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A New Eight-Order Iterative Method for Solving Nonlinear Equations with High Efficiency index

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Abstract

In this paper, we develop a new eighth-order method for simple roots of non- linear equations via weight function and interpolation methods. The method requires only three (3) function evaluation and a derivative evaluation with 81/4 ≈ 1.682 efficiency index. Numerical comparison between the proposed method with some other methods were presented, which shows that our method is promising.

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INTRODUCTION

One of the attractive area of numerical analysis is solving nonlinear equations . Mostly iterative method are used to find the solution of nonlinear equation. Throughout this paper, we consider iterative method to find a simple root α, i.e f(α) = 0 and f '(α) 6= 0 of a nonlinear equation

$$f(x) = 0 \quad (1)$$

where f : D ∈ R → R which is defined on an interval D. Newton method is probably the most widely used iterative method for finding the solution of (1) via

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, k \doteq 0, 1, 2, ... \quad (2)$$

where f '(xk) is the first order derivative. It is well known (Traub,1997) that the Newton method is quadratically convergent and requires only two (2) functions evaluation for each iteration step.

In the recent years, many new modified methods have been proposed to improve the convergence order and efficiency index of the classical iterative methods (Zhao et.al., 2012) There are some classical iterative methods such as Newton's method, Ostrowki's method (Ostrowsk,1973) which is defined by

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ x_{n+1} = y_n - \frac{f(x_n)}{f(x_n)+(f(x_n)-2f(y_n)}\frac{f(y_n)}{f'(x_n)} \end{cases} \quad (3)$$

Chebyshev-Halley method (Gutierrez et al., 1997) which is defined by

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ x_{n+1} = y_n - [1 + \frac{1}{2}\frac{Lf(x)}{1-\alpha Lf(x)}]\frac{f(x_n)}{f'(x_n)} \\ where Lf(x) = \frac{f''(x_n)f(x_n)}{f'(x_n)} \end{cases} \quad (4)$$

and Jarratts method (Argyros et al., 1994), which is defined by

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ x_{n+1} = y_n - [1 - \frac{3}{2}\frac{f'(y_n)-f'(x_n)}{3f'(y_n)-f'(x_n)}]\frac{f(x_n)}{f'(x_n)} \end{cases} \quad (5)$$

In addition, Chun and Ham (2007), Construct a fourth order modification of Newton's method for solving nonlinear equations. The method required two evaluation of the function and one of it's first derivative per iteration which is given by

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ x_{n+1} = y_n - \frac{2f(x_n)+(2\beta-1)f(y_n)}{2f(x_n)+(2\beta-5)f(y_n)}, where\ \beta \in R \end{cases} \quad (6)$$

Moreover, Jishe Feng (2009), obtained a new two-step iterative method for solving nonlinear equations which is define by

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ x_{n+1} = y_n - \frac{f(y_n)}{2f'(x_n)-f'(y_n)} \end{cases} \quad (7)$$

Khattri and Argyros (2010) Contribute a new iterative method for convergence order four for solving nonlinear equations given by

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ x_{n+1} = y_n - \frac{f(y_n)}{\alpha f'(x_n)+(1-\alpha)\frac{f(y_n)-f(x_n)}{y_n-x_n}} \end{cases} \quad (8)$$

Chun and Ham (2007) developed a family of sixth-order methods by weight function methods, which is written as:

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n = y_n - \frac{f(x_n)}{f(x_n)-2f(y_n)}\frac{f(y_n)}{f'(x_n)} \\ x_{n+1} = z_n - H(\mu_n)\frac{f(Z_n)}{f'(x_n)} \\ . \end{cases} \quad (9)$$

where $\mu_n = \frac{f(y_n)}{f(x_n)}$ H(t) represent a real valued function with H(0) = 1,H '(0) = 2,H ''(0) < ∞

Petkovic (2009) Construct a three point iterative method for solving nonlinear equations. It's order of convergence reached eight with only four functions evaluation. Per iteration , which means that the proposed method posses as high as possible computational efficiency in the sense of the Kung-Traup hypothesis (1974).

