Optimization
Iranian Journal of Optimization
Volume 8, Issue 2, 2016, 71-77
Research Paper
Online version is available on: www.ijo.iaurasht.ac.ir

Islamic Azad University
Rasht Branch
E-ISSN:2008-5427

Ranking of Bank Branches With Undesirable and Fuzzy Data: A DEA-Based Approach

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Received: 09 August 2016
Accepted: 26 October 2016

Abstract

Banks are one of the most important financial sectors in order to the economic development of each country. Certainly, efficiency scores and ranks of banks are significant and effective aspects towards future planning. Sometimes the performance of banks must be measured in the presence of undesirable and vague factors. For these reasons in the current paper a procedure based on data envelopment analysis (DEA) is introduced for evaluating the efficiency and complete ranking of decision making units (DMUs) where undesirable and fuzzy measures exist. To illustrate, in the presence of undesirable and fuzzy measures, DMUs are evaluated by using a fuzzy expected value approach and DMUs with similar efficiency scores are ranked by using constraints and the Maximal Balance Index based on the optimal shadow prices. Afterwards, the efficiency scores of 25 branches of an Iranian commercial bank are evaluated using the proposed method. Also, a complete ranking of bank branches is presented to discriminate branches.

Keywords:
Data envelopment analysis (DEA)
bank branches performance,
undesirable data
fuzzy data
ranking

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INTRODUCTION

Nowadays banks play the important role in the economy and financial systems of countries. Thus, the performance improvement of banks is one vital subject for managers and governments. Actually, managers and decision makers attempt to evaluate the efficiency of bank branches for future planning and decisions. In the literature, there are parametric and non-parametric methods to assess the efficiency of firms.

One of the popular approaches for estimating the efficiency of firms is the data envelopment analysis (DEA) non-parametric method, which is a mathematical technique for evaluating the performance of decision making units (DMUs) with multiple inputs and outputs. In traditional DEA models the efficiency of DMUs is usually assessed in such a way that desirable and precise measures present. Also, DMUs are divided into two subsets, efficient and inefficient, after estimating the efficiency. In the DEA literature, a lot of methods can be found for ranking and discriminating DMUs. Adler et al. (2002). reviewed ranking methods in DEA contexts. Furthermore, Alirezaee and Afsharian (2007) proposed a method for the complete ranking of DMUs by using constraints.Afterwards, Wu et al. (2010) argued Alirezaee and Afsharian's ranking (2007) is unstable and introduced the Maximal Balance Index. Also, Guo and Wu (2013)extended the proposed approach by Wu et al. (2010) for situations that undesirable outputs exist. However, there are situations in the real world that DMUs must be ranked and discriminated while undesirable and imprecise data exist.

In DEA contexts, there are studies with incorporating fuzzy measures. Hatami marbini et al. (2011) reviewed the fuzzy DEA literature. Moreover, Wang and Chin (2011) suggested a fuzzy DEA model by using a fuzzy expected value approach. In the current paper, Wang and Chin's approach (2011) is extended for situations that undesirable outputs exist. Actually, occasions can be found in the real world that fuzzy and undesirable outputs present. For instance, in the bank evaluation a factor like non-performing loans can be deemed as fuzzy and undesirable outputs. In many studies, the performance of banks is measured by using the DEA technique.Sherman and Gold (1985), Amirteimoori et al. (2014), Asmild and Tam (2007) and McEachern and Paradi (2007), Camanho and Dyson (1999), Ganganis et al. (2009), Mitropoulos et al. (2003), Pastor et al. (2006), Parkan (1987), Noulas et al. (2008), and Puri and Yadav (2014) are some of the surveys that evaluate the efficiencies of banks via different DEA models. Among aforementioned studies, Puri and Yadav (2014) proposed a fuzzy DEA model with undesirable outputs. To illustrate, they used α-cut approach for calculating the efficiency and cross-efficiency technique for ranking the efficient units. As mentioned in Wang and Chin (2011), α-cut approach requires considerable computational efforts.

