A New Method for the Residues Cost Allocation and Optimization of a Cogeneration System Using Evolutionary Programming

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Abstract –As any energy system produces functional products, such as work, heat, etc., it produces unintended remaining flows of matter or energy, too, which are called residues. One of the objectives of exergoeconomic analysis is to understand the cost formation process and the flow of costs in the system. In the conventional thermoeconomic methods, however, the problem of the cost of residues has not been perceived soundly. One of the complex problems in the cost assessment is residues cost allocation in a rational way. Two more important methods of the residues cost allocation are distribution of the cost of the residues proportionally to the exergy as well as to the entropy generation or negentropy. In this paper, a new method for the residues cost allocation is proposed. This new method uses the fuel-product (FP) table, a mathematical representation of the thermoeconomic model, as the input data. In order to represent the proposed method, a cogeneration system that produces 34MW of electricity and 18kg/s of saturated steam at 20bar is selected. For the optimization of this system, first, a code has been developed based on the real coding evolutionary algorithm and optimal solution is to be obtained; then, the proposed method is applied to the cogeneration system. For comparison of the results, two other methods have also been applied to the system. The results of the comparison show that the proposed method is more suitable and rational than the two other ones.

Keywords: Cogeneration system, Exergoeconomic, Optimization, Residues, Cost allocation, Negentropy, Evolutionary algorithm.

I. Introduction

The development of design techniques for an energy system with minimized costs is a necessity in a world with finite natural resources and the increase of the energy demand in developing countries [1]. Analysis of energy systems based on the second law of thermodynamics is called exergy analysis, which is the maximum useful work that we can obtain from the flow of matter or energy. Thermoeconomic (exergoeconomic) is a discipline which combines concepts of the exergy method with those of economic analysis [2]. Exergy analysis usually predicts the thermodynamic performance of an energy system and the efficiency of the system components by accurately quantifying the entropy generation of the components. Furthermore, exergoeconomic analysis estimates the unit cost of products such as electricity and steam and quantifies monetary loss due to irreversibility [3]. The objective of a thermoeconomic analysis might be: (a) to calculate separately the cost of each product generated by a system having more than one product; (b) to understand the cost formation process and the flow of costs in the system; (c) to optimize specific variables in a single component; or (d) to optimize the overall system [4]. A critical review of relevant publications regarding exergy and exergoeconomic analysis can be found in articles by Vieira *et al.* [5, 6], Sahoo [7], Zhang *et al.* [8], and Lazzaretto and Tsatsaronis [9].

Unavoidably, in any productive process, the achievement of functional products is inseparable from the generation of residues and waste disposals [10]. It is sufficiently important to allocate the cost of the products appropriately in the poly-generation systems. Therefore, it is required to know where the residues have been generated as well as their abatement costs [10]. In conventional thermoeconomic methods, such as exergetic cost theory, (ECT) [11], average cost theory (ACT) [12], specific cost exergy costing method (SPECO) [13] and modified productive structural analysis (MOPSA) [14, 15], the problem of the cost of residues has not been clarified soundly. As mentioned in Ref. [10], works based on the structural theory [16] and other thermoeconomic

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methodologies [9, 17] provide different approaches to residue analysis, but none of them give a general solution to the problem. The residue cost allocation is a complex problem due to its dependence on the nature of flows and the way they have been formed [10]. A more complete analysis for residues cost allocation has been performed by Torres *et al.* [10], in which they have presented the mathematical basis for the cost assessment and the formation process of residues. To this end, they have extended the ECT cost propositions to include a new concept: “the cost of the residues generated by a productive component”; they also have updated the equations provided by symbolic exergoeconomics to include the cost formation process of residues. Based on their work, a residue cost distribution ratio should be defined. For more details, see section 5, Appendix C and Ref. [10]. This residue cost distribution ratio can be made in several ways, depending on the type and nature of the residue. As stated in Ref. [10], there is not a general criterion to define the residue cost distribution ratios. Two more important methods of the residue cost allocation are, respectively, the distribution of the cost of the residues proportionally to the exergy [10] and distribution of the cost of the residues proportionally to the entropy generation or negentropy [18, 19]. The choice of the best residue distribution among possible alternatives is still an open research problem. In this paper, a new method for the residues cost allocation is proposed. This new method is based on the entropy distributed in the components which is different from the entropy generated along the process. The method uses the fuel-product (FP) table, a mathematical representation of the thermoeconomic model, as the input data. In the proposed method, the concepts of distribution of the cost of the residues proportionally to the exergy and distribution of the cost of the residues proportionally to the entropy generation are combined to achieve a more rational distribution of the cost of the residues. A simple cogeneration system, which delivers 34MW of electricity and 18*kg*/*s* of saturated steam at 20 *bar*, is selected for validation. For comparison of results, first, a real coding evolutionary algorithm was developed in MATLAB whose optimum solution is to be obtained, and then the proposed method and two other methods [10, 18, and 19] would have been applied. The results illustrates that the proposed method is more competent than the other two its rivals.

