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# Utilizing Firefly Algorithm-Optimized ANFIS for Estimating Engine Torque and Emissions Based on Fuel Use and Speed

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ABSTRACT

In this study, a method for predicting engine torque and emissions considering fuel consumption and engine speed parameters is presented. An adaptive neuro-fuzzy inference system (ANFIS) optimized with the Firefly algorithm is used. This strategy uses the global optimization capabilities of the Firefly algorithm, an algorithm inspired by biological phenomena, in combination with the ability of ANFIS to describe complicated non-linear relationships between inputs and outputs. The ANFIS system was trained on a dataset containing various engine operating conditions, with the Firefly algorithm fine-tuning the model parameters to ensure optimal effectiveness. The input parameters of the model consisted of fuel quantity and engine speed, while engine torque and nitrogen oxide emissions formed the output parameters. The results obtained showed high accuracy in predicting engine torque and emissions, confirming the effectiveness of the Firefly-optimized ANFIS model. This model makes an important contribution to engine performance monitoring and emissions management. It provides a powerful tool for real-time regulation and has the potential to improve fuel efficiency while reducing environmental impact. Future research efforts should extend the applicability of this model to a wider range of engine shapes and operating conditions.

## 1. Introduction

The ceaseless development in the field of engine technology shows an unrelenting striving for higher performance and environmental compatibility [10]. To achieve these two goals, a number of factors must be taken into account, of which fuel consumption and engine speed play a decisive role. These factors have a direct influence on important performance indicators such as engine torque and emissions [20, 26, 28, 29]. Therefore, the accuracy of predictions based on these factors becomes a cornerstone in this field, which requires thorough research on the subject [14, 30, 34, 39, 41].

Engine torque, a key indicator of engine efficiency, has a major impact on the immediate performance and long-term durability of an engine. Added to this is the importance of engine emissions, especially nitrogen oxide

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emissions, which have a significant impact on the environment. Given the increasing importance of environmental protection worldwide, research efforts to reduce emissions in the field of engine technology have gained considerable importance [4, 9, 21, 32, 35, 37]. In the past, a number of techniques using different methods and algorithms have been used to estimate engine torque and emissions, but with varying degrees of success. However, the advent of computational intelligence techniques was a crucial milestone that opened up a range of research opportunities. Of particular note in this context is the synergistic fusion of the Adaptive Neuro-Fuzzy Inference System (ANFIS) and the Firefly algorithm, which show remarkable potential for improved optimization [2, 6, 17, 19, 22, 23, 25, 27, 31 36].

ANFIS, a sophisticated hybrid model, combines the strengths of fuzzy logic and neural networks to accurately model complex relationships between input and output parameters. At the same time, the Firefly algorithm, a global optimization method inspired by the fascinating behaviour of fireflies, has shown impressive results in a variety of applications, including engine development [11, 13, 16, 18, 40].

Building on these advances, this paper develops a methodology that exploits the combined potential of the ANFIS model optimized with the Firefly algorithm. This innovative model was developed to predict engine torque and emissions as a function of fuel consumption and speed. However, the scope of this research goes beyond the mere development and validation of this model. It aims to investigate the model's potential as a pragmatic tool for optimizing engine performance and promoting environmental sustainability [17, 23, 27].

This paper presents a method for estimating engine torque and emissions using an adaptive neuro-fuzzy inference system (ANFIS) refined by the Firefly algorithm. The model uses engine speed and fuel consumption as input parameters, and its output parameters are engine torque and nitrogen oxide emissions. A data set covering a wide range of engine operating conditions was used to validate and train the model.

With the effective implementation of this model, we aim to make an important contribution to engine performance monitoring and emission control. This novel tool offers the potential for real-time regulation that could improve fuel efficiency and reduce environmental impacts.

Our results not only contribute to the existing body of knowledge in this important area but also provide impetus for further research. It is expected that the applicability of this model will be extended to a wider range of engine types and operating contexts.

## 2. Materials and Methods

This section describes the materials and methods used in this study. An adaptive neuro-fuzzy inference system (ANFIS) optimized by the Firefly algorithm is used to estimate engine torque and emissions, considering fuel consumption and speed as primary variables. The approach is explained systematically and comprehensively, covering aspects such as data acquisition, model formulation, and the optimization process.

### 2.1. Data Collection

The data set used in this study includes critical engine parameters: Fuel consumption and engine speed as input variables, and engine torque and nitrogen oxide (NOx) emissions as output variables. Fuel consumption, which indicates the amount of fuel consumed in a given period, reflects the efficiency of the engine. At the same time, engine speed, represented as RPM or vehicle speed, is a key factor for engine performance and efficiency. These input parameters are directly related to engine performance indicators, i.e., torque, a measure of engine power, and NOx emissions, an important indicator of environmental impact due to air pollution, acid rain, and potential health risks.

