Integer-order Versus Fractional-order Adaptive Fuzzy Control of Electrically Driven Robots with Elastic Joints

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Abstract
Real-time robust adaptive fuzzy fractional-order control of electrically driven flexible-joint robots has been addressed in this paper. Two important practical situations have been considered: the fact that robot actuators have limited voltage, and the fact that current signals are contaminated with noise. Through of a novel voltage-based fractional order control for an integer-order dynamical system and based on a Lyapunov's functions analysis, it is shown that the overall closed-loop system is robust, BIBO stable and the joint position tracking error is uniformly bounded. The satisfactory performance in lower energy consumption of the proposed fractional control scheme is verified in comparison with a standard integer-order controller by experimental results.

Keywords
Actuator Saturation, Direct Adaptive Fuzzy Control, Flexible-joint Robots, Fractional-order Control

1 Introduction

Many advanced control theories have been devoted to flexible joint robots using various control techniques such as nonlinear adaptive control [1], passivity-based control [2], adaptive back-stepping control [3], global position-feedback tracking control [4], Singular perturbation approach [5-6], predictive control [7], adaptive fuzzy approaches [8-9], hierarchical sliding mode control [10], dynamic surface control [11], and higher-order differential feedback control [12]. Majority of them have not considered the actuator electrical subsystem in the control design procedure. In other words, their control laws calculate the desired torque that should be applied to the manipulator joints.

Since most robotic systems use electrical motors as actuators, recently, some voltage-based controllers have been presented for electrically driven flexible joint robot (EDFJR) manipulators [13-14]. It proposed a robust control scheme in presence of uncertainties associated with both motor and robot dynamics. The controller design strategy is based on the actuators' electrical subsystems considering to voltage saturation nonlinearity. Hence, the knowledge of the actuator/manipulator dynamics model is not required as it is for many other control strategies. An extended form of this work has also been presented, [14], which assume availability of the motor signals. The advantage of these approaches is two-loop instead of three-loop control structure, which makes them superior to others. Nevertheless, their measurement requirements are substantial. As an extension in the field of EDFJR, [15] proposed a single-loop control scheme. This approach is superior since it is based on first-order dynamic model of EDFJR.

Studying the literature on the field of fractional calculus in recent years, confirms that equipping traditional control methods with fractional-order operators can enhance the closed-loop system performance by improving transient and steady-state response. Recently, a valuable finite-time robust controller has been developed by incorporating the adaptive back stepping concept into fractional order controller design [16]. The proposed control scheme is in the torque-level. Thus, it requires converting to the voltage-level for real-time implementation. In addition to this, it does not consider the role of actuator saturation in controller design, and stability analysis.

In this paper, we present a direct adaptive fuzzy fractional-order control for EDFJR by considering to saturation nonlinearity in the voltage-level. To the best of the author's knowledge, the proposed control scheme has not been investigated previously for EDFJR. The overall closed-loop system is proven to be BIBO stable and the joint position tracking errors are uniformly bounded based on the Lyapunov's direct method. Compared with the same integer-order control law, the proposed approach has lower energy consumption.

The rest of this paper is as follows. In section 2, some basic concepts about fractional calculus are presented. Section 3 introduces the model of an n-link flexible joint robot manipulator. Section 4 presents direct adaptive fuzzy fractional-order controller design. The stability analysis is also discussed in this section. In section 5, some experimental results are provided and finally, some conclusions are given in Section 6. Throughout this paper, we present the vectors and matrices in bold form; $\underline{\lambda}(\cdot)$ and $\overline{\lambda}(\cdot)$ indicate the smallest and largest eigen values, respectively, of a positive definite bounded matrix; and finally $\|\cdot\|$ indicate the Euclidian norm of a vector/matrix.

