{th}, 60^{th}, 75^{th}, 90^{th}, 105^{th}, 120^{th}, 135^{th}, and 150^{th} day) were collected at a 15^{th} day interval.

The mean FTDMY increased from 0.811 ± 0.004 kg on Td_1 (15^{th} day of lactation) to a peak yield of 1.025 ± 0.005 kg on Td_3 (45^{th} day of lactation) and subsequently declined to 0.379 ± 0.001 kg on Td_{10} (150^{th} day of lactation) and coefficient of variation was ranged from 20.40% to 28.68% (Table 1).

The estimated lactation curve parameters and standard errors (Table 2), as well as the observed and predicted FTDMY (Figure 1 and 2).

Table 1 Descriptive statistics of fortnightly test day (FTD) milk yield in kg

FTD	Mean	SDM	SEM	CV%	Minimum	Maximum
1 (15th day)	0.811f	0.22	0.004	27.12	0.133	1.850
2 (30th day)	1.011h	0.29	0.006	28.68	0.300	2.000
3 (45th day)	1.025h	0.27	0.005	26.34	0.054	2.000
4 (60th day)	0.896g	0.23	0.004	25.66	0.028	1.730
5 (75th day)	0.803f	0.19	0.004	23.66	0.058	1.600
6 (90th day)	0.716e	0.16	0.003	22.34	0.073	1.510
7 (105th day)	0.625d	0.13	0.002	20.80	0.046	1.390
8 (120th day)	0.539e	0.11	0.002	20.40	0.030	1.310
9 (135th day)	0.460b	0.10	0.002	21.73	0.100	1.220
10 (150th day)	0.379a	0.09	0.001	23.74	0.040	1.040

The means within the same column with at least one common letter, do not have significant difference (P>0.05).
SDM: standard deviation of the mean; SEM: standard error of the means; CV: coefficient of variation.

Table 2 Estimates of the model parameters

Parameters	Models				
	GF	IQP	EF	MLF	PRF
A	1.059±0.023	-0.023±0.098	1.129±0.261	2.063±0.0.082	-0.747±0.266
B	0.605±0.047	1.074±0.125	-0.135±0.194	-1.227±0.096	0.062±0.270
C	0.244±0.013	0.218±0.016	0.877±0.117	0.940±0.093	0.000±0.008
D	-	-	-	-	0.508±0.179
K	-	-	-	-	-0.413±0.241

GF: gamma function; IQP: inverse quadratic polynomial; EF: exponential function; MLF: mixed log function and PRF: polynomial regression function.

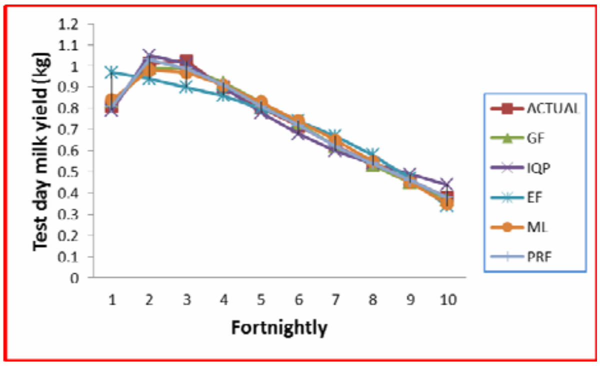

Figure 1 The observed and predicted FTDMYs from the various lactation curve functions

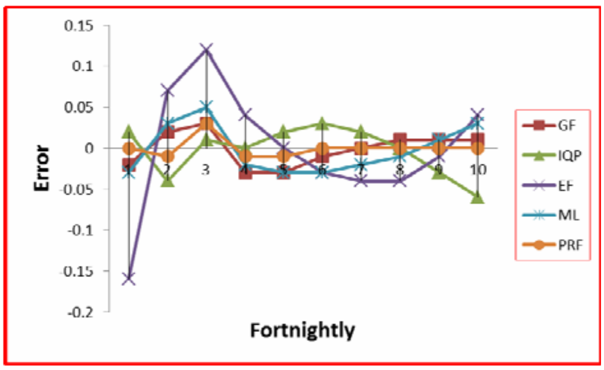

Figure 2 Residual errors between observed and predicted values from various lactation curve functions

Peak period, peak yield, persistency, and estimated total milk yield are presented, as well as predicted equations with R^2 and RMSE values for five different functions and correlation between observed and predicted milk yield (Tables 3 to 5).