Combining these parameters in one data set is essential to modelling the intricate interactions between them and ultimately improving fuel efficiency, optimizing engine torque, and reducing NOx emissions. Achieving this delicate balance not only promises economic benefits but is also in line with environmental sustainability goals.

To fully exploit the potential of this dataset, specifics such as the size of the dataset, the diversity of engines

tested, the range of operating conditions, and the statistical distribution of parameters are required. Regardless, the main objective remains the same: to use data-driven insights to promote performance improvements in engine technology while minimizing environmental impacts.
Input parameters:

Fuel consumption refers to the rate at which the engine consumes fuel and is usually measured in units such as liters per hour or gallons per hour. Fuel consumption is an important parameter for evaluating the efficiency of an engine.

RPM: This refers to engine speed and probably indicates the speed of the engine's crankshaft, usually expressed in revolutions per minute (RPM). Engine RPM: RPM is a critical factor that affects many aspects of engine performance, including power output and fuel efficiency.
Objective:

Engine torque: This is the torque produced by the engine, usually measured in newton meters (Nm) or foot-pounds (ft-lbs). It refers to the rotational force generated by the engine and is an important indicator of engine performance.

Nitrogen oxide emissions: This refers to the amount of nitrogen oxides (NOx) emitted by the engine during its operation. NOx emissions are important because of their negative impact on air quality and their contribution to climate change.

In the context of machine learning, the ultimate goal would be to create a model that is able to accurately predict the desired outcomes (torque and NOx emissions) based on the input parameters (fuel consumption and engine speed). The effectiveness of such a model can be measured by the extent to which its predicted values match the actual observed values for torque and NOx emissions.

## 2.2. Firefly Algorithm

Firefly Algorithm (FA) is a metaheuristic optimization algorithm inspired by nature, specifically emulating the behaviour of fireflies. Xin-She Yang pioneered the development of the Firefly Algorithm in 2008 [38]. Fireflies produce flashes of light to attract other fireflies. This mutual attraction behaviour is used in the algorithm to optimize a function.

The key aspects of the Firefly Algorithm are:

1. Fireflies are treated as gender-neutral within the algorithm, indicating that attraction between any two fireflies is not influenced by sex.
2. The algorithm operates on the principle that attractiveness corresponds to brightness and that both of these factors decline with increasing distance between the entities.
3. In the absence of a brighter entity, the movement of a firefly within the algorithm is determined by a random process.

The light intensity or luminosity of a firefly at a given position within the algorithm is ascertained by the corresponding value of the objective function at that same position.

The trajectory of a firefly '*i*', drawn towards another more attractive or brighter firefly '*j*', is dictated by a specific mathematical equation.

$$x_i(t+1) = x_i(t) + \beta_0 e^{-\gamma r^2}\left(x_i - x_j\right) + \alpha\varepsilon_i \quad (1)$$

Here:

- $x\_i(t+1)$ and $x\_i(t)$ are the position of the *i*th firefly at time $t+1$ and $t$ respectively.
- $\beta_0$ is the attractiveness at $r=0$.
- $e^{-\gamma r^2}$ is a Gaussian function that describes the decrease of light intensity.

- $\alpha$ is the randomization parameter.
- $\varepsilon_i$ is a random vector drawn from a uniform or Gaussian distribution.

The separation *r_ij* between two fireflies, *i* and *j*, located at points *x_i* and *x_j*, is typically computed using the Euclidean distance metric as follows:

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \quad (2)$$

The intensity or brightness (*I*) of a firefly situated at a particular position (*x*) is determined by the objective function *f*(*x*). To handle minimization problems and avoid the problem of division by zero, a standard approach is to employ Equation (3).

$$I(x) = 1/(f(x) + \varepsilon) \quad (3)$$

The Firefly algorithm can be tailored to meet the different needs of different optimization problems, resulting in multiple versions of the algorithm. Different types of optimization problems may require specific adaptations of the Firefly algorithm. A pseudocode representation that gives a general overview of the Firefly algorithm can be found in Table 1.

**Table 1.** Pseudocode Representation of the Firefly Algorithm

```
Define objective function f(x), x=(x1,...,xd)^T
Generate initial population of fireflies xi (i=1,2,...,n)
Light intensity Ii at xi is determined by f(xi)
Define light absorption coefficient gamma
While (t < MaxGeneration)
For i=1: n (all n fireflies)
    For j=1: n (all n fireflies)
        If (Ij > Ii), Move firefly i towards j
        Update attractiveness using intensity difference
        Evaluate new solutions and update light intensity
    End For j
End For i
Rank the fireflies and find the current best
End While
Postprocess results and visualization
```

## 2.3. Adaptive Neuro-Fuzzy Inference System (ANFIS) Application

The Adaptive Neuro-Fuzzy Inference System (ANFIS) [16] is a type of artificial neural network that combines the adaptability of neural networks with the inference and uncertainty management capabilities of fuzzy systems. ANFIS was developed in the early 1990s and is based on the Takagi-Sugeno fuzzy inference system. Since then, it has become an impressive tool for modeling and controlling complex systems [3, 18, 40].