II. Cogeneration System

A. Physical Model

Physical structure of an energy system represents how components are linked together and to the environment by means of a set of flows of matter, work or heat. Fig. 1 shows a schematic diagram of the cogeneration system which delivers 34MW of electricity and 18*kg*/*s* of saturated steam at20 *bar*. The system consists of a combustion chamber (CC), an air compressor (AC), a gas turbine (GT), a heat recovery steam generator (HRSG) and a stack. To achieve the optimization purpose, the following assumptions are made:

- The cogeneration system operates at a steady state.
- Air and the combustion gases are considered ideal gases with constant specific heats.
- The fuel is natural gas and assumed to be 100% methane. The methane is an ideal gas.
- Heat loss from the combustion chamber is considered to be 2% of the fuel lower heating value. All other components are considered adiabatic.
- The environment conditions are defined as T_0 = 25°C and P_0 = 1.013*bar*; these values are also used for enthalpy and exergy calculations.
- 5% pressure losses are assumed for the gases in HRSG and combustion chamber.

B. Thermodynamic Model

The thermodynamic model of an energy system, represented through a set of equations such as mass, energy and entropy balances for each component, is used to obtain some parameters, such as, pressure, temperature, enthalpy, entropy and exergy of flows. A detailed description of the model of the cogeneration system with all thermodynamic equations and variables can be found in the Appendix A.

C. Economic Model

In order to carry out a thermoeconomic analysis of an energy system, an economic model must be provided. The economic model takes into account the cost of the components, including amortization and maintenance, and the cost of fuel consumption. The purchase cost functions for each plant component can be found in the Appendix A, too.

D. Thermoeconomic Model

In order to perform a thermoeconomic analysis of an energy system, the thermoeconomic model is used. The productive structure, called productive or functional diagram, is a graphical representation of the fuel and product distribution given by the thermoeconomic model [20]. On the other hand, thermoeconomic model represents the productive purpose of each component. In this model, for each component, fuel and product are defined in terms of exergy flows. Table I represents definition of fuel and product for each component. In the productive or functional diagram [18, 19], the inputs of each component are the fuels and the outputs are the products. The exergy, done by each flow, is denoted as $E_{i,j}$ that represents the product of the i^{th} component used as fuel of the j^{th} component. Fig. 2 shows the productive diagram of the cogeneration system shown in Fig.1.

E. Definition of the Objective Function

The objective function is the total cost flow rate of the operation for the installation, obtained from,

$$C_T = C_F + \sum_{i=1}^{4} Z_i \qquad (1)$$

where C_T in ($/s) is the total cost flow rate of fuel and equipment and Z_i in ($/s) , the cost flow rate associated with capital investment and the maintenance cost for the *i*-th component (*i* = CC, AC, GT, HRSG, stack). Also, exergetic efficiency of the cogeneration system (ε_T) is defined as:

$$\varepsilon_T = \frac{\dot{W}_{\text{net}} + \dot{m}_s(e_7 - e_6)}{\dot{m}_F e_F} \qquad (2)$$

F. The Decision Variables

The key design variables, the decision variables, for the cogeneration system are:

- The air compressor pressure ratio (*PR*)
- The temperature of the combustion products entering the gas turbine (T_3)
- The isentropic air compressor efficiency (η_{AC})
- The isentropic gas turbine efficiency (η_{GT})

G. Physical Constraints (Feasibility Conditions)

The heat exchange between hot and cold streams in the HRSG requires that a feasible solution satisfies the following physical constraints:

$$\Delta T_{PP} = T_{5p} - T_7 > 0 \qquad (3)$$

$$T_4 \geq T_7 + \Delta T_{PP} \qquad (4)$$

$$T_5 \geq T_6 + \Delta T_{PP} \qquad (5)$$

$$T_{5P} > T_{6P} \qquad (6)$$

$$T_5 \geq 378.15\ \text{K} \qquad (7)$$

Fig. 3 shows the temperature profile of the HRSG. The last constraint is imposed on the exhaust gases temperature, which must not fall below 378.15ºK (105°C). This limitation is considered to prevent the condensation of the water vapor existing in the combustion products at the outlet section of economizer. The condensation of the water vapor in the presence of carbon dioxide may leads to the formation of carbonic acid which is a corrosive material and can damage the economizer surface [21]. Therefore, objective is to minimize Eq. (1) subject to the constraints imposed by the physical, thermodynamic and cost models of the installation. For more details, see Appendix A.

III. Optimization

The optimization has always been one of the most interested and essential parts of the design of energy systems. Usually, we are interested to know the optimum conditions of thermal systems. In recent years, exergoeconomic concepts have been used with search algorithms, such as genetic and evolutionary algorithm, to find out realistic optimal solution(s) of thermal systems [22–25]. Evolutionary programming (EP) is a powerful method of optimization when other techniques such as gradient descent or direct analytical discovery are not possible [7].

