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Bi-Level Portfolio Optimization Considering Fundamental Analysis in Fuzzy Uncertainty Environments

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A B S T R A C T

Portfolio construction and achieving the set of securities with the most desirability is one of the most critical problems in financial markets. Generally, there are two types of financial problems in literature, choosing the right stock portfolio from a set of possible options which is called portfolio selection, and calculating the portion of the purchase for each option which is called Portfolio optimization. In this paper, a new two-phase robust portfolio selection and optimization approach is proposed to deal with the uncertainty of the data. In the first phase of this approach, all candidate stocks' efficiency is measured using a data envelopment analysis (DEA) method. Financial criteria for evaluation chosen from fundamental perspectives. Then in the second phase, by applying the Fuzzy Weighted Goal programming (FWGP) model with criteria related to modern portfolio theory (return and risk) as well as the mentioned criteria, the portion of investment in each qualified stock is determined and the optimal investment portfolio is formed. Finally, the proposed approach is implemented in a real case study of the Dow Jones Industrial Market (DJIA). The resulting portfolios for the proposed models are compared against each other as well as against the Dow Jones Industrial Average index. The results show that for the data used and factors investigated some of the constructed portfolios, with a much smaller number of constituents, could potentially outperform DJIA in terms of their performance.

1. Introduction

The first concern of managers in any investment complex is the proper and optimal conversion of existing capital resources to the maximum possible income through investment at the right time. Therefore, concerning the economic situation, how and where to invest is a complex and risky job. In other words, all investors desire to reach the best possible choices by regarding the effective criteria in investment strategies in addition to their personal preferences, while minimizing the risk for a specific return [10, 11, 24, 25]. Financial issues, such as portfolio selection and optimization, are among the intriguing subjects of uncertainty planning. In recent years,

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we have witnessed an increase in the presentation of extensive research and various methods in the finance field, which include portfolio selection and portfolio management. The high importance of Profitability is the reason for studying portfolio topics and presenting a better model can lead to higher profits. Markowitz proposed the first model for the portfolio problems. Modern portfolio analysis received much attention from both researchers and investors after the publication of the seminal work of Markowitz, wherein he proposed the mean-variance portfolio selection model, which simultaneously maximizes portfolio return whilst minimizing portfolio risk [20].

The thing that increases the difficulty in solving portfolio models is the uncertainty in the problems. Uncertainty can be described as a complete lack of information about future events that can be reduced by gathering information, but it cannot be eliminated. The occurrence of economic crises in recent years has caused a special place for uncertainty in financial issues; also, many studies have been conducted to deal with uncertainty in these issues.

Financial issues, such as portfolio are also one of the cases that are examined in uncertain conditions because the investor cannot predict the exact portion of the portfolio return. The existence of uncertainty in the financial markets modeling would cause risk in the decision-making process. The approaches to dealing with uncertainty in the literature are limited and fuzzy, robust, and stochastic approaches can be named as the main approaches in this topic. The variety and complexity of investment methods have increased significantly in recent decades and this widespread growth created an extreme need for inclusive and integrated models.

The rest of the paper is organized as follows. In Section 2, a brief review of the initial and recent researches is presented. In Section 3 to 7, we present a brief Definition of modern portfolio theory (MPT), fundamental analysis, goal programming (GP), data envelopment analysis (DEA), and fuzzy approach. In Section 8, the proposed approach for the portfolio selection and the portfolio optimization is implemented for a real case study and finally, the conclusions of this study and some directions for future research are provided in Section 9.

2. Literature Review

The most significant research in this area has been by Markowitz [20] as modern portfolio theory. He presented the concept of variety in the portfolio selection problem. In the original Markowitz model, the portfolio selection problem is developed by risk, return, and balance between them. Markowitz model assumes that investors are interested only with returns attached to specific levels of risk when selecting their portfolios [24].

In further studies, based on the mean-variance model, portfolio allocation management was improved by adding several factors such as borrowing, loaning, short-term selling, transaction cost, to the original model [43, 46].

One of the techniques that use multi-objective models for optimization is the goal programming approach. Charnes [5] presented this method in 1955. For the first time, Lee and Lero [19] used it in financial problems. Subsequently, in several papers, this approach was used to select portfolios.

Ross and Roll [36] proposed the theory of arbitrage pricing or the multi-index model. Arbitrage pricing theory asserts that an asset's riskiness, hence its average long-term return, is directly related to its sensitivities to unanticipated changes in four economic variables inflation, industrial production, risk premiums, and the slope of the term structure of interest rates.

Konno & Yamazaki [17] proposed absolute deviation (AD) instead of variance as a risk measure for portfolio selection. The mean-absolute deviation model (MAD) is a linear programming (LP) model and reduces computational time .this risk measure quantifies the deviation from the expected return and by using the MAD model, it is not required to compute the covariance matrix.

Alexander and Baptista [2] relate value at risk (VaR) to mean-variance analysis and examine the financial implications of using a mean-VaR model for portfolio selection. When comparing two mean-variance efficient portfolios, the higher variance portfolio might have less VaR. Consequently, an efficient portfolio that globally minimizes VaR may not exist. finally, they show that it is plausible for certain risk-averse agents to end up selecting portfolios with larger standard deviations if they switch from using variance to VaR as a measure of risk.

Afshar Kazemi et al. [1] purpose a model to make an optimal portfolio by using data development analysis (DEA) and goal programming (GP) methods. Therefore, the data, which is related to 250 firms, have been collected from October 2009 to March 2010. The ratio efficiency of every industry's firm has been calculated by the DEA method and the most efficient ones selected, and 48 efficient firms have been determined. In the next phase, considering data related to the criteria, to determine goal number, linear programming has been used, and to ensure the attainment of the goals with the low priorities, the result has been brought to goal programming model after a little rebalancing. In the final phase, the investor has selected a portfolio using GP by considering priorities and his goals. The result shows the complete achievement of four goals and the incomplete achievement of two goals has 2.27 units positive derivation and it was made a various portfolio includes eight stocks among 250 stocks.