ANFIS is designed as a five-layer feed-forward neural network, with each layer performing a specific function [1, 15]:

***Layer 1 (Fuzzification Layer):*** This layer is equipped with adaptive nodes, often referred to as fuzzification nodes. Each of these nodes converts a given input into a degree of membership using a membership function. The shape of the membership function—gaussian, bell, or sigmoid—depends on the nature of the problem.

***Layer 2 (Rule Layer):*** This layer is populated by fixed nodes that perform a t-norm operator (usually multiplication) to simulate the AND operator in fuzzy logic and represent the firepower of a rule.

***Layer 3 (Normalization Layer):*** This layer also contains fixed nodes that normalize the ignition intensities derived from the previous layer. Each node calculates the ratio between the ignition intensity of a particular rule and the sum of the ignition intensities of all rules.

***Layer 4 (Defuzzification Layer):*** This layer consists of adaptive nodes, each of which calculates the contribution of a given rule to the final output. This contribution is the product of the normalized firing strength and a first-order polynomial, specifically for a Sugeno-type fuzzy inference system.

***Layer 5 (Summation Layer):*** The final layer of the ANFIS architecture, the summation layer, consists of fixed nodes that calculate the total output as the sum of all incoming signals, which are the outputs of the previous layer.

In this architecture, the parameters for the adaptive nodes (located in layers 1 and 4) are adjusted during the learning process, while the fixed nodes maintain their operating rules regardless of the learning progress. The learning process in ANFIS typically uses a hybrid learning algorithm that is a mixture of gradient descent and least squares estimation.

ANFIS essentially has the ability to map input features through a series of transformations to obtain a single-valued output or decision. This is best illustrated in the context of a first order Sugeno fuzzy model with two inputs and one output. Let us assume two inputs (x and y) and one output (f). A rule in such a system could look like this:

Rule: IF x is (fuzzy set) A and y is (fuzzy set) B THEN f is a function of x and y.

For example, the rule could be:

Rule: IF x is LOW and y is HIGH THEN f = px + qy + r

Here "LOW" and "HIGH" are fuzzy sets bound to inputs x and y, respectively. Membership functions convert unique input values into fuzzy values. The function f in the THEN part of the rule is a linear combination of the inputs x and y, with the coefficients p, q, and r determined during the learning phase.

Since ANFIS is able to encapsulate the associations and dependencies between inputs and outputs in a flexible yet interpretable way, this method is extremely promising for solving complicated and nonlinear problems by exploiting the learning ability of neural networks and the extensive response potential of fuzzy logic.

With this method, ANFIS can capture the relationships and dependencies between inputs and outputs in a highly interpretable yet flexible manner. The general structure of the ANFIS algorithm is shown in Figure **1**.

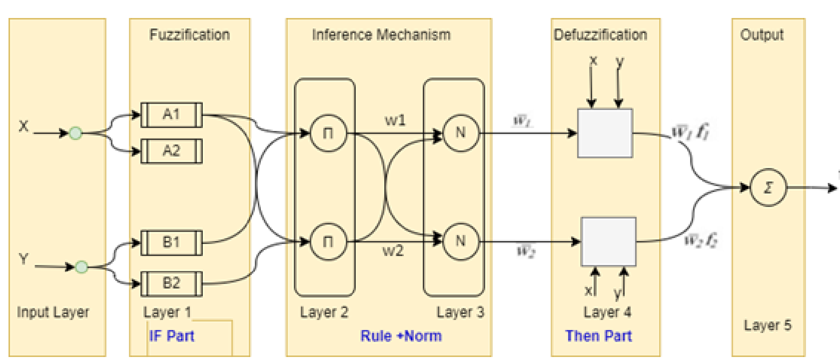


**Figure 1.** Structure of ANFIS algorithm [12]

## 2.4. Firefly Algorithm-Optimized ANFIS Model Development

The firefly algorithm (FA) is a metaheuristic method inspired by the luminous behavior of fireflies. It is a biomimetic optimization algorithm that has been successfully applied in various fields, including machine learning, where it is used to refine the parameters of other learning algorithms such as the Adaptive Neuro-

Fuzzy Inference System (ANFIS) [17], [23], [27].

