Robust Control of Electrically Driven Robots in the Task Space

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Received: October 3, 2017; Accepted: March 10, 2018

Abstract
In this paper, a task-space controller for electrically driven robot manipulators is developed using a robust control algorithm. The controller is designed using voltage control strategy. Based on the nominal model of the robotic arm, the desired signals for motor currents are calculated and then the voltage control law is proposed based on the current errors and motor nominal electrical model. Uncertainties such as parametric uncertainties, external disturbances and also imperfect transformation are compensated in the control law. The case study is a two-link robot manipulator equipped by permanent magnet DC motors. Simulation results verify the satisfactory performance of the proposed controller in reducing the tracking error and overcoming uncertainties.

Keywords
Robust Task-space Control, Imperfect Transformation, Uncertain Kinematics, Uncertain Dynamics

1. Introduction

Motion control of robot manipulators in the joint-space under uncertainties has been studied using various approaches, such as PID [1], feed-forward control [2], sliding-mode control [3], robust control [4-8], and neuro-fuzzy control [9-15]. The goal of control tasks is generally to move the tool center point (TCP) of the arm along a given path in the Cartesian coordinates (CC). That is, the desired trajectory of TCP is specified in CC, while the motions are actually obtained from the numerous actuators existing at the joints, which decide the required joint angles. However, despite of well behavior of the aforementioned strategies in joint-space, none of them can provide satisfactory tracking performances in workspace under the imperfect transformation from Cartesian to joint angles. Some of these reasons are as follow:

(I) The robot’s kinematics and dynamics change when a manipulator picks up different tools of unknown length, or unknown gripping points [16]. Therefore, the desired joint angles, their velocities, and accelerations are not produced precisely in joint-space under the imperfect transformation from task-space to joint-space.

(II) Tracking errors are appeared in task-space while actuators operate in joint-space. Thus, transformation of control space should be carried out to perform a control law [17]. As a fast result, the control inputs involve errors, if we use the imperfect transformation.

(III) The produced tracking errors in workspace are not detectable and compensable appropriately due to lack of feedbacks from the end-effector.

To deal with these problems, feedbacks from task-space are required to detect tracking error in CC. Based on this fact, the task-level controllers were then developed using assumption of perfect transformation in control spaces [18-20]. The considerable point is that, despite of efficiency, and

implement ability of task-level controller, the major problem in utilization of this type of controller is the existence of kinematic singularities. The task-level controllers may produce high joint torques while approaching to a singular configuration. Consequently, we are confronted by instability and large errors in the task-space [21].
Moreover, the control inputs involve errors if we use the imperfect transformation. Also, there is a problem that arises from task-space formulation of actuated robot manipulators including strong couplings between the joint motions, Jacobian matrix and its derivatives, as well as the inertia parameters of the payload carried by the manipulator end effectors. Therefore, designing a controller that solves the above problems has been the subject of many researches over the last decade.
Recently, a number of approximate Jacobian controllers have been presented to cope with the uncertain robot kinematics and dynamics using adaptive control laws. The proposed controllers do not require the exact knowledge of the kinematics and Jacobian matrix [22-25]. However, they are unable to handle unstructured uncertainty in the transformation, which is a missing link in almost all the proposed approaches. Therefore, such a proper controller cannot be easily developed to implement robotic applications in workspace. To tackle this problem, three good robust control strategies have been proposed aiming to prevent nominal performance degradations in presence of both parametric and unstructured uncertainties [26-28].
As a major point, common in almost all aforementioned strategies, previous approaches exclude the actuator dynamics, although, there is yet problem arises from adaptive control design as mentioned in [29-30]. In other words, extension of all the previously torque-based demonstrated control strategies is based on manipulator dynamics, and devoid of using actuator model in the controller structure and implementation. Thus, despite merit of the robust torque-based control laws from theoretical point of view, they might have some drawbacks from practical implementation point of view:
-A torque-based control law cannot be given directly to the torque inputs of an electrical manipulator. Because physical control variables are not the torque vector applied to robot links but rather electrical signals to actuators.
-Motors and drives dynamics are excluded in the torque-based control strategies, while the actuator dynamics are often a source of uncertainty, due e.g. to calibration errors, or parameter variation from overheating and changes in environment temperature [31].
-The control problem becomes hypersensitive when faster trajectories (motions along specified paths at high speeds) are demanded. The main reason of this sensitivity refers to dynamic problems arising from high velocities. Therefore, robot’s performance degrades quickly as speed increases.
In this paper, a robust control approach is developed to control actuated robotic arms under the imperfect transformation of control space with the presence of uncertainties including both parametric uncertainties and un-modeled dynamics in motor and robot dynamics. This is the main advantage, which makes this control approach superior to others.
The rest of this paper is organized as follows. Section 2 discusses about robot and actuator dynamics. Section 3 presents a robust joint-space controller under uncertainties and stability of the closed-loop system is then established using Lyapunov analysis. Section 4 develops a robust control approach for tracking control of robot manipulators in the task-space. The numerical results are

discussed in section 5 to show the effectiveness of the proposed control scheme. Finally, we give our concluding remarks in section 6. In what follows, we shall use the following notation. We denote by $\|x\|$ the Euclidean norm of a vector $x \in \Re^n$.

