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Research Paper

Optimal Trajectory Planning for an Industrial Mobile Robot using Optimal Control Theory

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Abstract
Optimal path planning with optimal journey time and the motor saturation limit are two main challenges in mobile industrial robot design. The motion speed and motor saturation limit are important factors determining the required torque. Calculating the optimal torque value reduces the construction and motor selection costs. This paper proposes the theory of optimal control open-loop base model for path planning by simultaneously minimizing the journey time, wheels' torque for industrial robots. In this study, nonlinear equations of robot motion were considered as a constraint in optimal control problems. Next, the cost function was proposed, including the torque of the left and right wheels and time-related terminal conditions and disturbance, in which the nonlinear equations of the industrial robot motion are assumed as constraints. The final equations were numerically solved, and the effectiveness of the proposed method was demonstrated by simulating and path design for industrial robots' motions along with considering motor saturation limit.

Keywords
Industrial Mobile Robot, Optimal Control, Optimal Time, Trajectory Planning

1. Introduction

Mobile industrial robots can be programmed to perform tasks in an industrial setting. Typically they are used in stationary and workbench applications. However, mobile industrial robots have introduced a new method for lean manufacturing. With advances in controls and robotics, current technology has been optimized, allowing for mobile tasks such as product delivery. This additional flexibility in manufacturing can save corporate time and money during the manufacturing process, resulting in a less expensive end product [1]. Figure 1 shows a mobile industrial robot.

Figure 1. An Industrial mobile robot example: OTTO 1500 self-driving vehicle for heavy-load material transport in warehouses, distribution centers, and factories [1]

The control problem of a mobile robot has been investigated in the past decades [2]. Korayem et al. investigated dynamic modeling and optimal path planning of a non-holonomic mobile robot in a complex environment [3]. Also, Nazemizadeh et al. presented an optimal control strategy for mobile robots' optimal trajectory planning using the indirect solution of the optimal control method [4]. They excluded the time parameter in their optimal control problem. Wu et al. designed a finite-time control law for a non-holonomic mobile robot in a dynamic model. Furthermore, they proposed a robust adaptive control strategy for controlling the mobile robot [5]. Cui et al. applied adaptive tracking and obstacle avoidance with unknown sliding to control a mobile robot. They introduced a sliding mode observer and an obstacle avoidance control law to estimate the sliding parameters online in a non-holonomic system to compensate for the unknown parameters [6]. Dos Santos et al. used a direct optimization method for the optimized path planning if the manipulators are in a constrained workspace [7]. Ramos optimally controlled time and energy minimization in the trajectory generation based on the kinematics of a mobile robot [8]. Tuncer and Yildirim proposed a new mutation operator for a genetic algorithm and applied it for mobile robot navigation in dynamic environments [9]. The above works were only addressed the motion planning of mobile robots and disregarded the optimality of journey time. Perrier et al. [10] designed the path of a non-holonomic moving robot. In their proposed method, they used only in kinematic modeling of a robot. However, in designing the path of a mobile robot, the actuators' torque limitations should be considered; otherwise, the path design errors increase. However, to increase the efficiency and effectiveness of mobile robots, designing their optimal path is paramount. Accordingly, various articles provide the optimal path design and consider various cost functions, such as minimum input efforts [11]. Korayem et al. [12] used a linear iterative programming method to design the path of a moving robot. This method has disadvantages such as linearization of equations and lack of proper convergence of the answer, making it unsuitable in practice. In [13], the optimal path of the non-holonomic mobile robot for the condition of the end-effector path was investigated. First, the dynamic modeling of the system was performed by considering non-holonomic constraints, and the optimal path of the mobile robot was designed for a specific end-effector path. Korayem and Nazemizadeh [14] investigated the path planning of a mobile robot in the presence of environmental obstacles and used potential functions for obstacle avoidance. In [15] and [16], the optimal path of a flexible mobile robot is designed using the optimal control method, and the path of the moving robot is known while the non-holonomic constraints of the robot are not considered. The present paper uses the optimal control theory for path planning, minimizing the journey time in the generated optimized trajectory. Generally, the optimal control problems are based on open-loop control, and a dynamic equation mainly plays the role of the constraints in the optimal control problem. In Section 2, the dynamic equations are derived while the constraints are discussed. Section 3 addresses the problem of optimal control and a new cost function for minimum journey time. In Section 4, the simulation and the results are presented.