Therefore, the current paper proposes an alternative method for evaluating the efficiency and ranking DMUs in the presence of undesirable and fuzzy data. At first, a new method is proposed for evaluating the efficiency of DMUs in the presence of undesirable and fuzzy data. To illustrate, Wang and Chin's (2011) approach is extended to situations that undesirable and fuzzy factors present. Then DMUs are fully ranked by using Guo and Wu's approach (2013). Indeed, a complete ranking of DMUs is achieved by using the Maximal Balance Index based on the optimal shadow prices. Afterwards, the suggested method is used to measure the performance of 25 branches of the Iranian commercial bank while undesirable and fuzzy factors are present. Then, Iranian bank branches are discriminated via the ranking procedure provided herein. The rest of this paper is organized as follows: In Section 2, we review the basic concepts and models that are used and generalized in this study. Section 3 provides the new approach for calculating the efficiency and discriminating DMUs where undesirable and inaccurate measures present. An application of the banking sector is given to clarify the approach in Section 4. Conclusions are revealed in Section 5.

PRELIMINARIES

In this section, some fundamental notions and models are discussed that are used and extended in the suggested approach. First, Guo and Wu's approach (2013) for full ranking of DMUs with undesirable outputs is explained. Second, A fuzzy DEA model, proposed by Wang and Chin (2011), is provided.

Complete ranking of DMUs with undesirable outputs

Suppose there are n DMUs with m inputs x_{ij}=(i=1,..., m), s desirable outputs y_{ij}=(r=1,..., s), and k undesirable outputs z_{kj}=(k=1,..., k). According to Guo and Wu (2013), the efficiency of DMU_0 can be calculated as follows:

$$
\begin{aligned}
Max \quad & E_o = \sum_{r=1}^{s} u_r y_{ro} \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{K} w_k z_{ko} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{K} w_k z_{kj} \le 0, \ \ j = 1, \dots, n, \\
& v_i, u_r, w_k \ge 0, \forall r, i, k.
\end{aligned} \tag{1}
$$

that *u, v* and *w* are shadow price vectors for inputs, desirable outputs and undesirable outputs. Then, Guo and Wu (2013) defined the profit constraint for as follows:

$$
\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{K} w_k z_{kj} \le 0, \ \ j = 1, \dots, n, \tag{2}
$$

in which $\sum_{r=1}^{s} u_r y_{rj}$ indicates total revenue for DMU_j and $\sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{K} w_k z_{kj}$ shows total cost for. The inputted profit is equal to zero when shadow prices are achieved from the technology. They called this situation as Balance situation.

For assuring the efficiency of DMU_0 is E_0 and the Balance Index is unique, Guo and Wu (2013) introduced the following model:

$$
\begin{aligned}
Max \quad & \sum_{i=1}^{m} v_i \kappa_i + \sum_{k=1}^{K} w_k \rho_k - \sum_{r=1}^{s} u_r q_r \\
s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{K} w_k z_{kj} \le 0, \ \ \forall j \\
& \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{K} w_k z_{ko} = 1 \\
& \sum_{r=1}^{s} u_r y_{ro} = E_0 \\
& v_i, u_r, w_k \ge 0, \forall r, i, k
\end{aligned} \tag{3}
$$

where K_i *(i=1,...,m)*, ρ_k *(k=1,...,K)*, and q_r *(r=1,…,s)* are the sum amount of I th input, K th undesirable output, and r th desirable output for all DMUs.