2. Preliminaries of Fractional Calculus

Definition1. The Caputo fractional derivative of the order α is defined as [17]

$$ {}_{t_0}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau. \quad t>t_0 \tag{1} $$

Where t_0 and t are the bounds of the operation, $n = \min\{k \in \mathbb{N} \,/\, k > \alpha > 0\}$, and $\Gamma(n)$ denotes the famous Gamma function, which is defined as

$$ \Gamma(n) = \int_{a}^{\infty} t^{n-1} e^{-\tau} d\tau \tag{2} $$

Definition2. Fractional integration of the order α of $f \in L^1([0,T])$, i.e. $\int_0^t |f(\tau)| d\tau < \infty$, is defined as

$$ I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{0}^{t} (t-\tau)^{\alpha-1} f(\tau) d\tau \tag{3} $$

For $t \in (0,T]$ [18].

Definition 3. The Caputo derivative of order $\alpha > 0$ of function $f \in C^n([0,T])$, i.e. f having continuous first *n* derivatives, is defined as $D^{\alpha} f(t) = I^{n-\alpha} D^n f(t)$, where $n = [\alpha]$ [18].

Property 1: The Caputo derivative of a constant is zero.

Property 2: For the Caputo derivative, we have $\lim_{\alpha \to n} {}_{t_0}^{C}D_t^{\alpha} f(t) = f^{(n)}(t)$.

Property 3: For the Caputo derivative, we have ${}_{t_0}^{C}D_t^{\alpha}({}_{t_0}^{C}D_t^{-\alpha} f(t)) = f(t)$.

The following Lemma plays also a key part to establish the subsequent control development and analysis.

Lemma 1: Let $x(t) \in \Re^{\beta}$. The Caputo fractional derivative of a differentiable function in quadratic form is as follows [19]

$$\frac{1}{2} {}_{t_0}^{C}D_t^{\alpha}(x^T(t)Px(t)) \le x^T(t)P\, {}_{t_0}^{C}D_t^{\alpha} x(t), \qquad \forall \alpha \in (0,1], \forall t \ge t_0$$

Where $P \in \Re^{\beta \times \beta}$ is a constant matrix.

3. Robot Dynamics

The dynamics of an electrically driven flexible-joint robot can be described by

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{K}(\mathbf{r}\boldsymbol{\theta}_m - \mathbf{q}) \tag{4}$$

$$\mathbf{J}\ddot{\boldsymbol{\theta}}_m + \mathbf{B}\dot{\boldsymbol{\theta}}_m + \mathbf{rK}(\mathbf{r}\boldsymbol{\theta}_m - \mathbf{q}) = \mathbf{K}_m\mathbf{I}_a \tag{5}$$

$$\mathbf{R}\mathbf{I}_a + \mathbf{L}\dot{\mathbf{I}}_a + \mathbf{K}_b\dot{\boldsymbol{\theta}}_m = \mathbf{v}(t) \tag{6}$$

where $\mathbf{q}$ is the n-vector of joint angles, $\mathbf{D}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$ is the n-vector of centrifugal and Coriol is forces, $\mathbf{g}(\mathbf{q})$ is the gravitational forces vector, $\boldsymbol{\theta}_m$ is the n-vector of motor angles, $\mathbf{I}_a$ is the n-vector of armature current, and $\mathbf{v}(t)$ is the n-vector control input voltage to the actuators. $\mathbf{J}$, $\mathbf{B}$, $\mathbf{r}$, $\mathbf{K}_m$, $\mathbf{L}$, $\mathbf{R}$, $\mathbf{K}_b$, and $\mathbf{K}$ are $n \times n$ constant diagonal matrices of actuator inertias, damping, gear-box ratio, torque constant, electrical inductance, electrical resistance, back-emf effects, and joint stiffness, respectively.

3. Direct Adaptive Fuzzy Fractional-order Control Design

Equations (4)-(6) represent a fifth-order highly nonlinear dynamic system that makes the control problem extremely difficult. To cope with this problem, a direct adaptive fuzzy fractional-order controller is developed based on the first order dynamic model of EDFJR by employing voltage as control input. The controller design procedure start by adding and subtracting ${}_{t_0}^{C}D_t^{\alpha} z(t)$, to the left hand side of actuator electrical subsystem in decentralized form as

$${}_{t_0}^{C}D_t^{\alpha} z(t) - {}_{t_0}^{C}D_t^{\alpha} z(t) + RI_a + L\dot{I}_a + K_b\dot{\theta}_m = \mathrm{v}(t) \tag{7}$$

