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Resonant Solitons Solutions to the Time M-Fractional Schrödinger Equation

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Abstract

In this research the time M-fractional resonant nonlinear Schrödinger differential equation with different forms of nonlinearities, containing Kerr-law and parabolic-law has been studied. For this objective, the modified Kudryashov method and the sine-Gordon expansion approach have been implemented to retrieve a series of resonant solitons solutions for the abovementioned model. The prospective of the schemes in founding soliton solutions of nonlinear time-fractional equations in the truncated M-fractional derivative sense is confirmed.

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INTRODUCTION

Fractional calculus is as old as the usual calculus. In the past several years, many of researchers have been trying to generalize the concept of the usual derivatives. Nowadays there are many definitions for the fractional derivative. Two the earliest of definitions are as follows(Capelas de Oliveira &et al, 2014; Figueiredo Camargo & et al, 2015; Herrmann, 2011; Katugampola, 2016; Khalil & et al, 2014; Kilbas & et al, 2006; Kilbas & et al, 2006; Podlubny, 1999).

Definition 1. ***(Riemann-Liouville definition)*** If n is a positive integer and $\alpha \in [n-1, n)$ the αth derivative of f is given by

$$D_a^{\alpha}(f)(x) = \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dx^n}\int_a^x \frac{f(t)}{(x-t)^{\alpha-n+1}}dt. \tag{1}$$

Definition 2. ***(Caputo definition)*** If n is a positive integer for $\alpha \in [n-1, n)$ the αth derivative of f is

$$D_a^{\alpha}(f)(x) = \frac{1}{\Gamma(n-\alpha)}\int_a^x \frac{f^n(t)}{(x-t)^{\alpha-n+1}}dt. \tag{2}$$

The presented definitions are attempted to satisfy the usual properties of the standard derivative (Khalil & et al., 2014). The only property inherited by all definitions of fractional derivative is the linearity property, but there are some disadvantages that caused their application confront with difficulty (Khalil & et al., 2014).

In 2014, Khalil et al. proposed the so-called conformable fractional derivative of order integer α to generalize the classical properties of calculus(Khalil & et al., 2014).One of the definitions that have been presented recently is conformable fractional derivative that removed some of drawbacks the presented definitions. More recently, in 2014, Katugampola has also proposed an alternative fractional derivative with classical properties, which refers to the Leibniz and Newton calculus, similar to the conformable fractional derivative (Katugampola, 2014). In 2017, Sousa and et al., introduced an M-fractional derivative involving a Mittag-Leffler function with one parameter that also satisfies the properties of integer-order calculus (Goreno, 2014; Vanterler et al., 2017). In this sense, Sousa and Oliveira introduced a truncated M-fractional derivative type that unifies four existing fractional derivative types mentioned above and which also satisfied the classical properties of integer-order calculus (Vanterler et al., 2018).

Definition 3. ***(Truncated Mittag-Leffler function)*** With $\beta > 0$, and $z \in \mathbb{C}$, the truncated Mittag-Leffler function of one parameter is defined by [6]

$$_{n}\mathbb{E}_{\beta}(z) = \sum_{k=0}^{n}\frac{z^k}{\Gamma(k\beta+1)}. \tag{3}$$

Definition 4. **(*Truncated M-fractional derivative)*** Given a function $f\colon [0,\infty) \to \mathbb{R}$. Then the truncated M-fractional derivative of f of order α is defined by

$$_{n}\mathcal{D}_M^{\alpha,\beta} f(x) = \lim_{\varepsilon\to 0}\frac{f\left(x\ {}_{i}\mathbb{E}_{\beta}(\varepsilon x^{-\alpha})\right) - f(x)}{\varepsilon} \tag{4}$$

for all $x > 0, \alpha \in (0, 1)$, where ${}_{n}\mathbb{E}_{\beta}(.)$, $\beta > 0$ is the Mittag-Leffler function with one parameter as defined by in Eq. 1. Note that if f is α-differentiable in some $(0, a), a > 0,$ and $\lim_{x\to 0^+} {}_{i}\mathcal{D}_M^{\alpha,\beta} f(x)$ exists, then one can define(Vanterler et al., 2017; Vanterler et al., 2018)

$$_{n}\mathcal{D}_M^{\alpha,\beta} f(0) = \lim_{x\to 0^+} {}_{n}\mathcal{D}_M^{\alpha,\beta} f(x).$$

If the M-fractional derivative of f of order α exists, then we simply say that f is α-differentiable. One can easily show that truncated M-fractional derivative satisfies all the following properties (Vanterler et al., 2017; Vanterler et al., 2018).

Let $\alpha \in (0,1)$ and f, g be functions α-differentiable at a point $x > 0$, Then

A. (Linearity Rule) For $a, b \in \mathbb{R}$ ${}_n\mathcal{D}_M^{\alpha,\beta}(af + bg) = a\left({}_n\mathcal{D}_M^{\alpha,\beta} f\right) + b\left({}_n\mathcal{D}_M^{\alpha,\beta} g\right)$,

B. For all $p \in \mathbb{R}$, ${}_n\mathcal{D}_M^{\alpha,\beta} x^p = \frac{p}{\Gamma(\beta+1)} x^{p-\alpha}$,

C. For all constant functions $f(x) = \lambda$, ${}_n\mathcal{D}_M^{\alpha,\beta} \lambda = 0$,

D. (Product Rule) ${}_n\mathcal{D}_M^{\alpha,\beta}(f.g) = g.\left({}_n\mathcal{D}_M^{\alpha,\beta} f\right) + f.\left({}_n\mathcal{D}_M^{\alpha,\beta} g\right)$,

E. (Quotient Rule) ${}_n\mathcal{D}_M^{\alpha,\beta}\left(\frac{f}{g}\right) = \frac{g.\left({}_n\mathcal{D}_M^{\alpha,\beta} f\right) - f.\left({}_n\mathcal{D}_M^{\alpha,\beta} g\right)}{g^2}$,

F. (Chain Rule) If function f, ordinary differentiable at $g(x)$, then ${}_n\mathcal{D}_M^{\alpha,\beta}(fog) = f'(g(x).\left({}_n\mathcal{D}_M^{\alpha,\beta} g\right)$,

G. ${}_n\mathcal{D}_M^{\alpha,\beta} f(x) = \frac{x^{1-\alpha}}{\Gamma(\beta+1)} \frac{df}{dx}$.

Definition 5. Let $\beta > 0$, $\alpha \in (m, m+1]$, for some $m \in \mathbb{N}$, and f, m, times differentiable (in the classical of sense) for $x > 0$. Then the local M-derivative of order n, of function f is defined by

$$ {}_n\mathcal{D}_M^{\alpha,\beta,m} f(x) = \lim_{\varepsilon \to 0} \frac{f^{(m)}(x\,{}_n\mathbb{E}_\beta(\varepsilon x^{-\alpha})) - f^{(m)}(x)}{\varepsilon}, \tag{5} $$

if and only if the limit exists (Vanterler et al., 2017; Vanterler et al., 2018).

