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Fuzzy bi-level linear programming problem using TOPSIS approach

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A B S T R A C T

This paper deals with a class of bi-level linear programming problem (BLPP) with fuzzy data. Fuzzy data are mainly considered to design the real-life BLPP. So we assume that the coefficients and the variables of BLPP are trapezoidal fuzzy numbers and the corresponding BLPP is treated as fuzzy BLPP (FBLPP). Traditional approaches such as vertex enumeration algorithm, Kth-best algorithm, Krush-Kuhn-Tucker (KKT) condition and Penalty function approach for solving BLPP are not only technically inefficient but also lead to a contradiction when the follower's decision power dominates to the leader's decision power. Also these methods are needed to solve only crisp BLPP. To overcome the difficulty, we extend Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) in fuzzy environment with the help of ranking function. Fuzzy TOPSIS provides the most appropriate alternative solution based on fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). An example is included how to apply the discussed concepts of the paper for solving the FBLPP.

1. Introduction

Bi-level linear programming problem (BLPP) is an optimization problem in which the constraints are also; and there exist two decision makers (DMs) namely leader and follower. The leader who moves first and want to optimize his/her own objective function and thereafter who reacts in leader's action becomes the follower. Also BLPP can be defined as two-person zero-sum Stackleberg game that solves decentralised planning problem with two executors in a hierarchical system. In real world system, there exists a complexity due to insufficient data or imprecise data or lack of information and then the problem cannot be solved by considering crisp BLPP. Therefore fuzzy set theory is introduced into the problem which becomes FBLPP that can be applied in large practical problem.

Bialas and Karwan [6] worked in two level linear programming problems. They examined the geometric characteristic of the problem and represented an algorithm for two level linear problems. Roy [17] applied fuzzy

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programming technique for Stackelberg game. Maity and Roy [15] solved fuzzy transportation problem using multi-choice goal programming. Maity et al. [14] represented multi-objective transportation problem with cost reliability under uncertain environment. Maiti and Roy [13] solved multi-choice stochastic bi-level programming problem in cooperative nature via fuzzy programming approach. Lee [12] represented fuzzy multilevel programming with max min operator. Anandalingam [2] formulated a mathematical programming of decentralized multi-level system. Shih et al. [24] discussed fuzzy approach for multi-level programming problem. Roy and Maiti [19] analyzed stochastic bi-level programming with multi choice for Stackelberg game via fuzzy programming. Roy and Mula [21] solved matrix game with rough payoffs using genetic algorithm. Shih [25] introduced an interactive approach for integrated multi-level system in fuzzy environment. Sultan et al. [26] developed a fuzzy approach for solving a three-level large scale LPP, where each level maximizes separately. Roy et al. [18] proposed an approach to solve multi-objective two-stage grey transportation problem using utility function with goals. Zheng et al. [28] found an interactive fuzzy decision making method for solving bi-level programming problem. Ren and Wang [16] represented an interval approach based on expectation optimization for fuzzy random bi-level linear programming problem. Roy et al. [20] introduced a conic scalarization approach to solve multi-choice multi-objective transportation problem with interval goal. Roy and Maity [22] proposed an approach in minimizing cost and time through single objective function in multi-choice interval valued transportation problem. Roy et al. [23] represented a new approach for solving intuitionistic fuzzy multi-objective transportation problem. Chen [7] extended the TOPSIS for group decision-making problem in fuzzy environment. Jahanshahloo et al. [11] described TOPSIS method for decision making problem with fuzzy data. Abo-Sinna et al. [1] incorporated TOPSIS for large scale multi-objective non-linear programming problems with block angular structure. Awasthi et al. [3] designed fuzzy TOPSIS for multi-criteria decision making approach for location planning in urban distribution where the centres are under uncertainty. Baky and Abo-Sinna [4] used TOPSIS approach for bi-level multi-objective decision making (MODM) problems. Dymova et al. [9] represented an approach to generalize the fuzzy TOPSIS method. Dey et al. [8] applied TOPSIS approach in linear fractional bi-level MODM problem based on fuzzy goal programming. Baky [5] introduced an interactive TOPSIS algorithm for solving multilevel non-linear MODM problem.