In this paper, in order for optimization of the cogeneration system shown in Fig. 1, a real coding evolutionary algorithm was developed in MATLAB and the optimum solution would be found. In this code, tournament selection was used as the selection mechanism. Also, arithmetical variable point crossover and real number uniform mutation were used for crossover and mutation, respectively. For more details, see Ref. [26]. To be more specific, for each decision variable, x_i, the lower ($x_{i,L}$) and the upper ($x_{i,U}$) , it is necessary to determine the limiting values. These values are presented in Table II. Fig. 4 shows the best fit of each generation versus the number of generations. Table III represents the optimum values of decision variables. Table IV represents sum of the investment cost flow rate (Z_T), fuel cost flow rate (C_F), total cost flow rate (C_T) and exergetic efficiency of the cogeneration system (ε_T) corresponding to the optimum conditions. Table V, also shows thermodynamic properties of the cogeneration system corresponding to the optimum conditions. Using tables I and V, values of fuel (*F*), product (*P*), irreversibility (*I*), exergetic efficiency (ε) and specific exergy destruction (kI), for each component, can be calculated. It should be noted that irreversibility of each component is difference between its fuel and product. The exergetic efficiency of a component is defined as the ratio between product and fuel. As we know, the unit exergy consumption (kB) evaluates the production performance of a component from a local point of view. The unit exergy consumption can be revised as [8],

$$\mathrm{kB} = \frac{F}{P} = \frac{P+I}{P} = 1 + \mathrm{kI} \quad (8)$$

where kI is defined as the ratio between the amount of irreversibility and the amount of product in a component, giving rise to the amount of exergy destroyed in a component to obtain one exergy unit of its product. The kI is defined as the specific exergy destruction [8]. Table VI shows the mentioned values for the cogeneration system.

IV. Productive and Dissipative Components

All components in an energy system can be divided into two groups, productive and dissipative components. The purpose of the productive components is to provide resources to other components or to obtain the final product of the system. The dissipative components, on the other hand, are used to eliminate, partially or totally, the wastes or residues that thrown away to the environment. The induced-draft fans of a steam generator and electrostatic precipitators for ash elimination in flue gases are two examples for dissipative components [10]. The last column in Table I indicates types of the components.

V. The Cost Formation Process of Residues

In the same way as there is a process for cost formation of the functional products, there also exists a cost formation process of the residues. The product cost of the i^{th} component, in a general form, is given by [10],

$$C_{P,\,i} = C_{F,\,i} + C_{R,\,i} + Z_i \quad (9)$$

where

$$C_{R,\,i} = \sum_{r \in \mathscr{V}_D} C_{ri} \quad (10)$$

In order to determine the C_{ri} values, we must define a residue cost distribution ratio ψ_{ir} such as

$$C_{ri} = \psi_{ir} C_{r0} \quad \text{with} \quad \sum_i \psi_{\mathrm{ir}} = 1 \quad (11)$$

As mentioned above, this allocation can be made in several ways, depending on the type and nature of the residue. However, there is not a general criterion to define the residue cost distribution ratios. The following system of linear equations, allows determining simultaneously the production cost of each component:

$$C_{P,\,i} - \sum_{j \in \mathscr{V}_P} y_{ij} C_{P,\,j} - \sum_{r \in \mathscr{V}_D} \psi_{ir} C_{P,r} = C_{e,i} + Z_i \qquad i = 1,\ldots,n \quad (12)$$

(12) can also be written in matrix notation as:

$$(U_D - \langle FP \rangle - \langle RP \rangle) C_P = C_e + Z \quad (13)$$

where $\langle FP \rangle$ is an $(n \times n)$ matrix whose coefficients are the cost distribution ratios of productive unit y_{ij} ($y_{ij} = E_{j,i} / P_j$) and $\langle RP \rangle$ is an $(n \times n)$ matrix whose coefficients ψ_{ij} are the cost distribution ratios of the dissipative unit.

VI. Review of the Two other Important Methods for the Residues Cost Allocation

In Refs. [18, 19], the residues cost allocation has been considered to be proportionally to the entropy generated along the process. This allocation works for closed cycle, like Rankine or refrigeration cycles, but it fails for other types of processes like gas turbines. Since in the closed cycles, the sum of the entropy generated in each productive process is equal to the entropy saved on the dissipative process, therefore, it is logical to distribute the cost of the wasted materials proportionally to the entropy generation. In case of open cycles, nonetheless, it is not true. For example, in the case of a simple gas turbine with a heat recovery steam generator, this process saves only a part of entropy generated in the global process. In Ref. [10], a simple method has been proposed to define the residue cost distribution ratios, which assumes that they are proportional to the exergy of the flows processed in the dissipative units, according to the productive structure of the plant,

$$\psi_{jr} = \frac{E_{j,r}}{F_r} \quad (14)$$

where $E_{j,r}$ represents the exergy of the flow produced in the j^{th} component and processed (dissipated) in the r^{th} component. The main advantage of this criterion is that the ratios could be obtained directly from the information provided by the productive diagram. It is important to remark that this option simplifies the software implementation of the costs computation, but it is neither the only way nor the best option for any type of dissipative unit. For more details, see Ref. [10].

To better perceive the idea behind the proposed method in this paper as well as compare its performance against the other two methods briefly explained here [10, 18 and 19], the three methods are described and applied to the cogeneration system.

A. Allocate the Cost of Residues Proportionally to the Entropy Generation along the Process

The To derive the production cost of each component from Eq. (13), it is necessary that the values of ψ_{jr}, *i.e.*, the coefficients in matrix $\langle RP \rangle$, be determined. In this section, allocation of the cost of residues proportionally to the entropy generation along the process will be described. To find values of ψ_{jr}, the values of entropy in table V will be used. Table VII shows description of the method. This method is denoted as option 1.

