Financial Efficiency Evaluation in a Network with Feedbacks

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Abstract

Financial efficiency evaluations in a network system, considering intermediate products, nowadays considers great importance for managers. In the process of doing business analysis, each stage of in a network is of particular importance. In this paper, with the help of Data Envelopment Analysis (DEA), the performance of a network with feedbacks are considered. In this assessment, financial efficiency is being assessed for a network with inputs, outputs, intermediate products, and feedbacks. As an example, the application of data from petrochemical companies is considered and implemented to a supply chain for evaluation and analysis the efficiencies.

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INTRODUCTION

Data Envelopment Analysis is a technique for calculating the relative efficiency of a set of decision-making units (DMUs) that is done using mathematical programming. The obtained efficiency is in the relative term because the performance is the result of the comparison of units with each other. When the unit is said to be efficient, this means that this unit does not waste the resources and does not have a lack in its production. In DEA technique the goal is to find an efficient boundary or production function, which is usually not available. Thus, by accepting the underlying DEA axioms, the boundary of the envelope covered the DMUs is considered as an approximation of the production function. The production function estimation was carried out in a non-parametric manner for the first time in 1957 by Farrell et al. Charnes et al., 1987, generalized Farrell definitions and modeling, the CCR model, which was capable of considering multiple inputs and outputs, to evaluate the efficiency of a set of decision-makers, assuming constant returns to scale form of the technology. Then Banker et al. (1984) introduced the BCC model with the assumption of variable returns to the scale form of the technology for efficiency evaluation. One of the early models for evaluating cost efficiency presented by Atanaspaulus et al. (1999) in DEA the literature. Then Comenough and Dyson (2005) developed it.

According to what the initial definitions of cost efficiency exists in the literature, it indicates the ability of a decision maker, given the input prices assumed to produce the same output level with the minimum cost. For the first time, Fare et. al. 1985 developed methods for empirically implementing the cost and revenue efficiencies. To evaluate the efficiency of various aspects, the problem of sensitivity analysis in DEA has been studied by many researchers. Arife and Kane 2008, Dellis et al. 2008, Issyk & Hassan 2002, Ray & Das 2010, Israire 2010, have introduced a cost efficiency model in DEA technique. Aparicio et al. 2015 have introduced a new perspective on cost and revenue efficiency models. This is a significant topic in performance evaluation that helps managers for better decision making.

Supply chain assessment is one of the key issues that today helps administrators make their decisions for guiding systems under their control. Because of this, it is also of particular importance in academic research. Among the most important issues studied and investigated in this field the work of Fazeipour Saen 2010 Shabanpur 2017, Bodaghi 2018 and Zolqdari et al. 2011 can be named. Therefore, in this paper cost and revenue efficiencies models for a two-stage supply chain with returned. Therefore, in this research, the supply chain with the outputs of the second stage that have been returned to the previous stage due to lack of desirable quality. This is a topic that occurs frequently in a large number of chains. For an application, data from petrochemical plants are reviewed to further explain the proposed models.

In the next section, we will review the framework for data envelopment analysis. In the third section, performance evaluation models for cost and revenue are presented for a two-stage supply chain with outputs returned to the previous stage. In the fourth section, the example is applied and in the fifth part the conclusions will be presented.

DATA ENVELOPMENT ANALYSIS

A decision making unit is a firm that uses a vector of input like $(x_1,...,x_m)$ to produce a vector of outputs like $(y_1,...,y_s)$. In DEA technique it is assumed to have n DMUs each uses m inputs to produce s outputs. The inputs and outputs data are considered to be semi-positive.