Where $z(t) = {}_{t_0}^{C}D_t^{\alpha} q(t)$. Let us define

$$r\theta_m - q = \delta \tag{8}$$

Where δ represents the effect of joint flexibility. Combining equations (7), (8), and introducing

$$\wp = RI_a + L\dot{I}_a + K_b r^{-1}\dot{\delta} + K_b r^{-1}\dot{q} - {}_{t_0}^{C}D_t^{\alpha} z(t) \quad (9)$$

Called as residual uncertainty, this leads to

$${}_{t_0}^{C}D_t^{\alpha} z(t) + \wp = \mathrm{v}(t) \quad (10)$$

From practical point of view, the range of actuator input may limit by some upper and lower bound [20-21]. Suppose that the input limitation is described as

$$\mathrm{v}(t)=\mathrm{sat}(u(t)) \quad (11)$$

Where $\mathrm{v}(t)$ is the actual actuator input, $u(t)$ is the controller output, $\mathrm{sat}(\cdot):\Re \to \Re$ represents saturation function, $\mathrm{sat}(u(t)) = \mathrm{sgn}(u(t))\min\{\xi_u, |u(t)|\}$, and $\xi_u > 0$ is the maximum admissible voltage of the motor. When controller output falls outside linear range of the actuator operation, actuator saturation occurs. The non-implemented control signal by the device, denoted as $\mathrm{dzn}(u(t),\xi_u)$, is then given by [20-21]

$$\mathrm{dzn}(u(t),\xi_u)=u(t)-\mathrm{sat}(u(t)) \quad (12)$$

Where $\mathrm{dzn}(u(t),\xi_u)$ represents dead-zone function. Now, substituting (11) into (10), and using (12), it follows that

$${}_{t_0}^{C}D_t^{\alpha} z(t) + \wp = u(t) - \mathrm{dzn}(u(t),\xi_u) \quad (13)$$

Remark 1: Equation (11) indicates that the motor voltage is bounded, i.e,

$$|v(t)| \le \xi_u \quad (14)$$

As a result, the variables I_a, $\dot{I}_a$, and $\dot{\theta}_m$ are upper bounded [20].

The considerable point is that the uncertain term $\wp$ cannot be evaluated directly, since the actual values of the motors' dynamic are unknown. In addition to these, there is problem arises from torque measurements as mentioned in [22]. Under these circumstances, using the Mamdani inference-engine, singleton-fuzzifier and center-average defuzzifier, a direct adaptive fuzzy fractional-order control is proposed in the form of

$$u(t) = \hat{\mathbf{y}}^{\mathrm{T}} \boldsymbol{\psi}(x_1, x_2) \quad (15)$$

Where $\hat{\mathbf{y}} \in \Re^M$ is the estimation of $\mathbf{y}$ used into a fuzzy system $\mathbf{y}^{\mathrm{T}}\boldsymbol{\psi}(x_1,x_2)$ which approximates the following function based on the universal approximation theorem of fuzzy systems as

$$\mathbf{y}^{\mathrm{T}} \boldsymbol{\psi}(x_1, x_2) + \varepsilon = {}_{t_0}^{C}D_t^{\alpha} z_d(t) + k_1 {}_{t_0}^{C}D_t^{\alpha} e(t) + k_p x_1 + k_d x_2 + \wp \quad (16)$$

Where $\boldsymbol{\psi}(x_1,x_2) \in \Re^M$ denotes fuzzy basis function vector fixed by the designer, the number M represents the number of linguistic fuzzy rules, ε is reconstruction error of fuzzy logic system;

$z_d(t) = {}_{t_0}^{C}D_t^{\alpha} q_d(t)$ with $q_d(t)$ denoting the desired joint position; k_1, k_p and k_d are positive scalar gains selected as control design parameters,

$$x_1 = e(t) + k_1 \, {}_{t_0}^{C}D_t^{-\alpha} e(t) \tag{17}$$

$$x_1 = e(t) + k_1 \, {}_{t_0}^{C}D_t^{-\alpha} e(t) \tag{18}$$

And

$$e(t) = q_d(t) - q(t) \tag{19}$$

Is the joint position tracking error. Now, applying Equations (15) and (16) to equation (13) provides the closed-loop system as

$${}_{t_0}^{C}D_t^{\alpha} x_2 + k_d x_2 + k_p x_1 = \tilde{\mathbf{y}}^{\mathrm{T}} \boldsymbol{\psi}(x_1, x_2) + \varepsilon + \mathrm{dzn}(\mathrm{u}(t), \xi_u) \tag{20}$$