A fuzzy DEA model

In traditional DEA models, all factors are deemed as specific numerical values. Nevertheless, the observed values of the input and output data in real applications are often imprecise or vague. The most commonly used fuzzy numbers are triangular and trapezoidal fuzzy numbers. Triangular and trapezoidal fuzzy numbers are often denoted as (a, b, d) and (a, b, c, d). Liu and Liu (2002) indicated that the expected value of a trapezoidal fuzzy variable (a, b, c, d) is as (a+b+c+d) and the expected value of a triangular fuzzy variable (a, b, d) is as (a+2b+c). Wang and Chin (2011) by using the expected value of fuzzy numbers proposed models to assess the efficiency of DMUs in the presence of fuzzy factors. At this point Wang and Chin's approach (2011) is illustrated briefly as follows:

Consider n DMUs that consume m inputs to produce s outputs. Assume $\tilde{x}_{ij}, (i = 1, \dots, m)$ and $\tilde{y}_{rj}, \ (r = 1, \dots, s)$ indicate inputs and outputs of the j th DMU (*j=1,…, n*), respectively that are characterized by trapezoidal fuzzy numbers $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^M, x_{ij}^N, x_{ij}^u)$ and $\tilde{y}_{rj} = (y_{rj}^l, y_{rj}^M, y_{rj}^N, y_{rj}^u)$ with $x_{ij}^l \ge 0$ and $y_{rj}^l \ge 0$ for r=1,..., s and j=1,..., n.

Wang and Chin (2011) considered the efficiency of DMUj as:

$$
\theta_j = \frac{\sum_{r=1}^{s} u_r (y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u)}{\sum_{i=1}^{m} v_i (x_{ij}^l + x_{ij}^M + x_{ij}^N + x_{ij}^u)} =
$$

$$
\frac{\sum_{r=1}^{s} u_r E(\tilde{y}_{rj})}{\sum_{i=1}^{m} v_i E(\tilde{x}_{ij})}, \ \ j = 1, \dots, n, \tag{4}
$$

From optimistic point of view, the efficiency was determined by the following model:

$$
\begin{aligned}
Max \quad & \theta_o^{best} = \sum_{r=1}^{s} u_r (y_{ro}^l + y_{ro}^M + y_{ro}^N + y_{ro}^u) \\
s.t. \quad & \sum_{i=1}^{m} v_i (x_{io}^l + x_{io}^M + x_{io}^N + x_{io}^u) = 1 \\
& \sum_{r=1}^{s} u_r (y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u) - \sum_{i=1}^{m} v_i (x_{ij}^l
\end{aligned}
$$

$$+ x_{ij}^M + x_{ij}^N + x_{ij}^u) \le 0 \quad j = 1,\dots,n$$

$$u_r, v_i \ge 0 \quad i = 1,\dots,m \ , \ r = 1,\dots,s. \tag{5}$$

Also, they suggested another model for evaluating the efficiency of DMUs from pessimistic viewpoint. Readers can refer to (2011) for more information.

FULL RANKING OF DMUS WHIT UNDESIRABLE AND FUZZY DATA

In this section an approach is proposed for obtaining complete ranking of DMUs where fuzzy data and undesirable outputs present. Suppose n DMUs, DMU_j j=(1,..., n), exist that use inputs $\tilde{x}_{ij}, (i = 1,\dots,m)$, produce s desirable outputs $\tilde{y}_{rj}, \ (r = 1,\dots,s)$, and emit k undesirable outputs $\tilde{z}_{kj}, (k = 1,\dots,K)$. Inputs, desirable outputs, and undesirable outputs are indicated by trapezoidal fuzzy numbers, i.e. $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^M, x_{ij}^N, x_{ij}^u)$, $\tilde{y}_{rj} = (y_{rj}^l, y_{rj}^M, y_{rj}^N, y_{rj}^u)$, and $\tilde{z}_{kj} = (z_{kj}^l, z_{kj}^M, z_{kj}^N, z_{kj}^u)$ with $x^l_{ij} \ge 0$, $z^l_{kj} \ge 0$, and $x^l_{ij} \ge 0$ for $i=1,\dots, m$, $r=1,\dots,s$, $k=1,\dots,K$ and $j=1,\dots n$. The following model is suggested for calculating the efficiency of DMUs in the presence of undesirable outputs and fuzzy data.

