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A New Approach for Solving Fuzzy Single Facility Location Problem under L_1 Norm

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ABSTRACT

The location allocation problem is one of the most attractive optimization problems and is widely used in the real world. Therefore, any attempt to bring this problem closer to real-world conditions would be significant and useful. In this paper, we utilize fuzzy logic due to the uncertainty of parameters in the real world. That is the weights (the amounts of demands of customers) and variables (the coordinates of the optimal place) are both considered fuzzy numbers. If these variables are considered definitively, due to various conditions and reasons, it may not be possible to acquire land or build a facility center in it, so we also considered this variable in a fuzzy way and a facility center area was obtained, that certainly, the decision maker can find the right place more easily. To solve the fuzzy problem a new approach based on presenting the problem in the form of equivalent expressions is proposed. This equivalent problem is solved using fuzzy arithmetic.

1. Introduction

The Single facility location problem studied by Fermat for the first time, is one of the branches of location problems [12]. This branch of the location problem has attracted the interest of many researchers. For example, the following studies can be mentioned in this field. In 2022, Omidi and Fathali [13] studied the inverse single facility location problem on a tree by balancing the distance of the server to clients. A branch-and-price algorithm for the robust single-source capacitated facility location problem under demand uncertainty was proposed by Jaehyeon and Park [16]. Srushti and Chow [15] deal to Air taxi skyport location problem with single-allocation choice-constrained elastic demand for airport access. Gökhan et al. [2] deal to the single facility location problem in multiple regions with different norms and then they devised a specially tailored branch-and-bound algorithm to

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solve the problem.

The location problem is one of the most important optimization and management science problems where is used for modeling real-world problems such as determining the location of administrative, military, educational, fire stations, bank branches, hospitals, and so on. This problem has attracted the attention of many researchers, and valuable results have been presented in this area [6, 25]. However, the existence of ambiguity and uncertainty in the real world are an undeniable fact. Therefore, in many cases, it is is impossible to determine the exact parameter values. In such cases, we use fuzzy theory, which is a suitable tool for expressing and describing uncertainty and accuracy in events [19, 20]. The fuzzy location problem has also received significant attention from many researchers, and numerous studies have been conducted in this area. In 2013 Kulkarni et al. attempted to present a state-of the- art review of papers on the simulation methodology for facility layout problems [10]. Jiuping and Song [26] studied the multi-objective dynamic layout problem for temporary construction facilities with unequal-area departments in a fuzzy environment. Taleshian and Fathali [22] studied the fuzzy-median problem with fuzzy weights and variables and presented a new algorithm for obtaining a fuzzy solution.

Taleshian et al. [24] showed that the majority property holds for the fuzzy 1-median problem on a tree. Then using a ranking function and the majority property, they presented a fuzzy algorithm to determine the median of a fuzzy tree. Taghi-Nezhad [18] considered the p-median problem in a fuzzy environment and proved the fuzzy vertex optimality theorem and its application. Jairo et al. [14] proposed an integrated approach of analytic hierarchy process and triangular fuzzy sets for analyzing the park-and-ride facility location problem. In addition, Shanshan et al. [27] presented a hybrid optimization approach for unequal-sized dynamic facility layout problems under fuzzy random demands. Selman et al. [9] introduced a novel approach to support the facility location process. Also, they presented an extension of the Additive ratio assessment (ARAS) method under an interval type-2 fuzzy environment for the first time. Moreover Taleshian et al. [23] proposed fuzzy algorithms for finding the absolute center and vertex center of a fuzzy tree. Also, Taghi-Nezhad et al. [21] studied the fuzzy facility location problem with point and rectangular destinations and presented two fuzzy algorithms named fuzzy critical point algorithm and fuzzy weighting average algorithm to obtain the best place for locating the facility center.

Atta et al. [3] deal to the capacitated maximal covering location problem with fuzzy coverage area and presented a new approach for solving this problem using metaheuristic approaches. Esmikhani et al. [7] examined the fuzzy robust facility layout problem equipped with cranes. Eydi and Shirinbayan [8] studied multi-modal and multi-product hierarchical hub location problem with fuzzy demands. Also, Aider et al. [1] presented a hybrid population-based algorithm for solving the fuzzy capacitated maximal covering location problem.