In DEA technique the definition of production possibility set (PPS) is as follows:

$$T=\{(x,y)|\ X>0\ \text{can produce}\ y>0\} \qquad (1)$$

According to the underlying DEA axioms, different DEA models can be considered. As one of the famous DEA models used for efficiency evaluation CCR model can be named.

$$\begin{aligned} &Min \quad \theta \\ &s\,t. \\ &\quad (\theta X_o, Y_o) \in T_{CCR} \end{aligned} \qquad (2)$$

Consider the following model with which cost efficiency score of the DMU_ounder assessment

is obtained can be obtained from the following model. X is the input variable and corresponding price is C. As the denominator is a fixed number the aim of this model is to minimize the nominator of the fraction which is $\sum_{i=1}^{m} c_i x_i$. Having the optimal solution of this model cost efficiency score is resulted.

$$CEo = Min \quad \frac{\sum_{i=1}^{m} c_i x_i}{\sum_{i=1}^{m} c_i x_{io}}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} = x_i \qquad i = 1,...,m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1,...,s,$$

$$x_i \geq 0 \qquad i = 1,...,m,$$

$$\lambda_j \geq 0 \qquad j = 1,...,n. \qquad (3)$$

In this model (x_{io}, y_{ro}) $i = 1,...,m, r = 1,...,s$ is the input and output vector of the DMUo under evaluation.

The aim of model (4) is to maximize $\sum_{r=1}^{s} w_r y_r$ in the nominator. The inverse of optimal objective function value is revenue efficiency score. Y is the output variable and corresponding price is w.

$$\frac{1}{REO} = Max \quad \frac{\sum_{i=1}^{s} w_r y_r}{\sum_{i=1}^{s} w_r y_{ro}}$$

$$s.t. \quad \sum_{j=1}^{n} x_{ij} \lambda_j \leq x_{io} \qquad i = 1,...,m,$$

$$\sum_{j=1}^{n} y_{rj} \lambda_j = y_r \qquad r = 1,...,s,$$

$$y_r \geq 0 \qquad r = 1,...,s,$$

$$\lambda_j \geq 0 \qquad j = 1,...,n \qquad (4)$$

Same as model (3) (x_{io}, y_{ro}) $i = 1,...,m, r = 1,...,s$ is the input and output vector of the DMUo under evaluation.

One of the important issues is adjusting these efficiency models such that they can be used in a supply chain. Because in the growing research in the various sciences, the supply chain is important. The supply chain is a network of processes, in which their ultimate goal is to provide customers with goods and services, including suppliers, manufacturers, distributors, wholesalers and retailers, which are coordinated together in order to satisfy customers satisfaction. The supply chain has every disciplines and trends and it has its own application that formulating it with optimal models is very important. By formulating the supply chain using optimization problems, managers can decide better for more efficient system guidance.

In the next section cost and revenue efficiency models are considered for seller and buyer stages of a supply chain.

FINANCIAL EFFICIENCIES WITH FEEDBACK IN NETWORK

The supply chain is a network of processes, in which their ultimate goal is to provide customers with goods and services, including suppliers, manufacturers, distributors, wholesalers and retailers, which are coordinated together in order to satisfy customers satisfaction. The topic of supply chain management gains an important attention for managers of different systems. Thus, here it is aimed to analyze a supply chain for better making decisions. According to what the initial definitions of cost efficiency in the capability of a decision-making unit, given the input prices, to produce the same output level with the minimum cost, is the cost efficiency score, Comanho and Dyson 2005. Consider the two-stage supply chain network system with inputs, outputs, and intermediate products. In this system, in addition to the inputs and outputs and intermediates, some parts of the outputs of the second stage are returned to the first stage due to lack of quality. Consider Fig 1. These kinds of outputs are returned to the previous stage due to quality shortcoming. These indices are the outputs of the second stage and the inputs of the first stage.

Consider the cost model for the supply chain (seller and buyer) depicted in Fig 1 as following.
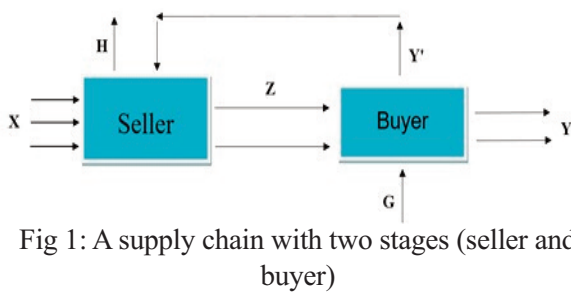
Fig 1: A supply chain with two stages (seller and buyer)

According to the basic definitions, a DMU is cost efficient if by given the inputs prices producing the same level of outputs, the same output or more can be generated with minimum cost, Aparicio et al. 2015.