For human judgement or time pressure or due to lack of information, BLPP is analysed in fuzzy environment, and we extend the BLPP for finding the best solution among the set of feasible solutions with the help of TOPSIS approach (proposed by Hwang and Yoon [10]). TOPSIS is one of the best, well known approaches where all the variables are controlled by the leader and find compromise solution with highest usefulness of majority and least uselessness for minority. Also the obtained solution is always non-dominated. Here we extend TOPSIS which is free from all limitations of other methods and apply in various fields such as supply chain management, military system, government policy where the DMs need a leader who provides the best preference of all alternatives.

The aim of the paper is to determine a compromise solution of BLPP with fuzzy variables and coefficients. We introduce an improved method which gives the solution closest to fuzzy positive ideal solution (FPIS) and farthest to fuzzy negative ideal solution (FNIS). We solve the BLPP in fuzzy system with a new approach where the DMs play a non-cooperative game independently among themselves. One of DMs may consider as leader according to the ranking order. Then the leader provides a better solution for controlling the other DMs sequentially according to their rank by TOPSIS approach. Hence we overcome the restriction that the leader only controls the lower level decision variables. FBLPP or fuzzy multi-level linear programming problem (FMLPP) is a useful process in energy network system, agriculture, economic system, conflict resolution. The aim of FBLPP may be different but most of cases it is applied in real-life system such as environmental studies, chemical engineering, biology, transportation, and game theory etc.

The rest of the paper is organized as follows. In Section 2, we define some basic definitions of fuzzy set and in Section 3, the theoretical background is given. In Section 4, a numerical application is given to illustrate the proposed approach. Finally, Section 5 presents the conclusion with future study.

2. Preliminaries

Here we review some basic definitions of fuzzy set which was introduced by Zadeh [27].

Definition 1: A fuzzy set $\tilde{A}$ in a universal set X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval [0, 1]. In fuzzy set, trapezoidal fuzzy number is a quadruplet defined as $\tilde{A} = (a_1, a_2, a_3, a_4)$ where $a_1 \le a_2 \le a_3 \le a_4$. Therefore for a trapezoidal fuzzy number $\tilde{A}$, the membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & if \quad x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & if \quad a_1 \le x \le a_2 \\ 1, & if \quad a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & if \quad a_3 \le x \le a_4. \end{cases}$$

Definition 2: A fuzzy set $\tilde{A}$ is normal if $Sup\, \mu_{\tilde{A}}(x) = 1$. A fuzzy set $\tilde{A}$ is convex iff for every pair of points x_1, x_2 in X, the membership function of $\tilde{A}$ satisfies the inequality $\mu_{\tilde{A}}(\delta x_1 + (1-\delta)x_2) \ge min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, where $\delta \in [0,1]$.

Definition 3: A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set of the real line $\mathbb{R}$ with continuous membership function.

Definition 4: The α-cut of a fuzzy set $\tilde{A}$ is a crisp subset of X and denoted by $\tilde{A}_\alpha = \{x : \mu_{\tilde{A}}(x) \ge \alpha\}$, where $\mu_{\tilde{A}}(x)$ is the membership function of $\tilde{A}$ and $\alpha \in (0,1]$. The α-cut of a trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ is given by $[a^l_\alpha, a^r_\alpha]$, where a^l_α is the lower bound and a^r_α is the upper bound of the interval and they are defined as, respectively $a^l_\alpha = (a_2 - a_1)\alpha + a_1$, $a^r_\alpha = a_4 - (a_4 - a_3)\alpha$.

Definition 5: Assume that $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. The distance between these trapezoidal fuzzy numbers $\tilde{a}$ and $\tilde{b}$ is $d(\tilde{a}, \tilde{b})$ which can be calculated as follows:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{4}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + (a_4 - b_4)^2]}.$$

If both $\tilde{a}$ and $\tilde{b}$ are two real numbers then the distance measurement is identical to the Euclidean distance.