This paper focuses on the problem of single facility location problem in which all parameters and variables are fuzzy numbers. Then, a method for solving this problem is proposed using fuzzy calculus. The paper is structured into six sections. In the next section, some necessary concepts and operations of fuzzy calculus are explained. In Section 3, the single facility location problem in a deterministic way is formulated and its equivalent form is obtained. In the Section 4, a new solution method based on fuzzy calculus is proposed. Applicability and efficiency of the presented algorithm is presented in Section 5 by solving a numerical example. Finally in Section 6, our findings are summarized.

2. Basic definitions

In this section, some necessary and essential definitions of fuzzy theory are presented [4, 5, 11, 17].

Definition 1. The fuzzy set $\tilde{A}$, whose reference set is the set of real numbers, is a fuzzy number where satisfies three following conditions:

(1) $\tilde{A}$ be a convex set

(2) The height of $\tilde{A}$ be equal to one

(3) The membership function $\mu_{\tilde{A}}(x)$ be continuous in a closed interval.

Fuzzy numbers have different types, but in this article, we only use triangular fuzzy numbers.

Definition 2. Triangular fuzzy numbers are presented as $\tilde{a} = \langle a_l, a_c, a_u \rangle$.

Definition 3. Let $\tilde{a} = \langle a_l, a_c, a_u \rangle$ and $\tilde{b} = \langle b_l, b_c, b_u \rangle$ be two nonnegative triangular fuzzy numbers and $k \in \mathbb{R}$, Operations between fuzzy numbers are defined as follows:

$$k\tilde{a} = \begin{cases} \langle ka_l, ka_c, ka_u \rangle, & k \geq 0 \\ \langle ka_u, ka_c, ka_l \rangle, & k < 0 \end{cases}$$

$$\tilde{a} + \tilde{b} = \langle a_l + b_l, a_c + b_c, a_u + b_u \rangle$$

$$\tilde{a} - \tilde{b} = \langle a_l - b_u, a_c - b_c, a_u - b_l \rangle$$

$\tilde{a} \leq \tilde{b}$ if and only if $(i)\ a_c \leq b_c, (ii)\ a_c = b_c\ and\ a_u - a_l > b_u - b_l\ (iii)\ a_c = b_c, a_u - a_l = b_u - b_l$ $and\ a_u + a_l < b_u + b_l$.

Remark 1. According to Definition 2.3, it is clear that $\tilde{a} = \tilde{b}$ implies $a_c = b_c,\ a_l = b_l$ and $a_u = b_u$.

Remark 2. Consider the following problem where the objective function is absolute value

$$\begin{aligned} &Min\ z = |cx| \\ &s.t. \\ &Ax \leq b \\ &x \geq 0 \end{aligned}$$

By adding the inequality $|cx| \geq y$ to the problem, the absolute value objective function becomes as follows:

$$\begin{aligned} &Min\ z = y \\ &s.t. \\ &Ax \leq b \\ &|cx| \leq y \\ &x, y \geq 0 \end{aligned}$$

Then, the inequality $|cx| \leq y$ can be replaced by the constraints $cx \leq y$ and $cx \geq -y$. So we have:

$$\begin{aligned} &Max\ z = y \\ &s.t. \\ &Ax \leq b \\ &cx \leq y \\ &cx \geq -y \\ &x, y \geq 0 \end{aligned}$$

Consequently, by using the variables y and the above constraints, the absolute value function is converted to linear programming and can be solved as linear programming.

3. Single facility location problem with L_1 norm

In this section first crisp single facility location problem with L_1 norm is defined and its equivalent form is presented. Then, single facility location problem with L_1 norm in fuzzy environment is defined.

3.1. Crisp single facility location problem with L_1 norm

The problem of finding the location of one facility center in which satisfies all demands with the minimum cost is called the single facility location problem. A single facility location problem with L_1 norm is presented in the following [12]:

$$Min\ f(x) = \sum_{j=1}^{n} w_j\left(\left|x_1 - a_{j1}\right| + \left|x_2 - a_{j2}\right|\right) = \sum_{j=1}^{n}\sum_{i=1}^{2} w_j\left|x_i - a_{ji}\right| \tag{1}$$

