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Reallocation of the Organization Forces by the Super Effective Model Despite the Limitations on Number and the Way of Transferring the Forces of Some Units

Maryam khatibi[1*] and Bijan Rahmani parchikolaei[2]

[1] M.Sc. Applied Mathematics of Islamic Azad University, Lahijan Branch
[2] Department of Mathematics, Nour Branch, Islamic Azad University, Nour, Iran

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Abstract

In this paper, the efficiency of the performance of some independent units of police organization was evaluated to optimize the total used inputs and the total produced outputs of each unit. For this purpose, with regard to the efficiency and inefficiency of each unit, based on the location of the units to one another, and also due to the limitations on number and the way of transferring the forces of some units, and using the super effective model, a model was presented to reallocate the inputs of the organization units in order to increase and improve the efficiency of inefficient units and to emphasize maintaining the efficiency of efficient units.

***Correspondence E_mail**: khatibi_maryam_m@yahoo.com

INTRODUCTION

The significant efforts of organizations to find a comprehensive evaluation model and to remove and eliminate the existing disadvantages in traditional evaluation methods has led to the introduction of the organizational improvement models in different sciences Tangen, (2004). In this regard, the research science in operations has the extensive application in the fields of decision-making Cooper et al. (2006). A branch of research science in operations is the coverage analysis of data (DEA) which as a non-parametric linear programming method enables the management to compare the units with the best decision-making unit (DMU) and for this purpose, presents some models in order to improve the efficient DMUs. This method calculates the ratio of outputs to inputs of each DMU in the form of a linear programming model and calls it a relative efficiency score Avkian, (2001).

Over time, many studies have been done in this area, for example: Roodposhti et al. (2010), Hosseinzadeh et.al. (2016), Shokrollahpour et.al. (2016), Siaby-Serajehlo et.al. (2016), Zare Haghighi et.al. (2016) and Vaez-Ghasmi and Moghaddas, (2017) have presented some papers in this area.

In many DEA models, a unit whose relative efficiency score is 1has the best performance and efficiency Noura et al. (2011).

Evaluation of performance provides a suitable context to create motivation and to facilitate the organization goals and the relationship between organizational resources and the amount of performed work and efforts and the degree of achievement of organizational goals can be measured through it. In addition, the evaluation leads to organizational units and individuals' proper feedback and also provides information for them about strengths and weaknesses and provides the conditions for their improvement Anvari Rostami et al. (2011).

Based on the performed studies and reviews, the DEA method is an appropriate model to evaluate the relative efficiency of police units. This method has great flexibility and it provides the weaknesses of each studied units by determining the efficient frontier. Also, it can present some recommendations about reducing the amount of inputs or increasing the output level of each unit in order to achieve efficient frontier Anvari Rostami et al. (2011).

Many studies have been done on this organization; for instance, Drake and Simper, (2005) studied the efficiency of 38 units of police forces through this method. Gorman and Ruggiero, (2008) showed the inefficiency of less than half of the evaluated units through DEA three-step method. Also, the articles by Anvari Rostami et.al. (2009), Wu et.al. (2010), Dadres and Valivand Zamani, (2013) can be cited. In most of implemented researches on police organizations in order to increase their efficiency, it was tried to increase outputs and decrease inputs simultaneously. But because in these organizations some special instructions and trainings are presented for some forces in order to professionalize the individuals, these organizations are not willing to dismiss or transfer these forces to other units or organizations and if they want to transfer their force, they do this transfer and replacement due to the their distance from desired units or organizations. Therefore, in this paper we have tried to present an effective and efficient model by considering these limitations in order to improve the efficiency of all inefficient units.

REVIEW ON DEA MODELS

The evaluation of efficiency of the independent units through the non-parametric methods has been studied by scholars over several decades. The proposed model by considering n decision-making units like $\{DMU_z : z=1,\dots,n\}$was proposed so that each unit with m different input produces s different output . Also, Xiz, $(i=1,\dots,m)$ is ith input and Y_{rz}, $(r=1,..,s)$ is rth output from z decision-making unit and if it is supposed that V_i , $(i=1,\dots,m)$ is the weight of inputs and Ur , $(r=1,..,s)$ is the weight of outputs, so the efficiency of zth unit is calculated through (1) relationship:

$$\frac{\sum_{r=1}^{s} u_r y_{rz}}{\sum_{i=1}^{m} v_i x_{iz}} \quad (1)$$

This perspective developed and a mathematic programming model was presented as non-linear and fraction model which has the capability of measuring the efficiency with several inputs and outputs. This developed model is called CCR which can be presented like model 2:

$$Max \quad \theta = \frac{\sum_{r=1}^{s} u_r y_{rz}}{\sum_{i=1}^{m} v_i x_{iz}}$$
$$s.t. \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \,, \qquad j = 1, \dots, n \,, \qquad (2)$$
$$v_i \geq 0 \,, \qquad i = 1, \dots, m \,,$$
$$u_r \geq 0 \,, \qquad r = 1, \dots, s \,.$$