The procedure for developing an ANFIS model optimized with the Firefly algorithm can be divided as follows:

Initialization: The Firefly population is initialized with randomly generated solutions within the search space. Each individual Firefly represents a possible solution that corresponds to a set of parameters for the ANFIS model.

Objective evaluation: The fitness or objective value of each Firefly (solution) is calculated as a function of the problem at hand. For ANFIS, this could be the error rate or another performance metric of the model.

Firefly movement: The position of each firefly is updated to reflect attraction by other fireflies. A particular firefly is attracted to another firefly with a higher light intensity (better fitness or target value). If there is no such firefly, it navigates randomly.

Update light intensity: The light intensity of each firefly is updated based on its new position.

Check termination: If the termination criteria are met (e.g., when the maximum number of iterations is reached or when the error rate falls below a certain threshold), the algorithm stops. Otherwise, the process returns to the third step.

Train the ANFIS model: The ANFIS model is trained using the optimal solution determined by the Firefly algorithm. This optimal solution represents the optimized parameters for the ANFIS model. Figure 2 illustrates the structure of the ANFIS model as optimized by the Firefly algorithm.

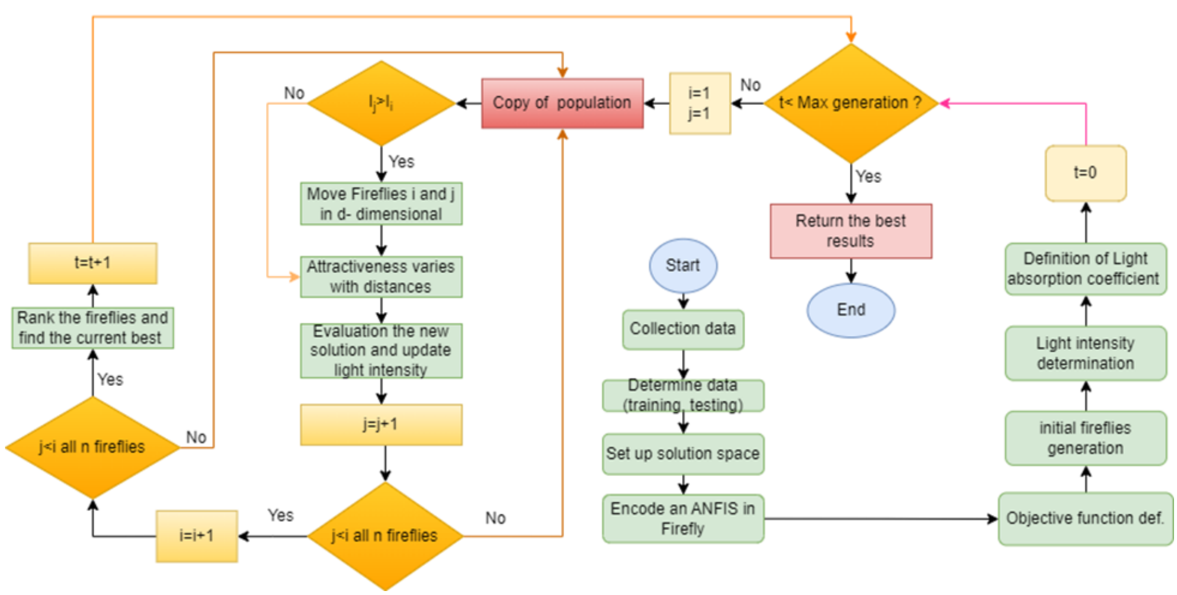


**Figure 2.** Flowchart of Firefly-ANFIS Algorithm

## 2.5. Model Evaluation

The effectiveness of the ANFIS model was improved by the Firefly algorithm in predicting engine torque and nitrogen oxide emissions. This was investigated using a number of evaluation measures. These include mean square error (MSE), root mean square error (RMSE), root mean square error (ME), and standard deviation of error (STD error). Taken together, these indices provide a robust assessment of the predictive power and overall performance of the model.

### Mean Squared Error (MSE):

The MSE is a widely used measure to evaluate the performance of a predictive model [8, 24]. It calculates the average of squared discrepancies between actual and predicted outcomes. Squaring ensures the elimination of negative signs and gives more weight to larger discrepancies. Lower MSE values are generally desirable, as they indicate that the model's predictions agree well with the actual values.

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2 \tag{1}$$

Here, N is the number of observations, represents the actual values, and represents the predicted values. The summation (Σ) is over all observations.

### Root Mean Squared Error (RMSE):

RMSE [7] is the square root of the MSE. It is also a measure of the differences between predicted and actual values, but because it is a square root, it is expressed in the same units as the values themselves, which often makes it easier to interpret than MSE. Like MSE, a lower RMSE means a better model.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2} \tag{2}$$

It is simply the square root of the MSE.