Where n is the number of demand centers, $A_j = \left(a_{j1}, a_{j2}\right),\ j = 1, \ldots, n$ are the coordinates of the demand centers, $X = (x_1, x_2)$ are the coordinates of the facility center, and $w_j,\ j = 1, \ldots, n$ is the demand of the j-th customer, and all parameters and variables of this model are non-negative. By performing mathematical operations and transforming the absolute value to constraints of model (1) the following model will be obtained:

$$\begin{aligned} & Min\ f(x) = \sum_{j=1}^{n}\sum_{i=1}^{2} w_j y_{ji} \\ & s.t. \\ & \quad x_i - a_{ji} \le y_{ji} \quad j = 1, \ldots, n, \quad i = 1,2, \\ & \quad x_i - a_{ji} \ge -y_{ji}, \quad j = 1, \ldots, n, \quad i = 1,2, \\ & \quad x_i \ge 0, \quad y_{ji} \ge 0, \quad j = 1, \ldots, n, \quad i = 1,2. \end{aligned} \tag{2}$$

And also by moving variables to the left and parameters to the right of constraints we have:

$$\begin{aligned} & Min\ f(x) = \sum_{j=1}^{n}\sum_{i=1}^{2} w_j y_{ji} \\ & s.t. \\ & \quad x_i - y_{ji} \le a_{ji}, \quad j = 1, \ldots, n, \quad i = 1,2, \\ & \quad x_i + y_{ji} \ge a_{ji}, \quad j = 1, \ldots, n, \quad i = 1,2, \\ & \quad x_i \ge 0, y_{ji} \ge 0, \quad j = 1, \ldots, n, \quad i = 1,2. \end{aligned} \tag{3}$$

Since all the parameters and variables of this model are non-negative, by converting the objective function Min to Max and the ≥ constraint to ≤ constraint, it can be written as follow:

$$\begin{aligned} & Max\ (-f(x)) = \sum_{j=1}^{n}\sum_{i=1}^{2} -w_j y_{ji} \\ & s.t. \\ & \quad x_i - y_{ji} \le a_{ji}, \quad j = 1, \ldots, n, \quad i = 1,2, \\ & \quad -x_i - y_{ji} \le -a_{ji}, \quad j = 1, \ldots, n, \quad i = 1,2, \\ & \quad x_i \ge 0, y_{ji} \ge 0, \quad j = 1, \ldots, n, \quad i = 1,2. \end{aligned} \tag{4}$$

Clearly, model (4) has all the conditions of the standard form of the dual simplex method, and it can be written and solved in the standard form of this method by adding slack variables:

$$\begin{aligned}
&Max\ (-f(x)) = \sum_{j=1}^{n}\sum_{i=1}^{2} -w_j y_{ji} \\
&\quad s.t. \\
&\quad\quad x_i - y_{ji} + s_{ji1} = a_{ji}, \qquad j = 1,\dots,n, \quad i = 1,2, \\
&\quad\quad -x_i - y_{ji} + s_{ji2} = -a_{ji}, \qquad j = 1,\dots,n, \quad i = 1,2, \\
&\quad\quad x_i \geq 0, y_{ji} \geq 0, s_{ji1} \geq 0, s_{ji2} \geq 0, \qquad j = 1,\dots,n, \quad i = 1,2.
\end{aligned} \tag{5}$$

3.2. Single facility location problem with L_1 norm in fuzzy environment

In the single facility location problem, it is assumed that all parameters of the problem are accurately determined, while the values observed in the real world are imprecise and ambiguous due to incomplete or unobtainable information. Therefore, in the following, we will consider the parameters and variables that cannot be determined precisely and definitively as fuzzy numbers. The parameter $W_j, j = 1,\dots,n$ which is the demand of customer place j is not certain and fixed in real conditions and it is possible to change it, so in model (5) we will consider it as fuzzy number. But $A_j = (a_{j1}, a_{j2}), j = 1,\dots,n$ is the exact location of customers, such as the location of stores or distribution centers, which are precisely determined and cannot be considered as fuzzy form. The variable $X = (x_1, x_2)$, is also the coordinate of the place where the facility center should be located. But if this variable is considered definitively, due to various conditions and reasons, it may not be possible to provide land or building a facility center there. So, this variable is considered as fuzzy number too. Hence, the optimal solution of the problem is obtained in fuzzy form and suggests an area and a range for the locating the facility center. Note that due to the relationship between y_{ji} and x_i, considering x_i to be fuzzy also causes y_{ji} to become fuzzy. Therefore, the fuzzy form of model (5) will be as follows:

$$\begin{aligned}
&Max\ (-f(x)) = \sum_{j=1}^{n}\sum_{i=1}^{2} -\widetilde{w}_j \tilde{y}_{ji} \\
&\quad s.t. \\
&\quad\quad \tilde{x}_i - \tilde{y}_{ji} + s_{ji1} = a_{ji}, \qquad j = 1,\dots,n, \quad i = 1,2, \\
&\quad\quad -\tilde{x}_i - \tilde{y}_{ji} + s_{ji2} = -a_{ji}, \qquad j = 1,\dots,n, \quad i = 1,2, \\
&\quad\quad x_i \geq 0, y_{ji} \geq 0, s_{ji1} \geq 0, s_{ji2} \geq 0, \qquad j = 1,\dots,n, \quad i = 1,2.
\end{aligned} \tag{6}$$

Following, a method for finding the solution of fuzzy problem 6 is presented.

4. The new method for fuzzy solving single facility location problem with L_1 norm

In this section, a new method for finding the optimal solution of problem (6) is presented whose steps are as follows:

Step 1: In problem (6), parameters and variables are all non-negative triangular fuzzy number. So, according to Definition 2.2, for each $i = 1,2$ and $j = 1,\dots,n$, let $\widetilde{w_j} = \langle w_j^l, w_j^c, w_j^u \rangle$, $\tilde{x}_i = \langle x_i^l, x_i^c, x_i^u \rangle$ and $\tilde{y}_{ji} = \langle y_{ji}^l, y_{ji}^c, y_{ji}^u \rangle$. Also, to preserve the triangular shape of these fuzzy number the constraints $x_i^c \geq x_i^l$, $x_i^u \geq x_i^c$, $y_{ji}^c \geq y_{ji}^l$ and $y_{ji}^u \geq y_{ji}^c$ is added to the problem (6). In order to coordinate Slack variables with fuzzy variables, slack variables are considered as triples $\tilde{s}_{ji1} = \langle s_{ji1}^l, s_{ji1}^c, s_{ji1}^u \rangle$ and $\tilde{s}_{ji2} = \langle s_{ji2}^l, s_{ji2}^c, s_{ji2}^u \rangle$. Now, by performing the calculations of Definition 3, we can write:

$$Max\ f_1(x) = \sum_{j=1}^{n}\sum_{i=1}^{2} -w_j^c y_{ji}^c \quad (7.1)$$

$$Min\ f_2(x) = \sum_{j=1}^{n}\sum_{i=1}^{2} (w_j^u y_{ji}^u - w_j^l y_{ji}^l) \quad (7.2)$$

$$Max\ f_3(x) = \sum_{j=1}^{n}\sum_{i=1}^{2} -(w_j^u y_{ji}^u + w_j^l y_{ji}^l) \quad (7.3)$$

$$s.t. \begin{cases} x_i^u - y_{ji}^l + s_{ji1}^u = a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \quad (7.4) \\ x_i^c - y_{ji}^c + s_{ji1}^c = a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \quad (7.5) \\ x_i^l - y_{ji}^u + s_{ji1}^l = a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \quad (7.6) \\ -x_i^l - y_{ji}^l + s_{ji2}^u = -a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \quad (7.7) \\ -x_i^c - y_{ji}^c + s_{ji2}^c = -a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \quad (7.8) \\ -x_i^u - y_{ji}^u + s_{ji2}^l = -a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \quad (7.9) \\ 0 \le x_i^l \le x_i^c \le x_i^u, & i = 1,2, \quad (7.10) \\ 0 \le y_{ji}^l \le y_{ji}^c \le y_{ji}^u, & j = 1,\ldots,n, \quad i = 1,2, \quad (7.11) \end{cases} \quad (7)$$

Step 2: To solve model (7), the objective function (7.1) and constraints (7.5), (7.8), (7.10) and (7.11) are considered independently of the other objective functions and constraints. So, the following problem is obtained:

$$Max\ f(x) = \sum_{j=1}^{n}\sum_{i=1}^{2} -w_j^c y_{ji}^c$$

$$s.t. \begin{cases} x_i^c - y_{ji}^c + s_{ji1}^c = a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \\ -x_i^c - y_{ji}^c + s_{ji2}^c = -a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \\ 0 \le x_i^c, & i = 1,2, \\ 0 \le y_{ji}^c, & j = 1,\ldots,n, \quad i = 1,2, \end{cases} \quad (8)$$

As it is clear the model (8) is smaller and simpler than model (7). Now, by solving the model (8) using the dual simplex method the optimal solution $x_i^c = x_i^{c*}$ and $y_{ji}^c = y_{ji}^{c*}$, *is obtained* for each $i = 1,2$ and $j = 1,\ldots,n$.