Where $\tilde{\mathbf{y}} = \mathbf{y} - \hat{\mathbf{y}}$ represents the difference between actual and estimated value of weighting vectors. The state space equation in the tracking space is then obtained using (20) as

$${}_{t_0}^{C}D_t^{\alpha} \mathbf{X} = \mathbf{AX} + \mathbf{B}(\tilde{\mathbf{y}}^{\mathrm{T}} \boldsymbol{\psi} + \varepsilon + \mathrm{dzn}(\mathrm{u}(t), \xi_u)) \tag{21}$$

In which

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -k_p & -k_d \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

A. Stability analysis

To proceed with subsequent stability analysis, the following lemma is required. First, the following two assumptions are enforced.

Assumption 1: The desired joint trajectories and their fractional-order derivatives up to a necessary order are continuous and uniformly bounded

Assumption 2: The reconstruction error ε is bounded, i.e. $|\varepsilon| < c_\varepsilon$ with known c_ε.

Now, we are ready to present the following lemma.

Lemma2. $|\mathrm{dzn}(u(t), \xi_u)|$ satisfies the following bounding inequality:

$$|\mathrm{dzn}(u(t), \xi_u)| \le \frac{\kappa \xi_u}{(1-\kappa)} \tag{22}$$

Where $\kappa = \max\left\{1 - \frac{\xi_u}{u(t)}\right\}$ is a constant smaller than 1.

Proof: Following the same procedure as [20], it can be easily shown that

$$|\mathrm{dzn}(u(t), \xi_u)| \le \kappa |u(t)| \tag{23}$$

According to Equations (15), (16), (18), and (20), the absolute of control signal $u(t)$ is bounded and given by

$$\left|u(t)\right| = \left|{}_{t_0}^{C}D_t^{\alpha} z(t) + \wp + \mathrm{dzn}(u(t), \xi_u)\right| \le \left|{}_{t_0}^{C}D_t^{\alpha} z(t) + \wp\right| + \left|\mathrm{dzn}(\mathrm{u}(t), \xi_u)\right| \tag{24}$$

This result together with Equation (23) yields

$$\left|\mathrm{dzn}(u(t), \xi_u)\right| \le \kappa\left|u(t)\right| \le \kappa\left|{}_{t_0}^{C}D_t^{\alpha} z(t) + \wp\right| + \kappa\left|\mathrm{dzn}(u(t), \xi_u)\right| \tag{25}$$

Now, according to equations (10), (11) and Remark 1, we have

$$\left|\mathrm{dzn}(u(t), \xi_u)\right| \le \frac{\kappa\xi_u}{(1-\kappa)}$$

And proof is completed ■

In order to find the adaptive law for the proposed fractional-order controller and proving the stability of the scheme, let us purpose the following positive definite function

$$V(\mathbf{X}, \tilde{\mathbf{y}}) = \mathbf{X}^T\mathbf{P}\mathbf{X} + \tilde{\mathbf{y}}^T\boldsymbol{\Gamma}\tilde{\mathbf{y}} \tag{26}$$

Where $\boldsymbol{\Gamma} \in \Re^{M\times M}$ is positive definite; and $\mathbf{P}, \mathbf{Q} \in \Re^{2\times 2}$ are the unique symmetric, positive definite matrices satisfying the matrix Lyapunov equation

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q} \tag{27}$$

Taking the Caputo fractional derivative to expression (26) and applying Lemma 1, it follows that

$${}_{t_0}^{C}D_t^{\alpha} V(\mathbf{X}, \tilde{\mathbf{y}}) \le 2\mathbf{X}^T\mathbf{P}\,{}_{t_0}^{C}D_t^{\alpha}\mathbf{X} + 2\tilde{\mathbf{y}}^T\boldsymbol{\Gamma}\,{}_{t_0}^{C}D_t^{\alpha}\tilde{\mathbf{y}} \tag{28}$$