$$\begin{cases} y = x - \frac{f(x)}{f'(x)} \\ z = y - p(t)\frac{f(y)}{f'(x)},\ t = \frac{f(y)}{f(x)} \\ x_{n+1} = z - \frac{f(z)}{f'(z)} \end{cases} \quad (10)$$

Zhao et.al. (2012) constructed a new family of eight-order methods for solving simple roots of nonlinear equations.

$$\begin{cases} y_n & = x_n - \frac{f(x_n)}{f'(x_n)}, \\ z_n & = y_n - G(\mu_n)\frac{f(y_n)}{f'(x_n)}, \\ x_{n+1} & = z_n - H(v_n)\frac{f(z_n)}{f[x_n,z_n]+f[y_n,z_n]-f[x_n,y_n]} \\ where\ \mu_n & = \frac{f(y_n)}{f(x_n)}\ and\ v_n = \frac{f(z_n)}{f(x_n)} \end{cases} \quad (11)$$

Ababneh (2016) proposed a new fourth order iterative methods second derivative free for solving nonlinear equations given by

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = y_n - 2\frac{f(y_n)}{f'(x_n)} + \frac{f(y_n)(f(x_n)+(\beta-2)f(y_n))}{f'(x_n)(f(x_n)+\beta f(y_n))} \\ (\frac{f(y_n)}{f'(x_n)})^2, \end{cases}$$
$$- \frac{f'(x_n)f(y_n)}{f(x_n)(f(x_n)+\beta f(y_n))} \quad (12)$$

where $\beta \in R$ is a constant and n=0,1,2,...

METHOD AND CONVERGENCE ANALYSIS

In this section, we present a new eight order iterative method for solving nonlinear equation via weight function and Newton interpolation.

Let $f : R \rightarrow R$ is eight times continuously differentiable on an interval $D \in R$ and has a simple zero $\alpha \in D$.

Consider the two point iterative method that was constructed by Ababneh (2016). In order to improve the convergence of (12), we added one Newton step and our proposed method is given as:

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \\ z_n = y_n - 2\frac{f(y_n)}{f'(x_n)} + \frac{f(y_n)(f(x_n)+(\beta-2)f(y_n))}{f'(x_n)(f(x_n)+\beta f(y_n))} - \frac{f'(x_n)f(y_n)}{f(x_n)(f(x_n)+\beta f(y_n))} \\ (\frac{f(y_n)}{f'(x_n)})^2, \\ x_{n+1} = z_n - \frac{f(z_n)K(t_n)H(u_n)}{2(f[x_n,z_n]-f[x_n,y_n])+f[y_n,z_n]+\frac{y_n-z_n}{y_n-x_n}(f[x_n,y_n]-f'(x_n))} \end{cases} \quad (13)$$

where $t_n = \frac{f(y_n)}{f(x_n)}\ and\ u_n = \frac{f(z_n)}{f(x_n)}$ The convergence analysis of the proposed method is presented in the following theorem.

Theorem1: Assume that the function f, K and H are sufficiently differentiable and f has a simple zero $\alpha \in D$. If the initial point x0 is sufficiently close to α, then the method defined in (13) converges to α with eight-order under the following conditions:

K(0)= −1, K '(0)= 0, K ''(0)= 2, and K '"(0)= 0

H(0)= −1, H '(0) = 0, and H ''(0) = 0

Proof:

Consider the Taylor expansion of the function f(xn) around α is given by

$$f(x_n) = f(\alpha)+\frac{1}{1!}f'(\alpha)(x_n-\alpha)+\frac{1}{2!}f''(\alpha)(x_n-\alpha)^2+ \frac{1}{3!}f'''(\alpha)(x_n-\alpha)^3+...+\frac{1}{8!}f^{(8)}(\alpha)(x_n-\alpha)^8+o(x_n-\alpha)^9 \quad (14)$$

Let en = xn − α be the error in n^{th} iteration with the assumption that f(α) = 0 and f0(α) 6= 0 , then we have

$$f(x_n) = f'(\alpha)[e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + c_5e_n^5 + c_6e_n^6 + c_7e_n^7 + c_8e_n^8 + o(e_n^9 \quad (15)$$

and

$$f'(x_n) = f'(\alpha)[1+2c_2e_n+3c_3e_n^2+4c_4e_n^3+ 5c_5e_n^4+6c_6e_n^5+7c_7e_n^6+8c_8e_n^7+9c_9e_n^8+o(e_n^9)] \quad (16)$$

where $c_n = \frac{f^{(n)}(\alpha)}{n!f'(\alpha)}$ for n=2,3,4,...

