

Analysis of Bifurcation Phenomenon in a Grid-Connected Electric Arc Furnace

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Abstract:

This paper aims to study the stability and dynamic behavior of a grid-connected electric arc furnace system, using bifurcation theory. This theory introduces a systematic method for stability analysis of dynamic systems, under changes in the system parameters. In fact, a parameter is constantly changed in each step, using MATLAB and/or AUTO software, and system eigenvalues are monitored simultaneously. Considering how the eigenvalues approach the system's imaginary axis on S plane in accordance with the changes in the targeted parameter, the occurred saddle-node and/or Hopf bifurcation of the system is extracted. In this paper, at first, electric arc furnace is modeled by the differential-algebraic equations and then based on the projection method, two-dimensional center manifold at the Hopf bifurcation happens in the electric arc furnace system is achieved analytically. Projection method is discussed in the bifurcation theory and is presented totally in this paper. Finally, the results are compared by the computer simulations.

Index Terms: Electric arc furnace, bifurcation theory, projection method, center manifold.

تحلیل پدیده انشعابات در یک کوره قوس الکتریکی متصل به شبکه قدرت

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خلاصه: هدف این مقاله، مطالعه پایداری و رفتار دینامیکی یک سیستم کوره قوس الکتریکی متصل به شبکه قدرت به کمک نظریه انشعابات است. این تئوری روش منظمی را برای تحلیل پایداری سیستم‌های دینامیکی تحت شرایط تغییر پارامترهای سیستم معرفی می‌کند. در حقیقت، در هر مرحله پارامتری از سیستم به طور پیوسته تغییر داده می‌شود و به طور همزمان مقادیر ویژه سیستم به کمک نرم‌افزار MATLAB و یا AUTO ردیابی می‌شود. با توجه به نحوه حرکت مقادیر ویژه و نزدیک شدن آنها به محور موهومی در صفحه S انشعاب گره-زینی و یا هاپف رخ داده در سیستم به ازای تغییر پارامتر مورد نظر استخراج می‌گردد. در این مقاله، ابتدا سیستم کوره قوس الکتریکی به کمک معادلات دیفرانسیل-جبری مدل‌سازی می‌شود و سپس به کمک روش تصویر، منیفلد مرکزی دو بعدی انشعاب هاپف که در سیستم کوره قوس الکتریکی رخ می‌دهد به صورت تحلیلی استخراج می‌شود. روش تصویر در نظریه انشعابات مطرح بوده و به طور کامل در این مقاله آورده شده است. در نهایت، نتایج کار با شبیه‌سازی کامپیوتری مقایسه شده است.

کلمات کلیدی: کوره قوس الکتریکی، تئوری انشعابات، روش تصویر، منیفلد مرکزی

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1. Introduction

Electric arc furnaces (EAFs) are used in a diverse range of industrial smelting processes. In general, these furnaces are supplied by an AC source. EAFs are highly nonlinear and nonperiodic dynamic systems and they tend to suffer from harmonic and inter-harmonic distortions [1, 2]. Conventional methods are unable to accurately describe an EAF's dynamic behavior but can be used to obtain estimates of the harmonics, inter-harmonics, flicker phenomenon and voltage fluctuations at the point of common coupling. These are all effects that adversely influence the power quality. In one of the first attempts to represent an EAF, a simple model was proposed that consisted of an inductor and a series resistor coupled to a current controller switch. However, this model does not account for the time-varying nature of the EAF [3]. In reference [4], a simplified current against voltage characteristic was used to represent the EAF's dynamic behavior in the modeling of power system transients. This model did not find widespread use since it does not incorporate time-varying effects. The power balance principle is used in references [1, 5] to allow the EAF's steady-state behavior to be represented using a differential equation.

In general, practical power systems operate in a quasistatic state, which varies smoothly with small changes in the system parameters. However, under certain operational conditions a small change in the system parameters can result in a significant qualitative change in the system behavior and therefore a change on the stability of the original system. Among several nonlinear mathematical theories [6], bifurcation analysis has been applied to investigate qualitatively the ways in which instabilities can take place in a power system as well as how the system equilibrium points become unstable [7]. Such qualitative changes take place in the system's behavior as a system parameter is varied slowly is called bifurcation. The parameter value at which bifurcation takes place is called bifurcation point. The bifurcation theory provides a set of mathematical techniques for nonlinear differential algebraic equations (DAEs). Thus it is adequate for studying electric power systems that are typically modeled as a set of nonlinear DAE. In particular, the bifurcation theory is widely recognized as an effective tool to study voltage stability [8-11]. The advantage of this technique consists in the determination of the system eigenvalues, i.e., it is not necessary to numerically solve the Jacobian matrix for the system, thus significantly reducing the computational effort and providing a qualitative tool to assess nonlinear oscillation in nonlinear power grid dynamical system. The presence of chaotic

motions in the two-degree freedom swing equations was discovered in [12]. Subsequent applications of bifurcation theory have been directed to the studies such as voltage collapse [13], subsynchronous resonance [14], ferroresonance oscillations [15], chaotic oscillations [16], and design of nonlinear controllers [17]. Furthermore, this theory has been applied to assess the dynamical behavior of nonlinear components such as induction motors [18], load models [19, 20], tap changing transformers [21], power system stabilizers [22] and static VAR compensators [22, 23]. However, to the best authors knowledge there is no application of this theory to assess the nonlinear oscillations due to AC-fed EAF analytically. In order to tackle the last problem, this paper introduces the application of bifurcations theory to assess the electric arc furnace dynamical performance due to quasistatic changes in the supply system parameters such as the grid thevenin impedance. It is investigated how bifurcations points lead the EAF to oscillatory instabilities. The main contribution of the present paper is applying the projection method analytically in the electric arc furnace system to achieve two-dimensional center manifold at the Hopf bifurcation.

2. Bifurcation phenomenon

The transient and steady state of a system represented by a set of differential equations can be solved by conventional numerical integration methods, by computing the trajectories and orbits using digital simulation. However, it is possible with bifurcations theory to predict the behavior of trajectories and orbits without resorting to the solution of the differential equations [24-29]. In this case, bifurcations analysis is applied to study the emergence of sudden changes in a system response arising from smooth, continuous variations on the system parameters. At certain points (bifurcations points) infinitesimal changes in system parameters can cause significant qualitative changes in equilibrium solutions. In other words, bifurcation is the emergence of phases in a non-equivalent topological format under changes in parameters [7]. Consider a parameter-dependent continuous-time system as following:

$$\dot{x} = f(x, \alpha) \quad (1)$$

Where $x \in \mathfrak{R}^n$ and $\alpha \in \mathfrak{R}^m$ are variables of phases and parameters. In this relation, f in comparison to x and α is steady. $x = x_0$ is a hyperbolic equilibrium point for the system in $\alpha = \alpha_0$. By a small change in parameter, the equilibrium point is changed slightly, but so that the hyperbolic is still constant.