B. Allocate the Cost of Residues Proportionally to the Exergy

In this section, allocating the cost of residues proportionally to the exergy has been described. In Ref. [10], a model for analyzing the cost formation process of the residues has been proposed, based on the symbolic exergoeconomic methodology. To this end, the first step is a fuel-product (FP) table to be constructed. FP table is a mathematical representation of the thermoeconomic model. FP table is constructed using exergy of each flow, which is presented in Table V. The FP table specifies the distribution of fuel and product through the power plant. On the other hand, each element of FP table is $E_{i,j}$, defined as previous. Table VIII gives FP table for the cogeneration system. Regarding Eq. (14) and the values of $E_{i,j}$ in Table VIII, values of ψ_{jr} would be calculated. Table IX characterizes this method. This method is denoted as option 2.

VII. The New Proposed Method for the Residues Cost Allocation

Fuzzy Against the two methods just described above, a new method would be proposed in this section. In the new method, using the FP table, distribution of entropy is calculated through the power plant in the following way. It is worth to mention that the distribution of entropy through the power plant is different from the entropy generation along the process. As mentioned above, FP table uses exergy of each flow (matter, work or heat). The first step for the proposed method is to calculate the FP table. The second step is calculation of the FP table using energy instead of exergy, i.e. enthalpy (H) instead of exergy (E) for matter flow and heat flow rate instead of exergy of heat flow rate. Here, the FP table (calculated with energy) is called $FP^{\langle H \rangle}$ table contrary to the FP table (calculated with exergy). From the thermodynamic point of view, we know that $T_0 S = H - E$, where T_0 is the environment temperature, hence, it is proposed an FP table be constructed by subtracting each element of the FP table from the corresponding element in the $FP^{\langle H \rangle}$ table, which is called $FP^{\langle S \rangle}$ table. Therefore, $FP^{\langle S \rangle}=FP^{\langle H \rangle}-FP$. It should be noted that $FP^{\langle S \rangle}$ table represents distribution of entropy through the power plant. Tables XI and XII show $FP^{\langle H \rangle}$ and $FP^{\langle S \rangle}$ tables, respectively. Notice that each element of $FP^{\langle H \rangle}$, FP and $FP^{\langle S \rangle}$ tables are denoted as $E_{i,j}^{H}$, $E_{i,j}$ and $E_{i,j}^{S}$, respectively. Also P_i and F_i in $FP^{\langle S \rangle}$ table are denoted as P_i^S and F_i^S, respectively. Therefore, it can be written as $E_{i,j}^{S} = E_{i,j}^{H} - E_{i,j}$. Table 12 shows how the values of the new method are derived. This method is denoted as option 3. In Appendix D, a numerical description of the proposed method is represented.

VIII. Results and Discussion

In order to validate the written evolutionary code for optimization, it is applied to the CGAM problem [27] and the values of optimum solution is compared with the results of Valero *et al.* [27]. The results are in good agreement together. As stated in section 2.7, the optimum solution should be satisfied the physical constraints, i.e. Eqs. (3)–(7). In the present work, the temperature difference at the pinch point was obtained to be 42.55K, *i.e.* $\Delta T_{PP} = 42.55\,\mathrm{K}$. Also, the values of the temperatures T_{5P} and T_{6P} were indicated to be 528.15 and 470.60K, respectively. Table 13 shows a more appropriate comparison between the three methods. This table shows that all values of option 3 (proposed approach) are between the values of the two other methods. This result represents a more suitable and rational residues cost distribution. So,

$$\begin{aligned} &\psi_{jr}^{\text{Option 1}} \le \psi_{jr}^{\text{Option 3}} \le \psi_{jr}^{\text{Option 2}} \\ &\psi_{jr}^{\text{Option 2}} \le \psi_{jr}^{\text{Option 3}} \le \psi_{jr}^{\text{Option 1}} \end{aligned} \tag{15}$$

According to the proposed method (option 3), 70.45% of the residual gases cost is charged to the combustion chamber and the rest to the air compressor. Table 14 shows exergoeconomic costs of components calculated for option 3. As mentioned in Ref. [10], it is useful and more convenient to break down the cost of the products into their three contributing elements, namely, energy resources, equipment costs and residue costs. Table 15 shows the breakdown of the exergoeconomic cost of the products according to Eq. (C. 22). The columns f_R and f_Z represent the contribution of the residues and equipment investment to the production cost. For more details see Appendix C. $FP^{\langle S \rangle}$ table has some important properties. These are as follows:

Property 1. For the i^{th} component, sum of the elements in each row is equal to the entropy generated along the process of the product:

$$P_i^S = \sum_j E_{ij}^S = [T_0 \dot{m}(s_{out} - s_{in})]_P \quad i = 1,2,3,4 \text{ and } j = 1,2,...,5 \qquad (16)$$

Property 2. For the i^{th} component, sum of the elements in each column is equal to the entropy generated along the process of the fuel:

$$F_j^S = \sum_i E_{ij}^S = [T_0 \dot{m}(s_{in} - s_{out})]_F \quad i, j = 1,2,3,4 \qquad (17)$$

Property 3. The available values in column corresponding to F_5^S have been used to define the residue cost distribution ratios. (See Table 12 and Appendix B).