Using expression (21) and (27) in (28), we have

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} V(\mathbf{X}, \tilde{\mathbf{y}}) \le & -\mathbf{X}^T\mathbf{Q}\mathbf{X} + 2\mathbf{X}^T\mathbf{P}\mathbf{B}\varepsilon + 2\tilde{\mathbf{y}}^T\boldsymbol{\psi}\mathbf{B}^T\mathbf{P}\mathbf{X} \\ & + 2\mathbf{X}^T\mathbf{P}\mathbf{B}\,\mathrm{dzn}(u(t), \xi_u) - 2\tilde{\mathbf{y}}^T\boldsymbol{\Gamma}\,{}_{t_0}^{C}D_t^{\alpha}\hat{\mathbf{y}} \end{aligned} \tag{29}$$

If we choose the adaptive law as

$${}_{t_0}^{C}D_t^{\alpha}\hat{\mathbf{y}} = \boldsymbol{\Gamma}^{-1}\boldsymbol{\psi}\mathbf{B}^T\mathbf{P}\mathbf{X} \tag{30}$$

Therefore, (29) can be further written as

$${}_{t_0}^{C}D_t^{\alpha} V(\mathbf{X}, \tilde{\mathbf{y}}) \le -\underline{\lambda}(\mathbf{Q})\left\|\mathbf{X}\right\|^2 + 2\overline{\lambda}(\mathbf{PB})\left\|\mathbf{X}\right\|\left|\varepsilon\right| + 2\overline{\lambda}(\mathbf{PB})\left\|\mathbf{X}\right\|\left|\mathrm{dzn}(u(t), \xi_u)\right| \tag{31}$$

*Remark 2***:** Suppose a sufficient number of basis functions are used and the approximation error can be ignored. In the case where $\left|u(t)\right| < \xi_u$, we have $\left|\mathrm{dzn}(u(t), \xi_u)\right| = 0$. Hence, (31) can be reduced to

$${}_{t_0}^{C}D_t^{\alpha} V(\mathbf{X}, \tilde{\mathbf{y}}) \le -\underline{\lambda}(\mathbf{Q})\left\|\mathbf{X}\right\|^2 \tag{32}$$

From inequality (32) we can conclude, by fractional integration, that $I_t^{\alpha}\mathbf{X}^T\mathbf{Q}\mathbf{X}<\infty$, which implies that $I_t^{\alpha}\mathbf{X}^T\mathbf{X}<\infty$, and thus, the RMS value of $\mathbf{X}$ converges to zero (see Proposition 1 in [18] for details).

Remark 3: Owing to the existence of ε in (31), it is very easy to prove using Lemma 2 and Assumption 2, that the following inequality hold

$$ {}_{t_0}^{C}D_t^{\alpha}V(\mathbf{X},\tilde{\mathbf{y}}) \le -\underline{\lambda}(\mathbf{Q})\|\mathbf{X}\|^2 + 2\overline{\lambda}(\mathbf{PB})\|\mathbf{X}\|\left(c_{\varepsilon}+\frac{\kappa\xi_u}{(1-\kappa)}\right) \tag{33} $$

As a result, ${}_{t_0}^{C}D_t^{\alpha}V(\mathbf{X},\tilde{\mathbf{y}})$ is negative definite as long as $\|\mathbf{X}\|$ is outside the compact set $\Omega_{\mathbf{X}}$ defined as

$$ \Omega_{\mathbf{X}} = \left\{\mathbf{X}\,\middle|\,\|\mathbf{X}\| \le \frac{2\overline{\lambda}(\mathbf{PB})}{\underline{\lambda}(\mathbf{Q})}\left(c_{\varepsilon}+\frac{\kappa\xi_u}{(1-\kappa)}\right)\right\} \tag{34} $$