The study of fractional differential equations has demonstrated very valuable over time. Solving fractional differential equations is very important, due to this fact, finding an exact solution and an approximate solution of fractional differential equations is clearly an important task. The purpose of this paper is studied the resonant soliton solutions of nonlinear Schrödinger's equation, is assumed by

$$ i\,{}_n^t\mathcal{D}_M^{\alpha,\beta}\psi + \eta\psi_{xx} + \delta F(|\psi|^2)\psi + \gamma\left\{\frac{|\psi|_{xx}}{|\psi|}\right\}\psi = 0, \tag{6} $$

where η is the coefficient of group-velocity dispersion, δ is the coefficient of non-Kerr nonlinearity, and γ presents the coefficient of resonant nonlinearity, by Modified Kudryashov method and sine-Gordon expansion approach, which ${}_n^t\mathcal{D}_M^{\alpha,\beta}\psi$, means truncated M-fractional derivative with respect to time-variable t. The resonant nonlinear Schrodinger's equation is a special type of nonlinear Schrodinger equations that is used to describe the dynamic of solitons and Madelung fluids in various nonlinear systems (Biswas, 2012; Ekici et al., 2017; Eslami et al, 2013; Hosseini et al., 2017; Hosseini et al., 2017; Ilie et al, 2018; Ilie et al., 2018; Ilie et al., 2018; Inc et al., 2017; Inc et al., 2018; Kudryashov, 2005; Kudryashov, 2013; Kudryashov, 2012; Mirzazadeh et al., 2014; Mirzazadeh et al., 2014; Podlubny, 1999; Triki et al., 2012; Triki et al., 2012; Triki et al., 2018; Zhou et al., 2016; Zhou et al., 2015).

MODIFIED KUDRYASHOV METHOD FOR TIME M-FRACTIONAL DIFFERENTIAL EQUATIONS

Consider the time M-fractional differential equation (Hosseini et al., 2017; Ilie et al., 2018)

$$ F\left(\psi, i\,{}_n^t\mathcal{D}_M^{\alpha,\beta}\psi, \psi_x, \psi_{xx}, \psi_{xxx}, \ldots\right) = 0, \tag{7} $$

where F is a polynomial and ${}_n^t\mathcal{D}_M^{\alpha,\beta}\psi$, means truncated M-fractional derivative with respect to time-variable t. The main steps of modified Kudryashov method are as the following form (Hosseini et al., 2017; Hosseini et al., 2017; Ilie et al., 2018; Kudryashov, 2005; Kudryashov, 2013; Kudryashov, 2012).

Step 1. Under traveling wave transformation

$$\psi(x,t;\alpha) = U(\xi)e^{i\mu}, \ \xi = \rho x + \frac{\upsilon\Gamma(\beta+1)}{\alpha} t^{\alpha}, \ \mu = \kappa x + \frac{\omega\Gamma(\beta+1)}{\alpha} t^{\alpha},$$

Eq. 7 can be reduced to nonlinear ordinary differential equation

$$G(U(\xi), U'(\xi), U''(\xi), \dots) = 0. \quad (8)$$

Step 2. Let us assume that the solution $U(\xi)$ of nonlinear Eq. 8 can be presented as the following

$$U(\xi) = a_0 + \sum_{i=1}^{N} a_i Q^i(\xi), \quad a_N \neq 0, \quad Q(\xi) = \frac{1}{1+da^{\xi}}, \quad (9)$$

where constant coefficients a_i will be determined latter, N is a positive integer that can be computed by means of balance principle.

Step 3. By substituting Eq. 9 into Eq. 8, we obtain a system of algebraic equations.

Step 4. Solving the generated system and setting obtained values in Eq. 9, finally produces resonant soliton solutions for the time M-fractional Eq. 7.

Resonant soliton solution of the time M-fractional resonant nonlinear Schrödinger equation via Kerr-law nonlinearity

The Kerr law nonlinearity states $F(s) = s$, which this kind of nonlinearity typically arises in the context of water waves or nonlinear fiber optics when the refractive index of the light is proportional to the intensity (Ekici et al., 2017; Ilie et al., 2018). Consider the time M-fractional resonant nonlinear Schrödinger's equation with nonlinear Kerr law as follows

$$i\,{}_{n}^{t}\mathcal{D}_{M}^{\alpha,\beta}\psi + \eta\psi_{xx} + \delta|\psi|^2\psi + \gamma\left\{\frac{|\psi|_{xx}}{|\psi|}\right\}\psi = 0, \quad (10)$$

which ${}_{n}^{t}\mathcal{D}_{M}^{\alpha,\beta}\psi$, means truncated M-fractional derivative with respect to time-variable t. Under traveling wave transformation

$$\psi(x,t;\alpha) = U(\xi)e^{i\mu}, \ \xi = x + \frac{2\eta\kappa\Gamma(\beta+1)}{\alpha} t^{\alpha}, \quad \mu = -\kappa x + \frac{\omega\Gamma(\beta+1)}{\alpha} t^{\alpha},$$

the equation 10 can be reduced to a nonlinear ordinary differential equation as the following form [13]

$$(\eta + \gamma)U'' - (\omega + \eta\kappa^2)U + \delta U^3 = 0. \quad (11)$$

According to modified Kudryashov method, let us assume that the solution $U(\xi)$ of nonlinear Eq. 11 can be as follows (Ilie et al., 2018).

$$U(\xi) = a_0 + \sum_{i=1}^{N} a_i Q^i(\xi), \quad a_N \neq 0, \quad Q(\xi) = \frac{1}{1+da^{\xi}}.$$