X. Conclusion

In Electrical A cogeneration system that produces 34MW of electricity and 18kg/s of saturated steam at20 bar, has been optimized by the real coding evolutionary algorithm and optimal solution has been obtained. Then, a new method has been proposed for the residues cost allocation. This method is based on the distribution of entropy through the components of the power plant. The main advantage of the distribution of the cost of the residues proportionally to the exergy (option 2) is that the residues cost distribution ratios could be derived directly from the information provided by the productive diagram, which simplifies the software implementation of the costs computation. The proposed method (option 3) has the advantage of the option 2, besides the property that the values of this option are between the values of the other two options, pointing out that the new option is more suitable and rational than the two other options. It is important to note that in order to extract the values of the new method, it is necessary to use $FP^{\langle S \rangle}$ table, whose interesting properties have already been mentioned.

Appendix A. Model of the Cogeneration System

A.1. the Physical and Thermodynamic Models

Here we present the equations that make up the physical model of the cogeneration system. These are the mass and energy balances for each component of the plant. Table A.1 represents some of the properties of the air and fuel.

Air Compressor (AC):

$$T_2 = T_1 \left\{ 1 + \frac{1}{\eta_{AC}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma_a - 1}{\gamma_a}} - 1 \right] \right\} \qquad (A.1)$$

$$P_1 = P_0 \quad \text{and} \quad T_1 = T_0 \qquad (A.2)$$

$$\dot{W}_{AC} = \dot{m}_a c_{p,a} (T_2 - T_1) \qquad (A.3)$$

Combustion Chamber (CC):

$$\dot{m}_g = \dot{m}_a + \dot{m}_F \qquad (A.4)$$

$$\dot{m}_a h_2 + \dot{m}_F \text{ LHV} = \dot{m}_g h_3 + \dot{Q}_{1,CC} \quad \text{with} \quad \text{LHV} = 50000 \text{ kJ/kg} \qquad (A.5)$$

$$\dot{Q}_{1,CC} = \dot{m}_F \text{LHV}(1 - \eta_{CC}) \quad \text{with} \quad \eta_{CC} = 0.98 \qquad (A.6)$$

$$P_3 = P_2 (1 - \Delta P_{CC}) \quad \text{with} \quad \Delta P_{CC} = 0.05 \text{ bar} \qquad (A.7)$$

Gas Turbine (GT):

$$T_4 = T_3 \left\{ 1 - \eta_{GT} \left[1 - \left(\frac{P_3}{P_4} \right)^{\frac{1 - \gamma_g}{\gamma_g}} \right] \right\} \qquad (A.8)$$

$$\dot{W}_{GT} = \dot{m}_g c_{p,g} (T_3 - T_4) \qquad (A.9)$$

$$\dot{W}_{net} = \dot{W}_{GT} - \dot{W}_{AC} \quad \text{with} \quad \dot{W}_{net} = 34 \text{ MW} \qquad (A.10)$$

Specific exergy, energy and entropy of the air streams: (i = 1, 2)

$$\text{exergy}: e_i = c_{p,a} \left[T_i - T_0 - T_0 \ln \frac{T_i}{T_0} \right] + R_a T_0 \ln \frac{P_i}{P_0} \qquad (A.11)$$

$$\text{energy}: \quad h_i = c_{p,a} (T_i - T_0) \qquad (A.12)$$

$$\text{entropy}: \quad s_i = c_{p,a} \ln \frac{T_i}{T_0} - R_a \ln \frac{P_i}{P_0} \qquad (A.13)$$

Specific exergy, energy and entropy of the gas streams: (i = 3,4,5,11) exergy: $e_i = c_{p,g}\left[T_i - T_0 - T_0 \ln\frac{T_i}{T_0}\right] + R_g T_0 \ln\frac{P_i}{P_0} + R_g T_0 \sum_j x_j^i \ln\frac{x_j^i}{x_j^0}$ and $j = O_2, N_2, CO_2, H_2O$ (A.14)

energy: $h_i = c_{p,g}(T_i - T_0)$ (A.15)

entropy: $s_i = c_{p,g} \ln\frac{T_i}{T_0} - R_g \ln\frac{P_i}{P_0}$ (A.16)

Heat Recovery Steam Generator (HRSG):

$T_{6P} = T_7 - \Delta T_{AP}$ with $\Delta T_{AP} = 15\,\text{K}$ (A.17)

$\dot{m}_g c_{p,g}(T_4 - T_{5P}) = \dot{m}_s(h_7 - h_{6P})$ (A.18)

with $\dot{m}_s = 18\,\text{kg/s}$ and $(h_7 - h_{6P}) = 1956\,\text{kJ/kg}$ (A.19)

$\text{temperature difference at the pinch} = \Delta T_{PP} = T_{5P} - T_7 > 0$ (A.20)

$T_5 = T_4 - \dot{m}_s(h_7 - h_6)/(\dot{m}_g c_{p,g})$ with $(h_7 - h_6) = 2689\,\text{kJ/kg}$ (A.21)

$P_0 = P_4(1 - \Delta P_{\text{HRSG}})$ with $\Delta P_{\text{HRSG}} = 0.05\,\text{bar}$ (A.22)

A.2. the Economic Model

When evaluating the costs of a plant, it is necessary to consider the annual cost of fuel and those associated with owning and operating each plant component. The purchased equipment costs (PEC) of the cogeneration system components as a function of thermodynamic parameters have been given in Ref. [27]. Based on the costs, the general equation for the cost rate (Z_i in \$/s) associated with capital investment and the maintenance cost for the *i*-th component is:

$$Z_i = \text{PEC}_i \ \text{CRF} \ \varphi \ / \ (N \times 3600) \quad \text{(A.23)}$$

here, PEC$_i$ is the purchase costs of the i^{th} component (\$), CRF , the annual capital recovery factor (CRF = 18.2 %), N , the number of the hours of plant operation per year (N = 8000 h), and φ , the maintenance factor (φ = 1.06).