2. Dynamic model of the wheeled mobile robot

This section derives the dynamic model of a mobile robot. Consider the mobile robot shown in Figure 2. The mobile robot moves on the ground using the friction contact between wheels and the ground. The presented model considers no slips in the driving and lateral directions. The dynamic motion modeling of the mechanical systems includes dynamic variables related to system motion caused by external forces and system inertia. The Lagrange method has been used to derive the dynamic equations of the robot.

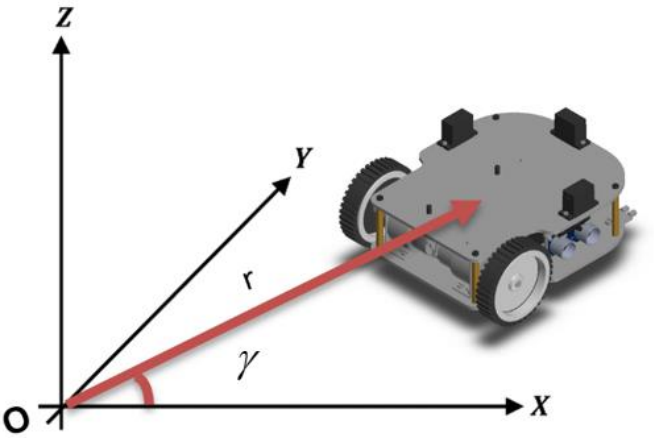

Figure 2. Schematic of the wheeled mobile robot

According to Lagrange formulation, the following differential equations are obtained [17]:

$$m_c\ddot{x} - \lambda_1 \sin\gamma = \frac{1}{r}(\tau_r + \tau_l)\cos\gamma \quad (1)$$

$$m_c\ddot{y} + \lambda_1 \cos\gamma = \frac{1}{r}(\tau_r + \tau_l)\sin\gamma \quad (2)$$

$$J\ddot{\gamma} = \frac{L}{2r}(\tau_r - \tau_l) \quad (3)$$

Where m_c and J denote the robot mass and moment of inertial, respectively. $x, y,$ and γ are generalized coordinates of the robot. L and r denote the distance between the wheels and the radius of each wheel, respectively. The torque of the left and right wheels are denoted by τ_l and τ_r, respectively. Finally, the Lagrange multiplier of the constrained system is denoted by λ. A dynamic model of the system with constraints can be expressed as follows:

$$M(q)\ddot{q} + V(q,\dot{q}) = E(q)u - A^T(q)\lambda \quad (4)$$

The parameters of the above equation are defined as: q is the vector of generalized coordinates and M is the positive definite mass and inertial matrix, V is the vector of Coriolis and centrifugal forces, E is transformation matrix from actuators space to the generalized coordinates space, u is the input vector and A^T is the matrix of kinematics constraints coefficients and λ is the vector of Lagrange multipliers. According to equation (4), matrices M, E, A^T and Vector u in the motion equation of the mobile robot can be expressed as follows:

$$M = \begin{bmatrix} m_c & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & J \end{bmatrix} \tag{5}$$

$$E = \frac{1}{r}\begin{bmatrix} \cos\gamma & \cos\gamma \\ \sin\gamma & \sin\gamma \\ \frac{L}{2} & -\frac{L}{2} \end{bmatrix} \tag{6}$$

$$A = [-\sin\gamma \quad \cos\gamma \quad 0] \tag{7}$$

$$u = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix} \tag{8}$$

The remaining matrices are zero. The system can be written in the state-space as follows [17]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\gamma} \\ \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} v\cos\gamma \\ v\sin\gamma \\ \omega \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m_c r} & \frac{1}{m_c r} \\ \frac{L}{2Jr} & -\frac{L}{2Jr} \end{bmatrix} \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix} \tag{9}$$