By using the homogeneous balance principle, we find $N = 1$, then the solution Eq. 11 is

$$U(\xi) = a_0 + a_1 Q(\xi). \quad (10)$$

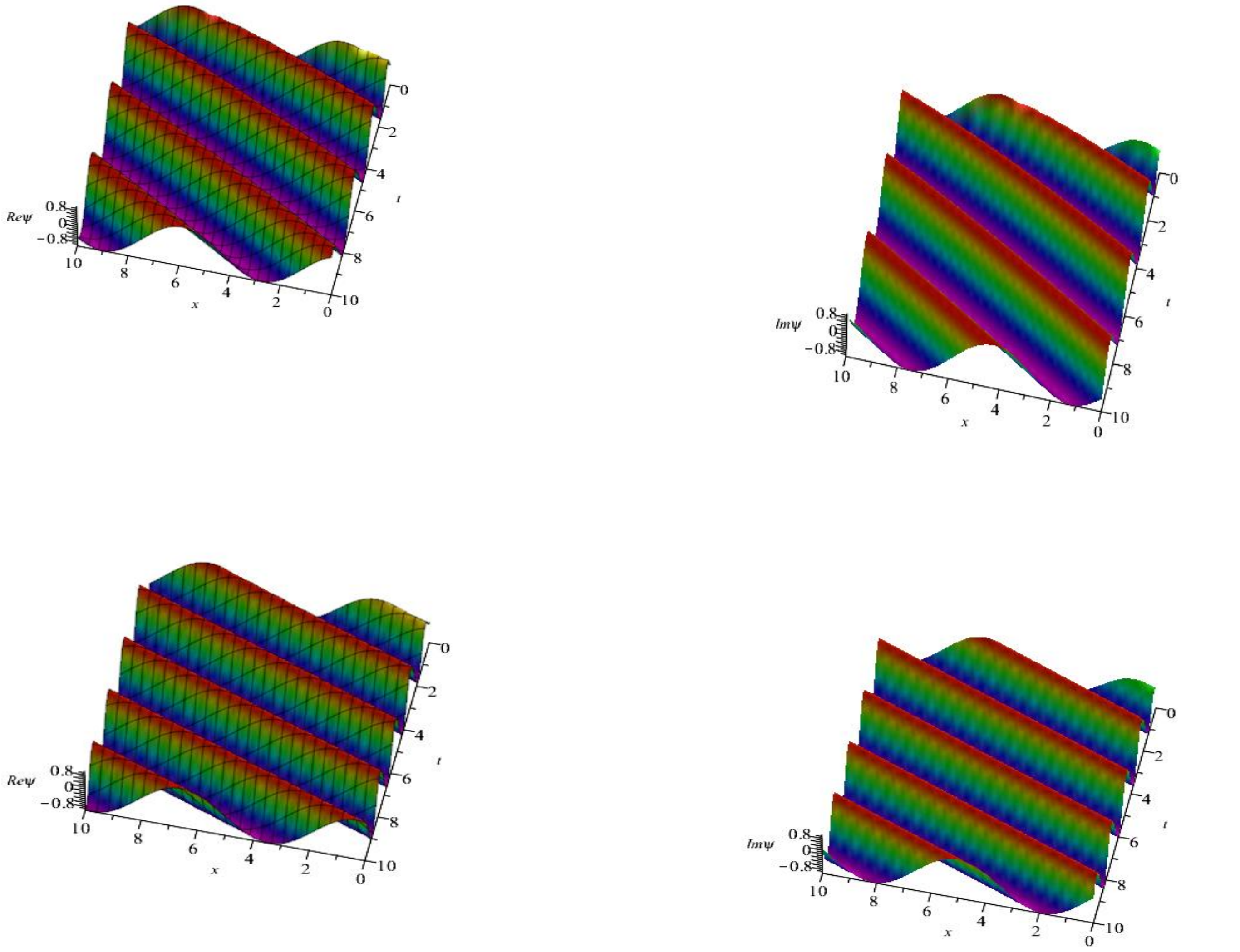
By substituting Eq. 12 along with its second order derivative into Eq. 11 and comparing the terms in the resulting equation, a nonlinear system will be gained (Ilie et al., 2018). By solving it, we find the soliton solution of Eq. 10, for different values of parameters a_0, a_1, as follows

Case 1. $a_0 = \pm\sqrt{-\frac{1}{2}\frac{\eta+\gamma}{\delta}}\ln a$, $a_1 = \mp 2\sqrt{-\frac{1}{2}\frac{\eta+\gamma}{\delta}}\ln a$, and the soliton solution of **Eq. 8 is**

$$\psi(x,t;\alpha,\beta) = \sqrt{-\frac{1}{2}\frac{\eta+\gamma}{\delta}}\ln a\left[\pm 1 + \frac{\mp 1}{1+da^{x+\frac{2\eta\kappa\Gamma(\beta+1)}{\alpha}t^{\alpha}}}\right] e^{i\left(-\kappa x + \frac{(-0.5(\ln a)^2(\eta+\gamma)-\eta\kappa^2)\Gamma(\beta+1)}{\alpha}t^{\alpha}\right)},$$

$$\lim_{\beta\to 1}\psi(x,t;\alpha,\beta) = \psi(x,t;\alpha),$$

where $\psi(x,t;\alpha)$ is a resonant soliton solution of resonant nonlinear Schrödinger's equation with nonlinear Kerr law, which is solved by Modified Kudryashov method (Ilie et al., 2018). The 3D graph of $Re(\psi(x,t;\alpha,\beta))$, and $Im(\psi(x,t;\alpha,\beta))$, are illustrated in Figs. 1.