$$\begin{aligned}
Max \quad & E_o = \sum_{r=1}^{s} u_r (y_{ro}^l + y_{ro}^M + y_{ro}^N + y_{ro}^u) \\
s.t. \quad & \sum_{i=1}^{m} v_i (x_{io}^l + x_{io}^M + x_{io}^N + x_{io}^u) + \sum_{k=1}^{K} w_k (z_{ko}^l + z_{ko}^M + z_{ko}^N + z_{ko}^u) = 1, \\
& \sum_{r=1}^{s} u_r (y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u) - \sum_{i=1}^{m} v_i (x_{ij}^l + x_{ij}^M + x_{ij}^N + x_{ij}^u) - \\
& \sum_{k=1}^{K} w_k (z_{kj}^l + z_{kj}^M + z_{kj}^N + z_{kj}^u) \le 0, \ j = 1,\dots,n, \\
& v_i, u_r, w_k \ge 0, \forall r, i, k.
\end{aligned} \tag{6}$$

Furthermore, the profit constraint for each DUMj can be defined as follows:

$$\begin{aligned}
& \sum_{r=1}^{s} u_r (y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u) - \sum_{i=1}^{m} v_i (x_{ij}^l + x_{ij}^M + x_{ij}^N + x_{ij}^u) \\
& - \sum_{k=1}^{K} w_k (z_{kj}^l + z_{kj}^M + z_{kj}^N + z_{kj}^u) \le 0, \ j = 1,\dots,n,
\end{aligned} \tag{7}$$

v_i, u_r, and w_k are shadow prices for inputs, desirable outputs and undesirable outputs. Also, $\sum_{r=1}^{s} u_r (y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u)$ shows the total revenue for DMU_j and $\sum_{i=1}^{m} v_i (x_{ij}^l + x_{ij}^M + x_{ij}^N + x_{ij}^u) + \sum_{k=1}^{K} w_k (z_{kj}^l + z_{kj}^M + z_{kj}^N + z_{kj}^u)$ represents the total cost for DMU_j. The inputted profit of the DMU is zero when shadow prices are derived from the technology. Similar to (Alirezaee et al., 2007; Guo et al., 2013 & Noulas et al., 2008), we take this situation as Balance situation where fuzzy data and undesirable outputs are observed. For obtaining the stable results and achieving a unique Balance Index, the Maximal Balance Index can be calculated as follows:

$$\begin{aligned}
Max \quad & \sum_{i=1}^{m} v_i \bar{\kappa}_i + \sum_{k=1}^{K} w_k \bar{\rho}_k - \sum_{r=1}^{s} u_r \bar{q}_r \\
s.t. \quad & \sum_{r=1}^{s} u_r (y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u) - \sum_{i=1}^{m} v_i (x_{ij}^l + x_{ij}^M + x_{ij}^N + x_{ij}^u) - \\
& \sum_{k=1}^{K} w_k (z_{kj}^l + z_{kj}^M + z_{kj}^N + z_{kj}^u) \le 0, \ \forall j \\
& \sum_{i=1}^{m} v_i (x_{io}^l + x_{io}^M + x_{io}^N + x_{io}^u) + \sum_{k=1}^{K} w_k (z_{ko}^l + z_{ko}^M + z_{ko}^N + z_{ko}^u) = 1 \\
& \sum_{r=1}^{s} u_r (y_{ro}^l + y_{ro}^M + y_{ro}^N + y_{ro}^u) = E_0 \\
& v_i, u_r, w_k \ge 0, \ \forall r, i, k
\end{aligned} \tag{8}$$