It is obvious that in a vertical form the hyperbolic condition of equilibrium point can be rejected in two ways. First is that for some amounts of the α parameter, a real simple eigenvalue gets closer to zero and we have: $\lambda_1 = 0$. Second, a pair of mixed simple eigenvalue gets close to the imaginary axis and we have: $\lambda_{1,2} = \pm i\omega_0$, $\omega_0 > 0$. In order to have more specific-values on the imaginary axis, more parameters are needed. The type of bifurcation related to the emergence of $\lambda_1 = 0$ is called saddle-node bifurcation. Corresponding bifurcation with the emergence of $\lambda_{1,2} = \pm i\omega_0$, $\omega_0 > 0$ is called Hopf bifurcation. Fold and Hopf bifurcations occur when $n \geq 1$ and $n \geq 2$, respectively.

2.1 Normalized saddle-node bifurcation

Consider the following one-dimension parameter-dependent continuous-time system [24]:

$$\dot{x} = \alpha + x^2 = f(x, \alpha) \quad (2)$$

This system has an equilibrium non-hyperbolic point $x_0 = 0$ with the absolute value of $f_x(0,0) = 0$ at $\alpha = 0$. The system's behavior is also obvious for other amounts of α which are shown in Fig. 1.

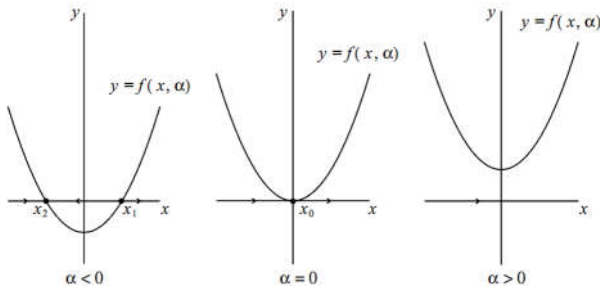


Fig. (1): Fold bifurcation

when $\alpha < 0$, there are two equilibrium points: $x_{1,2}(\alpha) = \pm\sqrt{-\alpha}$ where the left equilibrium point is stable and the right one is unstable. There is no equilibrium point in the system when $\alpha > 0$. When α starts from a negative point, moving to zero and then to the positive, the equilibrium points (stable and unstable) collide with each other and make an equilibrium point at $\alpha = 0$ with $\lambda = 0$ and then disappear. Now, add to system (2) higher-order terms that can depend smoothly on the parameter. It happens that these terms do not change qualitatively the behavior of the system near the origin $x = 0$ for parameter values close $\alpha = 0$. Actually, system of $\dot{x} = \alpha + x^2 + O(x^3)$ close to the origin is a topological equivalent of the $\dot{x} = \alpha + x^2$ system.

2.2 Normalized Hopf bifurcation

Consider the following two-dimension parameter-dependent continuous-time system [25]:

$$\begin{cases} \dot{x}_1 = \alpha x_1 - x_2 - x_1(x_1^2 + x_2^2) \\ \dot{x}_2 = x_1 + \alpha x_2 - x_2(x_1^2 + x_2^2) \end{cases} \quad (3)$$

For all amounts of α with the equilibrium point of $x_1 = x_2 = 0$ in the Jacobian matrix of $A = \begin{pmatrix} \alpha & -1 \\ 1 & \alpha \end{pmatrix}$,

this system has an absolute value of $\lambda_{1,2} = \alpha \pm i$. By introducing the mixed variable of $z = x_1 + ix_2$, system (3) can be changed to the following mixed form:

$$\dot{z} = (\alpha + i)z - z|z|^2 \quad (4)$$

Finally, showing $z = \rho e^{i\varphi}$ and doing some mathematical operations, the following polar form of system (4) is obtained.

$$\begin{cases} \dot{\rho} = \rho(\alpha - \rho^2) \\ \dot{\varphi} = 1 \end{cases} \quad (5)$$

As ρ and φ in equation (5) are independent from each other, while α passing through zero, bifurcations of the system's phases are simply analyzed using the polar form. The first equation (which should be considered for $\rho \geq 0$) has the equilibrium point of $\rho = 0$ for all amounts of α . If $\alpha < 0$, equilibrium point is linearly stable. In $\alpha = 0$, it is nonlinearly stable and for $\alpha > 0$, equilibrium point becomes nonlinear. Furthermore, for $\alpha > 0$ there is another stable equilibrium point of $\rho_0(\alpha) = \sqrt{\alpha}$. The second equation defines rotation at a constant speed. Therefore, the bifurcation diagram of the two-dimension system (3) is obtained through the accumulated effects of the defined movements in the two equations of (5) in the form of Fig. 2. The system always has an equilibrium point in the origin. This equilibrium point is a stable center for $\alpha < 0$ and unstable for $\alpha > 0$. At the critical amount of parameter, i.e. $\alpha = 0$, the equilibrium point is stable nonlinearly and is equivalent to the center topologically. When $\alpha > 0$, the equilibrium point is surrounded by a separate limit cycle which is unique and stable. The radius of this cycle equals to $\rho_0(\alpha) = \sqrt{\alpha}$. All circuits, both inside and outside the cycle considering the origin with passing time $t \rightarrow +\infty$ move to the convergence cycle. The system with the opposite sign for nonlinear phases can be analyzed in a similar way:

$$\begin{cases} \dot{x}_1 = \alpha x_1 - x_2 + x_1(x_1^2 + x_2^2) \\ \dot{x}_2 = x_1 + \alpha x_2 + x_2(x_1^2 + x_2^2) \end{cases} \quad (6)$$

And we have it in the mixed way:

$$\dot{z} = (\alpha + i)z + z|z|^2 \quad (7)$$

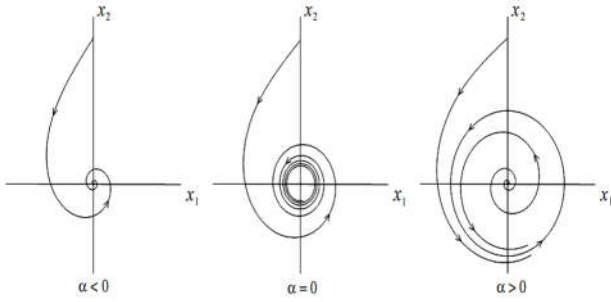


Fig. (2): Supercritical Hopf bifurcation

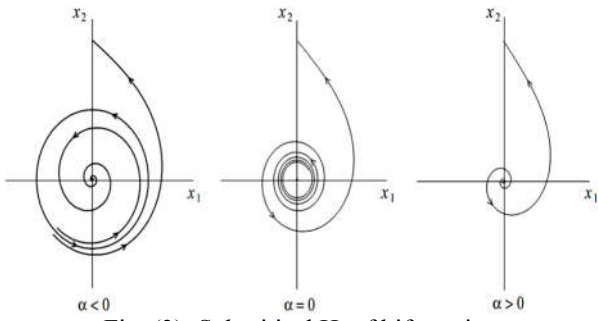


Fig. (3): Subcritical Hopf bifurcation

According to Fig. 3 in $\alpha = 0$, the system experiences Hopf bifurcation. Unlike system (3), there is an unstable limit cycle in system (6) where by passing from zero of α and changing it to the positive amount, the cycle disappears. The equilibrium point in origin for $\alpha \neq 0$ has a similar stability same as system (3). It means it is stable for $\alpha < 0$ and unstable for $\alpha > 0$. In critical point, the stability of system is the opposite of system (3) which is unstable nonlinearly. As it is clear, there are two Hopf bifurcations. As the limit cycle for positive amounts of α parameter is made (after bifurcation), the Hopf bifurcation of system (3) is super-critical. In the opposite side, system (6) is sub-critical because there is a limit cycle before bifurcation. In both bifurcations, as the bifurcation parameter increases in $\alpha = 0$, the equilibrium point loses its stability. In the first one (with the negative sign in cubic sentences), the stable equilibrium point is replaced by a limit cycle with a small domain. Therefore, the system is stayed next to the equilibrium point and it loses its stability in a soft and non-catastrophic form. In the second one (with a positive sign in cubic sentences), the attracting area of equilibrium point is restricted by an unstable limit cycle. When the parameter moves into its critical amount, the domain of this cycle is smaller and is removed by moving parameter from the critical point. So the system is moved away from its location next to the equilibrium point and stability is lost in a sharp and catastrophic form. If the system loses its stability softly, it can be easily controlled and if the parameter is negative again, the system is returned to its stable equilibrium point. But if the system loses its stability sharply, making the

bifurcation parameter negative, cannot return the system in the stable equilibrium point, because it may leave its attracting focus. It should be mentioned that recognition of the type of Hopf bifurcation is possible from stable equilibrium point in the critical amount of bifurcation. Now, the higher order phrases are added to systems (3) and (6). Demonstration of the systems in the vector form is as following:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \alpha & -1 \\ 1 & \alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \pm (x_1^2 + x_2^2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + O(\|x\|^4) \quad (8)$$

In which $x = (x_1, x_2)^T$, $\|x\|^2 = x_1^2 + x_2^2$ and sentences

$O(\|x\|^4)$ can be steady and dependent on α parameter. In this way, system (8) close to origin is equivalent of systems (3) and (6) topologically. So the higher order phrases do not affect the behaviors of system's bifurcation. In the following we are going to investigate just the Hopf bifurcation, since it happens in the EAF system.

2.3. Projection method for computation of center manifold

There is a useful method for center manifold computation which avoids the transformation of the system into its eigenbasis [24]. Instead, only eigenvectors corresponding to the critical eigenvalues of the Jacobian matrix shown by A and its transpose A^T are used to project the system into the critical eigenspace and its complement. Suppose system (1) is written as:

$$\dot{x} = Ax + F(x), \quad x \in \mathfrak{R}^n \quad (9)$$

where $F(x) = O(\|x\|^2)$ is a smooth function. In the case of Hopf bifurcation, A has a simple pair of complex eigenvalues on the imaginary axis: $\lambda_{1,2} = \pm j\omega_0$, $\omega_0 > 0$, and these eigenvalues are the only eigenvalues with $\text{Re } \lambda = 0$. Let $q \in \mathfrak{R}^n$ be a complex eigenvector corresponding to λ_1 : $Aq = j\omega_0 q, A\bar{q} = -j\omega_0 \bar{q}$. Introduce also the adjoint eigenvector $p \in \mathfrak{R}^n$ having the properties: $A^T p = -j\omega_0 p, A^T \bar{p} = j\omega_0 \bar{p}$ and satisfying the normalization: $\langle p, q \rangle = 1$ where $\langle p, q \rangle = \sum_{i=1}^n \bar{p}_i q_i$ is

the standard scalar product in \mathfrak{R}^n (linear with respect to the second argument). The critical real eigenspace T^c corresponding to $\pm j\omega_0$ is now two-dimensional and is spanned by $\{\text{Re } q, \text{Im } q\}$. The real eigenspace T^{su} corresponding to all eigenvalues of A other than $\pm j\omega_0$ is $(n-2)$ -dimensional.

Consider the lemma: $y \in T^{su}$ if and only if $\langle p, y \rangle = 0$. Here $y \in \mathfrak{R}^n$ is real, while $p \in \mathfrak{R}^n$ is complex. Therefore, the condition in this lemma implies two real constraints on y (the real and imaginary parts of $\langle p, y \rangle$ must vanish). This lemma allows decomposing any $xp \in \mathfrak{R}^n$ as:

$$x = zq + \bar{z}\bar{q} + y \quad (10)$$

where $zp \in \mathfrak{R}^1$ and $zq + \bar{z}\bar{q} \in T^c, y \in T^{su}$. The complex variable z is a coordinate on T^c . We have:

$$\begin{cases} z = \langle p, x \rangle \\ y = x - \langle p, x \rangle q - \langle \bar{p}, x \rangle \bar{q} \end{cases} \quad (11)$$

$$\begin{cases} \dot{z} = j\omega_0 z + \frac{1}{2}G_{20}z^2 + G_{11}z\bar{z} + \frac{1}{2}G_{02}\bar{z}^2 + \frac{1}{2}G_{21}z^2\bar{z} + \langle G_{10}, y \rangle z + \langle G_{01}, y \rangle \bar{z} + \dots \\ \dot{y} = Ay + \frac{1}{2}H_{20}z^2 + H_{11}z\bar{z} + \frac{1}{2}H_{02}\bar{z}^2 + \dots \end{cases} \quad (13)$$

where $G_{20}, G_{11}, G_{02}, G_{21} \in \mathfrak{R}^1, G_{01}, G_{10}, H_{ij} \in \mathfrak{R}^n$ and the scalar product in \mathfrak{R}^n is used. Complex number and vectors involved in (13) can be computed by the following formulas:

$$G_{ij} = \frac{\partial^{i+j}}{\partial z^i \partial \bar{z}^j} \langle p, F(zq + \bar{z}\bar{q}) \rangle \Big|_{z=0}, \quad i+j \geq 2, \quad (14)$$

$$\bar{G}_{10,i} = -\frac{\partial^2}{\partial y_i \partial z} \langle p, F(zq + \bar{z}\bar{q} + y) \rangle \Big|_{z=0, y=0}, \quad i=1,2,\dots,n, \quad (15)$$

$$\bar{G}_{01,i} = \frac{\partial^2}{\partial y_i \partial \bar{z}} \langle p, F(zq + \bar{z}\bar{q} + y) \rangle \Big|_{z=0, y=0}, \quad i=1,2,\dots,n, \quad (16)$$