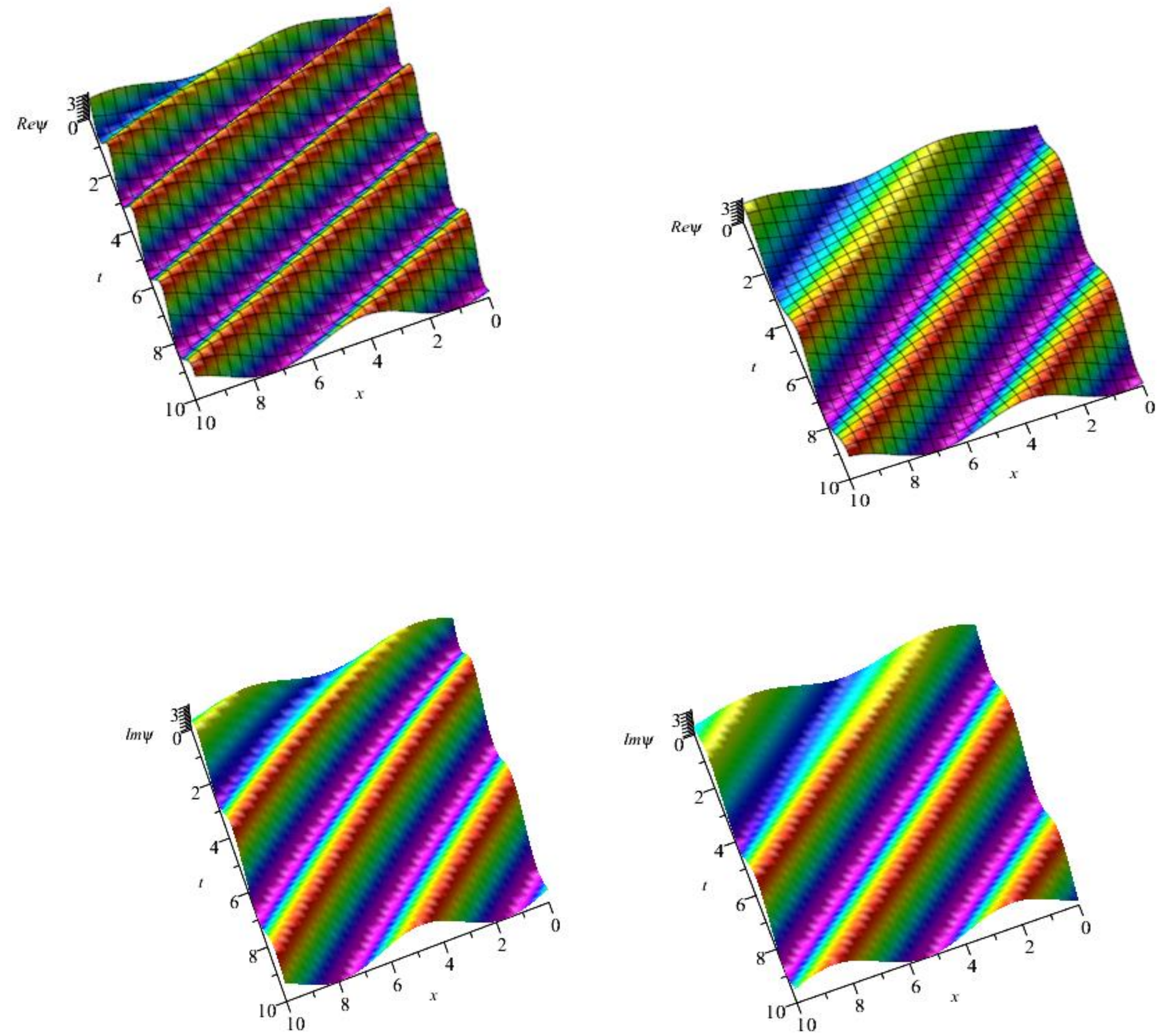
Figs. 1. Plots of the $Re(\psi(x,t;\alpha,\beta))$ and $Im(\psi(x,t;\alpha,\beta))$ corresponding to the values $(\alpha,\beta)=(0.8,1.5),(1,1.5)$ from left to right when $\eta=1,\beta=1,\kappa=1,\gamma=1,d=1,a=2.7$ respectively.

Case 2. a_0 , a_1 are arbitrary real number and $\beta=0$, and $\eta=-\gamma$ and the soliton solution of Eq. 8 is

$$\psi(x,t;\alpha,\beta)=\left[a_0+\frac{a_1}{1+da^{x-\frac{2\gamma\kappa\Gamma(\beta+1)}{\alpha}t^{\alpha}}}\right]e^{i\left(-\kappa x+\frac{\gamma\kappa^2\Gamma(\beta+1)}{\alpha}t^{\alpha}\right)},$$

$$\lim_{\beta\to 1}\psi(x,t;\alpha,\beta)=\psi(x,t;\alpha),$$

where $\psi(x,t;\alpha)$ is a resonant soliton solution of resonant nonlinear Schrödinger's equation with nonlinear Kerr law, which is solved by Modified Kudryashov method (Ilie et al., 2018). The 3D graph of $Re(\psi(x,t;\alpha,\beta))$, and $Im(\psi(x,t;\alpha,\beta))$, are showed in Figs. 2.



Figs. 2. Plot of the $Re(\psi(x,t;\alpha,\beta))$ **and** $Im(,(x,t;\alpha,\beta))$ corresponding to the values $(\alpha,\beta)=(0.8,2.5),(1,1.5)$ from left to right when $a_0=-2,a_1=-1,\kappa=1,\gamma=1,d=2,a=2.7$ respectively.

Resonant soliton solution of the time M-fractional resonant nonlinear Schrödinger equation with Parabolic-law nonlinearity

For parabolic-law nonlinearity, $F(s)=\delta s+\gamma s^2$ where β and γ are in general constants. This case appears in fiber optics (Ekici et al., 2017; Ilie et al., 2018). Consider the time M-fractional resonant nonlinear Schrödinger's equation with parabolic law nonlinearity as follows

$$i\,{}_{n}^{t}\mathcal{D}_{M}^{\alpha,\beta}\psi++\eta\psi_{xx}+(\delta|\psi|^2+\gamma|\psi|^4)\psi+\lambda\left\{\frac{|\psi|_{xx}}{|\psi|}\right\}\psi=0, \tag{13}$$

which ${}_{n}^{t}\mathcal{D}_{M}^{\alpha,\beta}\psi$, means truncated M-fractional derivative with respect to time-variable t. Under traveling wave transformation

$$\psi(x,t;\alpha)=U(\xi)e^{i\mu},\ \ \xi=x+\frac{2\eta\kappa\Gamma(\beta+1)}{\alpha}t^{\alpha},\quad \mu=-\kappa x+\frac{\omega\Gamma(\beta+1)}{\alpha}t^{\alpha},$$

the equation 13 can be reduced to a nonlinear ordinary differential equation as the following form (Ilie & et al., 2018)

$$(\eta+\lambda)U''-(\omega+\eta\kappa^2)U+\delta U^3+\gamma U^5=0.$$

Regarding to modified Kudryashov method and by using of transformation

$$U(\xi)=V^{\frac{1}{2}}(\xi),$$

Eq. 9 converts

$$(\eta+\lambda)(2VV"-(V')^2)-4(\omega+\eta\kappa^2)V^2+4\delta V^3+4\gamma V^4=0. \quad (14)$$

By means of the homogeneous balance principle, we obtain $N=1$, then the solution of Eq. 14 is

$$V(\xi)=a_0+a_1Q(\xi), \quad (15)$$

$$Q(\xi)=\frac{1}{1+da^{\xi}}.$$

By substituting Eq. 15 along with its second order derivative into Eq. 14 the resulting equation, a nonlinear system is expanded (Ilie et al., 2018). From solving nonlinear algebraic equations system, we attain