In this supply chain, in the "buyer" stage, the output "y'''" is returned to the "seller" stage due to lack of quality. Consider c,t,u and $\bar{v}$ to be the price vector which are fixed and known for the inputs x, z, g and y'.

$$CEo = Min \sum_{i=1}^{m} c_i \sum_{j=1}^{n} \lambda^1_j x_{ij} + \sum_{f=1}^{q} t_f \sum_{j=1}^{n} \lambda^2_j z_{fj}$$

$$+ \sum_{p=1}^{k} u_f \sum_{j=1}^{n} \lambda^2_j g_{pj} + \sum_{r=1}^{s'} \bar{v}_r \sum_{j=1}^{n} \lambda^1_j y'_{rj}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda^2_j z_{fj} = z_f \qquad f = 1,...,q,$$

$$\sum_{j=1}^{n} \lambda^2_j y_{rj} \geq y_{ro} \qquad r = 1,...,s,$$

$$\sum_{j=1}^{n} \lambda^1_j z_{fj} \geq z_{fo} \qquad f = 1,...,q,$$

$$\sum_{j=1}^{n} \lambda^1_j h_{ij} \geq h_{io} \qquad 1 = 1,...,p',$$

$$\sum_{j=1}^{n} \lambda^2_j y'_{rj} \geq y'_{ro} \qquad r' = 1,...,s',$$

$$\lambda^1_j \geq 0, \lambda^2_j \geq 0 \qquad j = 1,...,n. \qquad (5)$$

Cost efficiency score finally obtained from the following equations.

$$CEo = \frac{\sum_{i=1}^{m} c_i \sum_{j=1}^{n} \lambda^{1*}_j x_{ij} + \sum_{f=1}^{q} t_f \sum_{j=1}^{n} \lambda^{2*}_j z_{fj} + \sum_{p=1}^{k} u_f \sum_{j=1}^{n} \lambda^{2*}_j g_{pj} + \sum_{r'=1}^{s'} \bar{v}_r \sum_{j=1}^{n} \lambda^{1*}_j y'_{rj}}{\sum_{i=1}^{m} c_i x_{io} + \sum_{f=1}^{q} t_f z_{fo} + \sum_{p=1}^{k} u_f g_{po} + \sum_{r'=1}^{s'} \bar{v}_r y'_{ro}}$$

$$= \frac{\sum_{i=1}^{m} c_i x^*_i + \sum_{f=1}^{q} t_f z^*_f + \sum_{p=1}^{k} u_f g^*_p + \sum_{r'=1}^{s'} \bar{v}_r y^{'*}_r}{\sum_{i=1}^{m} c_i x_{io} + \sum_{f=1}^{q} t_f z_{fo} + \sum_{p=1}^{k} u_f g_{po} + \sum_{r'=1}^{s'} \bar{v}_r y'_{ro}}$$

$$(6)$$

Where

$$\sum_{j=1}^{n} \lambda^1_j x_{ij} = x_i \ (i = 1,...,m), \ \sum_{j=1}^{n} \lambda^2_j z_{fj} = z_f, (f = 1,...,q)$$

$$\sum_{j=1}^{n} \lambda^2_j g_{pj} = g_p, (p = 1,...,k), \sum_{j=1}^{n} \lambda^1_j y'_{rj} = y'_{r'}, (r' = 1,...,s')$$

$$(7)$$

This model is used for analysis the cost evaluation when returned outputs existed. More in-depth information can be obtained by stage analysis. Knowing the seller and buyer behaviors, managers can have information about inter-relationship in the supply chain. This information can be obtained through solving cost models for seller and buyer stages.