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2e_n^2 + 2(c_2^2 - c_3)e_n^3 + (7c_2c_3 - 4c_2^3 - 3c_4)e_n^4 + 2(3c_3^2 - 10c_2^2c_3 + 5c_2c_4 + 4c_2^4 - 2c_5)e_n^5 + (-16c_2^5 - 28c_2^2c_4 + 17c_3c_4 + 52c_2^3c_3 - c_2(33c_3^2 - 13c_5) - 5c_6)e_n^6 + o(e_n^7)$$

Let en,y = yn − α be the error in yn iteration where $y_n = x_n - \frac{f(x_n)}{f'(x_n)}$ and en = x_n − α then we have

en,y = c2e2n −2(c22 −c3)e3n −(7c2c3 −4c23 −3c4)e4n −2(3c23 −10c22c3 +5c2c4 +4c24 −2c5)e5n − $(-16c^5_2 - 28c^2_2c_4 + 17c_3c_4 + 52c^3_2c_3 - c_2(33c^2_3 - 13c_5) - 5c_6)e^6_n + o(e_n^7)$

also finding the Taylor expansion of f(yn) and simplifying it we have

$$f(y_n) = f'(\alpha)[e_{n,y} + c_2e_{n,y}^2 + c_3e_{n,y}^3 + c_4e_{n,y}^4 + c_3e_{n,y}^3 + c_4e_{n,y}^4 + c_5e_{n,y}^5 + c_6e_{n,y}^6 + c_7e_{n,y}^7 + c_8e_{n,y}^8 + o(e_{n,y}^9)] \quad (17)$$

substituting Eq. 15,16 and 17 in zn above at 13 then

$$e_{n,z} = (4c_2^3 + 2\beta 3_2 - c_2c_3)e_n^4 + (12\beta c_2^2c_3 - 26c_2^4 - 19\beta c_2^4 - 2c_2c_4 - 2\beta^2c_2^4 + 26c_2^2c_3 - 2c_3^2)e_n^5 + o(e_n^6)$$

but en,z = zn − α which is the error in the second point zn

and

$$z_n = y_n - 2\frac{f(y_n)}{f'(x_n)} + \frac{f(y_n)(f(x_n)+(\beta-2)f(y_n))}{f'(x_n)(f(x_n)+\beta f(y_n))} - \frac{f'(x_n)f(y_n)}{f(x_n)(f(x_n)+\beta f(y_n))}\left(\frac{f(y_n)}{f'(x_n)}\right)^2$$

therefore we have the error equation in z_n as

$$e_{n,z} = [2(2+2\beta)c_2^3 - c_2c_3]e_n^4 + [-(26+\beta(19+2\beta))c_2^4 + 2(13+6\beta)c_2^2c_3 - 2c_3^2 - 2c_2c_4]e_n^5 + o(e_n^6)$$

similarly for $f(z_n)$ we get

$$f(z_n) = f'(\alpha)[e_{n,z} + c_2e_{n,z}^2 + c_3e_{n,z}^3 + c_4e_{n,z}^4 + c_5e_{n,z}^5 + c_6e_{n,z}^6 + c_7e_{n,z}^7 + c_8e_{n,z}^8 + o(e_{n,z}^9)] \quad (18)$$