$$H_{ij} = \frac{\partial^{i+j}}{\partial z^i \partial \bar{z}^j} F(zq + \bar{z}\bar{q}) \Big|_{z=0} - G_{ij}q - \bar{G}_{ji}\bar{q}, \quad i+j=2 \quad (17)$$

$$\dot{z} = j\omega_0 z + \frac{1}{2}G_{20}z^2 + G_{11}z\bar{z} + \frac{1}{2}G_{02}\bar{z}^2 + \frac{1}{2}(G_{21} - 2\langle G_{10}, A^{-1}H_{11} \rangle + \langle G_{01}, (2j\omega_0 E - A)^{-1}H_{20} \rangle)z^2\bar{z} + \dots \quad (20)$$

where the scalar product in \mathfrak{R}^n is used. A nice feature of the above algorithm is that it gives the restricted system (20) directly in the complex form suitable for the Lyapunov coefficient computations. Write $F(x)$ in terms of multilinear functions $B(x, y)$ and $C(x, y, z)$:

$$\dot{z} = j\omega_0 z + \frac{1}{2}G_{20}z^2 + G_{11}z\bar{z} + \frac{1}{2}G_{02}\bar{z}^2 + \frac{1}{2}(G_{21} - 2\langle p, B(q, A^{-1}H_{11}) \rangle + \langle p, B(\bar{q}, (2j\omega_0 E - A)^{-1}H_{20}) \rangle)z^2\bar{z} + \dots \quad (23)$$

$$G_{20} = \langle p, B(q, q) \rangle, \quad G_{11} = \langle p, B(q, \bar{q}) \rangle, \quad G_{02} = \langle p, B(\bar{q}, \bar{q}) \rangle, \quad G_{21} = \langle p, C(q, q, \bar{q}) \rangle$$

$$H_{20} = B(q, q) - \langle p, B(q, q) \rangle q - \langle \bar{p}, B(q, q) \rangle \bar{q} \quad (24)$$

$$H_{11} = B(q, \bar{q}) - \langle p, B(q, \bar{q}) \rangle q - \langle \bar{p}, B(q, \bar{q}) \rangle \bar{q}$$

$$H_{02} = B(\bar{q}, \bar{q}) - \langle p, B(\bar{q}, \bar{q}) \rangle q - \langle \bar{p}, B(\bar{q}, \bar{q}) \rangle \bar{q}$$

Substituting of (24) into (23), results:

$$\dot{z} = j\omega_0 z + \frac{1}{2}g_{20}z^2 + g_{11}z\bar{z} + \frac{1}{2}g_{02}\bar{z}^2 + \frac{1}{2}g_{21}z^2\bar{z} + \dots \quad (25)$$

where $\langle \cdot, \cdot \rangle$ is the standard scalar product. In the coordinates of (11), system (9) has the form:

$$\begin{cases} \dot{z} = j\omega_0 z + \langle p, F(zq + \bar{z}\bar{q} + y) \rangle \\ \dot{y} = Ay + F(zq + \bar{z}\bar{q} + y) - \langle p, F(zq + \bar{z}\bar{q} + y) \rangle q - \langle \bar{p}, F(zq + \bar{z}\bar{q} + y) \rangle \bar{q} \end{cases} \quad (12)$$

System (12) is $(n-2)$ -dimensional, but one has to remember the two real constraints imposed on y . The system can now be written in a form (13) as follows:

The center manifold now has the representation:

$$y = V(z, \bar{z}) = \frac{1}{2}\omega_{20}z^2 + \omega_{11}z\bar{z} + \frac{1}{2}\omega_{02}\bar{z}^2 + O(|z|^3) \quad (18)$$

where $\langle p, \omega_{ij} \rangle = 0$. The vectors $\omega_{ij} \in \mathfrak{R}^n$ can be found from the linear equation:

$$\begin{cases} (2j\omega_0 E - A)\omega_{20} = H_{20} \\ -A\omega_{11} = H_{11} \\ (-2j\omega_0 E - A)\omega_{02} = H_{02} \end{cases} \quad (19)$$

These equations have unique solutions since the matrices in their left-hand sides are invertible in the ordinary sense because $0, \pm 2j\omega_0$ are not eigenvalues of A . The restricted equation can be written as:

$$F(x) = \frac{1}{2}B(x, x) + \frac{1}{6}C(x, x, x) + O(\|x\|^4) \quad (21)$$

Then we can express:

$$\langle G_{10}, y \rangle = \langle p, B(q, y) \rangle, \quad \langle G_{01}, y \rangle = \langle p, B(\bar{q}, y) \rangle \quad (22)$$

and write the restricted equation (20) in the form: where

Where

$$\begin{aligned} g_{20} &= \langle p, B(q, q) \rangle, \quad g_{11} = \langle p, B(q, \bar{q}) \rangle, \quad g_{02} = \langle p, B(\bar{q}, \bar{q}) \rangle \\ g_{21} &= \langle p, C(q, q, \bar{q}) \rangle - 2\langle p, B(q, A^{-1}B(q, \bar{q})) \rangle + \langle p, B(\bar{q}, (2j\omega_0 E - A)^{-1} B(q, q)) \rangle + \\ &\quad \frac{1}{j\omega_0} \langle p, B(q, q) \rangle \langle p, B(q, \bar{q}) \rangle - \frac{2}{j\omega_0} |\langle p, B(q, \bar{q}) \rangle|^2 - \frac{1}{3j\omega_0} |\langle p, B(\bar{q}, \bar{q}) \rangle|^2 \end{aligned} \quad (26)$$

Thus, the first Lyapunov coefficient which is achieved by [24]:

$$L_1(0) = \frac{1}{2\omega_0} \operatorname{Re} \left(\langle p, C(q, q, \bar{q}) \rangle - 2\langle p, B(q, A^{-1}B(q, \bar{q})) \rangle + \langle p, B(\bar{q}, (2j\omega_0 E - A)^{-1} B(q, q)) \rangle \right) \quad (27)$$

This formula seems to be the most convenient for analytical treatment of the Hopf bifurcation in n -dimensional systems with $n \geq 2$. It does not require a preliminary transformation of the system into its eigenbasis, and it expresses $L_1(0)$ using original linear, quadratic, and cubic terms, assuming that only the critical (ordinary and adjoint) eigenvectors of the Jacobian matrix are known. The kind of Hopf bifurcation (supper- or sub-critical) is dependent to sign of the first Lyapunov coefficient. If $L_1(0) < 0$, then system (1) close to the equilibrium point is equivalent of system (3) topologically (supper-critical bifurcation) and if $L_1(0) > 0$, then system (1) close to the equilibrium point is equivalent of system (6) topologically (sub-critical bifurcation).