All models predicted the FTDMY with high degree accuracy (R^2_{adj}) and the lowest RMSE. In the current study, the polynomial regression function produced the highest adjusted R^2 value of 99.4% and the lowest RMSR of 0.003 kg. The expected peak yield, persistency, and total milk yield were 1.03 kg, 60.8%, and 115.73%, respectively (Table 3). The error in FTDMY prediction using the polynomial regression function ranged from – 0.01 kg on the 30th day to 0.03 kg on the 45th day, with the expected peak yield (1.03 kg) observed on the 30th test day of lactation (Table 4). The correlation between the observed and estimated results was higher (99.8 %) than for other functions (Table 5). Catillo *et al.* (2002) reported similar R^2 values (99%) in Italian water buffaloes and Sahoo *et al.* (2015) in Murrah buffaloes (R^2 value 99.8% and RMSE 0.003 kg).

The gamma type function produced the second highest adjusted R^2 value of 98.8% and the lowest RMSR of 0.006 kg, with the projected peak yield occurring on the 45th day of lactation's test day.

The ascending phase and peak yield are not explained by gamma function. In Sirohi goats, this function explained a lower peak yield (0.99 kg) than the actual observed peak yield.

Table 3 Different lactation curve functions with parameters for prediction of fortnightly test days milk yield with goodness of fit

Function		PP	PY	Persistency %	TMY	Quality of prediction method	
		Observed					
		3	1.025	60.8	115.65		
		Predicted				Adjusted R2 (%)	RMSE (kg)
Gamma function	$Y_t = 1.059t^{0.608}e^{-0.244t}$	2	0.99	60.2	115.56	98.8	0.006
Inverse quadratic polynomial	$y_t = (-0.023 + 1.074t^{-1} + 0.218t)^{-1}$	2	1.05	61.6	115.87	97.4	0.009
Exponential function	$Y_t = 1.129 - 0.135e^{-0.7t} + .877t$	1	0.97	63.8	115.67	84.9	0.022
Mixed log function	$Y_t = 2.063 - 1.227t^{\frac{1}{2}} + 0.940logt$	2	0.98	61.5	115.73	97.6	0.008
Polynomial regression function	$Y_t = -0.747 + 0.062t + 0.00t^2 + 0.508logt - 0.413(logt)^2$	2	1.03	60.8	115.73	99.4	0.003

PP: peak period; PY: peak yield and TMY: total milk yield.
RMSE: root mean square error.

Table 4 Predicted fortnightly test days milk yield (FTDMY, kg) and error of different lactation curve functions

Test days (TD)	FTDMY (Mean)	GF	Error	IQP	Error	EF	Error	MLF	Error	PRF	Error
TD1	0.811	0.83	- 0.02	0.79	0.02	0.97	- 0.16	0.84	- 0.03	0.81	0.00
TD2	1.011	0.99	0.02	1.05	- 0.04	0.94	0.07	0.98	0.03	1.03	- 0.01
TD3	1.025	0.99	0.03	1.01	0.01	0.90	0.12	0.97	0.05	0.99	0.03
TD4	0.896	0.92	- 0.03	0.90	0.00	0.86	0.04	0.91	- 0.02	0.91	- 0.01
TD5	0.803	0.83	- 0.03	0.78	0.02	0.80	0.00	0.83	- 0.03	0.81	- 0.01
TD6	0.716	0.73	- 0.0.1	0.68	0.03	0.74	- 0.03	0.74	- 0.03	0.72	0.00
TD7	0.625	0.62	0.00	0.60	0.02	0.67	- 0.04	0.65	- 0.02	0.62	0.00
TD8	0.539	0.53	0.01	0.54	0.00	0.58	- 0.04	0.55	- 0.01	0.54	0.00
TD9	0.460	0.45	0.01	0.49	- 0.03	0.47	- 0.01	0.45	0.01	0.46	0.00
TD10	0.379	0.37	0.01	0.44	- 0.06	0.34	0.04	0.35	0.03	0.38	0.00

GF: gamma function; IQP: inverse quadratic polynomial; EF: exponential function; MLF: mixed log function and PRF: polynomial regression function.