Khanjarpanah et al. [15] proposed a novel logical and useful model for portfolio optimization. This model has some constraints such as flexibility and solid bounds in stock's weight in portfolio optimization problem and cardinality constraint is applied for portfolio problem. Then fuzzy programming is applied to handle the uncertainty of stock returns and with flexible and possibilistic programming, that both of these methods are categories of fuzzy programming, the proposed model converts to a crisp one. The proposed model for evaluating, performance testing, and logicality approved, is applied to some monthly returns of companies' stock of Tehran Stock Exchange and the results showed that in lower values of confidence level in proposed portfolio problem, it's possible to obtain a higher profit with low risk.

Peykani et al. [31] purposed a study to adopt data envelopment analysis (DEA) to construct portfolios, and compare their return rates with the market index to examine whether DEA portfolios created superior returns. In addition, this study investigated whether using the "size effect" as a stock selection strategy is appropriate in Taiwan. This study applied two DEA models to evaluate the efficiency of the firms and construct portfolios by selecting stocks with high efficiency. Furthermore, the return rates of the portfolios constructed by small-size firms, DEA models, and market indices were compared via empirical data analysis. The results showed that the size effect seems inappropriate as a stock selection strategy in the Taiwan stock market. However, the portfolios constructed by DEA models achieved noticeable superior returns.

Peykani et al. [32] proposed a two-phase robust portfolio selection and optimization approach to deal with the uncertainty. In the first phase of this approach, all candidate stocks' efficiency is measured using a robust data envelopment analysis (RDEA) method. Then in the second phase, by applying robust mean-semi variance-liquidity (RMSVL) and robust mean-absolute deviation-liquidity (RMADL) models, the amount of investment in each qualified stock is determined. Finally, the proposed approach is implemented in a real case study of TSE and results show that the proposed approach is effective for portfolio selection and optimization in the presence of uncertain data.

Tamiz & Azmi [44] in "Goal programming with extended factors for portfolio selection" proposed the use of several stock-related factors, called extended factors, for portfolio selection. These factors, including the traditional factors of risk and return, are represented as objectives in weighted goal programming (WGP) models. Several WGP models with passive and active target values and various weights for their unwanted deviational variables in their achievement functions have been developed. The resulting portfolios for the proposed models are compared against each other as well as against the Dow Jones Industrial Average index and portfolios obtained from the well-established Markowitz [20] and Konno and Yamazaki's [17] models. The experimental results strongly support the use of extended factors for portfolio selection problems and the assumption of meeting decision makers' preferences and utilities better than the portfolios based entirely on risk and return.

A review of the literature review is provided as follows in which the modelling method and criteria used in each model are given and the characteristics of our work have also been presented in the last row of Table 1 (FA: fundamental analysis, TA: technical analysis, MPT: Modern Portfolio Theory, U.C.: uncertainty):

Table 1. A review of literature review.

Researcher	Year	problem	method	model criteria			
				U.C.	MPT	TA	FA
Fu et al.	1997	Portfolio Selection	Genetic algorithm		✓	✓	
Chavarnakul & Enke	2008	Stock trading	Neural networks	✓		✓	
Jana et al.	2009	Portfolio Selection	Possibility theory, Fuzzy	✓	✓		
Lee & Yu	2010	Stock evaluation and portfolio	Conceptual model		✓		
Dastkhan et al.	2011	Portfolio selection	Fuzzy programming	✓	✓		
Esfahanipour & Mousavi	2011	Stock trading	Genetic programming			✓	
Gorgulho et al.	2011	Portfolio management	Genetic algorithm			✓	
Jasemi et al.	2011	Portfolio management	Conceptual model		✓	✓	
Yu and Lee	2011	Portfolio rebalancing	Fuzzy programming	✓	✓		
Afshar Kazemi et al.	2012	Portfolio Optimization	Data envelopment analysis, goal programming		✓		
Escobar et al.	2013	Stock recommendation	Fuzzy logic	✓		✓	
Gradojevic & Genc	2013	Stock market timing	Fuzzy logic	✓		✓	
Khodamoradi et al.	2013	Portfolio Selection	goal programming		✓		✓
Yunusoglu & Selim	2013	Stock evaluation and portfolio	Fuzzy expert system	✓		✓	
Mohammadi et al.	2014	Portfolio Optimization	goal programming		✓		
Shams & Alavi	2015	Portfolio Selection	Linear programming		✓	✓	✓
Tharavanij et al.	2015	Stock evaluation and portfolio	grid-search			✓	
Khanjarpanah et al.	2017	Portfolio Optimization	Fuzzy programming	✓	✓		
Tamiz & Azmi	2017	Portfolio selection	weighted goal programming		✓		✓
Khayamim et al.	2018	Portfolio rebalancing	Fuzzy programming, Market psychology	✓	✓	✓	
Mangaraj et al.	2018	Portfolio Optimization	Fuzzy goal programming	✓	✓		
Mehlawat et al.	2018	Portfolio selection & Optimization	Data envelopment analysis, Fuzzy optimization	✓	✓		✓
Falah & Farokh	2019	Portfolio Selection	Multi-objective fuzzy possibility programming	✓	✓		
Peykani et al.	2020	Portfolio selection & Optimization	Robust Data envelopment analysis	✓	✓		✓
This research	2022	Portfolio Selection, Portfolio Optimization	Data envelopment analysis, Fuzzy weighted goal programming	✓	✓		✓

3. Modern Portfolio Theory (MPT)

Markowitz's Modern portfolio theory (MPT) or mean-variance analysis [20] formed the first mathematical model for portfolio selection. In other words, it is a mathematical framework for assembling a portfolio of assets and formalization and extension of diversification in investing, the idea that owning different kinds of financial assets is less risky than owning only one type. Its key insight is that an asset's risk and return should not be assessed by itself, but by how it contributes to a portfolio's overall risk and return. It uses the variance of asset prices as a proxy for risk. Markowitz model assumes that investors are interested only with returns attached to specific levels of risk when selecting their portfolios.