3. Optimal control theory for path planning

In this section, the mobile robot's optimal control problem and trajectory optimization are addressed, and the optimum values of control inputs to generate the optimal paths involving the left and right wheels' torque and optimal time are obtained. In optimal motion planning of the mobile robot, the dynamic equations of the system are assumed as constraints of the optimal control problem, and it is aimed to determine the optimal state vector z and u, thus introducing a cost function:

$$\min_{u(t)} \left\{ \phi(t_f) + \int_0^{t_f} \left(L(z(t), u(t)) + disturbance\right) dt \right\} \tag{10}$$

And

$$s.t.: \ \dot{z}(t) = f(z(t), u(t))a$$
$$z(0) = z_0$$
$$z(t_f) = z_f$$

$$\frac{\partial \varphi}{\partial t_f}(t_f) + H(z^*(t_f), u^*(t_f), p^*(t_f)) = 0 \tag{11}$$

Where here $\phi(t_f)$ as follow and assume that disturbance is zero:

$$\phi(t_f) = \alpha t_f \tag{12}$$

Using the calculus of variations and the Pontryagin minimum principle, the Hamiltonian function is as follows [18]:

$$H(z,u,P) = L(u) + P^T f(z,u) \tag{13}$$

Where P is the co-stat vector. The optimality conditions are extracted as a set of differential equations: is as follows [18]:

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$$\dot{z}^* = \frac{\partial H}{\partial P} = \begin{bmatrix} v^*\cos\gamma^* \\ v^*\sin\gamma^* \\ \omega^* \\ \frac{1}{mr}(\tau_r^* + \tau_l^*) \\ \frac{L}{2Jr}(\tau_r^* - \tau_l^*) \end{bmatrix} \tag{14}$$

$$\dot{P}^* = -\frac{\partial H}{\partial z} = \begin{bmatrix} 0 \\ 0 \\ p_1^* v^* \sin\gamma^* - p_2^* v^* \cos\gamma^* \\ -p_1^* v^* \cos\gamma^* - p_2^* v^* \sin\gamma^* \\ -p_3^* \end{bmatrix} \tag{15}$$

$$\frac{\partial H}{\partial u} = \begin{bmatrix} 2\tau_r^* + \frac{p_4^*}{mr} + \frac{p_5^* L}{2Jr} \\ 2\tau_l^* + \frac{p_4^*}{mr} - \frac{p_5^* L}{2Jr} \end{bmatrix} = 0 \tag{16}$$

Using equation (16), the control law is obtained as follows:

$$\tau_r^* = -\frac{1}{2}\left(\frac{p_4^*}{mr} + \frac{p_5^*}{2Jr}\right) \tag{17}$$

$$\tau_l^* = -\frac{1}{2}\left(\frac{p_4^*}{mr} - \frac{p_5^*}{2Jr}\right) \tag{18}$$

In order to solve the optimal control problem with unknown time, the problem can be transformed into a standard form as suggested in [19]. The time should be parameterized as $s = \frac{1}{t_f} t$, so that $s \in [0,1]$, provided that $t \in [0, t_f]$ and the variable will be added. A new augmented state vector $m = (m_1 \quad m_2 \quad \cdots \quad m_{11}) \in \mathbb{R}^{11}$ will be defined as [19]:

$$m = [z^T \quad p^T \quad k]^T \tag{19}$$

In which $(m_1 \quad m_2 \quad m_3 \quad m_4 \quad m_5) = (x \quad y \quad \gamma \quad v \quad \omega)$ represents the position and linear and angular velocities, also $(m_6 \quad m_7 \quad m_8 \quad m_9 \quad m_{10}) = (p_1 \quad p_2 \quad p_3 \quad p_4 \quad p_5)$ represents the costate and $m_{11} = t_f$ the final time. Then, the temporal derivative of the augmented m state is $\frac{dm}{dt} = \frac{dm}{ds}\dot{s} = \frac{1}{t_f}\frac{dm}{ds}$.