Definition 6: Consider two positive fuzzy numbers $\tilde{m}$ and $\tilde{n}$ and the positive real number α, then α-cut of two fuzzy numbers are $\tilde{m}_\alpha = [m^l_\alpha, n^r_\alpha]$ and $\tilde{n}_\alpha = [n^l_\alpha, n^r_\alpha]$, $\alpha \in (0,1]$, respectively. Therefore by the interval arithmetic of fuzzy numbers $\tilde{m}$ and $\tilde{n}$ can be expressed as below:

(i) $(\tilde{m} + \tilde{n})_\alpha = [m^l_\alpha + n^l_\alpha, m^r_\alpha + n^r_\alpha]$;

(ii) $(\tilde{m} - \tilde{n})_\alpha = [m^l_\alpha - n^r_\alpha, m^r_\alpha - n^l_\alpha]$;

(iii) $(\tilde{m}.\tilde{n})_\alpha = [(m^l_\alpha.n^l_\alpha), (m^r_\alpha.n^r_\alpha)]$;

(iv) $(\frac{\tilde{m}}{\tilde{n}})_\alpha = [\frac{m^l_\alpha}{n^r_\alpha}, \frac{m^r_\alpha}{n^l_\alpha}]$;

(v) $(\tilde{m}.k)_\alpha = [m^l_\alpha.k, n^r_\alpha.k], k \ge 0;$

(vi) $(\frac{\tilde{m}}{k})_\alpha = [\frac{m^l_\alpha}{k}, \frac{n^r_\alpha}{k}], k > 0.$

Weighting concept: In BLPP, the weight is an important factor for solving it. Weighted method that generally assigns a set of weights to aggregate the multiple objectives into a single objective by a preference of structure provided by the DM. For different DMs with different ideas if the weights are represented by a fixed number, then values are derived from real situation. Therefore we consider the weight in such a way such that sum of all the variables is equal to one. Thus the weight can be defined as $w=(w_1,w_2,\ldots,w_n)$ with satisfying $\sum_{i=1}^{n} w_i = 1$.

3. Mathematical Structure of BLPP

A mathematical programming that contains an optimization problem in the constraints is called BLPP. This problem can be solved in different ways:

(1) A logical extension of mathematical programming.

(2) Generalization of a particular problem in game theory.

The player who takes decision, in first, is known as leader and the remaining player who reacts to the leader's decision is known as follower. In real-life system, a BLPP can be considered as:

Model 1

$$\max_{x_1} f_1 = c_{11}x_1 + c_{12}x_2,$$
$$\textit{where } x_2 \textit{ solves}$$
$$\max_{x_2} f_2 = c_{21}x_1 + c_{22}x_2,$$
$$\textit{subject to}, A_1x_1 + A_2x_2 \le b,$$
$$x_1 \ge 0, x_2 \ge 0, (x_1, x_2) \in S,$$

where S is the feasible region. c_{11}, c_{12} and c_{21}, c_{22} are $1\times m$ and $1\times n$ real matrices, respectively, and b is the column vector with component $(b_1, b_2, \ldots, b_l)$ and $x_1 = (x_{11}, x_{12}, \ldots, x_{1m})^t$, $x_2 = (x_{21}, x_{22}, \ldots, x_{2n})^t$. Also A_1 and A_2 are the $l\times m$ and $l\times n$ real matrices, x_1 and x_2 are decision variables with assuming x_1 is the decision variable of leader and x_2 is the decision variable of follower.