### Mean Error (ME):

ME is the average difference between the actual and predicted values without squaring the differences [5, 33]. Unlike MSE and RMSE, ME can be positive or negative and is therefore able to indicate whether predictions are generally too high (positive ME) or too low (negative ME). A ME, which is closer to zero, means a better model.

$$Error\ Mean = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i) \tag{3}$$

Similar to MSE, but without squaring the differences.

### Standard Deviation of Error (STD Error):

The standard deviation of error [5, 24] measures the spread of differences between actual and predicted values around their mean (ME) [5, 33]. A lower standard deviation of error means that the errors are more tightly clustered around their mean, indicating a more consistent prediction model. A high standard deviation of error, on the other hand, means that the errors are scattered, indicating less consistent performance of the model. These metrics are all part of the broader field of regression analysis, and they are basic tools for assessing the accuracy and reliability of models that predict continuous (as opposed to categorical) outcomes.

$$Error\ St.D = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x}_i)^2}{N-1}} \tag{7}$$

This is essentially the standard deviation of the differences between the actual and predicted values, where ME is the mean of these differences. The factor (N-1) in the denominator accounts for the degrees of freedom in estimating the standard deviation.

In all equations, the summation (Σ) is done over all observations from i = 1 to i = N.

## 3. Result and Discussion

The model showed exceptional accuracy in predicting engine torque and NOx emissions, highlighting the effectiveness of the Firefly algorithm in optimizing the parameters of the ANFIS model. This result underlines the ability of such models to provide reliable predictions for critical engine parameters. Most importantly, it improves real-time monitoring of engine performance and emission control, supporting greater fuel efficiency while reducing environmental impact.

In estimating engine torque, the performance of the model was robust, with a root mean square error (RMSE) of 41.7807, a mean error (ME) of 2.4076, and a standard deviation of error of 41.7693. This superior performance in estimating both engine torque and emissions can be attributed to the clever optimization of ANFIS parameters by the Firefly algorithm.

The graphical comparison of the estimated and actual values for engine torque and nitrogen oxide emissions highlights the remarkable ability of the model to generalize and maintain its robustness across different engine operating conditions. These promising results provide a compelling argument for using the model for real-time engine performance monitoring and emission control. This also confirms the successful application of the ANFIS model, optimized by the Firefly algorithm, in predicting engine torque and emissions based on fuel consumption and engine speed.

Figure 3 shows a visual demonstration of the training process of the Firefly algorithm. The diagram shows an iterative learning cycle in which the algorithm gradually adjusts its parameters to improve the model. The horizontal axis, or abscissa, of the diagram indicates the number of iterations or epochs, while the vertical axis, or ordinate, represents the performance metric, usually denoted by the error or loss function.

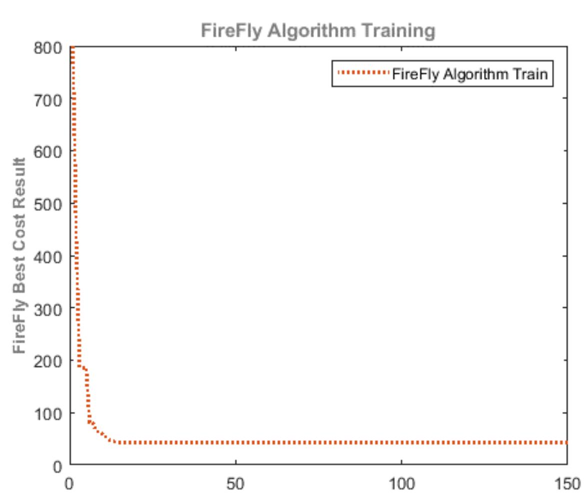


**Figure 3.** Firefly Algorithm Training

A downward trend can be seen in the successive iterations, indicating a systematic improvement in model performance by the Firefly algorithm. This consistent improvement demonstrates the algorithm's ability to progressively refine the model's predictive accuracy for engine torque and nitrogen oxide emissions.

Figure 4 provides a comprehensive visual representation of the Firefly algorithm's performance metrics during the training and testing phases. It illustrates six basic elements: the training quotient associated with the standard deviation of errors, the test quotient, the training error, the test error, the mean training error, and the mean test error.