Step 3: Now, by considering the objective function (7.2) and the remaining constraints, we will have the following model:

$$Min\ f(x) = \sum_{j=1}^{n}\sum_{i=1}^{2} (w_j^u y_{ji}^u - w_j^l y_{ji}^l)$$

$$s.t. \begin{cases} x_i^u - y_{ji}^l + s_{ji1}^u = a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \\ x_i^l - y_{ji}^u + s_{ji1}^l = a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \\ -x_i^l - y_{ji}^l + s_{ji2}^u = -a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \\ -x_i^u - y_{ji}^u + s_{ji2}^l = -a_{ji}, & j = 1,\ldots,n, \quad i = 1,2, \\ 0 \le x_i^l \le x_i^c \le x_i^u, & i = 1,2, \\ 0 \le y_{ji}^l \le y_{ji}^c \le y_{ji}^u, & j = 1,\ldots,n, \quad i = 1,2, \end{cases} \quad (9)$$

The optimal variable values obtained in the second step should be replaced in two last constraints of model (9).

The model (9) can be solved using the simplex method. If there exists a unique optimal solution for this model, the optimal values of theses variables and the optimal variable values obtained in the second step should be substituted in problem (7) to obtain the optimal value Z^*.

Step 4: By solving the deterministic models (8) and (9), the following situations may occur:

a) A unique optimal solution is obtained for variables $x_i^u = x_i^{u^*}$ and $x_i^l = x_i^{l^*}$. So, the optimal solution for model (7) is obtained by replacing $\tilde{x}_i = \langle x_i^l, x_i^c, x_i^u \rangle$, *and* $\tilde{y}_{ji} = \langle y_{ji}^l, y_{ji}^c, y_{ji}^u \rangle$ *the optimal value of the objective function is obtained as* $Z^* = \sum_{j=1}^{n} \sum_{i=1}^{2} -\tilde{w}_j \tilde{y}_{ji}$ *and t*he algorithm will stop.

b) A feasible solution is not exist for one or some of the deterministic models (8) and (9). In this case, model (7) does not have a feasible solution, and the algorithm will stop.

c) The solution for one or some of the deterministic models (8) and (9) is unbounded. In this case, model (7) does not have an optimal solution, and the algorithm will stop.

d) Multiple optimal solutions are obtained for the deterministic models (8) and (9). In this case, we will obtain a unique solution for model (7) by solving the third objective function of model (7) that is the objective function 7.3 and constraints from model (9).

5. Implementation of the algorithm and numerical example

In this section, with a numerical example our new algorithm is implemented.

Example 1: Let $A_1 = (1,1), A_2 = (3,7), A_3 = (4,2)$, be the coordinates of three customers with the demands $\widehat{W}_1 =< 2,2,3 >$, $\widehat{W}_2 =< 1,5,6 >$ and $\widehat{W}_3 =< 1,3,10 >$ respectively. So, the fuzzy problem is as follow:

$$
\begin{aligned}
&\text{Max}\big(-f(x)\big) = -<2,2,3>\tilde{y}_{11} - <1,5,6>\tilde{y}_{21} - <1,3,10>\tilde{y}_{31} - <2,2,3>\tilde{y}_{12} - <1,5,6>\tilde{y}_{22} - <1,3,10>\tilde{y}_{32} \\
&\quad s.t. \\
&\quad \tilde{x}_1 - \tilde{y}_{11} + \tilde{s}_{111} = 1, \\
&\quad \tilde{x}_1 - \tilde{y}_{21} + \tilde{s}_{211} = 3 \\
&\quad \tilde{x}_1 - \tilde{y}_{31} + \tilde{s}_{311} = 4, \\
&\quad \tilde{x}_2 - \tilde{y}_{12} + \tilde{s}_{121} = 1, \\
&\quad \tilde{x}_2 - \tilde{y}_{22} + \tilde{s}_{221} = 7, \\
&\quad \tilde{x}_2 - \tilde{y}_{32} + \tilde{s}_{321} = 2, \\
&\quad -\tilde{x}_1 - \tilde{y}_{11} + \tilde{s}_{112} = -1, \\
&\quad -\tilde{x}_1 - \tilde{y}_{21} + \tilde{s}_{212} = -3, \\
&\quad -\tilde{x}_1 - \tilde{y}_{31} + \tilde{s}_{312} = -4, \\
&\quad -\tilde{x}_2 - \tilde{y}_{12} + \tilde{s}_{122} = -1, \\
&\quad -\tilde{x}_2 - \tilde{y}_{22} + \tilde{s}_{222} = -7, \\
&\quad -\tilde{x}_2 - \tilde{y}_{32} + \tilde{s}_{322} = -2, \\
&\quad \tilde{x}_i \geq 0, \quad \tilde{y}_{ji} \geq 0, \quad \tilde{s}_{ji1} \geq 0, \quad \tilde{s}_{ji2} \geq 0, \quad j = 1, \dots, n, \quad i = 1,2
\end{aligned}
$$