Consider the following model for deriving the minimum cost of the seller stage for DMUo under evaluation. The aim of this model is to minimize the nominator where X and y' are inputs to the seller stage and corresponding prices are C and V^-.

$$S.CEo = Min \sum_{i=1}^{m} c_i \sum_{j=1}^{n} \lambda^1_j x_{ij} + \sum_{r=1}^{s'} \bar{v}_r \sum_{j=1}^{n} \lambda^1_j y'_{rj}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda^1_j z_{fj} \geq z_{fo} \qquad f = 1,...,q,$$

$$\sum_{j=1}^{n} \lambda^1_j h_{ij} \geq h_{io} \qquad 1 = 1,...,p'$$

$$(8)$$

In order to obtained the cost efficiency score of the seller stage, the following equation is used.

$$S.CEo = \frac{\sum_{i=1}^{m} c_i x^*_i + \sum_{r=1}^{s'} \bar{v}_r y^*_r}{\sum_{i=1}^{m} c_i x_{io} + \sum_{r=1}^{s'} \bar{v}_r y'_{ro}} \qquad (9)$$

In which

$$\sum_{j=1}^{n} \lambda^1_j x_{ij} = x_i. (i = 1, ..., m),$$

$$\sum_{j=1}^{n} \lambda^1_j y'_{rj} = y'_r. (r' = 1, ..., s') \qquad (10)$$

For analyzing the "buyer" stage the minimum cost model is as follows.

$$B.CEo = Min \quad \frac{\sum_{f=1}^{q} e_f z_f + \sum_{p=1}^{k} u_p g_p}{\sum_{f=1}^{q} e_f z_{fo} + \sum_{p=1}^{k} u_p g_{po}}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda^2_j y_{rj} \geq y_{ro} \qquad r = 1, ..., s$$

$$\sum_{j=1}^{n} \lambda^2_j y'_{rj} \geq y'_r \qquad r' = 1, ..., s'$$

$$\lambda^2_j \geq 0, \qquad j = 1, ..., n \qquad (11)$$

The cost efficiency of the buyer stage can be calculated from the following relation.

$$B.CEo = \frac{\sum_{f=1}^{q} e_f z^*_f + \sum_{p=1}^{k} u_p g^*_p}{\sum_{f=1}^{q} e_f z_{fo} + \sum_{p=1}^{k} u_p g_{po}} \qquad (12)$$

Where

$$\sum_{j=1}^{n} \lambda^2_j z_{fj} = z_f. (f = 1, ..., q), \sum_{j=1}^{n} \lambda^2_j g_{pj}$$

$$= g_p. (p = 1, ..., k) \qquad (13)$$

According to the basic definitions, a DMU is revenue efficient if by given the expected output prices using the same level of inputs, the same output or more can be generated with maximum revenue, Aparicio et al. 2015.

To estimate the revenue efficiency of this supply chain, we have the following linear model. For better analyzing the chain, seller and buyer stages should also be investigated. Thus, for revenue efficiency evaluation a supply chain itself is taken into consideration, alongside seller and buyer stages are also investigated. In all these models the buyer stage has such an output that due to lack of quality is returned to the seller stage.

Consider a supply chain as mentioned in Fig 1, the maximum revenue score of the supply chain, DMUo, obtained after solving model (14).

Consider w,t,s, and to be prices which are fixed and known for the outputs *y, z, h*, and *y'*. The aim of this model is to maximize the weighted sum of output variables, as

$$\sum_{r=1}^{s} w_r y_r + \sum_{f=1}^{q} t_f z_f + \sum_{l=1}^{p'} s_l h_l + \sum_{r'=1}^{s'} v_r y'_r$$