Table 5 Correlations between the observed milk yield and the different non-linear model used to the lactation curve of Sirohi goats

	Observed	GF	IQP	EF	MLF	PRF
Observed	1	0.996**	0.990**	0.939**	0.991**	0.998**
GF		1	0.980**	0.953**	0.997**	0.997**
IQP			1	0.903**	0.968**	0.991**
EF				1	0.967**	0.942**
MLF					1	0.993**
PRF						1

GF: gamma function; IQP: inverse quadratic polynomial; EF: exponential function; MLF: mixed log function and PRF: polynomial regression function.
** Correlation is significant at the 0.01 level.

The correlation between the observed and estimated result was the second highest (99.6%) of the functions studied. According to the current findings, Akpa *et al.* (2011) observed an R^2 value of 98.3% in Alpine goats, Fernandez *et al.* (2002) reported a 98% R^2 value in Murciano-Granadina goats, and Waheed and Khan (2013) found an R^2 value of 98.2% in Beetal goats. Contrary to the current findings, Bilgin *et al.* (2010) found lower R^2 values for this function in the Awassi, Morkraman, and Tushin breeds of sheep, with 92.4%, 91.9%, and 86.2%, respectively, and 86.8% R^2 in Akkaraman ewes by Keskin and Dag (2006).

In the Sirohi goat, the mixed log function produced the third highest adjusted coefficient of determination (97.6%) and the lowest RMSR (0.008 kg). The mixed log function projected a peak milk yield of 0.98 kg on the 30^{th} test day, which was marginally lower than the actual yield. Persistence (61.5%) and total milk yield (115.73%), on the other hand, were reported to be higher than observed. The error in FTDMY prediction using the mixed log function varied from −0.03 kg in the 15^{th}, 75^{th}, and 90^{th} days to 0.05 kg in the 45^{th} day. At the 45^{th} day, MLF had the largest prediction error in peak output. However, at the 15^{th}, 60^{th}, and 120^{th}

days, the mixed log function slightly overestimated test day milk. The observed and expected milk yields are correlated. As a result, Sahoo *et al.* (2015) investigated the MLF in Murrah buffaloes, finding that the R2 and RMSE were both 98% and 0.09 kg, respectively.

The inverse quadratic polynomial function produced (97.4%) adjusted R^2 and (0.009 kg) RMSE, with the maximum peak yield displayed (1.05 kg.). With a mixed log function, however, persistency (61.6%) and predicted total milk output (115.87%) were similar. The predicted error ranged from −0.06 kg in the 150th day to 0.03 kg in the 90th day. The observed and estimated milk yields were found to have a 99.0 % correlation. R^2 values (94% to 98%) in different parities of Italian water buffaloes were reported by Catillo *et al.* (2002).

In this study, the exponential function was found to have the least fit for fortnightly test day milk yield, with the lowest adjusted R^2 (84.9%) and a greater RMSR (0.022 kg) in Sirohi goats. In comparison to observed and other functions on the 30th day of lactation, the lowest peak yield (0.97 kg) was recorded on the 15th day (Table 3). The expected inaccuracy varied from −0.16 kg on day 15 to 0.12 kg on day 45. In the 15th day, the highest projected error of FTDMY was recorded. In the 15th, 60th, and 135th days, the exponential function overestimated the FTDMY. Similarly, when compared to other functions in this investigation, the correlations between observed and predicted total milk yield were the lowest. R^2 values (97% to 98%) in different parities of Italian water buffaloes were reported by Catillo *et al.* (2002). On the other hand, Sahoo *et al.* (2015) found R^2 value (95%) and RMSE (0.14 kg) in Murrah buffalo.

CONCLUSION

Five mathematical functions for modeling the lactation curve in Sirohi goat were compared for accuracy of predicting milk yields from test day's records. In the present findings, the result showed that the Ali and Schaeffer's model was the best model to predict milk yield at AICRP project area of Sirohi goat, with a relatively strong adjusted coefficient of determination and a low RMSR, out of the five lactation curve models examined. The results also revealed that all of these models accurately predicted the lactation curve.