Case 1. $a_0=0$, a_1 is arbitrary real number and

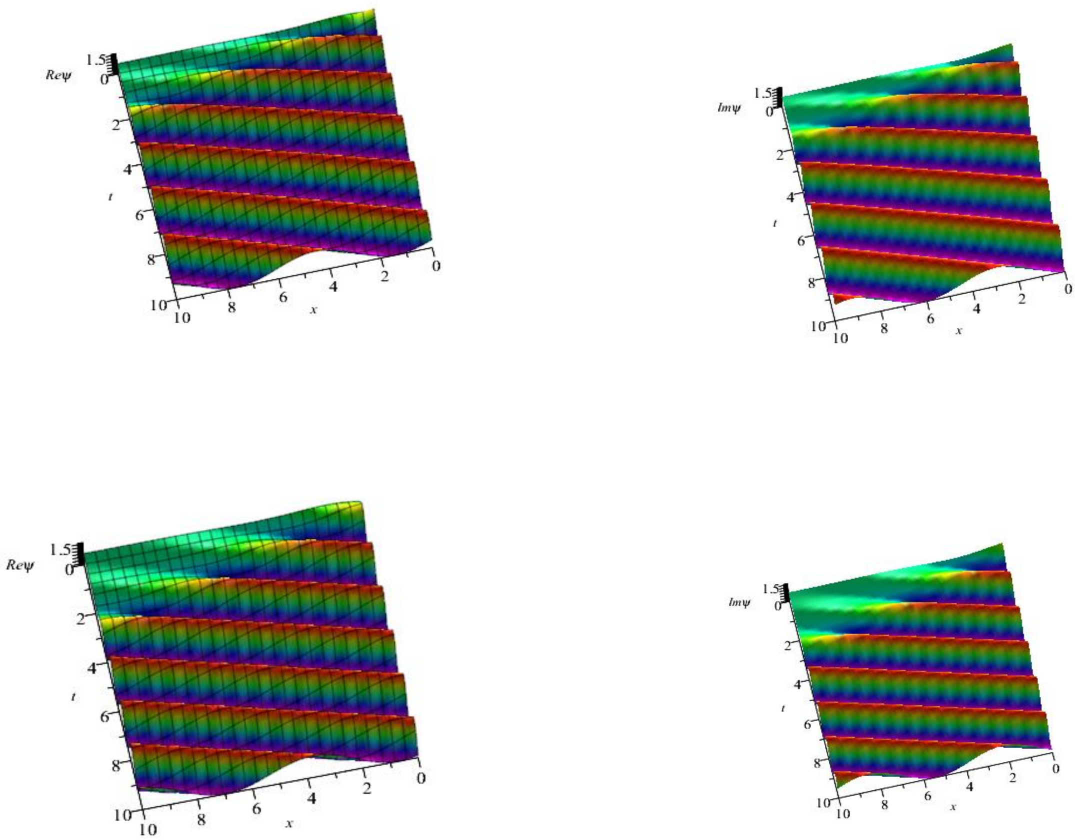
$$\delta=-\frac{4}{3}\gamma a_1, \quad \lambda=-\frac{3(\ln a)^2\eta+4\gamma a_1^2}{3(\ln a)^2}, \quad \omega=-\frac{1}{3}\gamma a_1^2-\eta\kappa^2$$

, then the soliton solution of Eq. 13 is

$$\psi(x,t;\alpha,\beta)=\sqrt{\frac{a_1}{1+da^{x+\frac{2\eta\kappa\Gamma(\beta+1)}{\alpha}t^{\alpha}}}}\,e^{i\left(-\kappa x+\frac{\omega\Gamma(\beta+1)}{\alpha}t^{\alpha}\right)},$$

$$\lim_{\beta\to 1}\psi(x,t;\alpha,\beta)=\psi(x,t;\alpha),$$

where $\psi(x,t;\alpha)$ is a resonant soliton solution of resonant nonlinear Schrödinger's equation with parabolic law nonlinearity, which is solved by Modified Kudryashov method (Ilie et al., 2018). The 3D plots of $Re(\psi(x,t;\alpha,\beta))$, and $Im(\psi(x,t;\alpha,\beta))$, are exhibited in Figs. 3.



Figs. 3. Plot of the $Re(\psi(x,t;\alpha,\beta))$ and $Im(\psi(x,t;\alpha,\beta))$ corresponding to the values $(\alpha,\beta)=(0.8,1.2),(1,0.5)$ from left to right when $a_1=-3,\eta=-2,\kappa=1,\gamma=2,d=4,a=2.7$ respectively.

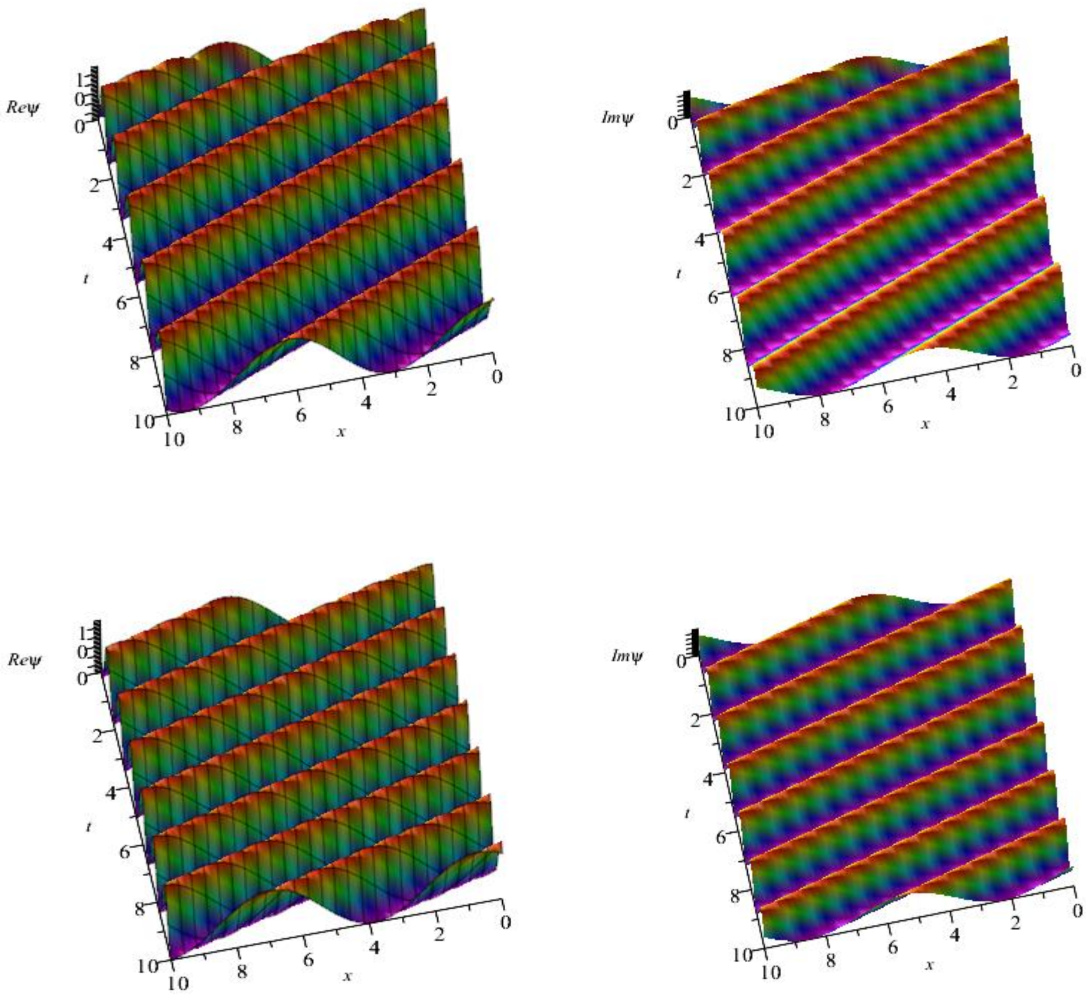
Case 2. $a_0=-a_1$, a_1 is an arbitrary real number and