We say that $x(\cdot):[0,T] \to \Re^n$ is in $L_2[0,T]$ if $\int_0^T \|x\|^2 dt < \infty$, $x(\cdot)$ is in $L_\infty[0,\infty)$ if $\|x\| < \infty$ for all $t \in [0,\infty)$.

2. Problem Formulation

Let us consider the following well-known differential equations of motion, which describes the electrical subsystem behavior of n permanent magnet DC motors driving n degrees of freedom robot. These equations are:

$$L\frac{di}{dt} + Ri + k_b\frac{dq}{dt} = u \tag{1}$$

where $L \in \Re^{n\times n}$ is a constant diagonal matrix of electrical inductance, $R \in \Re^{n\times n}$ is diagonal matrix of armature resistances, $k_b \in \Re^{n\times n}[volt/rad/\sec]$ is a diagonal constant matrix for the back-emf effects, $u \in \Re^n[volt]$ is the control input voltage applied for the joint actuators, and $i \in \Re^n[A]$ is the vector of motor armature currents. In addition, we assume that the joint-space dynamics of an n-link rigid-body robot manipulator can be described by the following second order nonlinear vector differential equation:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + T_d = Hi \tag{2}$$

Where $q \in \Re^n$ denotes a vector of generalized joint variables, $D(q) \in \Re^{n\times n}$ is the inertia matrix of manipulator which is symmetric and positive definite. $C(q,\dot{q})\dot{q} \in \Re^n$ is a vector function containing Coriolis and centrifugal forces, $g(q) \in \Re^n$ is a vector function containing gravitational forces. $T_d \in \Re^n$ is a vector, which includes both external disturbances and un-modeled dynamics, and $H \in \Re^{n\times n}$ is an invertible constant diagonal matrix characterizing the electro-mechanical conversion between the current vector and the torque vector. Equation (2) can be represented as

$$D_i(q)\ddot{q} + C_i(q,\dot{q})\dot{q} + g_i(q) + T_{di} = i \tag{3}$$

Where $D_i = H^{-1}D$, $C_i = H^{-1}C$, $g_i = H^{-1}g$ and $T_{di} = H^{-1}T_d$. According to [32], the robot dynamics described above has the following fundamental properties, which can be exploited to facilitate the control system design:

Property 1: The inertia matrix $D(q)$ is symmetric, positive definite and uniformly bounded for all $q \in \Re^n$. That means, there exist positive constants μ_1 and μ_2, $\mu_1 \le \mu_2$ such that

$$\mu_1 I_n \le D(q) \le \mu_2 I_n \qquad ,\forall q \in \Re^n \tag{4}$$

Property 2: The matrix $\dot{D}(q) - 2C(q,\dot{q})$ is skew-symmetric. That is

$$y^T \dot{D}(q) y = 2y^T C(q,\dot{q}) y \qquad ,\forall y, q, \dot{q} \in \Re^n \tag{5}$$

Property 3: The robot dynamics can be linearly parameterized as the multiplication of a known regress or matrix $W(q,\dot{q},\ddot{q}) \in \Re^{n\times m}$ with a parameter vector $P \in \Re^m$, i.e.

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = W(q,\dot{q},\ddot{q})P \tag{6}$$

Property 4: For all revolute manipulators, the norm of $C(q,\dot{q})$, $g(q)$, and T_d satisfies the following inequalities

$$\|C(q,\dot{q})\| \le \zeta_c \|\dot{q}\| \quad , \quad \|g(q)\| \le \zeta_g \quad , \quad \|T_d\| \le \zeta_t \tag{7}$$

Where ζ_c, ζ_g and ζ_t are positive real constants.

In the most of robotic applications, a desired path is specified for the end-effector in the task-space. Let us $X \in \Re^n$ to be a task-space vector, representing the position and orientation of the robot end-effector relative to a fixed user defined reference frame. Then, the forward kinematic and differential kinematic transformation between the robot links coordinates and the end-effector coordinates can be written as

$$X = h(q) \tag{8}$$

$$\dot{X} = J(q)\dot{q} \tag{9}$$

Where $h : \Re^n \to \Re^n$ is smooth and $J(q) \in \Re^{n\times n}$ is the so called analytical Jacobian matrix. The robot Jacobian describes a map from velocities in joint-space to velocities in operational space.

3. Robust Joint-Space Control

In this section, we design a robust control for electrically driven robot by applying the recursive procedure. It follows from (1) and (3) that, the overall system of actuated robot manipulator can be viewed as two-cascaded dynamical system, if i is considered as the input signal to robot dynamics of rigid body. One consequence of this definition is that the rigid-link manipulator input i cannot be commanded directly, and instead it must be realized as the output of the actuator dynamics through proper specification of the actuator control input u . Hence, in order to control the robot manipulator to track the desired trajectory, first a robust control scheme is designed to generate the fictitious control input i_d required to ensure that the system (3) evolves as desired. The next control objective is, naturally, to generate a suitable control voltage u so that the motor current i can follow the desired current command i_d , and thus q will follow the desired trajectory q_d . Based on this observation, a recursive control scheme is developed. By the last definitions, the first attempt is to define a switching rule, in the joint-space.