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REFERENCES

Akpa G.N., Asiribo E.O., Oni O.O. and Alawa P. (2001). The influence of non-genetic factors on the shape of lactation curves in Red Sokoto goats. *J. Anim. Sci*. **72,** 233-239.

Ali T.E. and Schaeffer L.R. (1987). Accounting for covariance among test days milk yields in dairy cows. *Canadian J. Anim. Sci.* **67,** 637-644.

Bilgin O.C., Esenbuga N. and Davis M. (2010). Comparison of models for describing the lactation curve of Awassi, Morkaraman and Tushin sheep. *Arch. Tierzucht.* **53,** 447-456.

Brody S. (1964). Bioenergetics and Growth. Reinhold Publishing Corporation, New York.

Catillo G., Macciotta N.P.P., Caretta A. and Cappio-Borlino A. (2002). Effect of age and calving season on lactation curves of milk production traits in Italian water buffaloes. *J. Dairy Sci*. **85,** 1298-1306.

FAOSTAT. (2020). Database of the Food and Agricultural Organization (FAO) of the United Nations. Food and Agriculture Organization of the United Nations (FAO), Rome, Italy.

Fernandez C., Sanchez A. and Garces C. (2002). Modeling the lactation curve for test-day milk yield in Muciano-Granadina goats. *Small Rumin. Res*. **46,** 29-41.

Gipson T.A. and Grossman M. (1989). Diphasicanalysis of lactation curves in dairy goats. *J. Dairy Sci*. **72,** 1035-1044.

Guo Z. and Swalve H.H. (1995). Modeling of the lactation curves as a sub-model in the evaluation of test day records. *Interbull Bulletin*. **11,** 4-7.

Kapadiya D.B., Prajapati D.B., Jain A.K., Mehta B.M., Darji V.B. and Aparnathi K.D. (2016). Comparison of Surti goat milk with cow and buffalo milk for gross composition, nitrogen distribution, and selected minerals content. *Vet. World*. **9,** 710-179.

Keskin I. and Dag B. (2006). Comparison of different mathematical models for describing the complete lactation of Akkramanewes in Turkey. *Asian-Australasian J. Anim. Sci*. **19,** 1551-1556.

Leon-Velarde C.U., McMillan I., Gentry R.D., Wilton J.W. (1995). Models for estimating typical lactation curves in dairy cattle. *J. Anim. Breed Genet.* **112,** 333-340.

Macciotta Nicolo P.P., Dimauro C., Salvatore, Rassu P.G., Steri R. and Pulina G. (2011). The mathematical description of lactation curves in dairy cattle. *Italian J. Anim. Sci*. **10,** 212-223.

Nazli T. (2017). The nutritional value and health benefits of goat milk components. MS Thesis. Ankara Univ., Ankara, Turkey.

NBAGR. (2018). National Bureau of Animal Genetic Resources, India. Available at: http://www.nbagr.res.in.

Nelder J.A. (1966). Inverse polynomials, a useful group of multifactor response functions. *Biometrics.* **22,** 1128-1141.

Ruiz R., Oregui L.M. and Hohenboken W.D. (2000). Comparison of models for describing the lactation curve of Latxa sheep and an analysis of factors affecting milk yield. *J. Dairy Sci.* **83,** 2709-2719.

Sahoo S.K., Singh A., Gupta A.K., Chakravarty A.K., Amnhore G.S. and Dash S.K. (2015). Comparative evaluation of different Lactation curve in function for prediction of Bi-Monthly

Test day milk yield in Murrak buffaloes. *Anim. Sci. Report.* **9,** 89-94.

Waheed A. and Khan M.S. (2013). Lactation curve of Beetal goats in Pakistan. *Arch. Tierzucht*. **56,** 892-898.

White I. M.S., Thompson R. and Brotherstone S. (1999). Genetic and environmental smoothing of lactation curves with cubic splines. *J. Dairy Sci.* **82,** 632-638.

Wilmink J.B.M. (1987). Adjustment of test day milk, fat and protein yield for age, season and stage of lactation. *Livest. Prod. Sci*. **16,** 335-348.

Wood P.D.P. (1967). Algebraic model of the lactation curve in cattle. *Nature*. **216,** 164-165.