Since $V(\mathbf{X},\tilde{\mathbf{y}})$ is positive-definite, this result, together with Theorem 7 from [23], it follows that $(\mathbf{X},\tilde{\mathbf{y}})$ are bounded. According to definition of x_2, the linear fractional-order differential equation ${}_{t_0}^{C}D_t^{\alpha}e(t)+k_1e(t)=x_2$ has the bounded input x_2. It is BIBO stable based on Routh-Hurwitz criteria [24]. Thereby $e(t)$, and ${}_{t_0}^{C}D_t^{\alpha}e(t)$ are bounded. According to assumption 1, $q_d(t)$ and ${}_{t_0}^{C}D_t^{\alpha}q_d(t)$ are bounded. Thus, the bounded variable $e(t)$ and ${}_{t_0}^{C}D_t^{\alpha}e(t)$ implies that $q(t)$ and ${}_{t_0}^{C}D_t^{\alpha}q(t)={}_{t_0}^{C}D_t^{\alpha}q_d(t)-{}_{t_0}^{C}D_t^{\alpha}e(t)$, $\alpha\in(0,1]$ are bounded. From (5) we have

$$ J\ddot{\theta}_m + B\dot{\theta}_m + r^2K\theta_m = rKq + K_mI_a \tag{35} $$

Which is a second-order linear differential equation with the bounded input. So, according to Routh-Hurwitz stability criteria, the variables θ_m, $\dot{\theta}_m$ and $\ddot{\theta}_m$ are bounded. Extending this result to all motors implies the boundedness of system states $\boldsymbol{\theta}_m$ and $\dot{\boldsymbol{\theta}}_m$. Then, our main results can be formulated as the following theorem.

Theorem 1: Let an integer-order system be described as in (4)-(6). By using the Mamdani inference-engine, singleton-fuzzifier and center-average defuzzifier and choosing the robust adaptive fuzzy fractional-order control (15) in the voltage-level and the update laws as equation (30), the tracking error, and the weighting vector are guaranteed to be uniformly bounded. This result together with Remark 1 implies that all states associated with each joint are bounded. As a result, the robotic system is robust and BIBO stable.

4. Experimental Results

For practical implementation of the proposed controller and comparison purpose, a single-link flexible joint electrically driven robot has been considered. A high flexible element made from polyurethane is utilized for power transmission. A geared permanent magnet DC motor made by Barber-Colman Company is the actuator of this system and rotates the flexible element. A pulse-width modulation (PWM) driver is used for exciting the motor. Its permitted input voltage range is

[-12V, +12V]. The other end of the flexible element is connected to a steel arm. To provide the controller with the feedbacks from the motor and joint angular positions, two potentiometers are installed. For Integer-order adaptive fuzzy controller, joint velocity is calculated from joint position measurements by using a discrete filter. The filter is completely described in [25]. We have set the filter parameters p_1 and p_2 to 4. Also, the sampling period of the data acquisition process is set to 10 msec. The measurement data are obtained by the data acquisition (DAQ) Advantech PCLD-8115D which has analog input ports. Using this DAQ, we can implement the controller programmed in MATLAB/SIMULINK practically through Real-Time Windows target libraries. A block diagram of the experimental setup is illustrated in Figure 1.

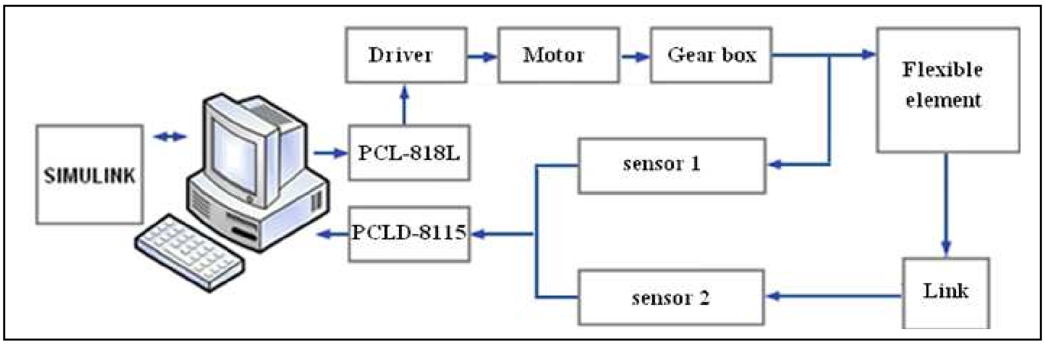

Figure1. Block diagram of the system

To explore the controller ability, performance of the proposed control method is compared with its integer-order form. Both control algorithms are based on the voltage control strategy. The desired trajectory $q_d(t)$ used in all experiments is given by