Note that outputs of DMUo for these two stages are $(x_{io}, z_{fo}, h_{lo}, y'_{r'}, y_{ro})$ $i=1,...,m, f=1,...,q, l=1,..., p', r'=1,...s', r=1,...,s$. The inverse of optimal objective function of model (14) is then returned as the revenue efficiency score. The same is true for revenue efficiency models of seller and buyer stages as well. Thus, the obtained efficiency lies in (0,1] interval. This is also performed for revenue efficiency of a supply chain as well as for the seller and buyer stages which makes it easier to compare the cost and revenue efficiencies to be compared among each other.

$$\frac{1}{REo} = Max \sum_{r=1}^{s} w_r \sum_{j=1}^{n} \lambda^2_j y_{rj} + \sum_{f=1}^{q} t_f \sum_{j=1}^{n} \lambda^1_j z_{fj}$$

$$+ \sum_{l=1}^{p'} s_l \sum_{j=1}^{n} \lambda^1_j h_{lj} + \sum_{r'=1}^{s'} v_r \sum_{j=1}^{n} \lambda^2_j y'_{rj}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda^1_j x_{ij} \leq x_{io} \qquad i = 1, ..., m,$$

$$\sum_{j=1}^{n} \lambda^2_j z_{fj} \leq z_{fo} \qquad f = 1, ..., q,$$

$$\sum_{j=1}^{n}\lambda^{2}_{j}g_{pj} \leq g_{po} \qquad p=1,...,k,$$

$$\sum_{j=1}^{n}\lambda^{1}_{j}y'_{r'j} \leq y'_{r'o} \qquad r'=1,...,s',$$

$$\lambda^{1}_{j} \geq 0, \lambda^{2}_{j} \geq 0 \qquad j=1,...,n \tag{14}$$

Therefore,

$$\frac{1}{RE_{o}} = Max \frac{\sum_{r=1}^{s}w_{r}y^{*}_{r}+\sum_{f=1}^{q}t_{f}z^{*}_{f}+\sum_{i=1}^{p'}s_{i}h^{*}_{i}+\sum_{r=1}^{s'}v_{r}y'^{*}_{r}}{\sum_{r=1}^{s}w_{r}y_{ro}+\sum_{f=1}^{q}t_{f}z_{fo}+\sum_{i=1}^{p'}s_{i}h_{io}+\sum_{r=1}^{s'}v_{r}y'_{r'o}} \tag{15}$$

Where

$$\sum_{j=1}^{n}\lambda^{2}_{j}y_{rj} = y_{r},(r=1,...,s),\sum_{j=1}^{n}\lambda^{1}_{j}z_{fj}=z_{f},(f=1,...,q)$$

$$\sum_{i=1}^{n}\lambda^{1}_{j}h_{ij}=h_{i},(1=1,...,p'),\sum_{i=1}^{n}\lambda^{2}_{j}y'_{r'j}=y'_{r'},(r'=1,...,s') \tag{16}$$

Consider the seller stage. For revenue evaluation of this stage the following model can be formulated. For the seller maximum revenue score is then obtained by the optimal objective function of model (17) which aim is to maximizes the weighted sum of output variables of this stage, which is $\sum_{f=1}^{q}t_{f}z_{f}+\sum_{i=1}^{p'}s_{i}h_{i}$.

$$\frac{1}{S.RE_{o}} = Max \sum_{f=1}^{q}t_{f}\sum_{j=1}^{n}\lambda^{1}_{j}z_{fj}+\sum_{i=1}^{p'}s_{i}\sum_{j=1}^{n}\lambda^{1}_{j}h_{ij}$$

s.t. $$\sum_{j=1}^{n}\lambda^{1}_{j}x_{ij} \leq x_{io} \qquad i=1,...,m$$

$$\sum_{j=1}^{n}\lambda^{1}_{j}y'_{r'j} \leq y'_{r'o} \qquad r'=1,...,s'$$

$$\lambda^{1}_{j} \geq 0, \qquad j=1,...,n. \tag{17}$$

In order to obtain the revenue efficiency of the seller stage, consider the following relation.

$$\frac{1}{S.RE_{o}} = \frac{\sum_{f=1}^{q}t_{f}z^{*}_{f}+\sum_{i=1}^{p'}s_{i}h^{*}_{i}}{\sum_{f=1}^{q}t_{f}z_{fo}+\sum_{i=1}^{p'}s_{i}h_{io}} \tag{18}$$