Markowitz also presents the efficient frontier, which is the set of optimal portfolios that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. The efficient frontier graphically represents portfolios that maximize returns for the risk assumed. Returns are dependent on the investment combinations that make up the portfolio. The Markowitz model, as follows:

$$
\begin{aligned}
&\min \delta_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij}\, x_i \\
&\max R_p = \sum_{i=1}^{n} r_i\, x_i \\
&\quad s.t.: \\
&\quad \sum_{i=1}^{n} x_i = 1 \\
&\quad x_i \geq 0 \qquad i = 1, \dots, n
\end{aligned}
\tag{1}
$$

where σ_{ij} is the covariance (Risk) between stocks i and j, x_i is the proportion invested in stock i, r_i the return of the ith stock, δ_p^2 portfolio return variance and R_p is portfolio expected return.

4. Fundamental Analysis

Fundamental analysis (FA) is the process of evaluating a public firm for its investment worthiness by looking at its business at the basic or fundamental financial level. It involves examining a firm's financials and operations, especially sales, earnings, growth potential, assets, debt, management, products, and competition. FA may also include analysing market behaviour that stresses the study of underlying factors of supply and demand [24].

In other words, FA is a method of measuring a security's intrinsic value by examining related economic and financial factors. in this field, we study anything that can affect the security's value, from macroeconomic factors such as the state of the economy and industry conditions to microeconomic factors and the main purpose is to arrive at a measurement criterion that an investor can compare with a security's current price to see whether the security is undervalued or overvalued.

The various fundamental factors can be grouped into two categories: quantitative and qualitative. The financial meaning of these terms is not much different from their standard definitions. Quantitative group related to information that can be shown in numbers and amounts. Revenue, profit, assets, and more can be measured with great precision. Qualitative group relating to the nature or standard of something, rather than to its quantity, this group is less tangible. They might include the quality of a company's key executives, its brand-name recognition, patents, and proprietary technology [12].

The most important element in the Quantitative group considers financial ratios which can be grouped into five categories:

- Liquidity Measurement Ratios: Liquidity ratios measure a company's ability to pay debt obligations and its margin of safety through the calculation of metrics including the current ratio, quick ratio, and operating cash flow ratio.
- Leverage ratios: A leverage ratio is any one of several financial measurements that assesses the ability of a company to meet its financial obligations. The leverage ratio aims to act as a compliment and a backstop to risk-based capital requirements. It should counterbalance the build-up of systemic risk by limiting the effects of risk weight compression during booms [51].
- Activity Ratios: activity ratios, also known as Efficiency ratios, are used by analysts to measure the performance of a company's short-term or current performance. In other words, an activity ratio measures a company's ability to use its assets to generate income.
- Profitability Indicator Ratios: Profitability ratios assess a company's ability to earn profits from its sales or operations, balance sheet assets, or shareholders' equity. In other words, Profitability ratios measure the company's use of its assets and control of its expenses to generate an acceptable rate of

return. We can name return on assets (ROA), return on equity (ROE) as the most important activity ratios.

- Investment Valuation Ratios: Valuation ratios are used for analyzing the attractiveness of an investment in a company. These measures primarily integrate a company's publicly traded stock price to give investors an understanding of how inexpensive or expensive the company is in the market. In general, the lower the ratio level, the more attractive investment in a company become. Popular valuation multiples include Price to earnings (P/E), Price to book (P/B), Price to sales (P/S), and Price to cash flow (P/CF) [25].

5. Fuzzy Approach

Since in many industrial and managerial problems, the decision-maker cannot accurately determine the values of the problem variable and this ambiguity can cause improper results, the fuzzy sets theory introduced. In fuzzy mathematical programming, the model parameters, constraint structure, and objective function are formulated with uncertainty. Then, using a defuzzification approach that can produce specific optimal solutions, the fuzzy model becomes a definite mathematical programming model [15].

A standard way to rank fuzzy numbers is to define a ranking function from a fuzzy set to a real set of numbers with an order. In other words, fuzzy logic systems produce acceptable but definite output in response to incomplete, ambiguous, distorted, or inaccurate input. Generally, in such methods, the fuzzy linear programming model becomes a classical linear programming model, and by solving this model, the answer to the main problem is determined.

We say that $\tilde{c}$, as a subset of $F(R)$, is a fuzzy number if it is normal, and a convex fuzzy set of R. These numbers depend on the possible values and the range of these values is [0, 1]. This weight is called the membership function in fuzzy logic and it demonstrates with $\mu_{\tilde{c}}(x)$. In addition, $\tilde{c}$ is a triangular fuzzy number when it is expressed as $\tilde{c} = (c^0, c^m, c^p)$ and its membership function is as follows [49] (see Figure 1):

$$\mu_{\tilde{c}}(x) = \begin{cases} f_c(x) = \frac{x-c^0}{c^m-c^0} & if\ c^0 < x < c^m \\ 1 & if\ x = c^m \\ g_c(x) = \frac{c^p-x}{c^p-c^m} & if\ c^m < x < c^p \\ 0 & if\ x \leq c^0\ or\ x \geq c^p \end{cases} \tag{2}$$

The solution method presented in this research for converting the proposed mixed linear fuzzy model to its equivalent definite model is the proposed model by Jimenez et al [14] who use a fuzzy ranking method to rank the fuzzy objective values and to deal with the inequality relation on constraints. It allows us to work with the concept of feasibility degree. The bigger the feasibility degree is the worst the objective value will be. This method offers the decision-maker the optimal solution for several different degrees of feasibility. This method builds a fuzzy subset in the decision space whose membership function represents the balance between the feasibility degree of constraints and the satisfaction degree of the goal. A reasonable solution is the one that has the biggest membership degree to this fuzzy subset [50].