In the cogeneration system, the objective function is the total cost rate (C_T); that is, sum of the investment cost rate (Z_T) and fuel cost rate (C_F), i.e.,

$$C_T = C_F + Z_T \quad \text{(A.24)}$$

where:

$$C_F = \dot{m}_F c_F \ \text{LHV} \quad \text{(A.24.a)}$$

and

$$Z_T = \sum_{k=1}^{n} Z_k \quad \text{(A.24.b)}$$

where n is the number of components.

Appendix B. Properties of the FP$^{\langle S\rangle}$ table

In this appendix, detailed information of the new proposed method is presented. Table B.1 shows numerical demonstration of the properties of this method. In this table, the bold-faced underlined values corresponding to combustor and compressor (in these components air/gas streams are their products) are summed and then divided by the summation (normalization); then, the residue cost distribution ratio (ψ) for these components is found, as shown in Table 12. In practice, these bold underlined values are the same values corresponding to column F_5^S of the FP$^{\langle S\rangle}$ table. The bold-faced values (not the bold-faced underlined values) in this table show entropy generated along the process of product for each component. For more details, see and focus on the values of this table. As mentioned above, Table 11 shows FP$^{\langle S\rangle}$ table. As of this table, the following equations can be written, called properties of the FP$^{\langle S\rangle}$ table.

Property 1. For the i^{th} component, sum of the elements of each row, i.e. $\sum_j E_{ij}^S$, represents P_i^S , i.e., it is equal to the entropy generated along the process of the product. Then, the following equation can be written:

$$P_i^S = \sum_j E_{ij}^S = [T_0 \dot{m}(s_{out} - s_{in})]_P \quad i = 1,2,3,4 \ \text{and} \ j = 1,2,...,5 \quad \text{(B.1.a)}$$

where, subscript P indicates that the entropy generated along the process of the product should be used. For example, Eq. (B.1.a) for the air compressor would be written in the following way:

$$P_2^S = E_{2,3}^S + E_{2,4}^S + E_{2,5}^S = T_0 \dot{m}_a (s_2 - s_1)$$

$$2916 = -6246 + 6077 + 3085 = 298.15 \times 101.4513 \times (0.0964 - 0.0000) \quad \text{(B.1.b)}$$

For more details, see Table B.1 and pay attention to the bold-faced values in this table. Also, it should be noted that the percent of relative error (μ) between P_i^S and

$[T_0\dot{m}(s_{out}-s_{in})]_P$ for each case is negligible.

Property 2. For *i*th component, sum of the elements in the each column, i.e. $\sum_i E_{ij}^S$, represents F_i^S , i.e., it is equal to the entropy generated along the process of the fuel. Then, the following equation can be written.

$$F_j^S = \sum_i E_{ij}^S = [T_0\dot{m}(s_{in}-s_{out})]_F \quad i, j = 1,2,3,4 \quad \text{(B.2.a)}$$

where, subscript *F* indicates the entropy generated along the process of the fuel should be used. For example, Eq. (B.2.a) for the gas turbine, is written in the following way:

$$F_3^S = E_{1,3}^S + E_{2,3}^S = T_0\dot{m}_g(s_3 - s_4)$$

$$-3516 = 2730 + (-6246) \cong 293.15\times 103.4317\times(1.0535-1.1675) = -3515$$

$$\mu = (3516-3515)/3515\times 100 = 0.03\%$$

(B.2.b)

For more details, see Table B.1. Also, it should be mentioned that the percent of the relative error (μ) between F_i^S and $[T_0\dot{m}(s_{in}-s_{out})]_F$, for all the cases, is less than 0.05%.

It is significant that while in the first component, i.e. combustion chamber, F_1^S is equal to –3664 kW, the RHS of the Eq. (B.2.a) for this case is equal to zero. But sum of the elements in the corresponding column, i.e. $\sum_{i=1}^4 E_{i1}^S$ in the FP$^{\langle S\rangle}$ table, are equal to zero and really $E_{0,1}^S = -3664$ kW, therefore, Eq. (B.2.a) is still validate. On the other hand, to obtain F_1^S , the value $E_{0,1}^S$ should not be considered.

Property 3. The available values in column corresponding to F_5^S define the residue cost distribution ratios. (See Table 12).