The aforementioned model assures that the efficiency of the DMU under evaluation, DMU_0, is E_0. Suppose, $\bar{x}_{ij} = (x_{ij}^l + x_{ij}^M + x_{ij}^N + x_{ij}^u)$, $\bar{z}_{kj} = (z_{kj}^l + z_{kj}^M + z_{kj}^N + z_{kj}^u)$ and $\bar{y}_{rj} = (y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u)$. Thus, $\bar{\kappa}_i$ (i=1,...,m), $\bar{\rho}_k$= (k=1,...,k) and $\bar{q}_r$ (r=1,...,s) represent the sum amount of i th $\bar{x}$, k th $\bar{z}$ and r th $\bar{y}$ for all DMUs. Indeed, DMUs can be ranked by using firstly the efficiency scores and secondly the Maximal Balance Index. It is clear, triangular fuzzy numbers and crisp data are special cases of trapezoidal fuzzy numbers. In the presence of triangular fuzzy numbers, $(x^l_{ij}+x^M_{ij}+x^N_{ij}+x^u_{ij})$ is substituted with $(x^l_{ij}+2x^M_{ij}+x^u_{ij})$ in (6), (7), and (8). Actually, $x^M_{ij}=x^N_{ij}$ in triangular fuzzy inputs. In the similar way, $(z^l_{kj}+z^M_{kj}+x^N_{kj}+x^u_{kj})$ and $(y^l_{rj}+y^M_{rj}+y^N_{rj}+y^u_{rj})$ are replaced with and $(y^l_{rj}+2y^M_{rj}+y^u_{rj})$, respectively. Actually, in the presence of triangular fuzzy measures, models (6), (7), and (8) can be rewritten as follows:

$$
\begin{aligned}
&Max \quad E_o=\sum_{r=1}^{s}u_r(y_{ro}^l+2y_{ro}^M+y_{ro}^u)\\
&s.t. \quad \sum_{i=1}^{m}v_i(x_{io}^l+2x_{io}^M+x_{io}^u)+\sum_{k=1}^{K}w_k(z_{ko}^l+2z_{ko}^M+z_{ko}^u)=1,\\
&\sum_{r=1}^{s}u_r(y_{rj}^l+2y_{rj}^M+y_{rj}^u)-\sum_{i=1}^{m}v_i(x_{ij}^l+2x_{ij}^M+x_{ij}^u)-\\
&\sum_{k=1}^{K}w_k(z_{kj}^l+2z_{kj}^M+z_{kj}^u)\le 0,\ \ j=1,\dots,n,\\
&v_i,u_r,w_k\ge 0,\forall r,i,k.
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
&\sum_{r=1}^{s}u_r(y_{rj}^l+2y_{rj}^M+y_{rj}^u)-\sum_{i=1}^{m}v_i(x_{ij}^l+2x_{ij}^M+x_{ij}^u)-\\
&\sum_{k=1}^{K}w_k(z_{kj}^l+2z_{kj}^M+z_{kj}^u)\le 0,\ \ j=1,\dots,n,
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
&Max \quad \sum_{i=1}^{m}v_i\bar{\kappa}_i+\sum_{k=1}^{K}w_k\bar{\rho}_k-\sum_{r=1}^{s}u_r\bar{q}_r\\
&s.t. \quad \sum_{r=1}^{s}u_r(y_{rj}^l+2y_{rj}^M+y_{rj}^u)-\sum_{i=1}^{m}v_i(x_{ij}^l+2x_{ij}^M+x_{ij}^u)-\\
&\sum_{k=1}^{K}w_k(z_{kj}^l+2z_{kj}^M+z_{kj}^u)\le 0,\ \ \forall j\\
&\sum_{i=1}^{m}v_i(x_{io}^l+2x_{io}^M+x_{io}^u)+\sum_{k=1}^{K}w_k(z_{ko}^l+2z_{ko}^M+z_{ko}^u)=1\\
&\sum_{r=1}^{s}u_r(y_{ro}^l+2y_{ro}^M+y_{ro}^u)=E_0\\
&v_i,u_r,w_k\ge 0,\ \forall r,i,k
\end{aligned} \tag{11}
$$