The model 2 is known as input-based CCR model. By converting the fractional model 2 to the linear model, the input-based multiple CCR model with form 3 will be obtained:

$$Max \quad \theta = \sum_{r=1}^{s} u_r y_{rz}$$
$$s.t. \quad \sum_{i=1}^{m} v_i x_{iz} = 1 \,,$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \,, j = 1, \dots, n \,, \qquad (3)$$
$$v_i \geq 0 \,, \quad i = 1, \dots, m \,,$$
$$u_r \geq 0 \,, \quad r = 1, \dots, s \,.$$

Cover models are dual multiple models which calculate the efficiency based on estimation of production function and the comparison of evaluated units with efficiency frontier (Mosadq Khah et al., 2011). The form of dual model 3 is as follows:

$$Min \quad \theta$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{iz} \,, \quad i = 1,2, \dots, m \,,$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rz} \,, \quad r = 1,2, \dots, s \,, \qquad (4)$$
$$\lambda_j \geq 0 \,, \qquad j = 1,2, \dots, n \,.$$

DETERMINATION OF INPUTS, OUTPUTS AND MODELING OF THE DATA

Through the evaluation of m=8 units of police organization, it was observed that a year's performance of these units can be modeled as two input-based BBC models. The first model is called the discovery model and the second model is called sending and following model. Each of these models has one input and five outputs. The input of each model is the personnel of each unit and the outputs of discovery model are violent crimes, number of discoveries of stolen objects, number of discovery of illegal entries, number of discovery of injuries or fatal accidents, and the number of discovery of murders, respectively. Also, the outputs of sending and following model are the number of answers to the contacts, number of investigating the complaints, number of 110 missions, number of available forces in order to deploy and send for missions and number of cases that the individuals are inspected, respectively. The amount of input and output of each model is presented in Table 1.

Suppose that X_i^n is the used input amount DMU_i, i=1,…,8 for implementing the model n, n=1,2 and y_i^n is produced output amount of each unit after performing the model n, n=1,2. Also, ω(n) is the number of outputs of model n. based on these assumptions, the input-based BCC model is formulated as follows:

$$\beta_r^n = min \quad \beta$$
$$s.t. \quad \sum_{i=1}^{m} \rho_i^n x_i^n \leq \beta \, x_r^n \,,$$
$$\sum_{i=1}^{m} \rho_i^n y_{ji}^n \geq y_{jr}^n \,, \qquad j = 1, \dots, \omega(n) \,, \qquad (5)$$
$$\sum_{i=1}^{m} \rho_i^n = 1 \,,$$
$$\rho_i^n \geq 0 \,, \ \forall i = 1, \dots, m \,.$$

After implementing the two models, the data in Table 2 were resulted.

In the Table, B and $\alpha^n=\beta^n x_i^n - x_i^n$ show the amount of efficiency and inefficiency of each unit, respectively. Due to the definitions of the scores of efficiency of inefficient and efficient units and due to the data from Table 2, it can be seen that some units are efficient and some units are inefficient.

Table 1: Input and outputs values

	x_i^1	y_1^1	y_2^1	y_3^1	y_4^1	y_5^2	x_i^2	y_1^2	y_2^2	y_3^2	y_4^2	y_5^2
DMU_1	12	51	32	2	1	1	14	25	14	8	10	0
DMU_2	5	3	2	2	0	0	9	29	7	8	7	3
DMU_3	5	3	4	0	0	0	12	17	9	8	9	0
DMU_4	4	1	3	0	0	0	15	25	16	6	15	4
DMU_5	11	42	2	0	0	1	8	18	6	12	9	1
DMU_6	8	27	2	0	0	0	12	11	5	3	7	1
DMU_7	5	1	1	0	0	0	11	25	10	10	7	1
DMU_8	7	20	20	0	0	0	6	27	9	8	9	0

Table 2: Input values, efficiency and inefficiency Scores

	x_i^1	β_i^1	α^1	x_i^2	β_i^2	α^2
DMU_1	12	1	0	14	1	0
DMU_2	5	1	0	9	1	0
DMU_3	5	0.86	-0.7	12	0.50	-6
DMU_4	4	1	0	15	1	0
DMU_5	11	1	0	8	1	0
DMU_6	8	1	0	12	1	0
DMU_7	5	0.80	-1	11	0.58	-5.04
DMU_8	7	1	0	6	1	0