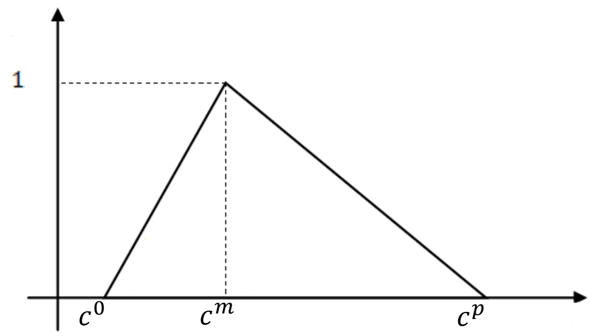

Figure 1. Triangular fuzzy number

According to Jimenez, the expected interval (EI) and expected value (EV) of triangular fuzzy number $\tilde{c}$ can be defined as follow:

$$EI(\tilde{c}) = [E_1^c, E_2^c] = \left[\int_0^1 f_c^{-1}(x)dx, \ \int_0^1 g_c^{-1}(x)dx\right] = \left[\frac{1}{2}(c^p + c^m), \frac{1}{2}(c^m + c^0)\right]$$

$$EV(\tilde{c}) = \frac{E_1^c + E_2^c}{2} = \frac{c^p + 2c^m + c^0}{4} \tag{3}$$

It is noted that the same equations can be used for a trapezoidal fuzzy number. Moreover, according to the ranking method of Jimenez for any pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree in which $\tilde{a}$ is bigger than $\tilde{b}$ is defined as follows:

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & if\ E_2^a + E_1^b < 0 \\ g_c(x) = \frac{c^0 - x}{c^0 - c^m} & if\ 0 \in \left[E_1^a - E_2^b, E_2^a - E_1^b\right] \\ 1 & if\ E_1^a - E_2^b > 0 \end{cases} \tag{4}$$

Consequently, using the definition of expected interval and expected value of a fuzzy number, the equivalent crisp α-parametric model can be written as follows:

$$\begin{aligned} &\min \quad EV(\tilde{c})x \\ &\text{s.t.} \\ &[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}]x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i}, \quad i = 1, \dots, l \\ &\left[\left(1 - \frac{\alpha}{2}\right)E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}\right]x \geq \frac{\alpha}{2}E_2^{b_i} + \left(1 - \frac{\alpha}{2}\right)E_1^{b_i}, \quad i = l+1, \dots, m \\ &\left[\frac{\alpha}{2}E_2^{a_i} + \left(1 - \frac{\alpha}{2}\right)E_1^{a_i}\right]x \leq \left(1 - \frac{\alpha}{2}\right)E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i}, \quad i = l+1, \dots, m \\ &x \geq 0 \end{aligned} \tag{5}$$

6. Goal Programming

In formulating and solving linear planning problems, the modelling process focuses on goals such as profit maximization or cost minimization, but many real-world decision-making situations, limiting the organization's goals to one, are not practical and desirable. Most organizations, in addition to the mentioned objective, have several other objectives in mind which cannot be formulated in simple leaner programming. To formulate and solve mentioned problems a valid method has been developed to complete the linear programming technique called goal programming [47].

The distinguishing feature of this method is in the goals that are prioritized from the point of view of decision-makers. It is not permanently possible to achieve any of the set goals. Therefore, the decision-maker is required to focus his efforts on the level of satisfaction of several goals. The general weighted goal programming (WGP) model is as follows:

$$min \sum_{i=1}^{m} \left(\frac{\alpha_i n_i}{k_i} + \frac{\beta_i p_i}{k_i}\right) \tag{6}$$

$$s.t:$$

$$f_i(x) + n_i - p_i = b_i \quad i = 1, \dots, m$$

$$x \in C_s$$

$$x, n_i, p_i \geq 0 \quad i = 1, \dots, m$$

where n_i is the $_i$th negative deviational variable, α_i the weighting factor for negative deviation variable i, p_i the ith positive deviational variable, β_i the weighting factor for positive deviational variable i, ki the normalization constant for deviational variable i, x the vector of the decision variables, $f_i(x)$ the ith objective function, bi is the ith target value, and C_s is an optional set of hard constraints.

The WGP allows for direct trade-offs between all unwanted deviational variables by placing them in a weighted, normalized single achievement function that is minimized. Each objective, $f_i(x)$, is given a target value, bi, that needs to be achieved. The negative deviational variable, n_i, measures the amount of underachievement of the target and the overachievement is measured by the positive deviational variable, pi. The unwanted deviation from the target value (i.e., negative deviation from return target or positive deviation from risk target) is then placed in the achievement function to be minimized. The values of α and β are set to reflect the decision maker's preferences [29].

For a reasonable investment decision where the targets assigned to criteria are imprecise due to fuzziness, the fuzzy multi-criteria decision-making (MCDM) approach is more meaningful for handling such problems.

Apart from a fuzzy multi-objective decision-making approach, the literature also shows several applications of fuzzy goal programming (FGP) in portfolio selection in different contexts. The central idea behind the fuzzy programming technique is that ill-defined problems are first formulated as fuzzy models. Since no solution procedure is available for fuzzy models, equivalent crisp models of the problems are to be designed using fuzzy set theory to facilitate their solutions by existing algorithms and solution codes.

7. Data Envelopment Analysis

Data envelopment analysis (DEA) is a nonparametric approach for calculating the relative efficiencies of a set of similar decision-making units (DMUs) by relating their outputs to their inputs and classifying the DMUs into managerially efficient and managerially inefficient [26-28, 35, 37, 38, 41, 42, 43]. It originated from Farrell's (1957) work, which was later popularized by Charnes et al. [5]. The CCR ratio model seeks to optimize the ratio of a linear combination of outputs to a linear combination of inputs.