But in real-life situation, for human judgement or time pressure or due to lack of information, the data are not always crisp. Therefore these problems cannot be solved in traditional approach as the data become fuzzy. So we transform this BLPP into fuzzy system where all the constraints, decision variables are taken as fuzzy; and the BLPP becomes FBLPP. In FBLPP, we consider two fuzzy decision variables $\tilde{x}_1 = (x_{11}, x_{12}, x_{13}, x_{14})$ and $\tilde{x}_2 = (x_{21}, x_{22}, x_{23}, x_{24})$ as trapezoidal fuzzy numbers. Assume here the coefficients such as $\tilde{c}_{11}, \tilde{c}_{12}, \tilde{c}_{21}, \tilde{c}_{22}$ and the column vector with components $\tilde{b}_1$ and $\tilde{b}_2$ are trapezoidal fuzzy numbers. Also $\tilde{A}_1$ and $\tilde{A}_2$ are the $l\times m$ and $l\times n$ fuzzy matrices. In BLPP, the leader moves first and optimizes its own objective function. The follower observes to the leader's action and then takes decision in such a way to optimize its own objective function. But here one can consider as leader who optimizes the upper level problem with a better solution. The other becomes as a follower and only solves the lower level problem. Therefore the mathematical structure of FBLPP is formulated as:

Model 2

$$\max_{x_1} \tilde{f}_1 = (\tilde{c}_{11} \otimes \tilde{x}_1) \oplus (\tilde{c}_{12} \otimes \tilde{x}_2), \tag{1}$$
$$\textit{where } x_2 \textit{ solves} \tag{2}$$
$$\max_{x_2} \tilde{f}_2 = (\tilde{c}_{21} \otimes \tilde{x}_1) \oplus (\tilde{c}_{22} \otimes \tilde{x}_2), \tag{3}$$
$$\textit{subject to}, (\tilde{A}_1 \otimes \tilde{x}_1) \oplus (\tilde{A}_2 \otimes \tilde{x}_2) \le \tilde{b}, \tag{4}$$
$$\tilde{x}_1 \ge \tilde{0}, \tilde{x}_2 \ge \tilde{0}, \tag{5}$$

where $\otimes$ denotes the fuzzy multiplication and $\oplus$ denotes the fuzzy addition.

The formulated problem is solved by the TOPSIS approach to find the leader's solution and hence to obtain the compromise solution. We extend TOPSIS from crisp system to fuzzy system.

A new modified approach to extend the TOPSIS in fuzzy environment is presented here. This method is easy to apply in decision-making problem under fuzzy system. Here we consider the rating of each alternative in trapezoidal fuzzy number. To avoid the complicated normalized formula which is used in classical TOPSIS method, we normalize the fuzzy number $\tilde{r}_i = (r_{i1}, r_{i2}, r_{i3}, r_{i4})$ by $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{4} x_{ij}^2}}$; $i = 1,2;\ j = 1,2,3,4$.

Hence the range of normalized trapezoidal fuzzy number belongs to [0, 1]. Considering the different importance of each criterion, we consider the weighted normalized fuzzy number $\tilde{v}_i = (v_{i1}, v_{i2}, v_{i3}, v_{i4})$ with weight vector $w_i = (w_{i1}, w_{i2}, w_{i3}, w_{i4})$ such that $\sum_{j=1}^{4} w_{ij} = 1, i = 1,2;$ and $\tilde{v}_i = w_i * \tilde{r}_i$; Therefore $\tilde{v}_i\ (i = 1,2)$ becomes weighted normalized trapezoidal fuzzy number and its ranges belong to [0, 1]. Next we define the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) as $F^{u*} = (f_{11}^+, f_{12}^+, f_{13}^+, f_{14}^+)$ and $F^{u-} = (f_{11}^-, f_{12}^-, f_{13}^-, f_{14}^-)$ respectively, where $f_{1j}^+ = \max\{f_{1j} : j = 1,2,3,4\}$; $f_{1j}^- = \min\{f_{1j} : j = 1,2,3,4\}$.