Table 1. Definition of fuel and product for each component

No.	Device	Fuel	Product	Type of component
1	Combustion Chamber	E_8	$E_3 - E_2$	Productive
2	Air Compressor	E_9	$E_2 - E_1$	Productive
3	Gas Turbine	$E_3 - E_4$	$E_9 + E_{10}$	Productive
4	Heat Recovery Steam Generator	$E_4 - E_5$	$E_7 - E_6$	Productive
5	Stack	E_5	E_{11}	Dissipative

Table 2. The lower and the upper limiting values for the decision variables of the cogeneration system

Variable	Value	
	Minimum	Maximum
PR	5	25
T_3 (K)	1200	1800
η_{AC}	0.7	0.9
η_{GT}	0.7	0.92

Table 3. Variables for optimal conditions derived from the evolutionary algorithm

Variable	*PR*	T_3 (K)	η_{AC}	η_{GT}
Value	17.7421	1477.60	0.84703	0.89583

Table 4. Sum of the investment cost flow rate (Z_T), fuel cost flow rate (C_F), total cost flow rate (C_T) and exergetic efficiency (ε_T) corresponding to optimal conditions from the evolutionary algorithm

Z_T ($/h)	C_F ($/h)	C_T ($/h)	ε_T (%)
216.7397	1425.9207	1642.6604	49.07

Table 5. Thermodynamic properties of the cogeneration system corresponding to optimal conditions

No.	Flow description	*p* (bar)	*T* (K)	$\dot{m}$ (kg/s)	*s* (kJ/kg · K)	*h* (kJ/kg)	*H* (kW)	*E* (kW)
0	Environment	1.013	298.15					
1	Air inlet compressor	1.013	298.15	101.4513	0.0000	0.00	0.00	0.00
2	Air outlet compressor	17.973	746.72	101.4513	0.0964	450.36	45689.78	42774.88
3	Gas inlet turbine	17.074	1477.60	103.4317	1.0535	1380.00	142731.61	112078.64
4	Gas inlet evaporator	1.066	819.09	103.4317	1.1675	609.50	63041.83	28873.44
5	Gas outlet economizer	1.013	418.98	103.4317	0.3981	141.37	14621.83	4182.52
6	Water inlet economizer	20	298.15	18.0000	0.3674	109.00	1962.00	79.20
7	Steam outlet evaporator	20	485.60	18.0000	6.3409	2798.00	50364.00	16470.26
8	Fuel combustion chamber	1.013	298.15	1.9804	0.0000	50000.00	99022.27	102686.10
9	Power air compressor						45689.78	45689.78
10	Power gas turbine						34000.00	34000.00
11	Gas outlet stack	1.013	418.98	103.4317	0.3981	141.37	14621.83	4182.52

Table 6. Fuel (F), product (P), irreversibility (I), exergetic efficiency (ε) and specific exergy destruction (kI) for each component

No.	Device	F (kW)	P (kW)	I (kW)	ε	kI
1	Combustion Chamber	102686.10	69303.76	33382.34	0.6749	0.4817
2	Air Compressor	45689.78	42774.88	2914.90	0.9362	0.0681
3	Gas Turbine	83205.20	79689.78	3515.42	0.9578	0.0441
4	Heat Recovery Steam Generator	24690.91	16391.06	8299.86	0.6638	0.5064
5	Stack	4182.52	4182.52	0.00	1.0000	0.0000

Table 7. Allocation of the cost of residues proportionally to the entropy generated along the process (Option 1)

No.	**Device**	Δs_i	$\psi_i = \frac{\Delta s_i}{\Delta s_T}$
1	Combustion Chamber	$s_3 - s_2 = 0.9572$	2.4044
2	Air Compressor	$s_2 - s_1 = 0.0964$	0.2421
3	Gas Turbine	$s_4 - s_3 = 0.1140$	0.2864
4	Heat Recovery Steam Generator	$s_5 - s_4 = -0.7695$	–1.9329
		$\Delta s_T = \sum_{i=1}^{4} \Delta s_i = 0.3981$	

Table 8. FP table for the cogeneration system

	F_0	F_1	F_2	F_3	F_4	F_5	Total
P_0		102686					102686
P_1				51450	15268	2586	69304
P_2				31755	9423	1596	42774
P_3	34000		45690				79690
P_4	16391						16391
R_5	4183						4183
		102686	45690	83205	24691	4182	

Table 9. Allocation of the cost of residues proportionally to the exergy (Option 2)

No.	**Device**	$\psi_i = \frac{E_{i,5}}{F_5}$
1	Combustion Chamber	0.6184
2	Air Compressor	0.3816
3	Gas Turbine	0.0000
4	Heat Recovery Steam Generator	0.0000

Table 10. FP$^{\langle H \rangle}$ table for the cogeneration system

	F_0^H	F_1^H	F_2^H	F_3^H	F_4^H	F_5^H	Total
P_0^H		99022					99022
P_1^H				54180	32920	9941	97041
P_2^H				25509	15500	4681	45690
P_3^H	34000		45690				79690
P_4^H	48402						48402
R_5^H	14622						14622
		99022	45690	79689	48420	14622	

Table 11. FP$^{\langle S \rangle}$ table for the cogeneration system

	F_0^S	F_1^S	F_2^S	F_3^S	F_4^S	F_5^S	Total
P_0^S		–3664					-3664
P_1^S				2730	17652	7355	27737
P_2^S				–6246	6077	3085	2916
P_3^S	0		0				0
P_4^S	32011						32011
R_5^S	10439						10439
		–3664	0	–3516	23729	10440	

Table 12. Allocation of the cost of residues based on the distribution of entropy (Option 3)

No.	**Device**	$\psi_i = \frac{E_{i,5}^S}{F_5^S}$
1	Combustion Chamber	0.7045
2	Air Compressor	0.2955
3	Gas Turbine	0.0000
4	Heat Recovery Steam Generator	0.0000