$$q_d(t) = 1.26 - 0.63\sin(\frac{2\pi}{5}t) \quad (36)$$

The control parameters for both fractional and integer-order form are the same, except that $\alpha = 0.5$ and $\mathbf{\Gamma} = 0.5 I_{(9)}$, where $I_{(\square)}$ denotes the identity matrix. Three membership functions are given to the fuzzy variables x_1 and x_2 in the operating range of the system, as shown by Figure 2. Thus, the whole space is covered by 9 fuzzy rules. The Mamdani-type linguistic fuzzy rules have also been completely explained in [15].

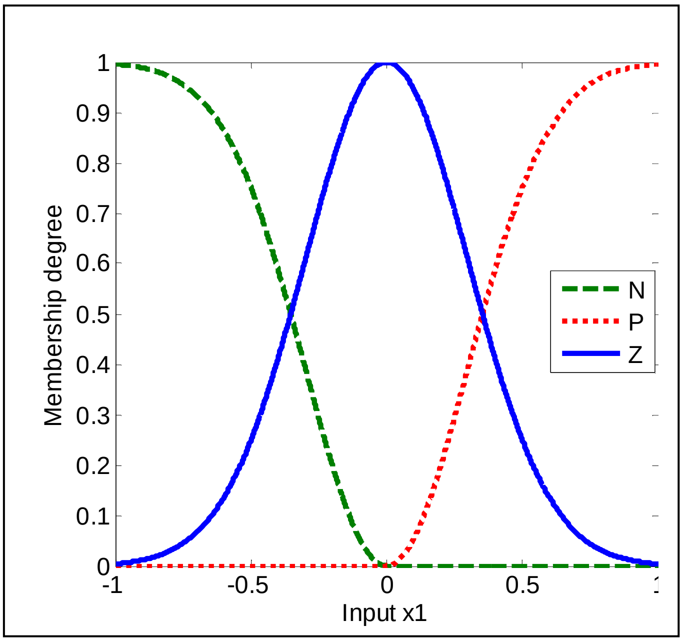

Figure2. Membership functions for the inputs x_1

Under these settings, experimental results are presented in Figures 3 to 5. The desired and actual joint angular positions are shown in Figure 3. Joint position tracking errors are illustrated in Figure 4. The applied voltages are also presented in Figure5. As can be seen, both controllers approximately have the same result in tracking of desired trajectory; except that fractional-order controller has lower energy consumption.