Table 1: Input data for 25 bank branches

#Branch	Staff	Costs
1	9	(8404,8405.36,8406)
2	8	(7469,7471.82,7473)
3	8	(7473,7475.05,7477)
4	11	(10333,10335.38,10336)
5	13	(12145,12147.96,12148)
6	8	(7495,7498.56,7499)
7	7	(6507,6509.38,6511)
8	10	(9325,9327.18,9329)
9	6	(5601,5603.00,5605)
10	10	(9410,9412.49,9414)
11	5	(4674,4676.78,4678)
12	8	(7477,7480.49,7482)
13	7	(6554,6556.98,6558)
14	7	(6586,6588.77,6589)
15	9	(8380,8381.34,8383)
16	8	(7469,7470.69,7472)
17	8	(7468,7470.16,7472)
18	7	(6555,6556.78,6558)
19	7	(6568,6569.35,6570)
20	9	(8400,8401.98,8403)
21	8	(7455,7456.87,7457)
22	10	(9340,9342.11,9344)
23	12	(11170,11174.53,11176)
24	7	(6560,6563.41,6564)
25	7	(6569,6570.93,6571)

Notice that we take into account undesirable outputs into consideration. The models can be generalized when undesirable inputs present.

EFFICIENCY EVALUATION OF IRANIAN BANK RANCHES

In the current section, 25 branches of an Iranian commercial bank are evaluated using the introduced procedure herein. The number of staff and costs are considered as input factors. The costs contain staff costs and operational costs of bank branches. Furthermore, deposits, income, and granted loan are taken as desirable output measures while non-performing loans are deemed as undesirable outputs. The deposit in each branch is the result of the attraction of the funds from customers. Income includes interest income and non-interest income. Granted loans are loans granted by governmental sectors. Non-performing loans are loans that are in default, according to the bank regulations. Input and output factors have been selected by consulting with the management of key branch. All input and output measures except the number of staff are regarded as triangular fuzzy data. Uncertainty occurs in available data of bank branches and the amounts of them have been determined via consulting with experts of each branch.

Data are depicted in Tables 1 and 2. At first, model (9) is calculated. The results can be found in column 2 of Table 3. As can be seen 11 branches have been distinguished as efficient. According to (10) and (11), Maximal Balance Index quantities for branches are obtained that are indicated in column 3 of Table 3. Afterwards, branches with similar efficiency scores are ranked and discriminated using the quantity of the Maximal Branch Index of branches. The results show DMU18 with the efficiency score of 1 has the highest rank while DMU22 with the efficiency score of 0.473 has achieved the least rank among all branches.