In addition, we also expand the two (2) weight functions $K(t_n)$ and $H(u_n)$ in the neighbourhood of 0 by Taylor expansion as shown below

$$K(t_n) = K(0) + \frac{1}{1!}K'(0)t_n + \frac{1}{2!}K''(0)t_n^2 + \frac{1}{3!}K'''(0)t_n^3 + O(t_n^4 \quad (19)$$

and

$$H(u_n) = H(0) + \frac{1}{1!}H'(0)u_n + \frac{1}{2!}H''(0)u_n^2 + O(u_n^3 \quad (20)$$

next, is to find tn and un which is given by

$t_n = \frac{f(y_n)}{f(x_n)}$ and $u_n = \frac{f(z_n)}{f(x_n)}$

$$t_n = c_2e_n + (-3c_2^2 + 2c_3) + (8c_2^3 - 10c_2c_3 + 3c_4)e_n^3 + o(e_n^4 \quad (21)$$

un =
(4c32+2βc32−c2c3)e3n+(−30c42+12βc22c3−2c2c4−21βc42−2β2c42−2c23+27c22c3)en4+o(e5n)

(22)

where tn and un are obtained by dividing $\frac{f(y_n)}{f(x_n)}$ and $\frac{f(z_n)}{f(x_n)}$ respectively substituting equations (15)-(22) into the proposed method then we have the error equation given by:

en+1 = (−c32c3+2βc52+4c32)e6n+(−2β2c62−50c62 −6c22c3−2c32c4+48c42c3+20βc42c3−31βc6 2)e7n +(217c32c23 −593c52c3 +348c72 −24β2c25c3 +80βc32c23 −348βc52c3 −3c32c5 −21c22c3c4 +33β2c72 $+ 75c^4_2c_4 + 259\beta c^7_2 + 30\beta c^4_2c_4 + 2\beta^3c^7_2 - 12c_2c^3_3)e^8n + o(e^9_n)$

(23)

It can be rewrite as

$$e_{n+1} = c_6 e_n^6 + c_7 e_n^7 + c_8 e_n^8 + o(e_n^9 \quad (24)$$

setting c6 and c7 to zero we have
c6 = 4c32 + 2βc52 − c32c3 = 0
c6 = [2(2 + β)c52 − c32c3] = 0

$$c_6 = [2(2+\beta)c_2^5 - c_2^3 c_3][1 - K(0)H \quad (25)$$

implies that, *H(0)* = −1 and *K(0)* = −1 similarly *c7 = −2β2c62 − 50c62 − 6c22c3 − 2c23c4 + 48c42c3 + 20βc42c3 − 31βc62 = 0* $c_7 = \beta[(-2\beta - 31)c_2^2 + 20c_3]c_2^4 + 2(24c_3 - 25c_2^2)c_2^4 - 2(3c_3^2 + c4)c_2^3 = 0$

$$c_7 = \beta[(-2\beta-31)c_2^2 K'(0)+10c_3(2-K'(0))]c_2^4+2(24c_3-25c_2^2)c_2^4 H'(0) -2(3c_3^2+c4)c_2^3 H''(0) = 0 \quad (26)$$

The Eq. 25 and 26 are obtained by setting c6 and c7 to zero on the error equation of the proposed method based on the condition of theorem 1 in order to have the exact eight convergence

Implies that , *K '(0)=0, K ''(0)=2, H '(0)=0 and H ''(0)=0* , then the error equation becomes

$$en+1 = (217c32c23-593c52c3+348c72-24\beta2c52c3+80\beta c23c23-348\beta c52c3-3c32c5-21c22c3c4+33\beta2c72+ 75c_2^4 c_4 + 259\beta c_2^7 + 30\beta c_2^4 c_4 + 2\beta3c_7^2 - 12c_2c_3^3)e_n^8 \quad (27)$$

therefore

$$e_{n+1} = [(217 + 80\beta)c_2^3 c_3^2 - (593 + 348\beta + 24\beta^2)c52c3 + (348 + 259\beta + 33\beta^2 + 2\beta^3)c_2^7 + (75 + 30\beta)c_2^4 c_4 - 21c_2^6 c_3 c_4 - 3c_2^3 c_5 - 12c_2 c_3^2]e_n^8 + o(e_n^9) \quad (28)$$

which is error equation of the proposed method.