To explain the fundamental premise of a DEA model, let there be j independent DMUs whose performance must be evaluated relative to each other. One begins with a given set of inputs parameters (M) and a given set of output parameters (N) which are common to all J firms. The relative efficiency then measures how well a given firm (in the group of J firms) converts its M inputs to the N outputs, which is calculated as the ratio of a specific aggregated output measure to a specific aggregated input measure. Such aggregated input (and output) measures are computed by taking a non-negative linear combination of the M inputs (and N outputs). Following this idea, the input-oriented relative performance (strength or efficiency) f_k of some firm k, $k = 1, \dots, J$, is then defined as the maximized value of the latter ratio, determined over all possible aggregating multipliers such that no firm in the group will attain a relative performance measure greater than unity. The CCR model is formulated as follows:

$$\begin{aligned} f_k := \max_{u,v} \quad & \frac{\sum_{n=1}^{N} (xo)_{nk} v_{nk}}{\sum_{m=1}^{M} (xi)_{mk} u_{mk}} \\ \text{s.t.} \quad & \frac{\sum_{n=1}^{N} (xo)_{nj} v_{nk}}{\sum_{m=1}^{M} (xi)_{mj} u_{mk}} \leqslant 1, \quad j = 1, \ldots, J \\ & u_{mk}, v_{nk} \geqslant 0, \quad m = 1, \ldots, M, \ \ n = 1, \ldots, N. \end{aligned} \tag{7}$$

For firm j, the level of input parameter m is $(xi)_{mj}$, $m = 1, \ldots, M$, while that of output parameter n is $(xo)n_j$, $n = 1, \ldots, N$. The input and output non-negative multipliers for firm k are denoted by the variables u_{mk} and v_{nk}, respectively. The following model yields the maximum achievable efficiency for firm k, denoted f_k, provided every other firm is also applying the same aggregating non-negative multipliers in computing their input to output conversion ratios. f_k is termed the DEA efficiency score of firm k. An efficiency score of less than one is indicative that it may be possible to decrease the level of input for the same level of output, while a score of 1 indicates the firm is DEA-efficient. By applying the following model to each firm independently, the respective (maximum) relative efficiency score for each firm is computed. The equivalent linear programming formulation of the following model is [21]:

$$\begin{aligned} & \hat{f}_k \coloneqq \max \sum_{n=1}^{N} (xo)_{nk} v_{nk} \\ & s.t: \\ & \quad \sum_{m=1}^{M} (xi)_{mk} u_{mk} = 1 \\ & -\sum_{m=1}^{M} (xi)_{mj} u_{mk} + \sum_{m=1}^{M} (xo)_{nj} v_{nk} \leq 0, \quad j = 1, \ldots, J \\ & \quad u_{mk}, v_{nk} \geq 0, \quad m = 1, \ldots, M, \quad n = 1, \ldots, N. \end{aligned} \tag{8}$$

It is easy to show that $\hat{f}_k = f_k$ holds under the non-negativity of the observed data. More specifically, if $(xi)_{mk} > 0$ for some $m = 1, \ldots, M$, then, $\hat{f}_k = f_k$ holds. Conversely, suppose $(xi)_{mk} \leq 0$ for all $m = 1, \ldots, M$. Then, the maximization in (7) is not well defined, and (8) is infeasible, in which case, we assign a performance strength of $\hat{f}_k = 0$.

The issue of negative data stems from the fact that in the application in this paper, the input and output data come from financial statements. That is possible for a given firm all input parameters have non-positive values, depending on how the input parameters are chosen from financial statements.

8. Problem Definition

In this research, according to the main objectives of this study, including increasing the efficiency of the model, comprehensive assessments of stocks from different financial aspects and criteria and increasing the ability to deal with the uncertainty of the data, using a two-level model, conclude portfolio selection and portfolio optimization in the form of two phases.

In the first phase, entitled portfolio selection, in the first step, using several critical factors in the capital market, the priority of investment in each industry and company, based on the expert's opinion, is extracted. In the second step of phase 1, financial criteria for evaluation of stocks are chosen from fundamental perspectives that contain Debt/asset (DA), Earnings per share (ES), Price/book value (PB), Operating cash flow ratio (OC) and Dividend yield (DY). These parameters examine a firm's fundamental performance through a range of performance perspectives. In the third step of phase 1, the data envelopment analysis (DEA) models are chosen to evaluate the stocks. In this paper, CCR-IO is selected. In the fourth step of phase 1, the DEA model for all the stocks will be run. In addition, by applying the DEA Model, all stocks will be ranked.

In the second phase, in the first step, Risk and return as uncertainty criteria in addition to the mentioned criteria are selected and weighted. In the second step of phase 2, Fuzzy Weighted Goal programming (FWGP) will be proposed as a portfolio optimization model. This step is the most important in the second phase. Finally, in the third step of phase 2, the FWGP model for all the top-ranked stocks in phase 1 will be run. A testing period of 15 days, in October 2010 (450 observations), is used for testing the resultant portfolios. A concept image of the proposed model is presented as Figure 2.