$$S_q = (\dot{q}_d - \dot{q}) + \alpha(q_d - q) \tag{10}$$

Where S_q is the switching rule, α is a diagonal positive definite gain matrix and $q_d \in \Re^n$ is a desired trajectory in joint-space. Let us define a vector $r \in \Re^n$ in the form of

$$r = \dot{q}_d + \alpha(q_d - q) \tag{11}$$

Thus

$$S_q = r - \dot{q} \tag{12}$$

Now, the problem is to design a desired current trajectory i_d so that a robust inner-controller u can be constructed to have $i \to i_d$ which further implies convergence of the output error as desired. To solve this problem, we define the desired current i_d as

$$i_d = \hat{D}_i \dot{r} + \hat{C}_i r + \hat{g}_i + \lambda S_q + \tau_{rl} \tag{13}$$

where $\hat{D}_i$, $\hat{C}_i$ and $\hat{g}_i$ are respectively estimates of D_i, C_i and g_i. The parameter λ is a positive definite matrix and control input τ_{rl} is considered for canceling both parametric uncertainties and unstructured uncertainties in mechanical subsystem of robot manipulator. Substituting (13) into (1), rearranging with some manipulation leads to dynamic of the output tracking loop as

$$D_i \dot{S}_q = W(q, \dot{q}, r, \dot{r})\tilde{P} + T_{di} - \lambda S_q - C_i S_q - \tau_{rl} - e_i \tag{14}$$

Where $e_i = i - i_d$ is the current error, $\tilde{P}$ is a vector of parametric errors defined as $\tilde{P} = P - \hat{P}$, $\hat{P}$ denotes an estimation of P and from (6)

$$W(q, \dot{q}, r, \dot{r})\tilde{P} = \left(D - \hat{D}\right)\dot{r} + \left(C - \hat{C}\right)r + \left(g - \hat{g}\right) \tag{15}$$

By the last result, the design procedure is now to design a control input u, to realize the perfect current vector in (13), such that, the current error can either converges to zero, or at least it is bounded by a constant. It returns to this fact that, a constant-bounded disturbance will not destroy the stability result under robust control i_d which is a result of uniform ultimate boundedness of tracking error using Lyapunov based theory of guaranteed stability of uncertain systems [32]. Toward this end, we may construct the control input in the form of

$$u = \hat{L}\frac{di_d}{dt} + \hat{R}i + \hat{K}_b \frac{dq}{dt} - \gamma e_i + \tau_{rm} \tag{16}$$

Where the gain matrix γ is selected to be positive definite and control input τ_{rm} is considered for canceling both parametric uncertainties and unstructured uncertainties in the motor dynamic equations. Substituting (16) in (1), the dynamics for the current tracking loop becomes

$$L\frac{de_i}{dt} + \gamma e_i = -\tilde{P}_i \Phi + \tau_{rm} \tag{17}$$

$$\tilde{P}_i = [L - \hat{L} \quad R - \hat{R} \quad K_b - \hat{K}_b] \tag{18}$$

$$\Phi = [\frac{di_d}{dt}^T \quad i^T \quad \dot{q}^T]^T \tag{19}$$

Where and $\hat{R}$, $\hat{L}$, $\hat{K}_b$ are estimates of R, L, K_b, respectively.

Stability Proof:

The asymptotic stability of the entire control system is analyzed based on lyapunov stability theory and using Barbalat's lemma. First, an important lemma is introduced and then the closed-loop stability is proved.

Lemma 1: Suppose that a symmetric matrix Q is partitioned as:

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \tag{20}$$

where Q_{11} and Q_{22} are square. Then, the matrix Q is positive definite if and only if Q_{11} is positive definite and $Q_{22} > Q_{12}^T Q_{11}^{-1} Q_{12}$.

Proof: Consider the Lyapunov function candidate

$$V = \frac{1}{2} S_q^T D_i(q) S_q + \frac{1}{2} e_i^T L e_i > 0 \tag{21}$$

The differentiation of V is

$$\dot{V} = \frac{1}{2} S_q^T \dot{D}_i(q) S_q^T + S_q^T D_i(q) \dot{S}_q + e_i^T L \dot{e}_i \tag{22}$$

From equations (14) and (17), and the fact that $\dot{D}_i(q) - 2C_i(q,\dot{q})$ is skew-symmetric, we have

$$\dot{V} = -\begin{bmatrix} S_q^T & e_i^T \end{bmatrix} Q \begin{bmatrix} S_q \\ e_i \end{bmatrix} + e_i^T(-\tilde{p}\Phi + \tau_{rm}) + S_q^T\left(W(q,\dot{q},r,\dot{r})\tilde{P} + T_{di} - \tau_{rl}\right) \tag{23}$$

With

$$Q = \frac{1}{2}\begin{bmatrix} 2\lambda & I \\ I & 2\gamma \end{bmatrix} \tag{24}$$

Based on lemma 1, for matrix Q to be a positive definite matrix, the requirement is

$$\gamma > \frac{1}{4}\lambda^{-1} \tag{25}$$

Condition (25) is met by choosing appropriate diagonal matrices of controller gains λ and γ. Now, due to establish $\dot{V} \leq 0$, it is sufficient that

$$e_i^T\left(-\tilde{p}\Phi + \tau_{rm}\right) + S_q^T\left(W(q,\dot{q},r,\dot{r})\tilde{P} + T_{di} - \tau_{rl}\right) \leq 0 \tag{26}$$