Based on step 1 of our presented algorithm we have:

$$Max\ \ f_1(x) = -\,2y_{11}^c - 5y_{21}^c - 3y_{31}^c - 2y_{12}^c - 5y_{22}^c - 3y_{32}^c$$

$$Min\ \ f_2(x)\ =\ 3y_{11}^u - y_{11}^l + 6y_{21}^u - y_{21}^l + 10y_{31}^u - y_{31}^l + 3y_{12}^u - y_{12}^l + 6y_{22}^u - y_{22}^l + 10y_{32}^u - y_{32}^l$$

$$Max\ \ f_3(x) = -\,3y_{11}^u - y_{11}^l - 6y_{21}^u - y_{21}^l - 10y_{31}^u - y_{31}^l - 3y_{12}^u - y_{12}^l - 6y_{22}^u - y_{22}^l - 10y_{32}^u - y_{32}^l$$

$s.t.$

$$\begin{aligned}
x_1^u - y_{11}^l + s_{111}^u &= 1,\\
x_1^u - y_{21}^l + s_{211}^u &= 3,\\
x_1^u - y_{31}^l + s_{311}^u &= 4,\\
x_2^u - y_{12}^l + s_{121}^u &= 1,\\
x_2^u - y_{22}^l + s_{221}^u &= 7,\\
x_2^u - y_{32}^l + s_{321}^u &= 2,\\
x_1^c - y_{11}^c + s_{111}^c &= 1,\\
x_1^c - y_{21}^c + s_{211}^c &= 3,\\
x_1^c - y_{31}^c + s_{311}^c &= 4,\\
x_2^c - y_{12}^c + s_{121}^c &= 1,\\
x_2^c - y_{22}^c + s_{221}^c &= 7,\\
x_2^c - y_{32}^c + s_{321}^c &= 2,\\
x_1^l - y_{11}^u + s_{111}^l &= 1,\\
x_1^l - y_{21}^u + s_{211}^l &= 3,\\
x_1^l - y_{31}^u + s_{311}^l &= 4,\\
x_2^l - y_{12}^u + s_{121}^l &= 1,\\
x_2^l - y_{22}^u + s_{221}^l &= 7,\\
x_2^l - y_{32}^u + s_{321}^l &= 2,\\
-x_1^l - y_{21}^l + s_{212}^u &= -3,\\
-x_1^l - y_{11}^l + s_{112}^u &= -1,\\
-x_1^l - y_{31}^l + s_{312}^u &= -4,\\
-x_2^l - y_{12}^l + s_{122}^u &= -1,\\
-x_2^l - y_{22}^l + s_{222}^u &= -7,\\
-x_2^l - y_{32}^l + s_{322}^u &= -2,\\
-x_1^c - y_{11}^c + s_{112}^c &= -1,\\
-x_1^c - y_{21}^c + s_{212}^c &= -3,\\
-x_1^c - y_{31}^c + s_{312}^c &= -4,\\
-x_2^c - y_{12}^c + s_{122}^c &= -1,\\
-x_2^c - y_{22}^c + s_{222}^c &= -7,
\end{aligned}$$