Now the Euclidean distance for each alternative from FPIS and FNIS which can calculated by $d_i^{k+} = \sqrt{\sum_{j=1}^{4} (v_{ij} - v_{ij}^{k+})^2}$ and $d_i^{k-} = \sqrt{\sum_{j=1}^{4} (v_{ij} - v_{ij}^{k-})^2}$ $i = 1,2;\ j,k = 1,2,3,4$; where $v_{ij}^{k+} = PIS$ and $v_{ij}^{k-} = NIS$; $i = 1,2;\ j = 1,2,3,4.$

The closeness coefficient of the alternative is defined to determine the ranking order of the alternatives which is calculated as $R_i^k = \frac{d_i^{k-}}{d_i^{k+} + d_i^{k-}}; i = 1,2, k = 1,2,3,4$. $K(x_{kl}, \tilde{v}_{il}) = e^{-\|x_{kl} - \tilde{v}_{il}\|^2 / \sigma^2}$

Therefore, an alternative must closest to FPIS and farthest to FNIS. According to the closeness coefficients, we determine the ranking order of all alternatives and select the best among the set of feasible alternatives. Hence we consider the best alternative as the leader and the leaving alternative is simultaneously as follower.

The algorithm for decision making problem in fuzzy system with TOPSIS approach is described as follows:

• ***Step 1:*** Formulate a fuzzy bi-level linear programming problem where all the variables are trapezoidal fuzzy numbers.

• ***Step 2:*** Transform the fuzzy problem into equivalent crisp sub-problems in both levels.

• ***Step 3:*** Solve only upper level sub problem with all constraints and obtain the value of two decision variables in fuzzy interval that optimizes the upper level objective function.

• ***Step 4:*** Normalized the fuzzy number and then construct the weighted normalized fuzzy number.

• ***Step 5:*** Obtain FPIS and FNIS; and then derive the Euclidean distance for each alternative from FPIS and FNIS.

• ***Step 6:*** Calculate the relative closeness coefficient to the alternative for determining the ranking order of the alternatives.

• ***Step 7:*** Rank the alternatives according to their closeness coefficients and select the best alternative which is closest to FPIS and farthest to FNIS with highest closeness coefficient. Also set the corresponding solution as upper level solution, i.e., for leader's solution.

• ***Step 8:*** Set the upper level variable at zero level and solve the lower level sub problem.

• ***Step 9:*** Find the solutions and set the best alternative by ranking method.

• ***Step 10:*** Obtain the new solution, in which upper level solution is already obtained in Step 3 and the lower level solution is calculated in Step 9.

• ***Step 11:*** Find the optimum value of objective function from Model 2 using α-cut of the variables which obtained in Step 10.

• ***Step 12:*** Stop.

4. Numerical example

We consider a production planning problem in coal field which is controlled by both state government and central government. There exist two DMs and they control two variables x_1 and x_2. The respective capacity and necessary data are given in Table 1. Therefore two DMs are state government and central government; and each of them is controlled only single decision variable. This problem becomes a bi-level linear programming problem. The goals of leader and follower are to maximize their own profits and revenues. In this bi-level system, one government becomes the leader and other government becomes the follower. Here we consider the leader who maximizes the revenue among themselves and the other is known as follower who optimizes the profit on production. According to the various conditions such as long time, vast area, land condition, environment pollution/condition, and the data involving in the problem are not precise. So to tackle the data, we choose them as trapezoidal fuzzy numbers.

Table 1: Data in trapezoidal fuzzy system

	Coal field 1	Coal field 2	Resource
Labour	(2.5,3,3.5,4)	(4,4.5,5,5.5)	(20,25,30,35)
Time	(1,1.5,2.5,3)	(2,3.5,4.5,5)	(10,15,15,20)
Transportation cost	(3.5,4,4,4.5)	(5,6,6.5,7)	(25,30,35,40)
Investment cost	(2,3,4,4.5)	(1,1,1.5,2)	(15,20,25,35)
Revenue	(2,3,4,4.5)	(1.5,2,2.5,3)	
Profit	(1,2,2.5,3)	(2.5,3,3.5,4)	

Utilizing Table 1, we formulate the mathematical structure of FBLLP as in Model 3.