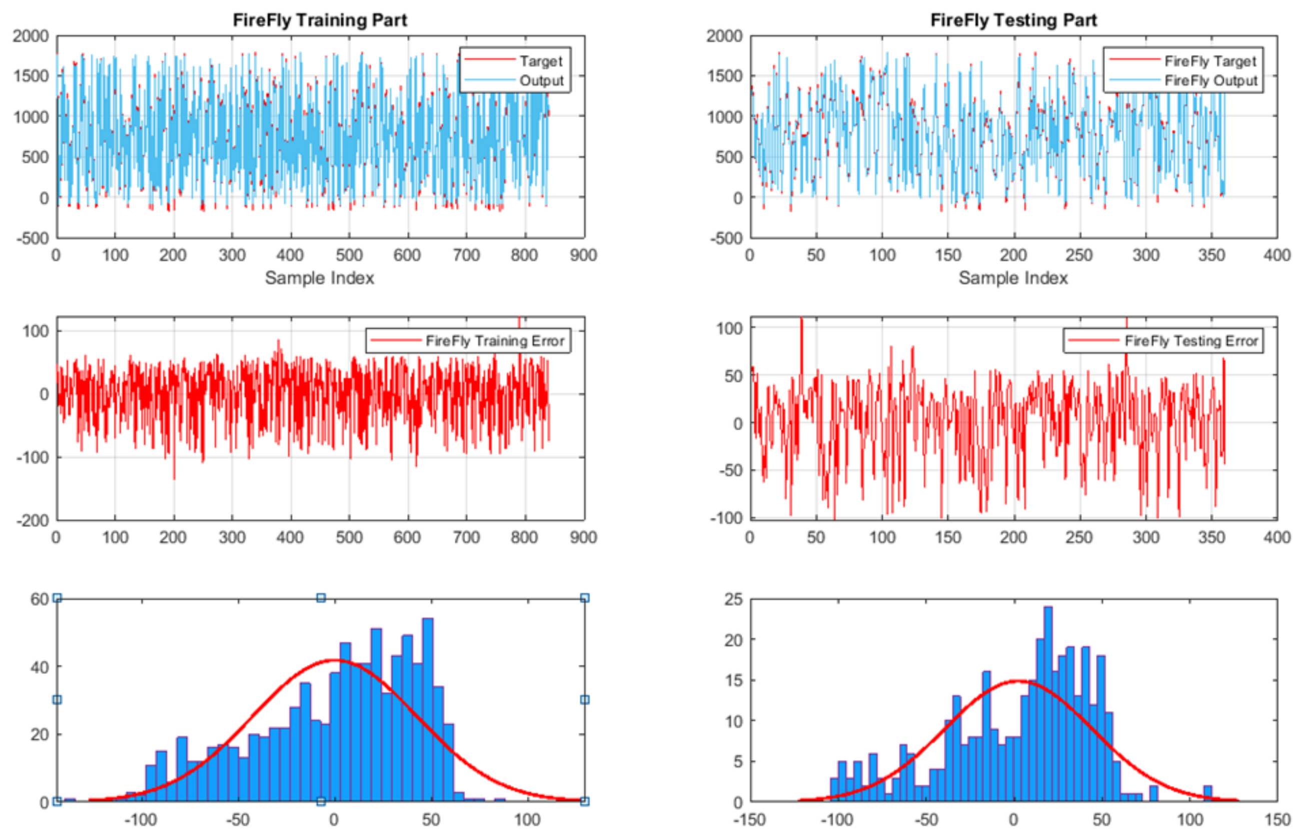


**Figure 4.** The performance of all datasets for FA model

The diagrams labelled 'Training Intercept' and 'Test Intercept' illustrate the predicted values of the model for the corresponding training and test data sets. The graphs labelled 'Training Error' and 'Test Error' also graphically represent the discrepancy between the model's predictions and show the deviation between the predicted and actual values for the respective training and test data sets.

The graphs labelled 'Mean Training Error' and 'Mean Test Error' represent the average prediction deviations of the model for the respective data sets and give a comprehensive insight into the performance of the model.

Finally, the graph labelled 'Error Std' displays the standard deviation of the prediction errors and serves as a measure of the model's error variability. Lower values of the standard deviation mean that the model's predictions are consistently close to the actual values.

In summary, Figure 4 provides a complex examination of the model's performance and shows that it is able to predict engine torque and nitrogen oxide emissions both accurately and consistently.