3. Electrical arc furnace mathematical model

In this paper, an EAF model based on an ordinary differential equation (ODE) capable of capturing its essential dynamic characteristics is used [1, 4]:

$$k_1 r^n + k_2 r \frac{dr}{dt} = \frac{k_3}{r^{m+2}} i^2 \quad (28)$$

where r is the arc furnace radius, n is the arc cooling effect. In fact, it varies from 0 to 2 with integer steps; where $n=0$ represents a nondependence of the arc temperature from the arc radius, $n=1$ represents a long arc and nonhot environment surrounding the arc and $n=2$ if the arc cooling is proportional to the electrodes cross-section. m is the inner arc temperature effect. It also varies from 0 to 2 with integer steps; where $m=0$ for a large and colder arc length and $m=2$ for a smaller and hotter arc length. Constants k_1 , k_2 and k_3 represent the arc cooling effect, the derivative proportion of the internal arc energy and the resistivity proportion of the arc column, respectively. These constants have a direct effect on the convergence speed to the system stability, the $v-i$ characteristic and on its equilibrium operation point. The electric circuit

representation of the EAF structure adopted in this paper is given in Fig. 4 [30-32]. In this case, R and L represent the resistance and reactance of the supply system, substation transformer winding, and distribution lines. Parameter L_H represents the reactance of flexible cables, the arc furnace transformer windings and electrodes. Parameters of the system are shown in table 1 [30]. In order to obtain a unified mathematical framework of supply system and EAF, the arc furnace model given by (28) must be coupled with the network equations. The resulting state space matrix equation is given by (29), which can be expressed in compact form as (1).

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \\ \dot{i}_H \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} & 0 & 0 \\ \frac{1}{C} & 0 & \frac{-1}{C} & 0 \\ 0 & \frac{1}{L_H} & \frac{-k_3 r^{-(m+2)}}{L_H} & 0 \\ 0 & 0 & \frac{k_3 r^{-(m+3)} i_H}{k_2} & \frac{-k_1 r^{(n-2)}}{k_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \\ i_H \\ r \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \end{bmatrix} V_s \quad (29)$$

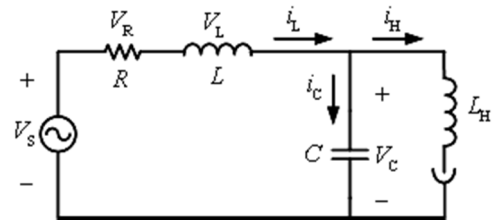


Fig. (4): Equivalent circuit of a EAF system connected to the power network

4. Bifurcation analysis of grid-connected EAF system

The equilibria of the system (29) are zeros of the field given by its right-hand side. After some mathematical computation two equilibria will be achieved as:

$$r_0 = 3.7625 \text{ cm}, \quad i_{L0} = i_{H0} = 8.6979 \text{ pu}, \quad v_{C0} = 0.1302 \text{ pu} \quad (30)$$

$$r'_0 = 0.4908 \text{ cm}, \quad i_{L0}' = i_{H0}' = 0.0193 \text{ pu}, \quad v_{C0}' = 0.9981 \text{ pu} \quad (31)$$

Jakobian matrix of the system (29) based on the equilibria has the representation of:

$$A = \begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} & 0 & 0 \\ \frac{1}{C} & 0 & \frac{-1}{C} & 0 \\ 0 & \frac{1}{L_H} & \frac{-k_3 r^{-(m+2)}}{L_H} & \frac{(m+2)k_3 r^{-(m+3)} i_H}{L_H} \\ 0 & 0 & \frac{2k_3 r^{-(m+3)} i_H}{k_2} & \frac{-(m+3)k_3 r^{-(m+4)} i_H^2}{k_2} - \frac{(n-1)k_1 r^{(n-2)}}{k_2} \end{bmatrix} \quad (32)$$

solving the characteristic equation $|\lambda I - A| = 0$ based on first and second equilibrium of (30) and (31), respectively results:

$$\lambda_{1,2} = -0.2386 \pm j14.1232, \lambda_3 = -0.4752, \lambda_4 = -96.1974 \quad (33)$$

$$\lambda_{1,2} = -0.2498 \pm j10.0921, \lambda_3 = -638.587, \lambda_4 = 25.2649 \quad (34)$$

According to the negative sign of all real values of eigenvalues for the first equilibrium point, one can deduce that it is stable and the second one is not. Thus, the system has just one stable equilibrium point. It is easy to check that a Hopf bifurcation takes place if the bifurcation parameter L reaches to value $L = L_0 = 0.4606 pu$. Actually, a pair of complex conjugate eigenvalues cross the imaginary axis at $\lambda_{1,2} = \pm j\omega_0$, $\omega_0 = 11.0224 > 0$. In this case two other

eigenvalues are $\lambda_3 = -0.1694$ and $\lambda_4 = -96.1974$ which are real and negative. In order to find the kind of Hopf bifurcation, according to (27), the first Lyapunov coefficient should be computed. First of all, the equilibrium point should be shifted to the origin, that is, $i_L = I_L + i_{L0}$, $v_C = V_C + v_{C0}$, $i_H = I_H + i_{H0}$, $r = \Re + r_0$ where i_{L0} , v_{C0} , i_{H0} and r_0 are given in (30). Substituting (29) results as:

$$\dot{i}_L = \left(\frac{-R}{L}\right)I_L + \left(\frac{-1}{L}\right)V_C + \left[\left(\frac{-R}{L}\right)i_{L0} + \left(\frac{-1}{L}\right)v_{C0} + \left(\frac{1}{L}\right)V_S\right] \quad (35)$$

$$\dot{v}_C = \left(\frac{1}{C}\right)I_L + \left(\frac{-1}{C}\right)I_H + \left[\left(\frac{1}{C}\right)i_{L0} + \left(\frac{-1}{C}\right)i_{H0}\right] \quad (36)$$

$$\dot{i}_H = \left(\frac{1}{L_H}\right)V_C + \left(\frac{-k_3}{L_H}\right)(\Re + r_0)^{-(m+2)} I_H + \left(\frac{-k_3}{L_H}\right)(\Re + r_0)^{-(m+2)} i_{H0} + \left[\left(\frac{1}{L_H}\right)v_{C0}\right] \quad (37)$$

$$\dot{\Re} = \left(\frac{k_3}{k_2}\right)(\Re + r_0)^{-(m+3)} (I_H + i_{H0})^2 + \left(\frac{-k_1}{k_2}\right)(\Re + r_0)^{(n-1)} \quad (38)$$

Taylor series expansion based on the origin is defined as:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} (x)^n = f(0) + \frac{1}{1!} f^{(1)}(0)x + \frac{1}{2!} f^{(2)}(0)x^2 + \frac{1}{3!} f^{(3)}(0)x^3 + \dots \quad (39)$$