Model 3

$$\max_{\tilde{x}_1} \ \tilde{f}_1 = ((2,3,4,4.5)\otimes(x_{11},x_{12},x_{13},x_{14}))\oplus((1.5,2,2.5,3)\otimes(x_{21},x_{22},x_{23},x_{24})),$$

where $\tilde{x}_2$ *solves,*

$$\max_{\tilde{x}_2} \ \tilde{f}_2 = ((1,2,2.5,3)\otimes(x_{11},x_{12},x_{13},x_{14}))\oplus((2.5,3,3.5,4)\otimes(x_{21},x_{22},x_{23},x_{24})),$$

subject to

$$((2.5,3,3.5,4)\otimes(x_{11},x_{12},x_{13},x_{14}))\oplus((4,4.5,5,5.5)\otimes(x_{21},x_{22},x_{23},x_{24}))\leq(20,25,30,35),$$
$$((1,1.5,2.5,3)\otimes(x_{11},x_{12},x_{13},x_{14}))\oplus((2,3.5,4.5,5)\otimes(x_{21},x_{22},x_{23},x_{24}))\leq(10,15,15,20),$$
$$((3.5,4,4,4.5)\otimes(x_{11},x_{12},x_{13},x_{14}))\oplus((5,6,6.5,7)\otimes(x_{21},x_{22},x_{23},x_{24}))\leq(25,30,35,40),$$
$$((2,3,4,4.5)\otimes(x_{11},x_{12},x_{13},x_{14}))\oplus((1,1,1.5,2)\otimes(x_{21},x_{22},x_{23},x_{24}))\leq(15,20,25,35),$$

where

$$x_{11}\geq 0, x_{11}\leq x_{12}, x_{12}\leq x_{13}, x_{13}\leq x_{14},$$
$$x_{21}\geq 0, x_{21}\leq x_{22}, x_{22}\leq x_{23}, x_{23}\leq x_{24},$$

where $\tilde{f}_1=(f_{11},f_{12},f_{13},f_{14})$ and $\tilde{f}_2=(f_{21},f_{22},f_{23},f_{24})$.

Since Model 3 is fuzzy LPP, we split the problem into four crisp sub-problems. Therefore the upper level sub-problems are defined in Model 4A as follows:

Model 4A

$$\max\ f_{11}=2x_{11}+1.5x_{21},$$
$$\max\ f_{12}=3x_{12}+2x_{22},$$
$$\max\ f_{13}=4x_{13}+2.5x_{23},$$
$$\max\ f_{14}=4.5x_{14}+3x_{24},$$

subject to

$$2.5x_{11}+4x_{21}\leq 20,\ 3x_{12}+4.5x_{22}\leq 25,\ 3.5x_{13}+5x_{23}\leq 30,\ 4x_{14}+5.5x_{24}\leq 35,$$

$$x_{11}+2x_{21}\le 10,\ 1.5x_{12}+3.5x_{22}\le 15,\ 2.5x_{13}+4.5x_{23}\le 15,\ 3x_{14}+5x_{24}\le 20,$$
$$3.5x_{11}+5x_{21}\le 25,\ 4x_{12}+6x_{22}\le 30,\ 4x_{13}+6.5x_{23}\le 35,\ 4.5x_{14}+7x_{24}\le 40,$$
$$2x_{11}+x_{21}\le 15,\ 3x_{12}+x_{22}\le 20,\ 4x_{13}+1.5x_{23}\le 25,\ 4.5x_{14}+2x_{24}\le 35,$$
$$x_{11}\ge 0, x_{11}\le x_{12}, x_{12}\le x_{13}, x_{13}\le x_{14},\ x_{21}\ge 0, x_{21}\le x_{22}, x_{22}\le x_{23}, x_{23}\le x_{24}.$$

Here we solve the upper level problem, that is Model 4A by LINGO 13.0 iterative scheme separately and find $X_1=\langle(6,6,6,6),(0,0,0,0.4)\rangle$, $X_2=\langle(0,6,6,6.67),(0,0,0,0)\rangle$, $X_3=\langle(0,0,6,6),(0,0,0,0.4)\rangle$ and $X_4=\langle(0,0,0,6.67),(0,0,0,0)\rangle$.