$\delta = \frac{4}{3}\gamma a_1, \qquad \lambda = -\frac{3(\ln a)^2\eta + 4\gamma a_1^2}{3(\ln a)^2}, \qquad \omega = -\frac{1}{3}\gamma a_1^2 - \eta\kappa^2$, then the soliton solution of Eq. 13 is

$$\psi(x,t;\alpha,\beta) = \sqrt{-a_1 + \frac{a_1}{1 + d a^{x + \frac{2\eta\kappa\Gamma(\beta+1)}{\alpha}t^{\alpha}}}}\, e^{i\left(-\kappa x + \frac{\omega\Gamma(\beta+1)}{\alpha}t^{\alpha}\right)},$$

$$\lim_{\beta\to 1}\psi(x,t;\alpha,\beta) = \psi(x,t;\alpha),$$

where $\psi(x,t;\alpha)$ is a resonant soliton solution of resonant nonlinear Schrödinger's equation with parabolic law nonlinearity, which is solved by Modified Kudryashov method (Ilie et al., 2018). The 3D graph of $Re(\psi(x,t;\alpha,\beta))$, and $Im(\psi(x,t;\alpha,\beta))$, are revealed in Figs. 4.



Figs. 4. Plot of the $Re(\psi(x,t;\alpha,\beta))$ **and** $Im(\psi(x,t;\alpha,\beta))$ corresponding to the values $(\alpha,\beta) = (0.8,0.2), (1,0.5)$ from left to right when $a_1 = -2, \eta = 1, \kappa = 1, \gamma = -4, d = 2, a = 2.7$ respectively.

SINE-GORDON EXPANSION METHOD FOR THE TIME M-FRACTIONAL DIFFERENTIAL EQUATIONS

From previous section, assume that the solution of Eq. 11 can be articulated as follows (Ekici et al., 2017; Hosseini et al., 2017; Hosseini et al., 2017; Ilie et al., 2018)

$$U(\xi) = A_0 + \sum_{i=1}^{N} \tanh^{i-1}(\xi)\,[B_i \operatorname{sech}(\xi) + A_i \tanh(\xi)],$$

or

(16)

$$U(w(\xi)) = A_0 + \sum_{i=1}^{N} \cos^{i-1}(w(\xi))\,[B_i \sin(w(\xi)) + A_i \cos(w(\xi))].$$

Calculating the positive integer N using the homogeneous balance technique, setting Eq. 16 into Eq. 8, and comparing the terms, produces a nonlinear algebraic system which by solving it, we achieve the resonant soliton solutions of the time M-fractional Eq. 7 (Ekici et al., 2017; Hosseini et al., 2017; Hosseini et al., 2017; Ilie et al., 2018).

Resonant soliton solution of the time M-fractional resonant nonlinear Schrödinger equation via Kerr-law nonlinearity

As stated by previous section and sine-Gordon method, let us presume that the solution $U(\xi)$ of nonlinear ordinary differential equation 11 can be as follows Eq. 16. By using the homogeneous balance principle, we find $N = 1$, then the solution (16) is the following form

$$U(\xi) = B_1 \operatorname{sech}(\xi) + A_1 \tanh(\xi) + A_0,$$

and therefore

(17)

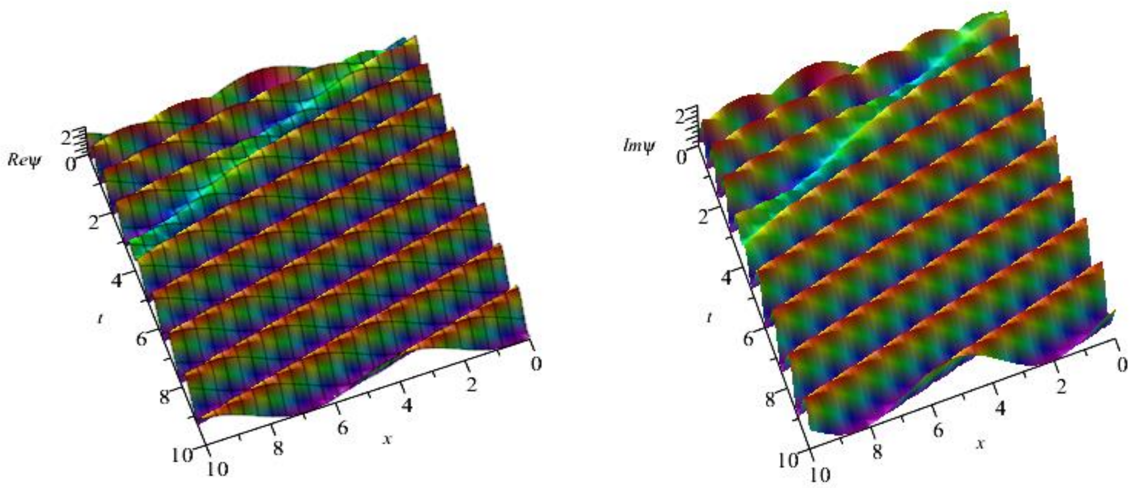
$$U(w(\xi)) = B_1 \sin(w(\xi)) + A_1 \cos(w(\xi)) + A_0,$$