Based on (39), Taylor series for functions in the right hand side of equations (37) and (38), results in:

$$\begin{aligned} \dot{i}_H &= \left(\frac{1}{L_H}\right)V_C + \left(\frac{-k_3 r_0^{-(m+2)}}{L_H}\right)I_H + \left(\frac{k_3(m+2)(r_0)^{-(m+3)} i_{H0}}{L_H}\right)\Re + \left(\frac{k_3(m+2)(r_0)^{-(m+3)}}{L_H}\right)\Re I_H + \\ &\left(\frac{-k_3(m+2)(m+3)(r_0)^{-(m+4)} i_{H0}}{2L_H}\right)\Re^2 + \left(\frac{-k_3(m+2)(m+3)(r_0)^{-(m+4)}}{2L_H}\right)\Re^2 I_H + \\ &\left(\frac{k_3(m+2)(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}}{6L_H}\right)\Re^3 + \left(\frac{k_3(m+2)(m+3)(m+4)(r_0)^{-(m+5)}}{6L_H}\right)\Re^3 I_H + \\ &\left(\frac{-k_3(m+2)(m+3)(m+4)(m+5)(r_0)^{-(m+6)} i_{H0}}{24L_H}\right)\Re^4 + \left(\frac{-k_3 r_0^{-(m+2)} i_{H0}}{L_H}\right) + \left(\frac{v_{C0}}{L_H}\right) + O(\Re^4 I_H + \Re^5) \quad (40) \\ \dot{\Re} &= \left(\frac{2k_3 r_0^{-(m+3)} i_{H0}}{k_2}\right)I_H + \left(\frac{-k_3(m+3)(r_0)^{-(m+4)} i_{H0}^2 - k_1(n-1)r_0^{(n-2)}}{k_2}\right)\Re + \left(\frac{-2k_3(m+3)(r_0)^{-(m+4)} i_{H0}}{k_2}\right)\Re I_H \\ &+ \left(\frac{k_3(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}^2 - k_1(n-1)(n-2)r_0^{(n-3)}}{2k_2}\right)\Re^2 + \left(\frac{k_3(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}}{k_2}\right)\Re^2 I_H \end{aligned}$$

$$\begin{aligned}
 & + \left(\frac{k_3 r_0^{-(m+3)}}{k_2} \right) I_H^2 + \left(\frac{-k_3 (m+3)(r_0)^{-(m+4)}}{k_2} \right) \Re I_H^2 + \left(\frac{k_3 (m+3)(m+4)(r_0)^{-(m+5)}}{2k_2} \right) \Re^2 I_H^2 + \left(\frac{k_3 r_0^{-(m+3)} i_{H0}^2}{k_2} \right) \\
 & + \left(\frac{-k_3 (m+3)(m+4)(m+5)(r_0)^{-(m+6)} i_{H0}^2}{6k_2} \right) \Re^3 + \left(\frac{-k_3 (m+3)(m+4)(m+5)(r_0)^{-(m+6)} i_{H0}}{3k_2} \right) \Re^3 I_H + \\
 & \left(\frac{k_3 (m+3)(m+4)(m+5)(m+6)(r_0)^{-(m+7)} i_{H0}^2}{24k_2} \right) \Re^4 + \left(\frac{-k_1 r_0^{(n-1)}}{k_2} \right) + O(\Re^3 I_H^2 + \Re^4 I_H + \Re^5) \quad (41)
 \end{aligned}$$

Now, multilinear functions defined as (21) should be computed. If $\xi_1 = I_L$, $\xi_2 = V_C$, $\xi_3 = I_H$, $\xi_4 = R$ then:

$$B_i(x, y) \Big|_{i=1,2,3,4} = \sum_{j,k=1}^4 \frac{\partial^2 F_i(\xi)}{\partial \xi_j \partial \xi_k} \Big|_{\xi=0} x_j y_k \quad (42)$$

$$C_i(x, y, z) \Big|_{i=1,2,3,4} = \sum_{j,k,g=1}^4 \frac{\partial^3 F_i(\xi)}{\partial \xi_j \partial \xi_k \partial \xi_g} \Big|_{\xi=0} x_j y_k z_g \quad (43)$$

Functions $B(x, y)$ and $C(x, y, z)$ are equal to:

$$B(x, y) = [0 \quad 0 \quad B_3(x, y) \quad B_4(x, y)]^T \quad (44)$$

$$C(x, y, z) = [0 \quad 0 \quad C_3(x, y, z) \quad C_4(x, y, z)]^T \quad (45)$$

Setting $i = 3, 4$ in equation (42) and (43) yields:

$$B_3(x, y) = \left(\frac{k_3 (m+2)(r_0)^{-(m+3)}}{L_H} \right) (x_3 y_4 + x_4 y_3) + \left(\frac{-k_3 (m+2)(m+3)(r_0)^{-(m+4)} i_{H0}}{L_H} \right) (x_4 y_4) \quad (46)$$

$$\begin{aligned}
 B_4(x, y) = & \left(\frac{2k_3 r_0^{-(m+3)}}{k_2} \right) (x_3 y_3) + \left(\frac{-2k_3 (m+3)(r_0)^{-(m+4)} i_{H0}}{k_2} \right) (x_3 y_4 + x_4 y_3) + \\
 & \left(\frac{k_3 (m+3)(m+4)(r_0)^{-(m+5)} i_{H0}^2 - k_1 (n-1)(n-2) r_0^{(n-3)}}{k_2} \right) (x_4 y_4) \quad (47)
 \end{aligned}$$

$$\begin{aligned}
 C_3(x, y, z) = & \left(\frac{-k_3 (m+2)(m+3)(r_0)^{-(m+4)}}{L_H} \right) (x_3 y_4 z_4 + x_4 y_3 z_4 + x_4 y_4 z_3) + \\
 & \left(\frac{k_3 (m+2)(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}}{L_H} \right) (x_4 y_4 z_4) \quad (48)
 \end{aligned}$$

$$\begin{aligned}
 C_4(x, y, z) = & \left(\frac{-2k_3 (m+3)(r_0)^{-(m+4)}}{k_2} \right) (x_3 y_3 z_4 + x_3 y_4 z_3 + x_4 y_3 z_3) + \left(\frac{2k_3 (m+3)(m+4)(r_0)^{-(m+5)} i_{H0}}{k_2} \right) \times \\
 & (x_3 y_4 z_4 + x_4 y_3 z_4 + x_4 y_4 z_3) + \left(\frac{-k_3 (m+3)(m+4)(m+5)(r_0)^{-(m+6)} i_{H0}^2}{k_2} \right) (x_4 y_4 z_4) \quad (49)
 \end{aligned}$$

Jacobian matrix based on the stable equilibrium point is:

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ A_{21} & 0 & A_{23} & 0 \\ 0 & A_{32} & A_{33} & A_{34} \\ 0 & 0 & A_{43} & A_{44} \end{bmatrix} \quad (50)$$

in which $A_{11} = \frac{-R}{L}$, $A_{12} = \frac{-1}{L}$, $A_{21} = \frac{1}{C}$, $A_{23} = \frac{-1}{C}$

$$, \quad A_{32} = \frac{1}{L_H}, \quad A_{33} = \frac{-k_3 r_0^{-(m+2)}}{L_H},$$

$$A_{34} = \frac{(m+2)k_3 r_0^{-(m+3)} i_{H0}}{L_H}, \quad A_{43} = \frac{2k_3 r_0^{-(m+3)} i_{H0}}{k_2} \quad \text{and}$$

$A_{44} = \frac{-(m+3)k_3 r_0^{-(m+4)} i_{H0}^2}{k_2} - \frac{(n-1)k_1 r_0^{(n-2)}}{k_2}$. Eigenvector related to the eigenvalue $\lambda = j\omega_0$ is equal to $Aq = j\omega_0 q$, $q = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$ and adjoint eigenvector related to the eigenvalue $\lambda = j\omega_0$ is equal

to $A^T p' = -j\omega_0 p'$, $p' = [p'_1 \ p'_2 \ p'_3 \ p'_4]^T$. Now, the eigenvectors should be normalized as $\langle p', q \rangle = \sum_{i=1}^n \bar{p}'_i q_i = 1$ where \bar{p}'_i is conjugate complex of p'_i . Now, N is defined as $N = \langle p', q \rangle = \bar{p}'_1 q_1 + \bar{p}'_2 q_2 + \bar{p}'_3 q_3 + \bar{p}'_4 q_4$. If p is set to $p = 1/\bar{N} p'$ then $\langle p, q \rangle = \langle \frac{1}{\bar{N}} p', q \rangle = \frac{1}{\bar{N}} \langle p', q \rangle = 1$.

Now, we are going to find components of equation (27):

$$B(q, \bar{q}) = [0 \ 0 \ B_3(q, \bar{q}) \ B_4(q, \bar{q})]^T \quad (51)$$

$$B(q, q) = [0 \ 0 \ B_3(q, q) \ B_4(q, q)]^T \quad (52)$$

$$C(q, q, \bar{q}) = [0 \ 0 \ C_3(q, q, \bar{q}) \ C_4(q, q, \bar{q})]^T \quad (53)$$

$$E = A^{-1} B(q, \bar{q}) = [E_1 \ E_2 \ E_3 \ E_4]^T \quad (54)$$

$$G = (2j\omega_0 I - A)^{-1} B(q, q) = [G_1 \ G_2 \ G_3 \ G_4]^T \quad (55)$$

where I is 4×4 identity matrix and:

$$B_3(q, \bar{q}) = \left(\frac{k_3(m+2)(r_0)^{-(m+3)}}{L_H} \right) (q_3 \bar{q}_4 + q_4 \bar{q}_3) + \left(\frac{-k_3(m+2)(m+3)(r_0)^{-(m+4)} i_{H0}}{L_H} \right) (q_4 \bar{q}_4) \quad (56)$$

$$B_4(q, \bar{q}) = \left(\frac{2k_3 r_0^{-(m+3)}}{k_2} \right) (q_3 \bar{q}_3) + \left(\frac{-2k_3(m+3)(r_0)^{-(m+4)} i_{H0}}{k_2} \right) (q_3 \bar{q}_4 + q_4 \bar{q}_3) + \left(\frac{k_3(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}^2 - k_1(n-1)(n-2)r_0^{(n-3)}}{k_2} \right) (q_4 \bar{q}_4) \quad (57)$$

$$B_3(q, q) = \left(\frac{2k_3(m+2)(r_0)^{-(m+3)}}{L_H} \right) (q_3 q_4) + \left(\frac{-k_3(m+2)(m+3)(r_0)^{-(m+4)} i_{H0}}{L_H} \right) (q_4^2) \quad (58)$$

$$B_4(q, q) = \left(\frac{2k_3 r_0^{-(m+3)}}{k_2} \right) (q_3^2) + \left(\frac{-4k_3(m+3)(r_0)^{-(m+4)} i_{H0}}{k_2} \right) (q_3 q_4) + \left(\frac{k_3(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}^2 - k_1(n-1)(n-2)r_0^{(n-3)}}{k_2} \right) (q_4^2) \quad (59)$$

$$C_3(q, q, \bar{q}) = \left(\frac{-k_3(m+2)(m+3)(r_0)^{-(m+4)}}{L_H} \right) (2q_3 q_4 \bar{q}_4 + q_4^2 \bar{q}_3) + \left(\frac{k_3(m+2)(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}}{L_H} \right) \times (q_4^2 \bar{q}_4) \quad (60)$$

$$C_4(q, q, \bar{q}) = \left(\frac{-2k_3(m+3)(r_0)^{-(m+4)}}{k_2} \right) (q_3^2 \bar{q}_4 + 2q_3 q_4 \bar{q}_3) + \left(\frac{2k_3(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}}{k_2} \right) \times (2q_3 q_4 \bar{q}_4 + q_4^2 \bar{q}_3) + \left(\frac{-k_3(m+3)(m+4)(m+5)(r_0)^{-(m+6)} i_{H0}^2}{k_2} \right) (q_4^2 \bar{q}_4) \quad (61)$$

$$B_3(q, E) = \left(\frac{k_3(m+2)(r_0)^{-(m+3)}}{L_H} \right) (q_3 E_4 + q_4 E_3) + \left(\frac{-k_3(m+2)(m+3)(r_0)^{-(m+4)} i_{H0}}{L_H} \right) (q_4 E_4) \quad (62)$$

$$B_4(q, E) = \left(\frac{2k_3 r_0^{-(m+3)}}{k_2} \right) (q_3 E_3) + \left(\frac{-2k_3(m+3)(r_0)^{-(m+4)} i_{H0}}{k_2} \right) (q_3 E_4 + q_4 E_3) + \left(\frac{k_3(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}^2 - k_1(n-1)(n-2)r_0^{(n-3)}}{k_2} \right) (q_4 E_4) \quad (63)$$

$$B_3(\bar{q}, G) = \left(\frac{k_3(m+2)(r_0)^{-(m+3)}}{L_H} \right) (\bar{q}_3 G_4 + \bar{q}_4 G_3) + \left(\frac{-k_3(m+2)(m+3)(r_0)^{-(m+4)} i_{H0}}{L_H} \right) (\bar{q}_4 G_4) \quad (64)$$

$$B_4(\bar{q}, G) = \left(\frac{2k_3 r_0^{-(m+3)}}{k_2} \right) (\bar{q}_3 G_3) + \left(\frac{-2k_3(m+3)(r_0)^{-(m+4)} i_{H0}}{k_2} \right) (\bar{q}_3 G_4 + \bar{q}_4 G_3) + \left(\frac{k_3(m+3)(m+4)(r_0)^{-(m+5)} i_{H0}^2 - k_1(n-1)(n-2)r_0^{(n-3)}}{k_2} \right) (\bar{q}_4 G_4) \quad (65)$$