This show that the convergence order of our proposed method is eight for any real value of the parameter *β*

Due to the above theorem 1, we can select the two (2) weight functions K(tn) and H(un) arbitrary as follows:

$$K(t_n) = 1 + 2(3+\beta)t_n^2, H(u_n) = 1 + 3u_n^2 \quad (29)$$

$$K(t_n) = 1 + \frac{6+2\beta}{1+3t_n^2}, H(u_n) = \frac{1+4u_n^2}{1+u_n^2} \quad (30)$$

$$(t_n) = \frac{1+2t_n-3\beta t_n^2}{1+4t_n+8t_n^2}, H(u_n) = 4 - \frac{3}{1+u_n^2} \quad (31)$$

NUMERICAL RESULTS

In this section, we test the performance of our proposed method using (29), (30) and (31) named KS1, KS2 and KS3 respectively with existing eight order method. These are compare with existing eight order methods such as:(i) Thukral and Petkovic (TPM) (Thukral & Petković, 2010).

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n = y_n - \frac{f(y_n)}{f'(x_n)}\frac{f(x_n)+bf(y_n)}{f(x_n)+(b-2)f(y_n)} \\ x_{n+1} = z_n - \frac{f(z_n)}{f'(x_n)}[Q(\frac{f(y_n)}{f(x_n)}) + v(x_n, y_n, z_n)] \end{cases} \quad (32)$$

where $Q(\frac{f(y_n)}{f(x_n)}) = \frac{f(x_n)^2}{f(x_n)^2 - 2f(x_n)f(y_n) - f(y_n)^2}$

$$v(x_n, y_n, z_n) = \frac{f(z_n)}{f(y_n) - 9f(z_n)} + 4\frac{f(z_n)}{f(x_n)}$$

and Q(0) = 1, Q '(0) = 2, Q ''(0) = 10 − 4b, Q '''(0) = 12b2 − 72b + 72

(ii) (Bi et al., 2009) (BEM):

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \\ z_n = y_n - h(u_n)\frac{f(y_n)}{f'(x_n)} \\ x_{n+1} = z_n - \frac{f(x_n)+(\gamma+2)f(z_n)}{f(x_n)+\gamma f(z_n)}\frac{f(z_n)}{f[z_n,y_n]+(z_n-y_n)f[z_n,x_n,x_n]} \end{cases} \quad (33)$$

where γ ∈ R is constant $u_n = \frac{f(y_n)}{f(x_n)}$ and h(t) represents a real valued function with h(0) = 1 , h ‘(0) = 2 , h ‘’(0) = 10 and h ‘’’(0) < ∞ and

(iii) Salimi et al., 2018):

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \\ z_n = y_n - 2\frac{f(y_n)}{f'(x_n)} + \frac{f(y_n)(f(x_n)+(\beta-2)f(y_n)}{f'(x_n)(f(x_n)+\beta f(y_n))} - \frac{f'(x_n)f(y_n)}{f(x_n)(f(x_n)+\beta f(y_n))} \\ (\frac{f(y_n)}{f'(x_n)})^2, \\ x_{n+1} = z_n - \frac{f(z_n)\eta(t_n)\psi(u_n)}{f[z_n,y_n]+(z_n-y_n)f[z_n,x_n,x_n]} \end{cases} \tag{34}$$

with weight function

$$\eta(t_n) = 1 - 4(2+\beta)t_n^3, \psi(u_n) = 1 + 2u_n$$
$$\eta(t_n) = 1 - \frac{4\beta+8}{1+2t_n}t_n^3, \psi(u_n) = \frac{1+3u_n}{1+u_n},$$
$$\eta(t_n) = \frac{1+t_n-4\beta t_n^3}{1+t_n+8t_n^3}t_n^3, \psi(u_n) = 3 - \frac{2}{1+u_n},$$

Where $t_n = \frac{f(y_n)}{f(x_n)}$, $u_n = \frac{f(z_n)}{f(x_n)}$,and β ∈ R

We apply the methods to solve some benchmark test functions drawn from (Chun & Ham, 2007).