$$
\begin{aligned}
&-x_2^c - y_{32}^c + s_{322}^c = -2,\\
&-x_1^u - y_{11}^u + s_{112}^l = -1,\\
&-x_1^u - y_{21}^u + s_{212}^l = -3,\\
&-x_1^u - y_{31}^u + s_{312}^l = -4,\\
&-x_2^u - y_{12}^u + s_{122}^l = -1,\\
&-x_2^u - y_{22}^u + s_{222}^l = -7,\\
&-x_2^u - y_{32}^u + s_{322}^l = -2,\\
&0 \le x_1^l \le x_1^c \le x_1^u,\\
&0 \le x_2^l \le x_2^c \le x_2^u,\\
&0 \le y_{11}^l \le y_{11}^c \le y_{11}^u,\\
&0 \le y_{21}^l \le y_{21}^c \le y_{21}^u,\\
&0 \le y_{31}^l \le y_{31}^c \le y_{31}^u,\\
&0 \le y_{12}^l \le y_{12}^c \le y_{12}^u,\\
&0 \le y_{22}^l \le y_{22}^c \le y_{22}^u,\\
&0 \le y_{32}^l \le y_{32}^c \le y_{32}^u,
\end{aligned}
$$

Now based on step 2 of our presented algorithm the following model should be solve first:

$$
\begin{aligned}
&Max\ \ f_1(x) = -\,2y_{11}^c - 5y_{21}^c - 3y_{31}^c - 2y_{12}^c - 5y_{22}^c - 3y_{32}^c\\
&s.t.\\
&x_1^c - y_{11}^c + s_{111}^c = 1,\\
&x_1^c - y_{21}^c + s_{211}^c = 3,\\
&x_1^c - y_{31}^c + s_{311}^c = 4,\\
&x_2^c - y_{12}^c + s_{121}^c = 1,\\
&x_2^c - y_{22}^c + s_{221}^c = 7,\\
&x_2^c - y_{32}^c + s_{321}^c = 2,\\
&-x_1^c - y_{11}^c + s_{112}^c = -1,\\
&-x_1^c - y_{21}^c + s_{212}^c = -3,\\
&-x_1^c - y_{31}^c + s_{312}^c = -4,\\
&-x_2^c - y_{12}^c + s_{122}^c = -1,\\
&-x_2^c - y_{22}^c + s_{222}^c = -7,\\
&-x_2^c - y_{32}^c + s_{322}^c = -2,\\
&x_1^c,\ \ x_2^c \ge 0,\\
&y_{11}^c,\ \ y_{21}^c,\ \ y_{31}^c \ge 0,\\
&y_{12}^c,\ \ y_{22}^c,\ \ y_{32}^c \ge 0
\end{aligned}
$$

The optimal solution of this problem is obtained as follow:
$x_1^c = 3$, $x_2^c = 2$, $y_{11}^c = 2$, $y_{21}^c = 0$, $y_{31}^c = 1$, $y_{12}^c = 1$, $y_{22}^c = 5$, $y_{32}^c = 0$ and $f_1(x) = 34$.

Now based on step 3 of our new algorithm and using the above obtained solution we have:

$$Min\ \ f_2(x) = 3y_{11}^u - y_{11}^l + 6y_{21}^u - y_{21}^l + 10y_{31}^u - y_{31}^l + 3y_{12}^u - y_{12}^l + 6y_{22}^u - y_{22}^l + 10y_{32}^u - y_{32}^l$$

$$x_1^u - y_{11}^l + s_{111}^u = 1,$$
$$x_1^u - y_{21}^l + s_{211}^u = 3,$$
$$x_1^u - y_{31}^l + s_{311}^u = 4,$$
$$x_2^u - y_{12}^l + s_{121}^u = 1,$$
$$x_2^u - y_{22}^l + s_{221}^u = 7,$$
$$x_2^u - y_{32}^l + s_{321}^u = 2,$$
$$x_1^l - y_{11}^u + s_{111}^l = 1,$$
$$x_1^l - y_{21}^u + s_{211}^l = 3,$$
$$x_1^l - y_{31}^u + s_{311}^l = 4,$$
$$x_2^l - y_{12}^u + s_{121}^l = 1,$$
$$x_2^l - y_{22}^u + s_{221}^l = 7,$$
$$x_2^l - y_{32}^u + s_{321}^l = 2,$$
$$-x_1^l - y_{11}^l + s_{112}^u = -1,$$
$$-x_1^l - y_{21}^l + s_{212}^u = -3,$$
$$-x_1^l - y_{31}^l + s_{312}^u = -4,$$
$$-x_2^l - y_{12}^l + s_{122}^u = -1,$$
$$-x_2^l - y_{22}^l + s_{222}^u = -7,$$
$$-x_2^l - y_{32}^l + s_{322}^u = -2,$$
$$-x_1^u - y_{11}^u + s_{112}^l = -1,$$
$$-x_1^u - y_{21}^u + s_{212}^l = -3,$$
$$-x_1^u - y_{31}^u + s_{312}^l = -4,$$
$$-x_2^u - y_{12}^u + s_{122}^l = -1,$$
$$-x_2^u - y_{22}^u + s_{222}^l = -7,$$
$$-x_2^u - y_{32}^u + s_{322}^l = -2,$$
$$0 \le x_1^l \le 3 \le x_1^u,$$
$$0 \le x_2^l \le 2 \le x_2^u,$$
$$0 \le y_{11}^l \le 2 \le y_{11}^u,$$
$$0 \le y_{21}^l \le 0 \le y_{21}^u,$$
$$0 \le y_{31}^l \le 1 \le y_{31}^u,$$
$$0 \le y_{12}^l \le 1 \le y_{12}^u,$$
$$0 \le y_{22}^l \le 5 \le y_{22}^u,$$
$$0 \le y_{32}^l \le 0 \le y_{32}^u,$$