REALLOCTION DESPITE THE LIMITATIONS

According to the definitions of efficient scores, in most DEA models, the effective model is the one whose efficiency score is 1. The super efficient scores were presented by Anderson-Peterson in 1993. Based on this method, an efficient model in an input-based super efficient model has the efficient scores which are greater or equal to one. This model is formulated as follows:

$$\beta_r^{n\,super} = \min \beta^{super}$$
$$s.t. \sum_{\substack{i=1 \\ i\neq r}}^{m} \rho_i^n x_i^n \leq \beta^{super} x_r^n ,$$
$$\sum_{\substack{i=1 \\ i\neq r}}^{m} \rho_i^n y_{ji}^n \geq y_{jr}^n , \quad j = 1, \dots, \omega(n) , \qquad (6)$$
$$\sum_{\substack{i=1 \\ i\neq r}}^{m} \rho_i^n = 1 ,$$
$$\rho_i^n \geq 0 , \ \forall j = 1, \dots, m , i \neq r .$$

Using this model, we present a plan which eliminates all the inefficacies of inefficient units so that no is created in efficiency of efficient units. For this purpose, first we calculate the super efficient scores of efficient units according to Table 6 and then replace the infinite super efficient scores with the highest amount among the super efficient scores for each model separately. These scores will be shown with * index in Table 3.

Table 3: Input values, supper efficiency and inefficiency Scores

	x_i^1	β_i^{1super}	α^1	x_i^2	β_i^{2super}	α^2
DMU₁	12	1.27*	3.24	14	1.83*	11.62
DMU₂	5	1.27*	1.35	9	1.83*	7.47
DMU₃	5	0.86	-0.7	12	0.50	-6
DMU₄	4	1.25	1	15	1.83*	12.45
DMU₅	11	1.27*	2.97	8	1.83*	6.64
DMU₆	8	1.02	0.16	12	0.58	-5.04
DMU₇	5	0.80	-1	11	1.83*	9.13
DMU₈	7	1.24	1.68	6	1.83*	4.98

Based on the data from unit 6, from the 8 available personnel in first performance, 4 people are specialist in both performances and the 4 remaining people just have the essential specialty (proficiency) in first performance and from the 12 available people in second performance, 3 people have the needed specialty in both performances and the 9 remaining people just have the necessary proficiency in second performance. Also, in unit 7, from the 5 available people in the first performance, 2 people in both performances and 3 people in second performance have the needed specialty and from 11 available people in second performance, 6 people are professional and specialist in both performances and the 5 remaining people only have the needed specialty in second performance. Another limitation is that the distance of unit 3 from units 1 and 4 is very high and there is no possibility for transferring the force between these two units. Now, due to the limitations and restrictions, at first we put the units in three separate categories based on their efficiency scores. The units of first category are those units which are inefficient in both performances, the second category consists of the units which are super efficient in both performances and the third category comprises the units which are super efficient in one performance and inefficient in another performance. It is necessary that the units which belong to first category eliminate their extra individuals in both performances in order to increase their efficiency and the units which belong to second category do not need to reallocate their forces because of their super efficient feature. But the units which belong to third category need reallocation of some of their forces. Therefore, we implement this reallocation as follows:

Step 1: we identify the minimum efficiency score among the two performances of units which belong to second category.

Step 2: the degree of possible reallocation may be limited by ABS[$\beta_r^n x_i^n - x_i^n$] and the degree of super efficiency [$\beta_r^{n\ super}\ x_i^n - x_i^n$] . Now, we reallocate the number of extra individuals of that identified performance according to the limitations of the super efficient performance of that (the same) unit.

Step 3: the step 1 and 2 are repeated again so that no reallocation can be possible among the performances of each unit.

Step 4: when no reallocation between the performances of each unit is possible, we identify the minimum remaining inefficiency score among the two performances of inefficient units.

Step 5: we reallocate the number of extra individuals of that unit in that identified performance according to the limitations of unit which has the maximum available super efficiency score in the same performance. The degree of reallocation is calculated in a manner similar to step 2.

Step 6: The steps 4 and 5 will be repeated so that no reallocation is needed.

Now the resulting data after implementing the plan are according to Table 4:

Table 4: Input values, efficiency and inefficiency scores after implementation of design

	x_i^1	β_i^1	α^1	x_i^2	β_i^2	α^2
DMU_1	14.86	1	0	14	1	0
DMU_2	5	1	0	9	1	0
DMU_3	4.3	1	0	6	1	0
DMU_4	4	1	0	16.98	1	0
DMU_5	11.7	1	0	8	1	0
DMU_6	8.14	1	0	7.02	1	0
DMU_7	4	1	0	18	1	0
DMU_8	7	1	0	6	1	0

According to the data of Table 4, it can be observed that despite the available limitations, all the inefficacy of inefficient units was eliminated so that there is no fault and damage in the inefficacy of efficient units.

CONCLUSION

In this paper, a plan was presented in order to increase the efficacy of several units of police organization. Because some units provided special instructions and education in order to professionalize some people, they were reluctant to transfer and reallocate these people and also due to their distance from some units, if they wanted to transfer their other forces, this replacement was impossible, but despite all these limitations, this plan was so flexible that after its implementation, all the inefficient units achieved the needed efficacy and the purposes of management were satisfied.

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