$f_1(x) = x3 + 4x2 - 10$,
α=1.365230013414096845760806829 0 and x_0=1

$f_2(x) = x2 - ex - 3x + 2$,
α=0.2575302854398607604553673049 4 and x_0=0

$f_3(x)= xex2-sin2(x)+3cos(x)+5$,
α=1.207647827130918927009416758 4 and x_0=−1

$f_4(x)= sin(x)ex + log(x2 + 1)$, α = 0 and x_0 =2

$f_5(x)= (x - 1)3 - 2$,
α=2.259921049894873164767210607 3 and x_0=3

$f_6(x)= (x + 2)ex - 1$,
α=−0.4428544010023885831413280000 0 and x_0=2

$f_7(x)= sin2(x) - x2 + 1$,
α=1.404491648215341226035086817 8 and x_0=2

where α is a root $f_k(x)$= 0 for k= 1,2,....7 and x_0 is an initial approximation.

The numerical results reported here have been carried out in Matlab R2014a to test our proposed methods KSD1,KSD2,and KSD3 and also compare them with methods TPM,BEM, SNSP1, SNSP2 and SNSP3. We terminate the iteration when ever |f(xn)| <10−7

Table 1 and 2 shows the difference of the root α and the approximate xn . The absolute values of the function |f(xn)| , number of iteration and the computational order of convergence (COC) is also calculated in the tables. Where the COC is defined by (Weerakoon & Fernando, 2000).

$$\rho \approx \frac{ln|(x_n+1-\alpha)/(x_n-\alpha)|}{ln|(x_n-\alpha)/(x_n-1-\alpha)|}$$

DISCUSSION

The results presented in the Tables 1 and 2 shows that our methods KSD1, KSD2 and KSD3 converges more rapidly than some methods proposed by TPM, BEM, SNSP1, SNSP2, and SNSP3. It also shows that the new methods introduced in this paper have at least equal performance in terms of number of iteration when compared to the other existing eight-order methods. The total number of function evaluation in each iteration are almost the same expect on functions f3 and f6.

CONCLUSION

A new eight-order iterative method for solving nonlinear equations with high efficiency index have been constructed for approximating a simple root of a given nonlinear equation. The method uses only four functions evaluation in each iteration and a numerical comparison with some other known method shows that our proposed method have higher convergence order.

Table 1: Comparism of iterative methods TPM, BEM, SNSP1, SNSP2, SNSP3, with the new methods KSD1, KSD2, KSD3