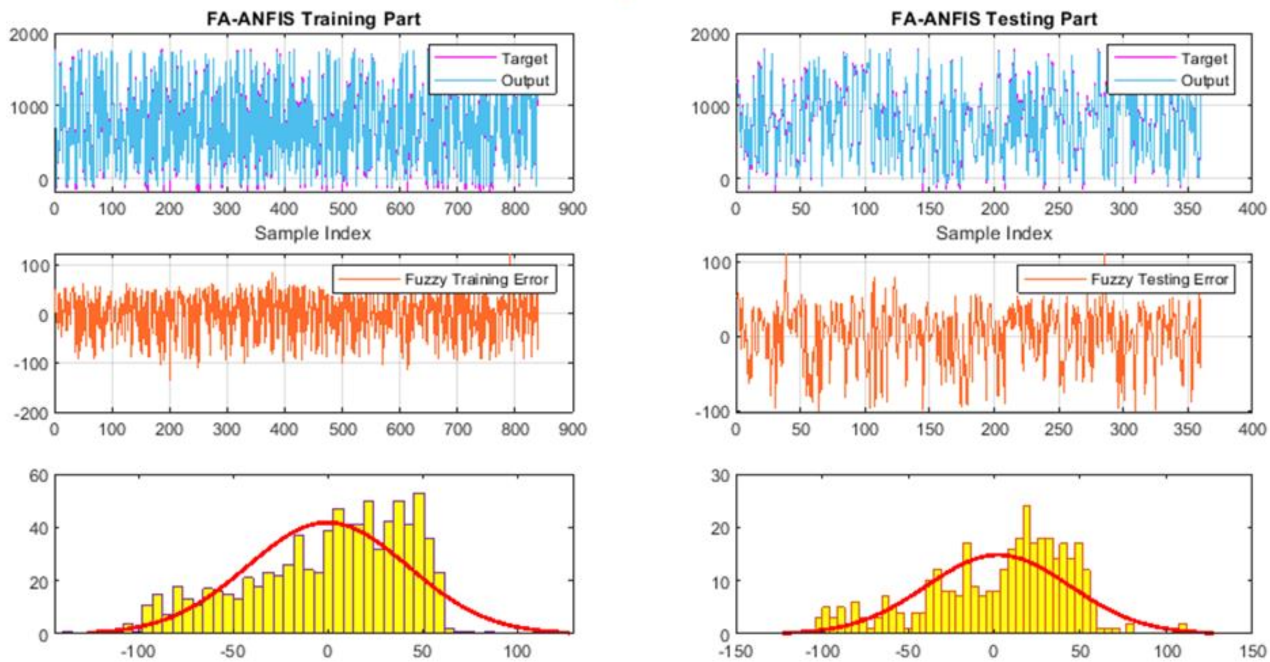


**Figure 5.** The performance of all datasets for ANFIS-FA model

Figure **5** shows a detailed evaluation of the performance of the Adaptive Neuro-Fuzzy Inference System (ANFIS) during the training and testing phases. This figure contains a detailed visualization of the development of the model during these periods, supplemented by the corresponding error metrics.

This figure provides a comprehensive representation of the training and testing procedures of the ANFIS model and shows the results of these procedural steps. In addition, the degree of deviation is graphically represented by showing both the training and testing errors, providing valuable insight into the predictive reliability and accuracy of the model.