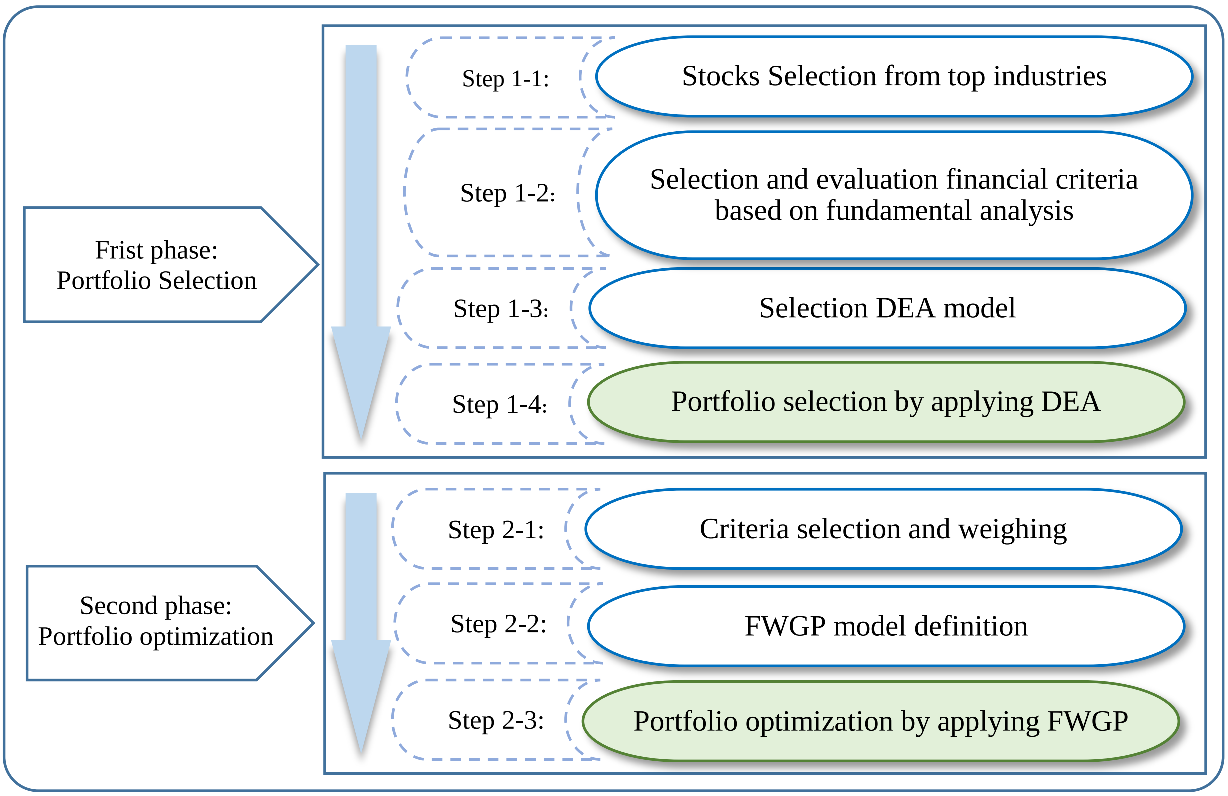

Figure 2. Concept image of the proposed model

➢ First Phase: Portfolio Selection

In this phase the performance of all stocks that investors can invest in them, are evaluated and measured. At the end of this phase, only the stocks that pass the filter of the investor are qualified to be a candidate that can be invested in the second phase. Financial criteria for evaluation of stocks are chosen from fundamental perspectives.

Various experiments carried out for the model and their results are provided. The data are from the 30 stocks of the DJIA index (Table 2). The experiments use a constructing period of 102 days, from September 2010 to December 2010 (3060 observations).

Table 2. Financial criteria' data

DEA	Input (1)	Input (2)	output (1)	output (2)	output (3)
Stocks \ Criteria	Debt/asset	Price / book value	Earnings per share	Operating cash flow ratio	Dividend yield
Alcoa	0.5634	1.27	-1.06	0.337	0.91
American Express Co.	0.8811	3.35	1.54	0.485	1.70
Boeing	0.9627	19.23	1.87	0.200	2.42
Bank of America	0.9091	0.67	-0.29	0.306	0.35
Caterpillar	0.8377	4.07	1.43	0.292	2.21
Cisco Systems	0.4399	2.95	1.33	0.638	0.00
Chevron	0.4333	1.68	5.24	0.647	3.51
El DuPont de Nemours	0.7822	4.36	1.92	0.490	3.46
Walt Disney	0.4390	1.47	1.76	2.125	0.97
General Electric	0.8400	1.38	1.03	0.138	3.01
The Home Depot	0.5257	2.45	1.57	0.481	3.00
Hewlett-Packard	0.6416	2.83	3.14	0.342	0.75
IBM	0.7834	7.55	10.10	0.632	1.81
Intel Corporation	0.2102	2.69	0.77	1.630	3.12
Johnson & Johnson	0.4327	3.51	4.40	0.606	3.38
JP Morgan Chase	0.9500	1.04	2.24	0.358	0.54
Kraft Foods	0.6102	1.55	2.03	0.406	3.64
The Coca-Cola Co.	0.4774	5.29	2.93	0.497	2.85
McDonald's Corp.	0.5253	4.79	4.11	4.462	3.11
3M Co.	0.5000	4.60	4.52	1.293	2.48
Merck & Co.	0.4489	1.92	5.65	0.149	4.15
Microsoft Corp.	0.4634	4.32	2.10	1.116	2.34
Pfizer Inc.	0.5728	1.63	1.23	0.643	4.13
Procter & Gamble	0.5203	2.85	4.11	0.649	3.01
AT&T	0.6190	1.62	2.12	1.015	5.81
The Travelers C.	0.7649	0.02	6.33	11.260	0.59
United Technologies	0.6015	3.24	4.12	0.221	2.26
Verizon Communications	0.6287	2.26	1.29	1.279	5.95
Wal-Mart Stores	0.5698	2.86	3.70	0.430	2.21
Exxon Mobil Corp.	0.5006	2.92	3.98	0.573	2.59

Now, according to the existing models in the field of data envelopment analysis, the CCR input-oriented (CCR-OO) model was selected for this research. According to the studies, debt-to-asset ratio and price-to-book value ratio are selected as model inputs and earnings per share criteria, operating cash flow ratio, and dividend return as the output of the DEA model.

Now, before applying the DEA model, we evaluate the number of selected stocks for the model based on the following equation. According to this equation, for selecting the number of DMUs in the model, the number of inputs and outputs should be considered:

$$\text{DMUs} \geq 3 \times (\text{N.O. inputs} + \text{N.O. outputs})$$

Ignoring the following equation may repeal the validation of the model and the ability to identify inefficient units. The reason for the mentioned problem is a large number of units with efficiency equal to one.

Considering that the research model has 2 input and 3 output variables, the minimum number of stocks required is 15 shares to satisfy the following equation and, in this research, 30 stocks are used to describe the model.

Finally, according to the previous information, the DEA model for the mentioned stocks is in Lingo.18.0 software designed and applied in a system with 8GB installed RAM and Intel core i5-8100 CPU Processor. The efficiency ratio of each stock was obtained as described in Table 3.

According to Table 3, 8 stocks used in the DEA model (green cases) have a performance of more than 0.8, which is considered as the selected stock portfolio in the first phase of the research (Figure 3).