$$\frac{dm^*}{ds} = m_{11}^* \begin{bmatrix} m_4^*\cos(m_3^*) \\ m_4^*\sin(m_3^*) \\ m_5^* \\ \frac{1}{m_c r}(\tau_r^* + \tau_l^*) \\ \frac{L}{2Jr}(\tau_r^* - \tau_l^*) \\ 0 \\ 0 \\ -m_6^* m_4^*\sin(m_3^*) + m_7^* m_4^*\cos(m_3^*) \\ m_6^*\cos(m_3^*) + m_7^*\cos(m_3^*) \\ m_8^* \end{bmatrix} \quad (20)$$

4. Simulations and results

The mathematical details used to solve the optimal control problem are presented in the previous section. In the following, the problem is solved via a numerical approach. The characteristics of the robot are given in Table1.

Table 1. Parametric values of mobile robot

Property	Symbol	Value
Mass of the robot	m_c	$94(kg)$
Moment of inertia of the robot	J	$6.61(kg.m^2)$
The radius of each wheel	r	$0.08(m)$
The distance between the wheels	L	$0.4(m)$

The simulation conditions are selected according to Table 2 to show the effectiveness and capability of the proposed method.

Table 2. The initial condition for simulation

State	x	y	γ
Start point	0	0	0
Target point	5	5	45

The optimal path generated by the optimal control is shown in Figure 3.

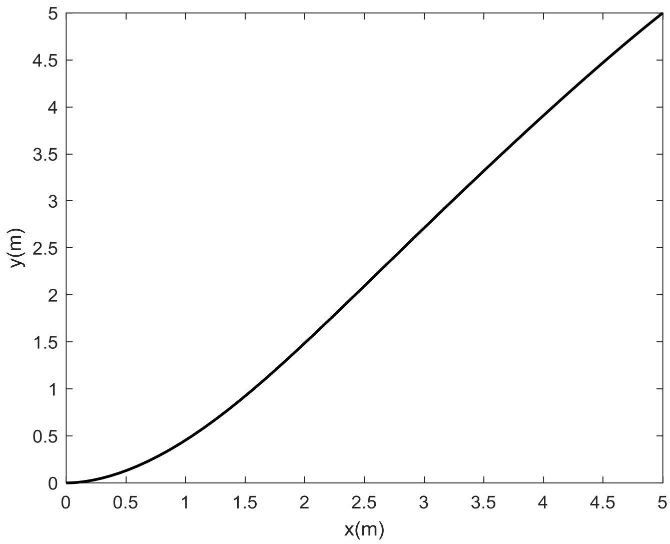
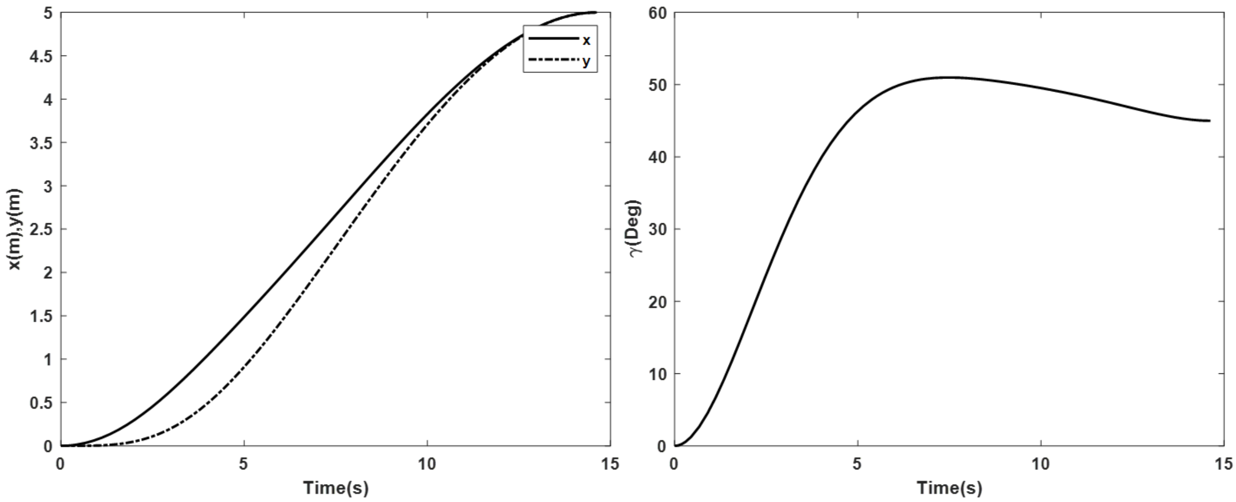