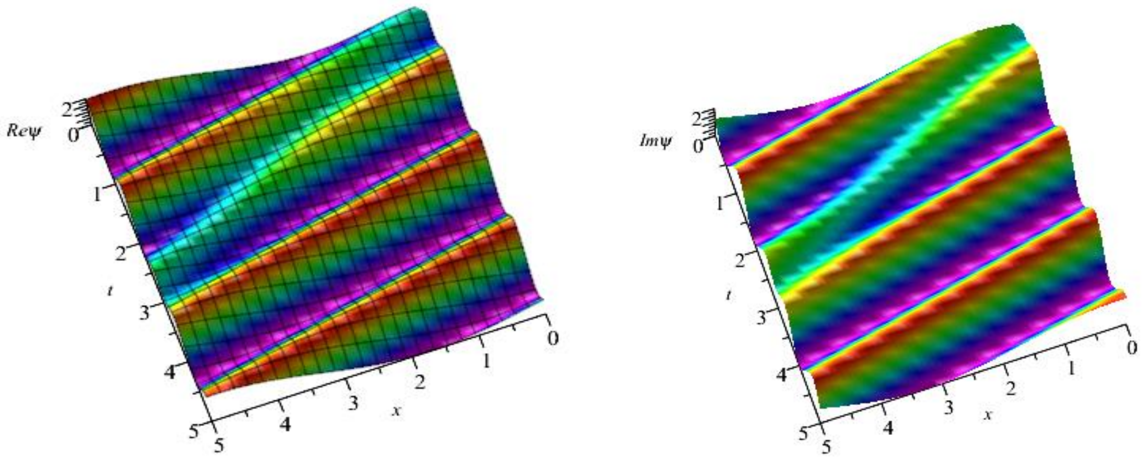
where either A_1or B_1 may be zero, but both A_1 or B_1cannot be zero simultaneously. By substituting Eq. 17 along with its second order derivative into Eq. 11 and comparing the terms in the resulting equation, a nonlinear system will be gained (Ekici et al., 2017; Hosseini et al., 2017; Hosseini et al., 2017; Ilie et al., 2018). By solving it, we obtain the soliton solutions of time fractional resonant nonlinear Schrödinger equation via Kerr law nonlinearity 10, as

Case 1. $$\psi(x,t;\alpha,\beta) = \pm e^{i\left(-\kappa x+\frac{(-\eta\kappa^2-2\eta-2\gamma)\Gamma(\beta+1)}{\alpha}t^{\alpha}\right)}\sqrt{-\frac{2\eta+2\gamma}{\delta}}\tanh\left(x+\frac{2\eta\kappa\Gamma(\beta+1)}{\alpha}t^{\alpha}\right),$$

$$\lim_{\beta\to 1}\psi(x,t;\alpha,\beta) = \psi(x,t;\alpha),$$

where $\psi(x,t;\alpha)$ is a resonant soliton solution of resonant nonlinear Schrödinger's equation with Kerr low nonlinearity, which is solved by way of sine-Gordon technique [13]. The 3D plots of $Re(\psi(x,t;\alpha,\beta))$, and $Im(\psi(x,t;\alpha,\beta))$, are demonstrated in Figs. 5.





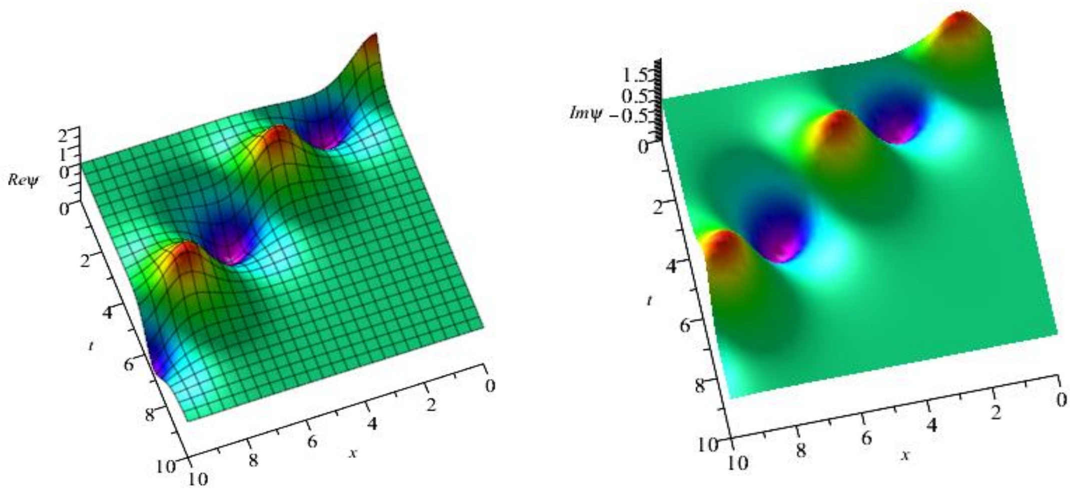
Figs. 5. Plot of the $Re(\psi(x,t;\alpha,\beta))$ **and** $Im(\psi(x,t;\alpha,\beta))$ corresponding to

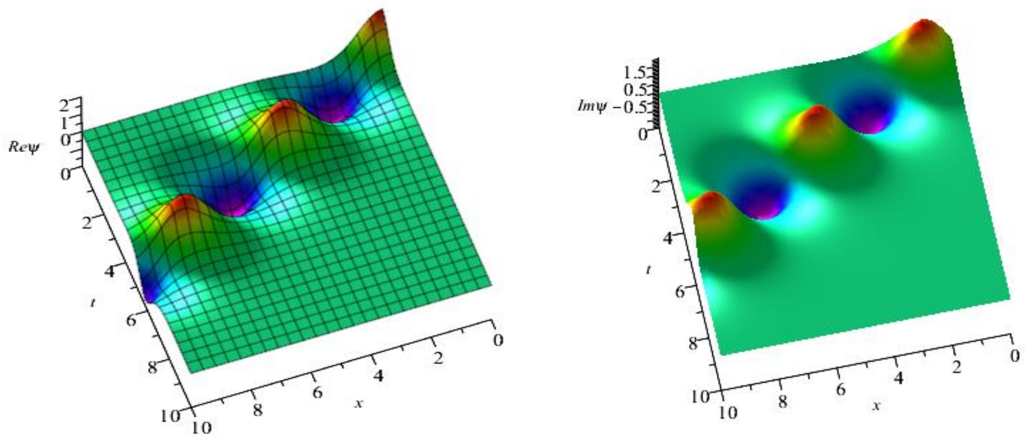
the values $(\alpha,\beta)=(0.8,1.5),(1,0.5)$ from left to right when $\eta=1,\beta=1,\kappa=-1,\gamma=1$ respectively.