Table 3. DEA efficiency Result

#	Stocks	efficiency	#	Stocks	efficiency	#	Stocks	efficiency
1	Alcoa	100%	11	The Home Depot	55%	21	Merck & Co.	90%
2	American Express	23%	12	Hewlett-Packard	12%	22	Microsoft Corp.	40%
3	Boeing	23%	13	IBM	18%	23	Pfizer Inc.	88%
4	Bank of America	58%	14	Intel Corporation	100%	24	Procter & Gamble	53%
5	Caterpillar	29%	15	Johnson & Johnson	63%	25	AT&T	100%
6	Cisco Systems	20%	16	JP Morgan Chase	15%	26	The Travelers Co.	100%
7	Chevron	81%	17	Kraft Foods	65%	27	United Technologies	35%
8	El DuPont de Nemours	42%	18	The Coca-Cola Co.	43%	28	Verizon Comm.	100%
9	Walt Disney	56%	19	McDonald’s Corp.	76%	29	Wal-Mart Stores	36%
10	General Electric	76%	20	3M Co.	40%	30	Exxon Mobil Corp.	47%

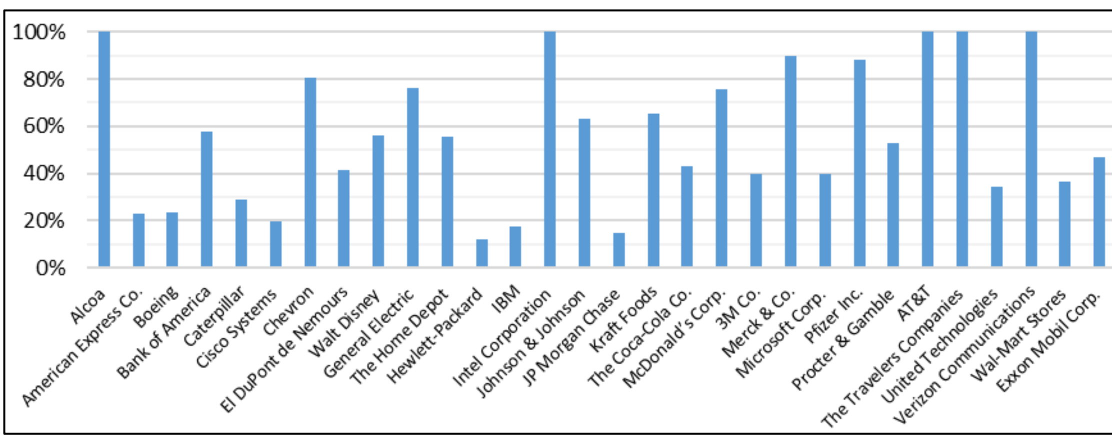

Figure 3. DEA efficiency chart

➢ Second Phase: Portfolio optimization

In this phase, we run a portfolio optimization with an FWGP approach with the above-mentioned criteria in addition to risk and return. According to mentioned equation, the model used for this research is the weighted ideal planning (WGP) model.

$$\min \frac{\alpha_{RE}}{k_{RE}} n_{RE} + \frac{\alpha_{DA}}{k_{DA}} n_{DA} + \frac{\alpha_{ES}}{k_{ES}} n_{ES} + \frac{\alpha_{PB}}{k_{PB}} n_{PB} + \frac{\alpha_{OC}}{k_{OC}} n_{OC} + \frac{\alpha_{DY}}{k_{DY}} n_{DY} + \frac{\alpha_{RI}}{k_{RI}} p_{RI} + \frac{\alpha_{DA}}{k_{DA}} p_{DA}$$

$$s.t. \tag{9}$$

$$\sum_{j=1}^{30} \widetilde{RI}_j x_j + n_{RI} - p_{RI} = b_{RI}$$

$$\sum_{j=1}^{30} \widetilde{RE}_j x_j + n_{RE} - p_{RE} = b_{RE}$$

$$\sum_{j=1}^{30} DA_j x_j + n_{DA} - p_{DA} = b_{DA}$$

$$\sum_{j=1}^{30} ES_j x_j + n_{ES} - p_{ES} = b_{ES}$$

$$\sum_{j=1}^{30} PB_j x_j + n_{PB} - p_{PB} = b_{PB}$$

$$\sum_{j=1}^{30} OC_j x_j + n_{OC} - p_{OC} = b_{OC}$$

$$\sum_{j=1}^{30} DY_j x_j + n_{DY} - p_{DY} = b_{DY}$$

$$\sum_{j=1}^{30} x_j = 1, \quad x_j \geq 0 \quad j = 1, \dots, 30$$

The final outputs of the stock selection model, which are 8 stocks, are considered as the inputs of the FWGP model. The significant point in this phase is to consider the variable Risk and Return as a variable with fuzzy uncertainty in the GP model. The reason that we use triangular fuzzy numbers to demonstrate uncertainty in the model is the period used for extracting uncertainty data from the target market, which is 3 terms. The values of the mentioned criteria, as well as the weighted used in the model, are given In Table 4:

Table 4. FWGP Input data

Criteria	$\widetilde{RI}$			$\widetilde{RE}$			DA	PB	ES	OC	DY	Eff
Penalty factor	0			6			1	3	5	4	2	-
incentive factor	7			0			1	0	0	0	0	-
Goal	0.000			0.300			0.615	3.379	2.840	1.123	2.609	100%
Alcoa	0.013	0.025	0.037	0.002	0.003	0.005	0.563	1.270	-1.060	0.337	0.910	100%
Chevron	0.010	0.015	0.020	0.038	0.043	0.049	0.433	1.680	5.240	0.647	3.510	81%
Intel Co.	0.018	0.019	0.020	-0.14	-0.113	-0.08	0.210	2.690	0.770	1.630	3.120	100%
Merck & Co.	0.008	0.013	0.018	0.055	0.071	0.093	0.449	1.920	5.650	0.149	4.150	90%
Pfizer Inc.	0.008	0.015	0.022	0.024	0.036	0.051	0.573	1.630	1.230	0.643	4.130	88%
AT&T	0.009	0.010	0.011	0.116	0.129	0.14	0.619	1.620	2.120	1.015	5.810	100%
Travelers Co.	0.004	0.013	0.030	0.037	0.049	0.057	0.765	0.020	6.330	11.26	0.590	100%
Verizon Com.	0.003	0.012	0.200	0.147	0.152	0.155	0.629	2.260	1.290	1.279	5.950	100%

- The risk variable is applied as a triangular fuzzy number with the Jimmens defuzzification approach in the model.
- Considering that the desired values related to risk and return criteria are at the lowest and highest possible, the incentive factor for the risk criterion and the penalty factor for the return criterion are set to zero.
- The only debt-to-assets ratio (DA) variable has both penalty and incentive factors, which means that any amount more or less than the goal number causes disadvantage of the desired stock score.