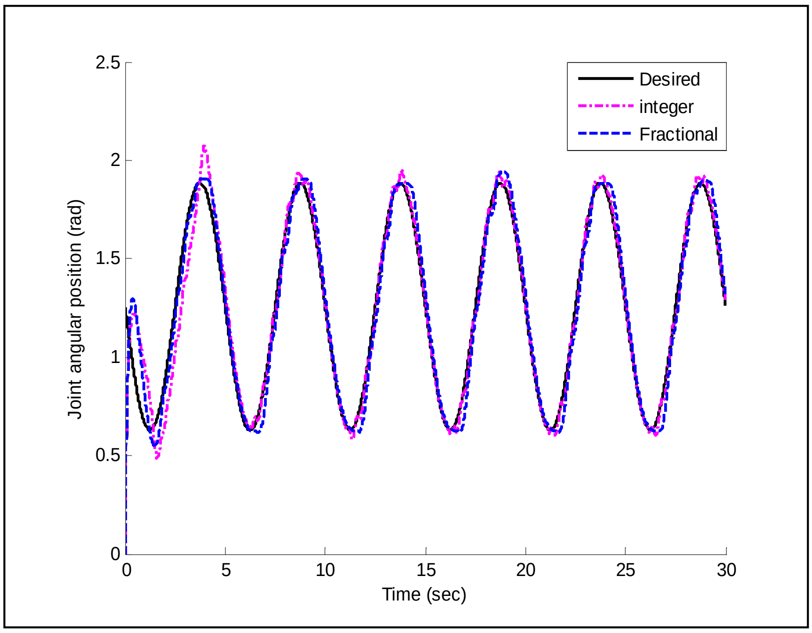

Figure3. Output tracking performance

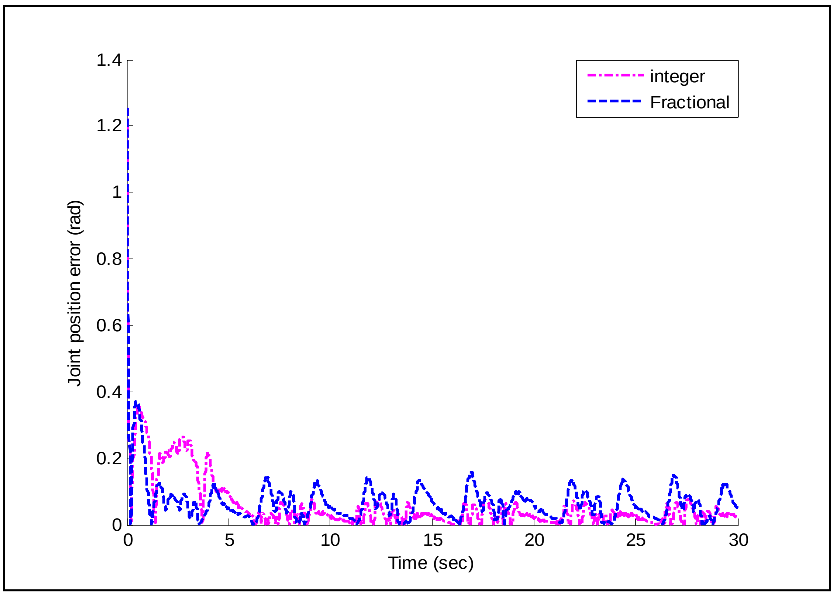

Figure4. Joint position tracking error

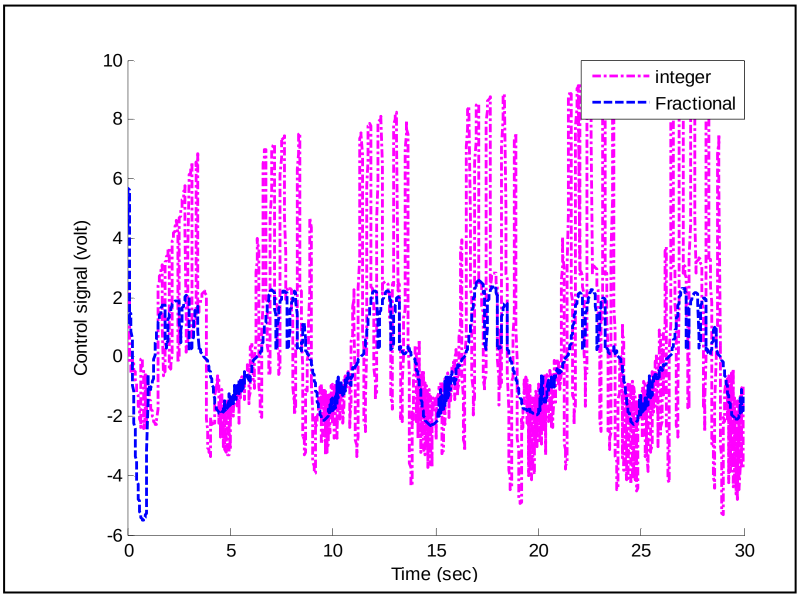

Figure5. Control signal

5. Conclusion

This paper presents a direct adaptive fuzzy fractional-order control scheme for electrically driven flexible joint robots considering uncertainties in both actuator and manipulator dynamics. The controller design is not dependent on the mechanical dynamics of the actuators and manipulators, thus is free from problems associated with torque control strategy in the design and implementation. Based on Lyapunov stability concept, it is shown that the proposed controller can guarantee stability of closed-loop system and satisfactory tracking performances. Experimental results show that tracking performance is satisfactory such that the effects of joint flexibility are well under control. The voltages of motors are permitted under the maximum values.

6. References

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