Now, equation (27) is rewritten:

$$L_1(L_0) = \frac{1}{2\omega_0} \text{Re}(\langle p, C(q, q, \bar{q}) \rangle - 2\langle p, B(q, E) \rangle + \langle p, B(\bar{q}, G) \rangle) \quad (66)$$

Table (1): System parameters [30]

C	L	R	L_H	V_s	m	n	k_1	k_2	k_3
0.1 pu	0.1 pu	0.1 pu	0.1 pu	1 pu	2	2	0.08	0.005	3

Using software MATLAB the computations can be done. The result is $L_1(L_0) = 1.5732 \times 10^{-5}$. The Lyapunov coefficient is clearly positive. Thus, the Hopf bifurcation is sub-critical. Therefore, there is an unstable limit cycle in (29), which disappears when the bifurcation parameter L crosses $L_0 = 0.4606$ pu from fewer values to more ones. The equilibrium point is stable for $L < L_0$ and unstable for $L > L_0$. It is nonlinearly unstable at the critical parameter value. The region of attraction of the equilibrium point is bounded by the unstable cycle, which shrinks as the parameter approaches its critical value and disappears. Thus, the system is pushed out from a neighborhood of the equilibrium, giving us a sharp or catastrophic loss of stability. In the following the EAF system is simulated by software AUTO to compare the results with the analytic investigation. Furthermore, it is possible to find out possible bifurcation points in the limit cycles in the case of bifurcation parameter changes. The result is shown in Fig. 5. This figure relates the current magnitude in the inductor, i_L , to the bifurcation parameter L , which in this case is the system inductance. A Hopf bifurcation point was found when the inductance reached a $L_0 = 0.4606$ pu value, as illustrated in Fig. 5. The solid and open circles indicate the stable and unstable trajectories in the periodic orbit. In fact, when the inductance reached a $L'_0 = 0.3251$ pu value, fold bifurcation of cycles happens, as shown in the zoom plot of Fig. 6. b by the labels UHB (unstable/subcritical Hopf bifurcation) and SHB (stable/supercritical Hopf bifurcation) points. Therefore, two limit cycles (one stable and the other unstable) collide and disappear at the bifurcation parameter L'_0 . Fig. 7, generally shows fold bifurcation of limit cycles. Assume that at $\alpha = 0$ the cycle has a simple multiplier $\mu_1 = 1$ and its other multiplier satisfies $0 < \mu_2 < 1$. The restriction of Poincare map P_α to the invariant manifold W_α^c is a one-dimensional map, having a fixed point with

$\mu_1 = 1$ at $\alpha = 0$. This generically implies the collision and disappearance of two fixed points of P_α as α passes through zero. Under our assumption on μ_2 , this happens on a one-dimensional attracting invariant manifold of P_α . Thus, a stable and a saddle fixed point are involved in the bifurcation. Each fixed point of the Poincare map corresponds to a limit cycle of the continuous-time system. Therefore, two limit cycles (stable and saddle) collide and disappear in the system at this bifurcation.

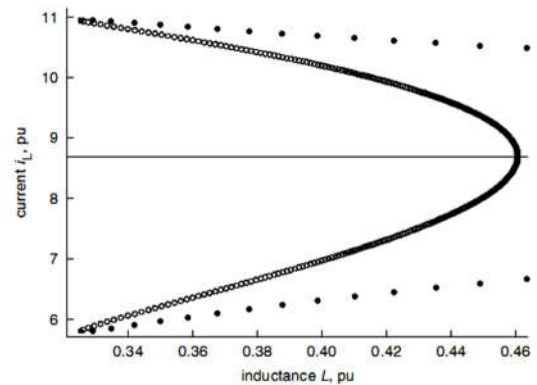
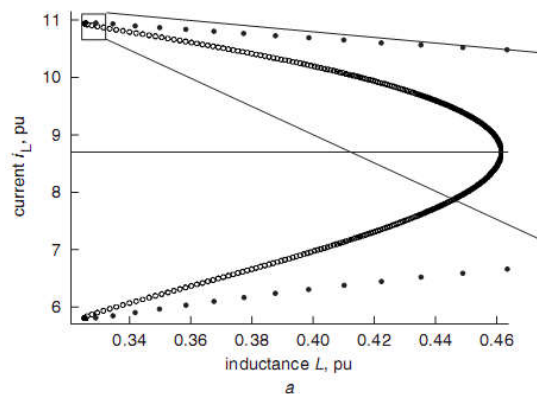


Fig. (5): Hopf bifurcation behaviour



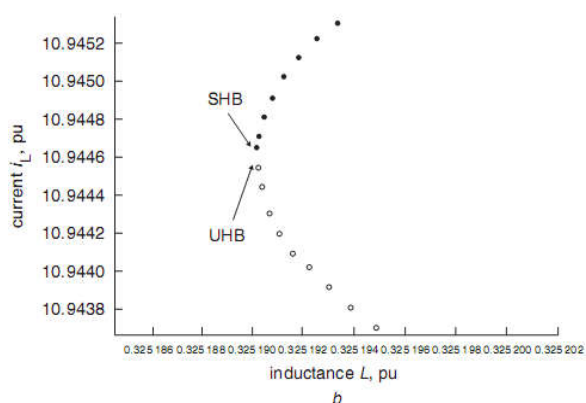


Fig. (6): Hopf bifurcation behaviour

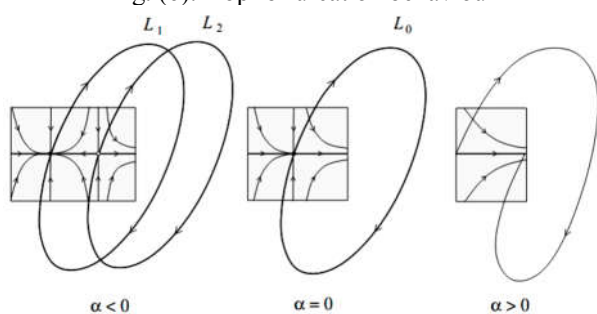


Fig. (7): Fold bifurcation of limit cycles

5. Conclusion

In the present paper, an electric arc furnace model based on the instantaneous power balance was used. Stability analysis of the arc furnace based on bifurcations theory has been presented in the present contribution. Based on both analytic investigation and computer simulation was observed that a bifurcation parameter led to a sub-critical Hopf bifurcation which in turn resulted in stable and unstable zones. It was shown that Hopf bifurcation is the basis for the emergence of unstable periodic oscillations. It has been demonstrated that bifurcation theory allows a qualitative assessment and identification of the expected system dynamics at different operation points without resorting to time-domain simulations. In fact, projection method was applied to find out the kind of Hopf bifurcation. Also a fold bifurcation of cycles took place for some value of the bifurcation parameter. It should be remarked that the Hopf bifurcation and also fold bifurcation of cycles were obtained for inductance values well within the system operation range.

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