To satisfy (26), the robust control inputs are then given by

$$\tau_{rl} = \delta_{rl}(t)\frac{S_q}{\|S_q\|} \quad \text{for} \quad \|S_q\| \neq 0 \tag{27}$$

$$\tau_{rm} = -\delta_{rm}(t)\frac{e_i}{\|e_i\|} \quad \text{for} \quad \|e_i\| \neq 0 \tag{28}$$

where $\delta_{rl}(t)$ and $\delta_{rm}(t)$ are positive scalar functions of time, namely a bounding function to show the upper bound of uncertainties, obtained from

$$\left\|W(q,\dot{q},r,\dot{r})\tilde{P} + T_{di}(t)\right\| \leq \delta_{rl}(t) \quad , \quad \left\|\tilde{P}_i\Phi\right\| \leq \delta_{rm}(t) \tag{29}$$

Up to now, we have proved that S_q and e_i are uniformly bounded, i.e. $S_q, e_i \in L_\infty$. From the computation

$$\int_0^\infty \begin{bmatrix} S_q^T & e_i^T \end{bmatrix} Q \begin{bmatrix} S_q \\ e_i \end{bmatrix} dt \leq -\int_0^\infty \dot{V} dt = V_0 - V_\infty < \infty \tag{30}$$

We have S_q, $e_i \in L_2$. Therefore, boundedness of $\dot{S}_q$ and $\dot{e}_i$ can be achieved by observing (14) and (17), since all of terms are bounded. Therefore, the control system has asymptotic stability and convergence of tracking error can be concluded by Barbalat's Lemma. This completes the proof of the closed-loop system stability.

Remark: It must be emphasized that, the robust control strategy, not designed at the torque-based of the rigid body often requires a bounding function on the time derivative of some robust torque control. Thus, it is best that the outer robust control be differentiable and it's time derivative bounded by a function of reasonable magnitude. Thus, in the outer robust control phase, we may make a modification that redefines the control (27) to be

$$\tau_{rl} = -\frac{\mu\|\mu\|^{v}}{\|\mu\|^{1+v} + \varepsilon^{1+v}}\delta_{rl}(t) \tag{31}$$

where $\mu \square\ \delta_{rl}(t)e_i$. The choice of design parameters v , ε and how to well design a robust controller are extensive discussions, which can be founded in [32].

4. Robust Task-Space Control

Robust control approaches have been extensively developed to control robot manipulators in joint-space. It is obvious that, even, though they can present perfect tracking control in joint-space, they cannot provide satisfactory performances in CC under imperfect transformation of control space. In addition, extension of many task-space control approaches has been by the assumption of perfect transformation, which is not real with presence of uncertainties. To cope with this problem, a hybrid-switching rule is proposed as follows:

$$S_x = \beta\hat{J}^{-1}(\hat{q}_d)(X_d - X) + \hat{J}^{-1}(\hat{q}_d)\dot{X}_d - \dot{q} \tag{32}$$

where $\beta > 0$ determines the behavior of the error dynamics, $\hat{J}^{-1}(\hat{q}_d)$ is the inverse of imperfect Jacobian matrix, $X_d \in \Re^n$ is the desired trajectory in CC and $\hat{q}_d \in \Re^n$ is a transformed desired trajectory to joint-space which is calculated by imperfect inverse kinematics as

$$\hat{q}_d = inv\hat{h}(X_d) \tag{33}$$

Also from (8)

$$\dot{\hat{q}}_d = \hat{J}^{-1}(\hat{q}_d)\dot{X}_d \tag{34}$$

As can be seen from (32), the control system switches on $S_x = 0$ to achieve a zero tracking error vector such that X converges to X_d as time increases to infinity. Let us define a vector $\hat{r} \in \Re^n$ in joint-space of the form

$$\hat{r} = \hat{J}^{-1}(\hat{q}_d)\left(\dot{X}_d + \beta(X_d - X)\right) \tag{35}$$

Then (32) can be rewritten as

$$S_x = \hat{r} - \dot{q} \tag{36}$$

Moreover, the time derivative of $\hat{r}$ is

$$\dot{\hat{r}} = \dot{\hat{J}}^{-1}(\hat{q}_d)\left(\beta(X_d - X) + \dot{X}_d\right) + \hat{J}^{-1}(\hat{q}_d)\left(\beta(\dot{X}_d - \dot{X}) + \ddot{X}_d\right) \tag{37}$$

The time derivative of S_x is also obtained from (36)

$$\dot{S}_x = \dot{\hat{r}} - \ddot{q} \tag{38}$$

Now we design a desired current

$$i_d = \hat{D}_i \dot{\hat{r}} + \hat{C}_i \hat{r} + \hat{g}_i + \lambda S_x + \tau_{rl} \quad (39)$$

With the parameters, defined as the same as before. Substituting (39) into (1), rearranging, and some mathematical calculations one obtains the dynamic of the output-tracking loop as

$$D_i \dot{S}_x = W(q, \dot{q}, \hat{r}, \dot{\hat{r}})\tilde{P} + T_{di} - \lambda S_x - C_i S_x - \tau_{rl} - e_i \quad (40)$$

The remaining task here is to adjust the control input u in (1), exactly as the same as those defined by (16), to ensure convergence of the actual current to the perfect one.