To solve the mentioned FWGP model the Jimenez approach and Lingo.18.0 software have been used and the results of solving the model under various α are as given in Table 5 and Figure 5.

Table 5. FWGP Result

#	Stocks	α = 0.2	α = 0.4	α = 0.6	α = 0.8
1	Alcoa	0%	0%	0%	0%
2	Chevron	0%	0%	0%	0%
3	Intel Corporation	0%	33%	58%	61%
4	Merck & Co.	33%	39%	41%	34%
5	Pfizer Inc.	0%	0%	0%	0%
6	AT&T	0%	0%	0%	0%
7	The Travelers Companies	2%	0%	1%	5%
8	Verizon Communications	65%	28%	0%	0%
-	Z	0.442	0.452	0.466	0.489

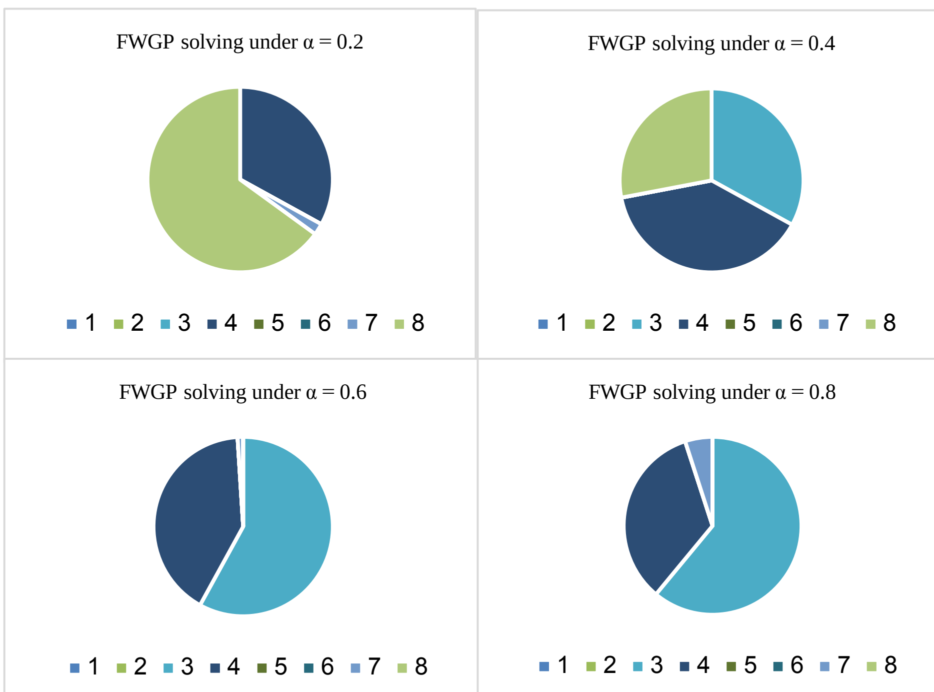

Figure 5. FWGP Result pie chart

In Table 5, the weight value of each stock is obtained in two different values of the confidence level. The value of the objective function, which is a combination of return and risk and fundamental criteria, is also shown in this table. It should be noted that the cardinality constraint as well as the maximum number of selected stocks (3 in this case) has been applied to the issue.

In the following, various experiments carried out for the models stated above and their results are provided. The data, as mentioned, are from the 30 stocks of DJIA index. The experiments use a constructing period of 102 days, from September 2010 to December 2010 (3060 observations). To assess the efficiency of the research model outcomes, Markowitz, Konno and Yamazaki, GP, DEA and DJIA profitability methods were used as given in Table 6.

Table 6. The overall results for experiments

method	No. of stocks selected	Overall risk	Overall return (%)
DJIA	30	0.0061	0.121
Markowitz	6	0.0043	0.063
Konno and Yamazaki	24	0.0061	0.104
GP	3	0.0079	0.077
DEA	8	0.0057	0.041
FWGP ($\alpha = 0.2$)	3	-	0.1569
FWGP ($\alpha = 0.4$)	3	-	0.1225
FWGP ($\alpha = 0.6$)	3	-	0.0966
FWGP ($\alpha = 0.8$)	3	-	0.1010

9. Conclusion and Future Research Directions

In this study, a novel approach for the portfolio construction problem is proposed to deal with data uncertainty, increasing conservatism levels of the investment process, and assessing the comprehensive of stocks. Accordingly, this study presents using a two-level model, concluding portfolio selection and portfolio optimization in the form of two phases. It is worth mentioning, the past information is the basic data entered in this research, similar to other research, and the environmental changes and fluctuations of stocks price can be strong in a different timeline, so to encounter to mentioned problems, the issue is analysed in an uncertain space. to be specific, the amount of risk and return criteria is considered as a variable with fuzzy uncertainty. To deal with this uncertainty, we used the Jimmens's defuzzification approach in the model.

In portfolio selection, using financial criteria which are chosen from fundamental perspectives and contain Debt/asset (DA), Earnings per share (ES), Price/book value (PB), Operating cash flow ratio (OC) and Dividend yield (DY), the DEA CCR-IO model for all the stocks will be run and by applying the DEA Model, all stocks will be ranked.

In Portfolio optimization, Risk and return as uncertainty criteria in addition to the mentioned criteria are selected and weighted. Then, the proposed FWGP model is applied for all the top-ranked stocks in portfolio selection. Finally, a testing period of 15 days, in October 2010 (450 observations), was used for testing the resultant portfolios.

The results of the experimentations in this paper show promising signs for exploiting portfolio selection and optimization problems. This paper shows that for the data used and factors investigated some of the constructed portfolios, with a much smaller number of constituents, could potentially outperform DJIA in terms of their performance measured by risk and return.

For future studies, uncertainty programming approaches such as robust mathematical programming and chance-constrained programming can be applied to deal with another type of data uncertainty. Moreover, an extended DEA approach for dealing with data uncertainty can be employed for proposing uncertain portfolio selection models [24-34].

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