Where

$$\sum_{j=1}^{n}\lambda^{1}_{j}z_{fj} = z_{f},(f=1,...,q),$$

$$\sum_{j=1}^{n}\lambda^{1}_{j}h_{ij}=h_{i},(1=1,...,p') \tag{19}$$

For the buyer stage, model (20) reveals the revenue efficiency score when optimal objective function of this model is derived. This model tries to maximize the weighted sum of output variables of the second stage, which is $\sum_{f=1}^{g}e_{f}z_{f}+\sum_{r'=1}^{s'}v_{r}y'_{r}$.

$$\frac{1}{B RE_{o}} = Max \sum_{f=1}^{q}e_{f}\sum_{j=1}^{n}\lambda^{2}_{j}y_{rj}+\sum_{r'=1}^{s'}v_{r}\sum_{j=1}^{n}\lambda^{2}_{j}y'_{rj}$$

s.t. $$\sum_{j=1}^{n}\lambda^{2}_{j}z_{fj} \leq z_{fo} \qquad f=1,...,q$$

$$\sum_{j=1}^{n}\lambda^{2}_{j}g_{pj} \leq g_{po} \qquad p=1,...,k$$

$$\lambda^{2}_{j} \geq 0, \qquad j=1,...,n. \tag{20}$$

Revenue efficiency of the buyer stage can be obtained via:

$$\frac{1}{B RE_{o}} = \frac{\sum_{f=1}^{q}e_{f}z^{*}_{f}+\sum_{r'=1}^{s'}v_{r}y'^{*}_{r}}{\sum_{f=1}^{q}e_{f}z_{fo}+\sum_{r=1}^{s'}v_{r}y'_{ro}} \tag{21}$$

Where

$$\sum_{j=1}^{n}\lambda^{2}_{j}y_{rj} = y_{r},(r=1,...,s),$$

$$\sum_{j=1}^{n}\lambda^{2}_{j}y'_{rj} = y'_{r}.(r'=1,...,s') \tag{22}$$

APPLICATION

In this application a seller and buyer supply chain in industry in the petrochemical industry is considered. The "seller" in this example is the producer of petrochemicals and "buyer" is the downstream industry that uses petrochemicals. Polypropylene is made from polymerization of propylene under relatively low pressure and temperature conditions in the presence of the famous Ziegler-Nata catalyst. The existence of this catalyst is an isotypic polymer that can crystallize to about 90%. Polypropylene is a thermoplastic polymer used in a wide range of applications including film and sheet, blow molding, injection molding, food packaging, textile, laboratory and medical equipment, tubes, industrial and building applications, and automotive components. In addition, the polymer produced from propylene monomer is generally resistant to chemical solvents, acids and acids.

Polypropylene has distinct and distinguished features in comparison with other polymers, which include:

• The relatively inexpensive price of propylene monomer in comparison with other monomers of polymers

• Low PP price compared to other PP polymers PP and special light weight

• Flexibility and wide spectrum of PP production with variable physical and chemical properties

• Increase new applications and improve the properties of new production grades

• Increasing the use of PP in medical devices and the development of specific PP-applications

• Increased PP consumption as an alloy with other polymers

Homopolymer:

The simplest types of polymers are hemopolymers made up of polymer chains composed of single repeat units. That is, if A is a repeat unit, a homopolymeric chain will be arranged as AAA in the molecular molecular sequence of the polymer.

Copolymer:

Copolymers are polymers that produce polymorphisms of two or more different monomers that are compatible with each other, creating different structures in this way.

Random Copolymer:

These copolymers are prepared by polymerizing a mixture of monomeropropylene and comonomer ethylene randomly arranged in the polymer chain. How to place two monomers in a completely randomized chain, and changing the properties of the polymer is also due to the same arrangement.

Some of the main examples of the consumer (buyer) of polyethylene are as follows.