Figure 3. The optimal path generated by the mobile robot in the horizontal plane

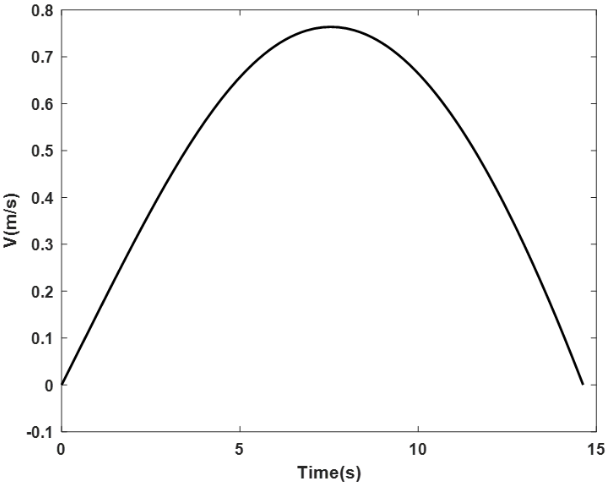
The optimal time calculated in the simulation is 14.62 seconds. The time history of the positions and the robot's orientation are shown in Figure 4.



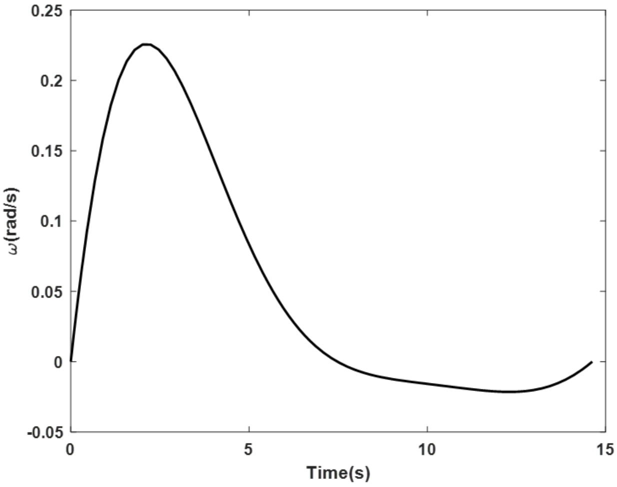
(a) (b)

Figure 4. The time history of the (a) positions and (b) orientation of the robot

The linear and angular velocities of the robot's mass center as a function of time are shown in Figure 5.



(a)



(b)

Figure 5. Center of mass (a) linear velocity and (b) angular velocity of the mobile robot

Also, the time history of the angular velocity of the left and right wheels is presented in Figure 6.

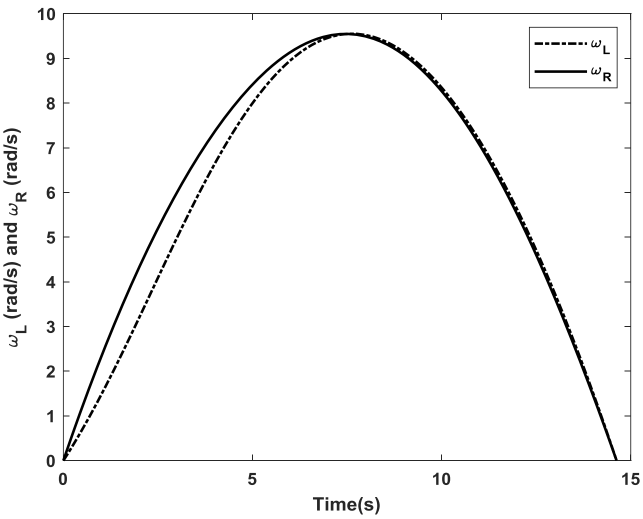


Figure 6. The time history of the angular velocity of left and right wheels of the mobile robot