Stability Proof:

Letting a candidate Lyapunov function be defined as

$$V = \frac{1}{2} S_x^T D_i(q) S_x + \frac{1}{2} e_i^T L e_i > 0 \quad (41)$$

Through a similar calculation of previous section, the differentiation of V is

$$\dot{V} = -\begin{bmatrix} S_x^T & e_i^T \end{bmatrix} Q \begin{bmatrix} S_x \\ e_i \end{bmatrix} + e_i^T(-\tilde{p}\Phi + \tau_{rm}) + S_x^T\left(W(q, \dot{q}, \hat{r}, \dot{\hat{r}})\tilde{P} + T_{di} - \tau_{rl}\right) \quad (42)$$

where Q is positive definite due to the suitable selection of gain matrices λ and γ. Again, in order to establish asymptotic stability, it is sufficient that

$$e_i^T(-\tilde{p}\Phi + \tau_{rm}) + S_x^T\left(W(q, \dot{q}, \hat{r}, \dot{\hat{r}})\tilde{P} + T_{di} - \tau_{rl}\right) \le 0 \quad (43)$$

The robust control inputs are then given by

$$\tau_{rl} = \delta_{rl}(t) \frac{S_x}{\|S_x\|} \quad \text{for} \quad \|S_x\| \neq 0 \quad (44)$$

$$\tau_{rm} = -\delta_{rm}(t) \frac{e_i}{\|e_i\|} \quad \text{for} \quad \|e_i\| \neq 0 \quad (45)$$

where $\delta_{rl}(t)$ and $\delta_{rm}(t)$ are given by (29). These conditions together with Barbalat's lemma complete the proof of the closed-loop system stability.

5. Computer Simulation

In this section, we present the simulation results for the proposed control scheme. The simulation task is carried out based on a two degree-of-freedom planer robot driven by dc motors. The dynamic model of the robot system can be described in the form of Equation (2) as

$$D(q)=\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\ ,\ C(q,\dot{q})\dot{q}=\begin{bmatrix} -2m_2l_1l_{c2}sin(q_2)\left(\dot{q}_1\dot{q}_2+0.5\dot{q}_2^2\right) \\ m_2l_1l_{c2}sin(q_2)\dot{q}_1^2 \end{bmatrix}$$

$$g(q)=\begin{bmatrix} \left(m_1l_{c1}+m_2l_1\right)gcos(q_1)+m_2l_{c2}gcos(q_1+q_2) \\ m_2l_{c2}gcos(q_1+q_2) \end{bmatrix}$$

$$d_{11}=m_2\left(l_1^2+l_{c2}^2+2l_1l_{c2}cos(q_2)\right)+m_1l_{c1}^2+I_1+I_2 \tag{46}$$

$$d_{21}=d_{12}=m_2l_{c2}^2+m_2l_1l_{c2}cos(q_2)+I_2$$

$$d_{22}=m_2l_{c2}^2+I_2$$

Where q_1 and q_2 are the angle of joints 1 and 2, m_1 and m_2 are the mass of links 1 and 2 respectively, l_1 and l_2 are the length of links 1 and 2, Ii is the link's moment of inertia given in center of mass, l_{ci} is the distance between the center of mass of link and the ith joint, and g is the gravity acceleration. The manipulator dynamic parameters are defined as $l_1=l_2=0.75$m, $l_{c1}=l_{c2}=0.375$m, $m_1=m_2=0.5$kg and $I_1=I_2=0.0234$; Also, the exact-actuator dynamic model parameters are selected as R=diag(1,1), K_b=diag(1,1) and L=diag(0.025, 0.025) and H=diag (10,10). In order to observe the effect of the actuator dynamics, the endpoint is required to track a rapidly-varying task space trajectory, characterized by 0.2m radius circle centered at (0.8m, 1.0m) in 2 seconds. The forward kinematic equation is given by

$$\begin{aligned} X &= l_1cos(q_1)+l_2cos(q_1+q_2) \\ Y &= l_1sin(q_1)+l_2sin(q_1+q_2) \end{aligned} \tag{47}$$

The manipulator Jacobian matrix $J(q)$ mapping from task-space to joint-space is given as

$$J(q)=\begin{bmatrix} -l_1sin(q_1)-l_2sin(q_1+q_2) & -l_2sin(q_1+q_2) \\ l_1cos(q_1)+l_2cos(q_1+q_2) & l_2cos(q_1+q_2) \end{bmatrix} \tag{48}$$

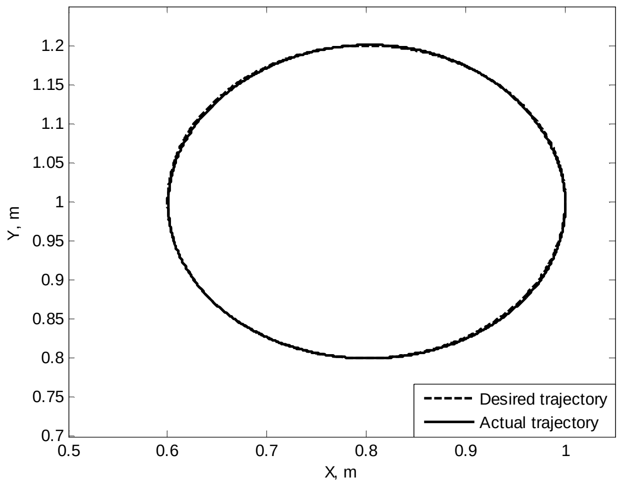
The link's length is estimated by a gain of 1.1 from real values defined as before. The initial tracking error is considered zero in all simulations. The external disturbances and unstructured uncertainties are assumed as