Now by solving this problem the optimal solution is obtained as follow:
$x_1^l = 1.73$, $x_1^u = 4$, $x_2^l = 1.5$, $x_2^u = 4.388$ and $f_2(x) = 196$.

Hence, based on our presented algorithm, the fuzzy solution of the fuzzy problem and the optimal value of objective function are $\tilde{x}_1^* =< 1.73,3,4 >$, $\tilde{x}_2^* =< 1.5,2,4.388 >$ and $\tilde{f}^* =< 12,34,64.716 >$ respectively.

The variable $\tilde{x}^* = (\tilde{x}_1^*, \tilde{x}_2^*)$ is the best place that the facility center should be established. But if these variables is considered definitively, due to various conditions and reasons, it may not be possible to acquire land or build a facility center in it, so we also considered this variable in a fuzzy way and a facility center area was obtained, that certainly, the decision maker can find the right place more easily. The optimal solution is shown as a heat map on Figure 1. As you can see, the optimal solution is not a single point and it consists of a color spectrum from black to yellow, which respectively black represents the highest degree of membership and yellow represents the lowest degree of membership for the optimal solution.

As it is clear, by solving this example the applicability and simplicity of implementation of our presented algorithm is demonstrated. Also, you can see that the optimal solution and the optimal value obtained using our presented algorithm are in fuzzy form and this shows the accuracy of our method.

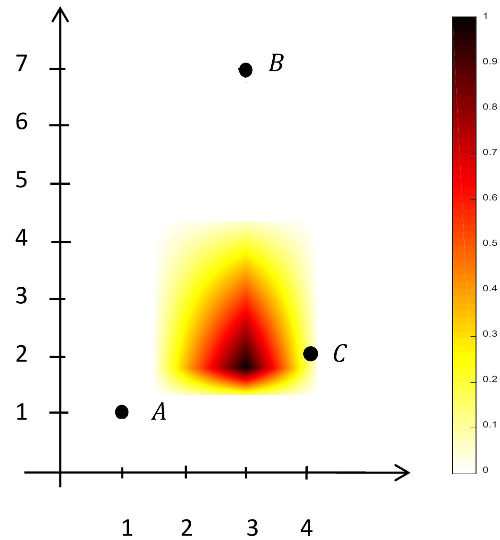

Figure 1: Locations of custumers and optimal solution

6. Conclusion

In this article due to the uncertainty of parameters in the real world for the single facility location problem the fuzzy logic is utilized. In this paper in addition to the weights (the amounts of demands of costumers), variables (the coordinates of optimal place) are also considered fuzzy. Hence, the decision maker can find the right place for locating facility center more easily. Because if these variables are considered definitively, due to various conditions and reasons, it may not be possible to acquire land or build a facility center in it, so we also considered this variable in a fuzzy way and a facility center area was obtained.

Then, using fuzzy logic operations and calculations, a new algorithm for decomposing the fuzzy model into three smaller and simpler deterministic models is proposed. By solving these decomposed models and obtaining their optimal solutions, the optimal solution for the fuzzy model is obtained. Finally, to demonstrate the implementation, efficiency, and simplicity of the proposed algorithm, a numerical example is solved. To further develop and improve the results of this article, the main fuzzy model of the problem can be solved using a metaheuristic method, and the results can be compared with the results obtained from the algorithm proposed in this paper.

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