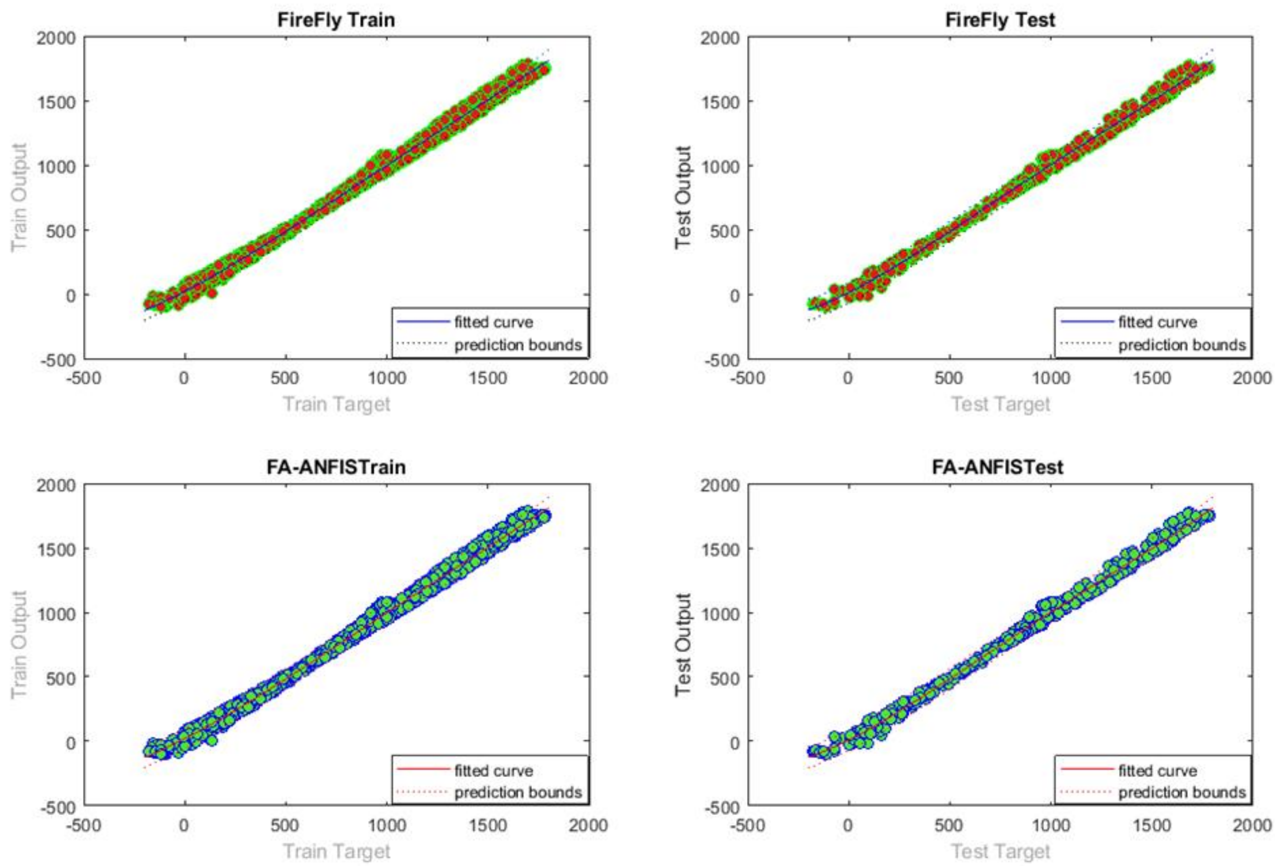


**Figure 6.** Fuzzy and firefly algorithm training, testing R values fitted curve and prediction bounds

Figure 6 shows a comprehensive comparative evaluation of the Firefly algorithm and the Adaptive Neuro-Fuzzy Inference System (ANFIS) during their respective training and testing phases. In particular, this figure shows the correlation coefficients (R-values), which give an indication of the extent and direction of correlation between predicted and observed values for both methods.

The section of the figure labeled 'Fitted Curve' visually represents the model predictions for both the Firefly algorithm and ANFIS and provides a descriptive representation of the agreement between the predicted results and the observed data.

In addition, the 'prediction bounds' plotted on the graph indicate the likely range into which the actual values are expected to fall given the model predictions. These bounds illustrate the degree of uncertainty or variability associated with the predictions and thus serve as an intuitive measure of their reliability.

**Table 2**. Statistical Measures for Employed Model

| | **Firefly** | | **FA-ANFIS** | |
|---|---|---|---|---|
| | Training | Testing | Training | Testing |
| MSE | 1798.2071 | 1744.8841 | 1798.1614 | 1745.623 |
| RMSE | 42.4053 | 41.7718 | 42.4047 | 41.7807 |
| Error Mean | -0.010689 | 2.3893 | -24187e-13 | 2.4076 |
| Error STD | 42.4306 | 41.7615 | 42.43 | 41.7693 |

Table 2 provides a rigorous numerical comparison of the performance metrics associated with the models studied. The metrics clearly show that the Adaptive Neuro-Fuzzy Inference System (FA-ANFIS) optimized with the Firefly algorithm outperforms the other models when the Root Mean Square Error (RMSE) evaluation criterion is applied. This result underlines the superior efficiency and precision of the FA-ANFIS model in predicting engine torque and nitrogen oxide emissions. Considering these promising results, the scope for further research and deployment of this model is very large and could lead to significant advances in real-time engine performance monitoring and emission control.

## 4. Conclusion

In summary, this research demonstrates the significant potential of an optimized adaptive neuro-fuzzy inference system (ANFIS) in accurately predicting key engine parameters such as torque and nitrogen oxide emissions as a function of fuel consumption and engine speed. The resilience and flexibility of the Firefly algorithm, a bio-inspired heuristic, were used to optimize the ANFIS model, resulting in a remarkable improvement in prediction accuracy.

While the ANFIS base model provided a remarkable approximation of the inherent input-output relationships, the integration of the Firefly algorithm significantly improved the predictive capacities of the model, confirming the effectiveness of the Firefly algorithm in optimizing the model.

The refined ANFIS model that emerged from this research is proving to be a useful tool for real-time monitoring and control of engine performance. Based on fuel consumption and engine speed as primary inputs, the model, with its competent predictions of engine torque and emission parameters, creates the basis for developing strategies to improve fuel efficiency, optimize torque, and reduce emissions.

Fundamentally, this research contributes to the existing body of knowledge on engine performance optimization and emissions control and it opens up potential pathways for achieving economic and environmental sustainability in sectors such as transportation and power generation.

Future research efforts could explore the application of this methodological approach to different engine models and the inclusion of additional performance metrics. In addition, improving the efficiency of the Firefly algorithm and exploring other optimization techniques could help improve the performance of ANFIS models. Continued research and innovation in this area are critical to meeting the growing demand for efficient, environmentally friendly engine technologies.

**Conflict of interest:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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