Case 2. $\psi(x,t;\alpha,\beta) = \pm e^{i\left(-\kappa x+\frac{(-\eta\kappa^2+2\eta+2\gamma)\Gamma(\beta+1)}{\alpha}t^{\alpha}\right)}\sqrt{\frac{2\eta+2\gamma}{\delta}}\,\mathrm{sech}\left(x+\frac{2\eta\kappa\Gamma(\beta+1)}{\alpha}t^{\alpha}\right),$

$$\lim_{\beta\to 1}\psi(x,t;\alpha,\beta)=\psi(x,t;\alpha),$$

where $\psi(x,t;\alpha)$ is a resonant soliton solution of resonant nonlinear Schrödinger's equation with Kerr low nonlinearity, which is solved by way of sine-Gordon technique (Ilie et al., 2018). The 3D plots of $Re(\psi(x,t;\alpha,\beta))$, and $Im(\psi(x,t;\alpha,\beta))$, are explained in Figs. 6.



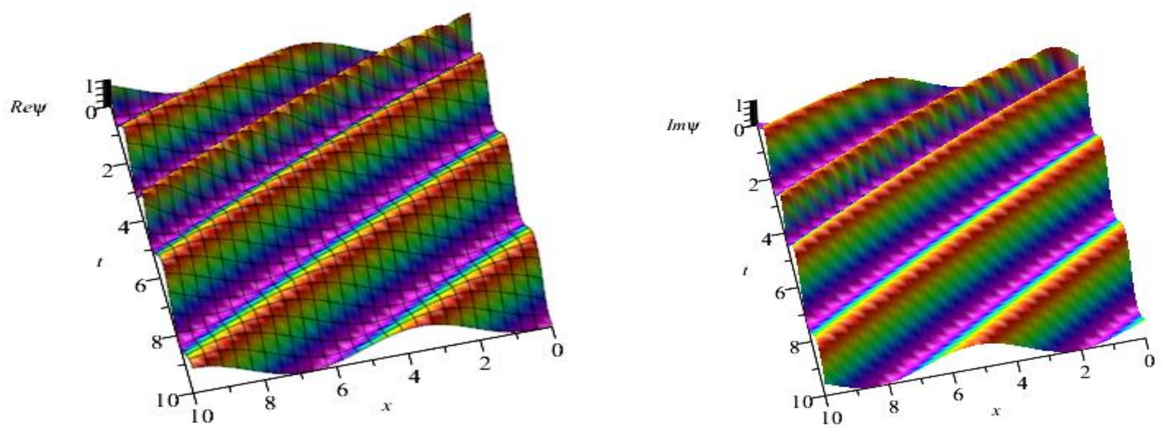


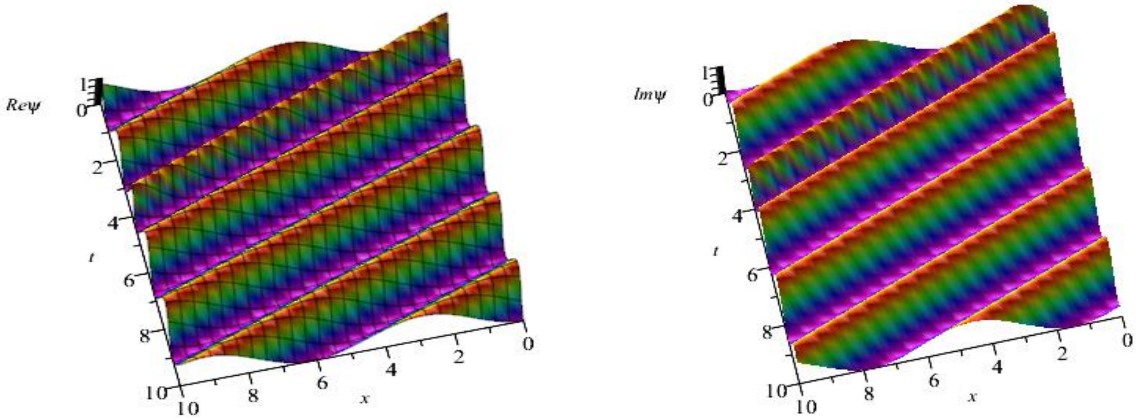
Figs. 6. Plot of the $Re(\psi(x,t;\alpha,\beta))$ **and** $Im(\psi(x,t;\alpha,\beta))$ corresponding to the values $(\alpha,\beta)=(0.8,0.5),(1,0.5)$ from left to right when $\eta=1,\beta=1,\kappa=-1,\gamma=1$ respectively.

Case 3. $\psi(x,t;\alpha,\beta)=$
$\pm e^{i\left(-\kappa x-\frac{(2\eta\kappa^2+\eta+\gamma)\Gamma(\beta+1)}{2\alpha}t^{\alpha}\right)}\sqrt{\frac{\eta+\gamma}{-2\delta}}\left[\tanh\left(x+\frac{2\eta\kappa\Gamma(\beta+1)}{\alpha}t^{\alpha}\right)\pm\text{sech}\left(x+\frac{2\eta\kappa\Gamma(\beta+1)}{\alpha}t^{\alpha}\right)\right],$

$$\lim_{\beta\to 1}\psi(x,t;\alpha,\beta)=\psi(x,t;\alpha),$$

where $\psi(x,t;\alpha)$ is a resonant soliton solution of resonant nonlinear Schrödinger's equation with Kerr low nonlinearity, which is solved by way of sine-Gordon technique (Ilie et al., 2018). The 3D plots of $Re(\psi(x,t;\alpha,\beta))$, and $Im(\psi(x,t;\alpha,\beta))$, are clarified in Figs. 7.





Figs. 7. Plot of the $Re(\psi(x,t;\alpha,\beta))$ **and** $Im(\psi(x,t;\alpha,\beta))$ corresponding to the values $(\alpha,\beta)=(0.8,1.5),(1,1.5)$ from left to right when $\eta=1,\beta=1,\kappa=-1,\gamma=1$ respectively

CANCLUSION

As the wide application of fractional derivatives in applied sciences, in the present article, we have strained to detect resonant soliton solutions to the time M-fractional forms of resonant nonlinear Schrödinger equations with different nonlinearity. For this purpose, we have used modified Kudryashov method and sine-Gordon expansion approach. The modified Kudryashov method and the sine-Gordon expansion approach were applied as an effectual procedure for solving the time M-fractional resonant nonlinear Schrödinger equations with Kerr law nonlinearity. The modified Kudryashov method was used as an effective arrangement to solve the time M-fractional resonant nonlinear Schrödinger equations via parabolic law nonlinearity. It would be stated the validity of the results reported in this article was investigated by putting each soliton solution back into its corresponding equation.

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