$$T_d(t)=\left[40\sin(2\pi t)\quad 10\right]^{\mathrm{T}}\quad N.m \tag{49}$$

At first, to clarify the significance of the actuator dynamics in the closed-loop stability, we applied a torque-based robust control investigated in [28] as

$$u=\hat{D}\dot{\hat{r}}+\hat{C}\hat{r}+\hat{g}+\lambda S_x+\tau_{rl} \tag{50}$$

With $\beta=100$, $\lambda=10$ and no knowledge of the manipulator dynamic. As shown in Figure 1, the end-point trajectory converges smoothly to the desired trajectory. Now, if a torque-based robust controller is applied to the same electrically driven robot with the same set of controller parameters, some modifications are needed so that the robot and the controller would be compatible. The purpose of



Figuer1. The end-effector position in the task-space

Using the same set of parameters is to have an effective comparison. Let us introduce a conversion matrix K_τ which satisfies $K_\tau u(\infty) = Hi(\infty) = \tau(\infty)$ so that the controller (50) becomes in the voltage-level

$$u(\infty) = K_\tau^{-1}\left(\hat{D}\dot{\hat{r}} + \hat{C}\hat{r} + \hat{g} + \lambda S_x + \tau_{rl}\right) \tag{51}$$

As can be seen from Figure 2, the controller designed in torque-based is not able to give acceptable performance under the conditions that the actuator dynamics is important such as the fast motion trajectory simulated here. As a fast result, consideration of the actuator dynamics is very important when faster trajectory is required. Now, to clarify the significance of the proposed controller two simulation set will be investigated.

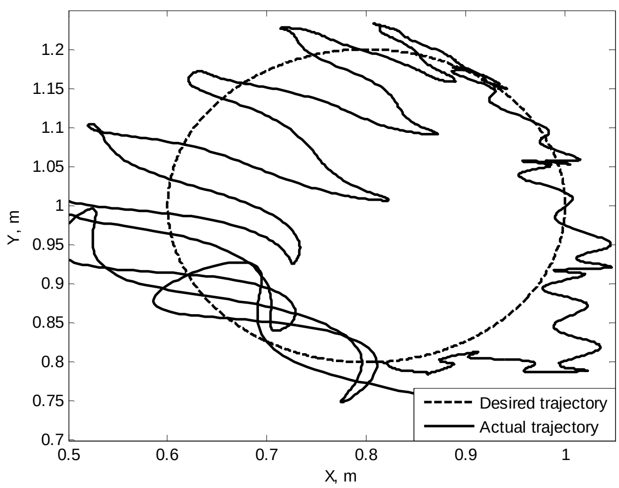
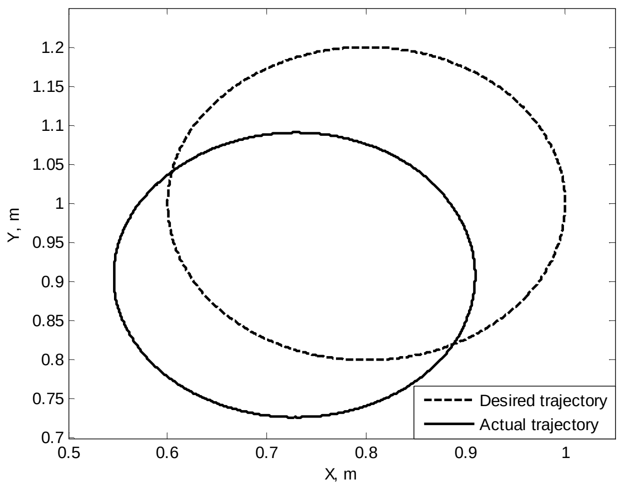


Figure2. The end-effector position in the task-space

Simulation 1: The joint-space control given by (13) and (16) is simulated to track a circle in CC. The most conservative choices of $\hat{D}_i$, $\hat{C}_i$ and $\hat{g}_i$ are selected here equal to zero. Namely, assume no knowledge about the system. We set the controller with $\alpha = 120$, $\gamma = 6$ and $\lambda = 4$. Figure 3 shows the tracking performance of the robot endpoint and its desired trajectory in the CC. The norm of tracking error in CC indicates a maximum value of 135mm, while the norm of joint errors is negligible with a maximum value of 1.05mm as shown in Figures 4 and 5 respectively. Therefore, despite of good tracking performance of the robust joint-space control strategies; they cannot provide satisfactory performance in CC under imperfect transformation of control space. The efforts to the two joints are given in Figure 6.As shown in this Figure, the control signals are bounded and after a short transient state, motor voltages are smooth. The performance in the current tracking loop is quite good as shown in Figure 7. According to this Figure, there is no sudden change in currents and motor currents are bounded. To show the role of the robust control input, we repeated the previous simulation in absence of τ_{rl}, as shown in Figure 8. As shown in this Figure, the performance of the joint-space control strategies is degraded by the imperfect transformation which clarifies the considerable influence of the proposed robust control method.