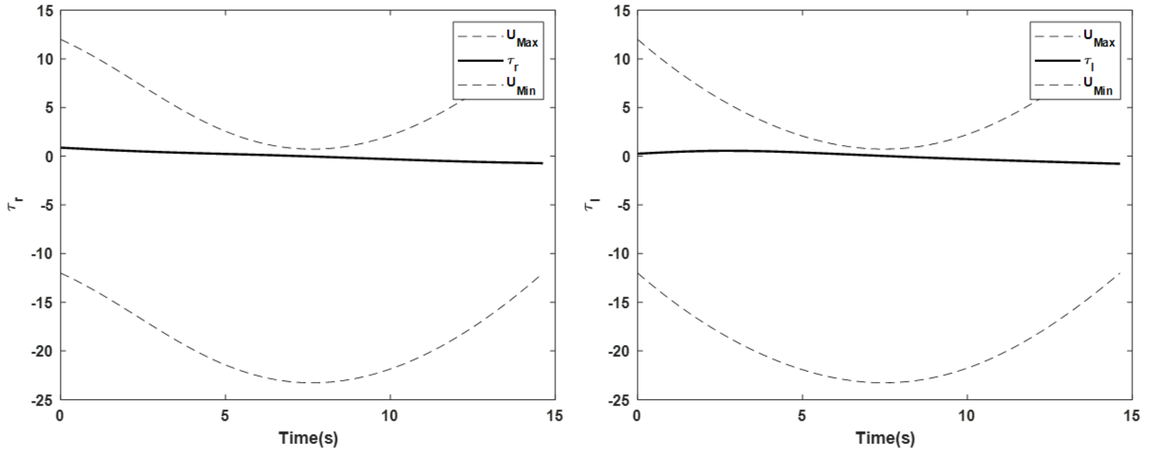
The lower and upper limits of the torques for Dc actuators of the mobile robot are obtained as:

$$U^{+} = K_1 - K_2 \upsilon$$
$$U^{-} = -K_1 - K_2 \upsilon \quad (21)$$

Where is υ the angular velocity of the wheels, that in the simulations the actuator constants are considered as:

$$K_1 = [12 \quad 12]$$
$$K_2 = [1.18 \quad 1.18] \quad (22)$$

Figure 7 depicts the optimum torque values for the wheel actuators. According to the obtained path, the torque and velocity of the right wheel are expectedly higher than that of the left wheel.



(a) (b)

Figure 7. Optimal Torques of wheeled (a) right wheeled (b) left wheeled

5. Conclusion

In the present research, the optimal path planning with optimal time based on the open-loop optimal control theory was investigated for a mobile industrial robot. First, the robot's nonlinear equations were expressed, that this equation is assumed constraints in optimal control problem. In order to minimize the time, the appropriate cost function was presented for optimal motion control of the system. It includes the torque inputs of wheels and time-related terminal conditions and disturbance formulated in optimal control problems presented. In order to solve the optimal control problem, the minimum principle of Pontryagin was employed. In the numerical simulation of the optimal control problem, selecting the α coefficient in the condition terminal is vital since it directly affects time optimization. In this simulation, its α value is considered equal to 0.9. The α coefficient acts as a weight function in optimization.

It can be selected the optimal motor to build the robot according to the torque obtained to generate the optimal path at the minimum time without surpassing the motors' saturation limit. Simulation results demonstrate the proposed method's power and capability in selecting a suitable motor for building robots.

6. References

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