In the implementation of the model, polypropylene producing companies are used as vendors and from downstream industries such as textiles, packaging and automobiles as purchasers. The statistical information is as follows.

Table 1: Examples

Application	Industry
Ruff and rope	Textile and fiber industries
Yarn and filament	
Nonwoven	
BOPP	Packaging Industry
Bed and blow	
Lid and bottle cap	
Carton plast	
Buckets and boxes	
Thin wall utensils	
Parts inside and outside the car	Automotive Industry

Table 2: Statistical data information

Data	Name	Minimum lower bound	Maximum lower bound	Minimum of Upper bound	Maximum of Upper bound	Minimum of core	Maximum of core
X1	propylene	160000	180000	200000	210000	180000	190000
X2	ethylene	2000	4000	7800	8000	7000	7500
X3	the catalyst	250	350	400	450	300	400
H	Wastes of F grades	1500	1700	19000	20000	8000	8500
Z1	Homopolymer	112000	122000	137000	147000	133000	134000
Z2	Copolymer	40000	45000	50000	52500	46500	47500
Z3	Casual copolymer	8100	9300	10000	10900	9475	9875
G	Raw materials other than those mentioned	5000	5500	14000	15000	11000	12500
Y1	Textile and fiber industries grades	51000	53000	55000	64500	52500	54500
Y2	Industry packaging grades	68000	78000	81000	86000	89000	80000
Y3	Automotive Industry grades	34000	43000	46000	48000	39000	40000
Y'	Quality returns of the product	510	570	615	685	585	590

According to the statistical information of data given in Table 2 and provided models for cost and revenue analysis the results are gathered in Table 3. In the case that the supply chain is taken into considerations as a black box by performing the DEA models, mentioned in preliminaries model (3) and (4) the results of cost and revenue scores are also listed in Table 3. As it can be seen, there is difference when a DMU is considered by its stages and the case that it considers as a black box.

Table 3: Results

DMUs	Cost efficiency				Revenue efficiency			
	Seller	Buyer	Supply Chain	Blackbox	Seller	Buyer	Supply Chain	Blackbox
1	0.875	0.658	0.754	0.873	0.546	0.964	0.865	0.925
2	0.484	0.925	0.834	0.735	0.934	1	0.925	0.834
3	0.658	1	0.863	1	0.923	0.487	0.873	-/783
4	0.837	1	0.924	1	1	0.871	0.927	1
5	0.783	0.761	0.810	0.818	0.378	1	0.587	0.785
6	1	0.835	0.746	0.684	0.498	0.587	0.879	1
7	0.562	0.683	0.678	0.785	0.983	0.876	1	0.893
8	0.839	0.925	1	0.835	1	0.489	0.761	0.873
9	0.931	0.759	1	0.834	0.478	0.780	1	0.893
10	0.472	0.830	0.764	0.782	0.834	0.487	0.892	0.920

An important difference is that the classifications of DMUs are different while the whole process is being considered as black box compared to the situations when the existing intermediate relationships are also viewed.

CONCLUSION

Supply chain management is especially important in business processes. Because of the challenges faced by strategic executives of organizations, supply chain management and its analysis gain an important attention. In the process of business acquisition analysis, each stage of a supply chain is of particular importance, and today, using different technologies and tools, all of the stages can be analyzed separately. In this paper, with the help of data envelopment analysis, the performance evaluation of a supply chain has been addressed. The features of this chain are that it has the output that is returned to the previous stages due to lack of quality. In different application all over the world, this may have happened frequently that such outputs are returned to the previous stage due to lack of minimum quality. In this paper, these kinds of outputs are considered in a supply chain while cost and revenue evaluations are being performed. Mathematical modeling of such issue has been designed in this paper to examine the cost and revenue efficiencies of a supply chain with input, output, intermediate products, and outputs to the previous stage by using the data envelopment analysis technique. As an example, data from petrochemical companies are considered and a supply chain evaluation under the mentioned situations is performed.

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