Table 2: Output data for 25 bank branches

#Branch	Deposits	Income	Granted loans	Non-Performing loans
1	(205070,205073.65, 205075)	(11990,11990.24, 11993)	(151088,151090.33, 151093)	(5988,5989.61, 5990)
2	(235492,235494.90, 235496)	(6284,6285.21, 6287)	(147547,147549.84, 147550)	(7512,7513.22, 7515)
3	(237013,237015.48, 237018)	(12158,12160.74, 12161)	(163024,163025.07, 163027)	(46960,46962.01, 46964)
4	(209724,209725.54, 209727)	(2788,2788.35, 2789)	(105005,105006.47, 105009)	(18705,18707.25, 18708)
5	(385417,385418.60, 385420)	(16667,16668.96, 16670)	(98959,98961.20, 98964)	(8476,8477.13, 8479)
6	(138238,138239.61, 138240)	(6780,6782.93, 6783)	(108155,108156.11, 108158)	(2734,2735.53, 2737)
7	(183835,183837.47, 183838)	(3404,3406.11, 3407)	(82560,82561.82, 82562)	(12614,12616.65, 12618)
8	(287007,287008.02, 287009)	(9988,9989.26, 9990)	(94015,94016.22, 94017)	(24087,24089.01, 24090)
9	(181129,181130.30, 181131)	(1028,1029.18, 1031)	(105531,105531.49, 105532)	(3421,3423.28, 3425)
10	(240364,240365.74, 240367)	(24473,24475.74, 24476)	(196128,196129.53, 196130)	(43298,43300.63, 43303)
11	(126794,126795.38, 126797)	(6195,6197.12, 6198)	(38357,38359.39, 38360)	(3877,3878.46, 3879)
12	(207637,207639.32, 207641)	(2602,2603.87, 2604)	(96637,96638.59, 96639)	(4807,4809.09, 4810)
13	(239988,239989.56, 239991)	(7152,7153.80, 7155)	(88133,88133.12, 88134)	(9494,9495.05, 9497)
14	(154348,154349.49, 154351)	(2065,2066.30, 2068)	(78916,78916.66, 78917)	(15323,15325.67, 15326)
15	(189020,189022.39, 189024)	(5515,5516.38, 5517)	(69399,69399.98, 69401)	(1988,1989.11, 1991)
16	(122329,122330.03, 122333)	(6796,6798.10, 6799)	(72600,72602.11, 72603)	(3965,3966.60, 3968)
17	(194806,194808.91, 194810)	(7442,7443.78, 7445)	(59239,59239.65, 59240)	(3647,3648.65, 3649)
18	(113540,113542.11, 113543)	(9933,9935.13, 9936)	(70377,70378.51, 70380)	(1916,1918.09, 1920)
19	(193148,193149.73, 193150)	(2468,2470.78, 2471)	(86643,86644.57, 86645)	(7820,7822.50, 7821)
20	(221505,221507.57, 221508)	(2527,2528.34, 2529)	(107904,107905.48, 107906)	(19845,19847.67, 19848)
21	(266868,266870.01, 266871)	(23224,23226.95, 23227)	(62528,62530.34, 62532)	(8106,8109.63, 8111)
22	(161850,161851.78, 161853)	(2420,2422.26, 2423)	(78858,78858.77, 78859)	(25091,25094.37, 25095)
23	(208355,208356.87, 208357)	(4511,4512.15, 4514)	(114262,114264.01, 114265)	(9010,9012.63, 9015)
24	(240393,240394.60, 240396)	(5163,5163.16, 5165)	(118890,118892.72, 118893)	(3645,3647.43, 3648)
25	(174337,174340.99, 174341)	(420,421.22, 425)	(137342,137343.55, 137344)	(37853,37855.17, 37858)

Table 3: Results

#Branch	Efficiency	Maximal Balance Index	Rank
1	1	-22.32	7
2	1	-11.8359	9
3	1	-10.6411	10
4	0.558	-5.4494	23
5	0.9011	-18.5775	13
6	1	-97.3073	3
7	0.771	-8.559	17
8	0.8423	-5.9255	15
9	1	-36.8727	5
10	1	-12.1063	8
11	0.7474	-11.8994	19
12	0.7578	-7.4482	18
13	1	-8.5036	11
14	0.6509	-8.5634	22
15	1	-138.9247	2
16	0.6562	-22.5177	21
17	0.8719	-36.9074	14
18	1	-153.6456	1
19	0.8035	-8.5661	16
20	0.7197	-6.631	20
21	1	-28.483	6
22	0.473	-5.9636	25
23	0.5423	-5.2436	24
24	1	-60.8832	4
25	0.9715	-9.7267	12

CONCLUSION

There are occasions in the real world that firms must be discriminated and ranked while undesirable and fuzzy data exist. For instance, non-performing loans can be considered as an undesirable output factor in the banking sector in a way that there is imprecise information about some factors like loans, income, costs, etc.

Thus, the present paper has been proposed a new approach for estimating the efficiency and full ranking of DMUs where undesirable and fuzzy measures present. Actually, a fuzzy DEA model has been extended for evaluating the efficiency of DMUs in the presence of undesirable and fuzzy factors. Then, the efficiency scores and Maximal Balance Index have been used for full ranking of DMUs. Also, the efficiency scores of branches of an Iranian bank have been evaluated and branches have been complete ranked via the proposed method herein.

ACKNOWLEDGEMENT

Financial support by Lahijan Branch, Islamic Azad University Grant No. 1235, 17-20-5/3507 is gratefully acknowledged.

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