Now using TOPSIS method, we normalize this solution. After that we construct the weighted normalized number $\tilde{v}_i=(v_{i1},v_{i2},v_{i3},v_{i4})$ with weight vector $w_i=(w_{i1},w_{i2},w_{i3},w_{i4})$ such that $\sum w_{ij}=1$ and $\tilde{v}_i=w_i\times\tilde{r}_i$; $i=1,2$; $j=1,2,3,4$. Hence, we consider $w_{ij}=\frac{1}{4}; i=1,2$; $j=1,2,3,4$.

Here X_j^* and $X_j^{'}$ give the maximum and minimum values of the objective functions f_{1j}, $j=1,2,3,4$. That is X_j^* and $X_j^{'}$ are the PIS and NIS respectively.

Now

$$X_1^*=\langle(6,6,6,6),(0,0,0,0.4)\rangle, X_2^*=\langle(6,6,6,6),(0,0,0,0.4)\rangle,$$
$$X_3^*=\langle(6,6,6,6),(0,0,0,0.4)\rangle,\ X_4^*=\langle(0,6,6,6.67),(0,0,0,0)\rangle;$$
$$X'_1=\langle(0,0,0,6.67),(0,0,0,0)\rangle,\ X'_2=\langle(0,0,6,6),(0,0,0,0.4)\rangle,$$
$$X'_3=\langle(0,0,0,6.67),(0,0,0,0)\rangle, X'_4=\langle(6,6,6,6),(0,0,0,0.4)\rangle.$$

Now we obtain PIS and NIS from Table 2.

Table 2: Positive ideal solution and Negative ideal solution

	f_{11}	f_{12}	f_{13}	f_{14}
X_1^*	12*	18*	24*	28.2
X_2^*	0	18	24	30.015*
X_3^*	0	0	24	28.2
X_4^*	0	0	0	30.015

Therefore $PIS=F^{u^*}=(f_{11}^*,f_{12}^*,f_{13}^*,f_{14}^*)=(12,18,24,30.015)$ and $NIS=F^{u-}=(f_{11}^-,f_{12}^-,f_{13}^-,f_{14}^-)=(0,0,0,28.2)$.

Next we obtain the Euclidean distance for each alternative from PIS and NIS as $d_i^{k+}=\sqrt{\sum_{j=1}^{4}(v_{ij}-v_{ij}^{k+})^2}$, $d_i^{k-}=\sqrt{\sum_{j=1}^{4}(v_{ij}-v_{ij}^{k-})^2}$, $i=1,2$; $k=1,2,3,4;$ where, $v_{ij}^{+}=PIS$, $v_{ij}^{-}=NIS$. $d_1^{1+}=0$, $d_1^{1-}=0.255$, $d_1^{2+}=0.134$, $d_1^{2-}=0.157$, $d_1^{3+}=0.196$, $d_1^{3-}=0.193$, $d_1^{4+}=0.217$, $d_1^{4-}=0.255$, $d_2^{1+}=0$, $d_2^{1-}=0.25$, $d_2^{2+}=0.25$, $d_2^{2-}=0.25$, $d_2^{3+}=0$, $d_2^{3-}=0.25$, $d_2^{4+}=0$, $d_2^{4-}=0.25$.

Calculating the relative closeness to the alternatives with ranking function such as $R_i^k=\frac{d_i^{k-}}{d_i^{k+}+d_i^{k-}}$, $i=1,2; k=1,2,3,4;$, we get $R_1^1=1$, $R_1^2=0.53951, R_1^3=0.4961, R_1^4=0.5402$ an $R_2^1=1, R_2^2=0.5, R_2^3=1, R_2^4=1$.

Since $R_1^1>R_1^4>R_1^2>R_1^3$, and $R_2^1=R_2^3=R_2^4=1>R_2^2$, we conclude that $(6,6,6,6)$ is a better alternative than others for leader. Therefore we consider the upper level DM is the leader as $\tilde{x}_1$ and the lower level DM is the follower as $\tilde{x}_2$. Now $\tilde{x}_1 = (6,6,6,6)$ is the upper level solution.