Figuer3. The end-effector position in the task-space

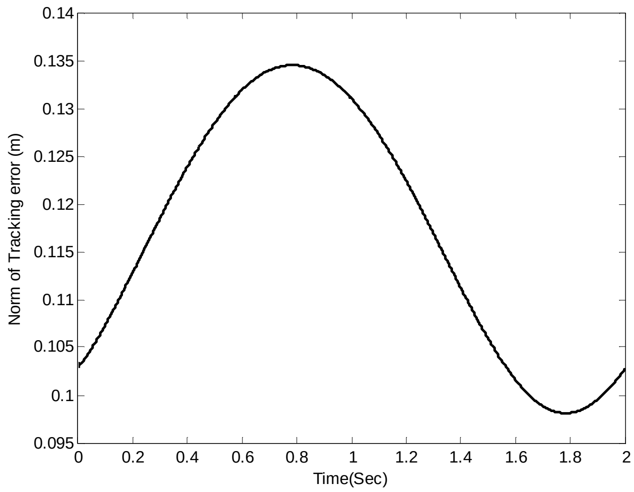
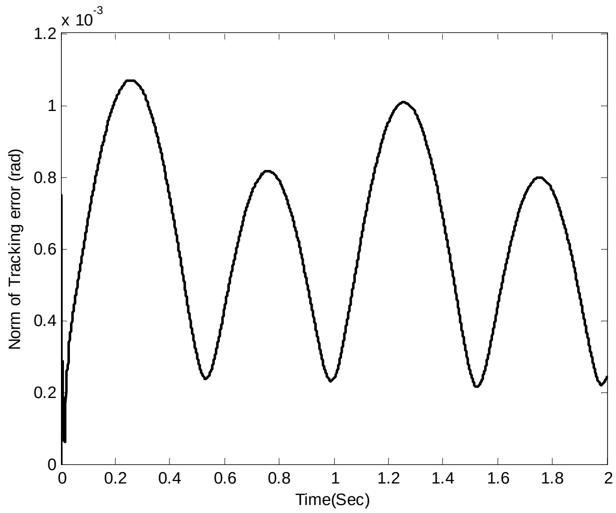


Figure4. Tracking error in task-space



Figuer5. Tracking error in joint-space

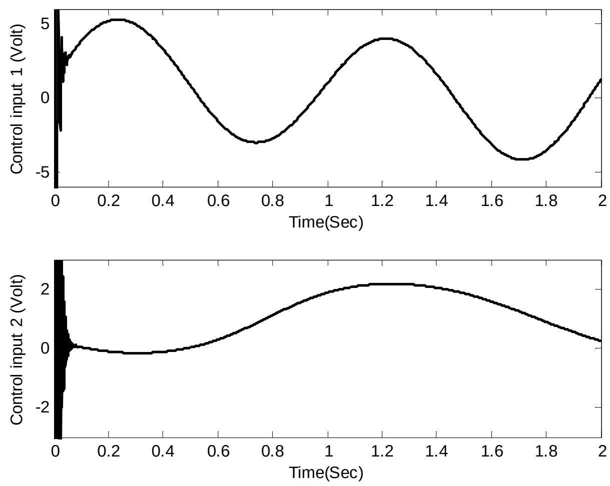


Figure6. The control efforts for both joints

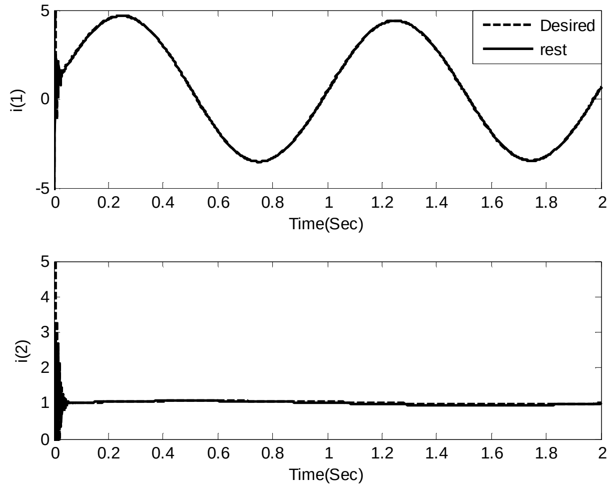


Figure7. Tracking in current loop

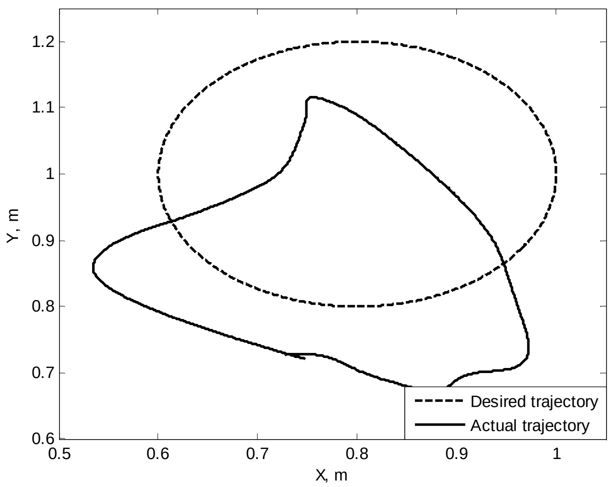


Figure8. The end-effector position in absence of τ_{rl}

Simulation 2: The task-space control given by (16) and (39) is simulated where the parameters are as the same as before but $\beta = 100$. The control system tracks well the circle as shown in Figure 9. As can be seen from Figure 10, end-effector positions converge nicely to the desired value in task-space. Furthermore, the technical limits such as motor voltages and performance in the current tracking loop are illustrated in Figures11 and 12, respectively. As shown in these Figures, motors' voltages and currents are bounded and the controller can be implemented practically due to these reasonable signals. Moreover, it seems that the performance of the current control loop is acceptable and current tracking errors are very small. The simulation results clearly show the effectiveness of the proposed control scheme to robustly stabilize the system, while achieving robust performance subjects to uncertainties in kinematic equations.