METHODS	β	$\lvert x_n - \alpha \rvert$	$\lvert f(x_n) \rvert$	ITERATION	COC
		$f1(x) = x^3 + 4x^2 - 10$, x0 = 1			
TPM	*1*	2.6581e-12	4.3896e-12	4	8.0000
BEM	*-1*	1.4943e-13	2.4656e-12	3	8.0000
SNSP1	*-1*	3.6429e-9	6.015e-8	3	8.0000
SNSP2	*0*	3.6492e-91.8385e-	6.0261e-8	3	8.0000
SNSP3	*1*	9	3.0360e-8	3	8.0000
KSD1	*-1*	3.3799e-13	5.5813e-12	3	8.0000
KSD2	*0*	6.6041e-13	1.0905e-12	3	8.0000
KSD3	*1*	1.6667e-13	2.7522e-12	3	8.0000
		$f2(x) = x^2 - e^x - 3x + 2$, x0 = 1			
TPM	*1*	1.6881e-12	6.3787e-12	3	8.0000
BEM	*0*	5.6510e-14	2.1360e-13	2	8.0000
SNSP1	*-1*	1.7388e-13	6.5713e-13	3	8.0000
SNSP2	*0*	1.7390e-13	6.5721e-13	3	8.0000
SNSP3	*1*	8.6921e-14	3.2836e-13	3	8.0000
KSD1	*-1*	4.2386e-18	7.4303e-17	3	8.0000
KSD2	*0*	4.2350e-18	7.4317e-17	3	8.0000
KSD3	*1*	2.0446e-18	7.72660e-17	3	8.0000
		f3(x) = xex − sin2(x) + 3cos(x) + 5, x0 = -2			
TPM	*1*	5.3260e-12	1.0816e-10	7	8.0000
BEM	*1*	2.8089e-12	5.7040e-11	6	8.0000
SNSP1	*-1*	2.01130e-12	4.0877e-11	6	8.0000
SNSP2	*0*	1.4648e-12	2.9746e-11	6	8.0000
SNSP3	*1*	6.5533e-13	1.3310e-11	6	8.0000
KSD1	*-1*	1.3895e-12	2.8219e-11	6	8.0000
KSD2	*0*	7.0329e-15	1.4440e-13	6	8.0000
KSD3	*1*	1.3838e-15	2.6520e-14	6	8.0000
		$f4(x) = \sin(x)e^x + \log(x^2 + 1)$, x0 = 2			
TPM	*1*	5.4499e-13	5.4500e-13	5	8.0000
BEM	*-1*	2.9273e-14	2.9000e-14	4	8.0000
SNSP1	*-1*	9.4697e-9	9.4698e-9	3	8.0000
SNSP2	*0*	1.7737 e-8	1.7737 e-8	3	8.0000
SNSP3	*1*	3.3210e-8	3.3210e-8	3	8.0000
KSD1	*-1*	1.6318e-8	1.6318e-8	3	8.0000
KSD2	*0*	1.7147e-9	1.7147e-9	3	8.0000
KSD3	*1*	5.1183e-11	5.1183e-11	3	8.0000

Table 2: Comparism of iterative methods TPM, BEM, SNSP1, SNSP2, SNSP3, with the new methods KSD1, KSD2, KSD3

METHODS	**B**	$\lvert x_n - \alpha \rvert$	$\lvert f(x_n) \rvert$	**ITERATION**	**COC**
		$f5(x) = (x-1)^3 - 2, x0 = 3$			
TPM	*1*	6.5281e-14	3.100e-13	5	8.0000
BEM	*0*	1.3323e-15	7.0000e-15	4	8.0000
SNSP1	*-1*	2.1758e-9	1.0362e-8	3	8.0000
SNSP2	*0*	4.5065e-9	2.1461e-83.8802e-	3	8.0000
SNSP3	*1*	8.1479e-93.9324e-	8	3	8.0000
KSD1	*-1*	9	1.8727e-8	3	8.0000
KSD2	*0*	3.2512e-10	1.5483e-9	3	8.0000
KSD3	*1*	5.8899e-12	2.8048e-11	3	8.0000
		$f6(x) = (x+2)e^x - 1, x0 = 2$			
TPM	*1*	9.0154e-11	1.4805e-10	7	8.0000
BEM	*1*	7.7767e-11	1.2770e-10	6	8.0000
SNSP1	*-1*	4.5075e-14	7.4e-14	6	8.0000
SNSP2	*0*	1.0902e-13	1.79e-13	6	8.0000
SNSP3	*1*	1.7886e-13	2.94e-13	6	8.0000
KSD1	*-1*	8.8596e-14	1.45e-13	6	8.0000
KSD2	*0*	1.4988e-15	2e-15	6	8.0000
KSD3	*1*	7.6275e-9	1.2526e-8	6	8.0000
		$f7(x) = \sin^2(x) - x^2 + 1, x0 = 1$			
TPM	*1*	3.8414e-14	9.5000e-14	5	8.0000
BEM	*-1*	8.8817e-16	3.0000e-15	4	8.0000
SNSP1	*-1*	1.6168e-9	4.0136e-9	3	8.0000
SNSP2	*0*	3.1668e-9	6.5721e-9	3	8.0000
SNSP3	*1*	5.6023e-9	0.0e-15	3	8.0000
KSD1	*-1*	2.9333e-9	7.2819e-9	3	8.0000
KSD2	*0*	2.1399e-10	5.3123e-10	3	8.0000
KSD3	*1*	3.5705e-12	0.0e-16	3	8.0000

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