Then we solve the lower level problem taking only lower level variable; and upper level variable at zero level. Now we solve the lower level problem as

Model 4B

$$
\begin{aligned}
& max \;\; f_{21} = 2.5x_{21}, \\
& max \;\; f_{22} = 3x_{22}, \\
& max \;\; f_{23} = 3.5x_{23}, \\
& max \;\; f_{24} = 4x_{24}, \\
& subject\ to \\
& 4x_{21} \le 20, \;\; 4.5x_{22} \le 25, 5x_{23} \le 30, \;\; 5.5x_{24} \le 35, \\
& 2x_{21} \le 10, \;\; 3.5x_{22} \le 15, 4.5x_{23} \le 15, \;\; 5x_{24} \le 20, \\
& 5x_{21} \le 25, \;\; 6x_{22} \le 30, 6.5x_{23} \le 35, \;\; 7x_{24} \le 40, \\
& x_{21} \le 15, \;\; x_{22} \le 20, \;\; 1.5x_{23} \le 25, 2x_{24} \le 35, \\
& x_{21} \ge 0, \; x_{21} \le x_{22}, \; x_{22} \le x_{23}, \; x_{23} \le x_{24}.
\end{aligned}
$$

Now solving Model 4B by LINGO 13.0 iterative scheme and we get $\tilde{x}_2{}^1 = (0.4, 3.33, 3.33, 4)$, $\tilde{x}_2{}^2 = (0, 3.33, 3.33, 4)$, $\tilde{x}_2{}^3 = (0, 0, 3.33, 4)$, $\tilde{x}_2{}^4 = (0, 0, 3.33, 4)$. Again by ranking method in TOPSIS we find PIS and NIS. Also by the previous algorithm, we show that $R_2^1 > R_2^2 > R_2^3 = R_2^4$. Hence the best alternative is $\tilde{x}_2 = (0.4, 3.33, 3.33, 4)$.

Therefore the new solution is $\tilde{x}_1 = (6,6,6,6)$ and $\tilde{x}_2 = (0.4, 3.33, 3.33, 4)$.

Now we obtain the maximum values of $\tilde{f}_1$ and $\tilde{f}_2$ using $\alpha -$ cut, by choosing the value of $\alpha = \frac{1}{2}$. Therefore

$$\tilde{f}_1 = \left((2,3,4,4.5)\otimes(6,6,6,6)\right)\oplus\left((1.5,2,2.5,3)\otimes(0.4,3.33,3.33,4)\right) = [18.26, 35.58];$$
$$\tilde{f}_2 = \left((1,2,2.5,3)\otimes(6,6,6,6)\right)\oplus\left((2.5,3,3.5,4)\otimes(0.4,3.33,3.33,4)\right) = [14.13, 30.25].$$

Here we say that this result is better than the other obtained result in previous approach which solves BLPP in fuzzy environment. Also we overcome the condition whenever the follower’s decision power dominates the leader’s decision power.

5. Conclusion

In this paper, we have analyzed a BLPP in fuzzy environment with taking trapezoidal fuzzy data. TOPSIS algorithm has been extended in fuzzy environment for solving FBLPP. Also fuzzy TOPSIS has been used for not only to find the leader between two decision makers by ranking method but also find the closest to the best ideal solution. Hence our aim is to represent a new practical approach which may be useful in various fuzzy multi-level linear programming problems. This approach is very simple as well as very suitable and practical to solve FBLPP. It overcomes the condition in choosing the upper level decision maker and the lower level decision maker. That is to find the leader and follower between two decision makers. We take upper level decision maker which gives a better solution in solving first objective function by ranking method with the help of TOPSIS approach. Next the upper level variable control the lower level decision maker to search the optimum value with taking itself as zero at that time. Though TOPSIS was primarily developed for real data, but here we extend TOPSIS method to decision making problem with fuzzy data. The advantage of this method is that this method can be extended to solve large scale problems. In future, one may extend the presented model with more than two decision makers in BLPP. Also this work will be extended to solve multilevel programming problem with intuitionistic fuzzy number or the parameters are interval valued intuitionistic fuzzy number. Another scope is also to consider non-linear multi-level programming problem.

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