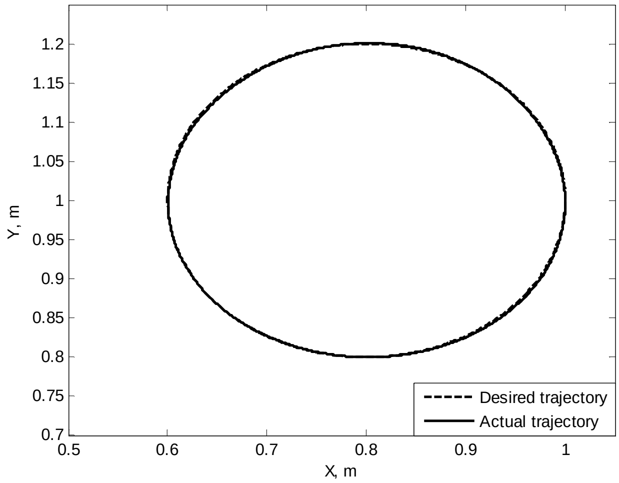


Figure9. The end-effector position in the task space

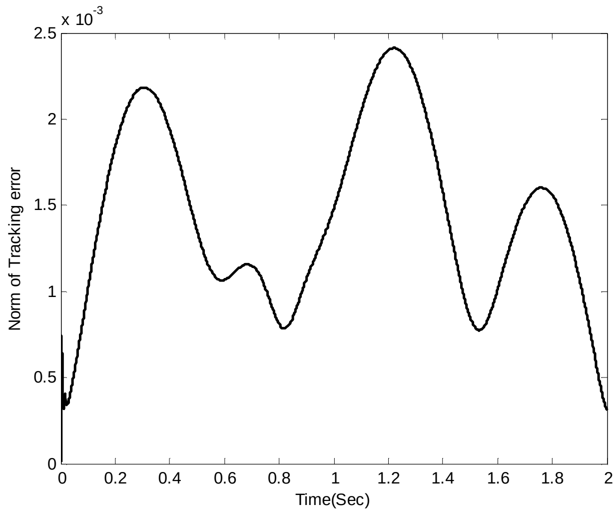


Figure10. Tracking error in task-space

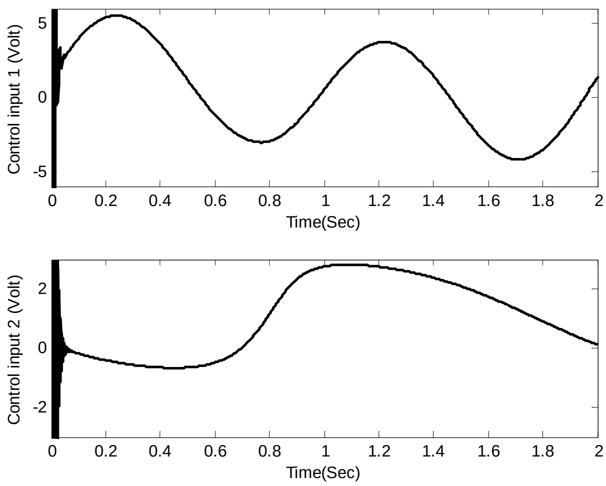


Figure11. The control efforts for both joints

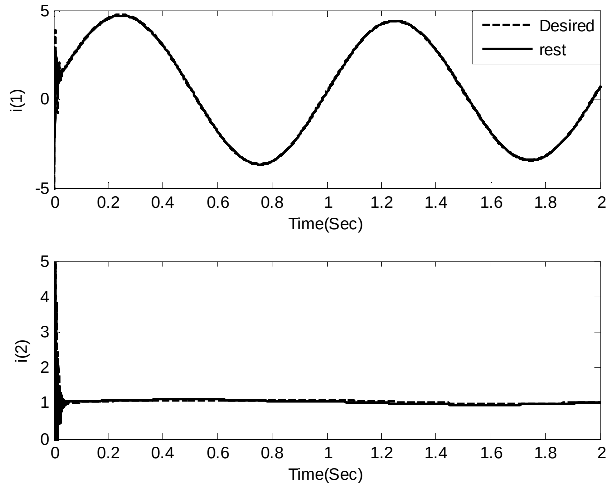


Figure12. Tracking in current loop

6. Conclusion

A voltage-based controller for robust task-space control of robot manipulators has been developed in this paper. The controller is designed using nominal models of the robot manipulator and motors. Uncertainties originated from the mismatch between nominal and actual models have been compensated using feedbacks from both joint-space and task-space via a robust control approach. The desired signals for motor currents have been obtained using nominal model of the manipulator and then the voltage control law is proposed based on the current errors and motor nominal electrical model. It has been shown that the closed-loop system is asymptotically stable based on Lyapunov stability analysis. Simulation results show that the performance of the proposed controller is comparable to that of torque-based controllers.

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