Table 13. Residue cost distribution ratios

No.	**Device**	**Option 1**	**Option 2**	**Option 3**
1	Combustion Chamber	2.4044	0.6184	0.7045
2	Air Compressor	0.2421	0.3816	0.2955
3	Gas Turbine	0.2864	0.0000	0.0000
4	Heat Recovery Steam Generator	–1.9329	0.0000	0.0000

Table 14. Exergoeconomic costs of components

No.	**Device**	c_P **(¢/kWh)**	C_F **($/h)**	C_R **($/h)**	Z **($/h)**	C_P **($/h)**
1	Combustion Chamber	2.1598	1425.92	67.84	3.09	1496.85
2	Air Compressor	3.3737	1308.15	41.87	93.09	1443.11
3	Gas Turbine	2.8631	2182.57	0.00	99.04	2281.61
4	Heat Recovery Steam Generator	4.0827	647.67	0.00	21.53	669.20
5	Stack	2.6231	109.71	0.00	0.00	109.71

Table 15. Cost decomposition

No.	Device	C_P ($/h)	C_P^e ($/h)	C_P^r ($/h)	C_P^z ($/h)	f_R (%)	f_Z (%)
1	Combustion Chamber	1496.85	1425.92	67.84	3.09	4.53	0.21
2	Air Compressor	1443.11	1056.71	123.18	263.22	8.53	18.24
3	Gas Turbine	2281.61	1843.06	141.81	296.74	6.21	13.01
4	Heat Recovery Steam Generator	669.20	546.92	42.08	80.19	6.29	11.98
5	Stack	109.71	92.65	7.13	9.94	6.50	9.06

Table A.1. Properties of the air and fuel

Atmospheric pressure	$P_0 = 1.013$ bar
Atmospheric temperature	$T_0 = 25°C$
Reference substances: Air (relativity humidity = 60%)	$x^0_{O_2} = 0.2059,\ x^0_{N_2} = 0.7748,\ x^0_{CO_2} = 0.0003,\ x^0_{H_2O} = 0.019$
Specific energy and exergy of the fuel (methane):	$e_F = 51850$ kJ/kg , $h_F = LHV = 50000$ kJ/kg
Air	$c_{p,a} = 1.004$ kJ/(kg·K), $\gamma_a = 1.4$, $R_a = 0.287$ kJ/(kg·K)
Combustion gases	$c_{p,g} = 1.17$ kJ/(kg·K), $\gamma_g = 1.33$, $R_g = 0.29$ kJ/(kg·K)
The molecular weights of methane and air	$M_{CH_4} = M_F = 16.043$ kg/kmol, $M_a = 28.648$ kg/kmol
Reaction of complete combustion:	

$$f\,CH_4 + x^0_{O_2}O_2 + x^0_{N_2}N_2 + x^0_{CO_2}CO_2 + x^0_{H_2O}H_2O \Rightarrow (f + x^0_{CO_2})CO_2 + (2f + x^0_{H_2O})H_2O + (x^0_{O_2} - 2f)O_2 + x^0_{N_2}N_2$$

Table B.1. Numerical representation of the properties of the new method*

No.	Device		$P_i^H = \sum_j E_{i,j}^H,\ P_i = \sum_j E_{i,j},\ P_i^S = \sum_j E_{i,j}^S$	Descriptions
1	Combustion Chamber	$E_{1,j}$	$E_{1,3} + E_{1,4} + E_{1,5} = P_1$	$T_0(\dot{m}_g s_3 - \dot{m}_a s_2) = 29752$ kW
		$E_{1,j}^H$	54180 + 32920 + 9941 = 97041 kW	
		$E_{1,j}$	51450 + 15268 + 2586 = 69304 kW	$P_1^S = 27737$ kW
		$E_{1,j}^S$	2730 + 17652 + 7355 = 27737 kW	$\mu = 6\%$
2	Air Compressor	$E_{2,j}$	$E_{2,3} + E_{2,4} + E_{2,5} = P_2$	$T_0\dot{m}_a(s_2 - s_1) = 2916$ kW
		$E_{2,j}^H$	25509 + 15500 + 4681 = 45690 kW	
		$E_{2,j}$	31755 + 9423 + 1596 = 42774 kW	$P_2^S = 2916$ kW
		$E_{2,j}^S$	– 6246 + 6077+ 3085 = 2916 kW	$\mu = 0\%$
3	Gas Turbine	$E_{3,j}$	$E_{3,0} + E_{3,2} = P_3$	$T_0\dot{m}_g(s_3 - s_4) = -3515$ kW
		$E_{3,j}^H$	34000 + 45690 = 79690 kW	
		$E_{3,j}$	34000 + 45690 = 79690 kW	$F_3^S = -3516$ kW
		$E_{3,j}^S$	0 + 0 = 0 kW	$\mu = 0.03\%$
4	Heat Recovery Steam Generator	$E_{4,j}$	$E_{4,0} = P_4$	$T_0\dot{m}_s(s_7 - s_6) = 32058$ kW
		$E_{4,j}^H$	48402 = 48402 kW	$P_4^S = 32011$ kW and $\mu = 0.15\%$
		$E_{4,j}$	16391 = 16391 kW	$T_0\dot{m}_g(s_4 - s_5) = 23727$ kW
		$E_{4,j}^S$	32011 = 32011 kW	$F_4^S = 23729$ kW and $\mu = 0.008\%$

* Mass flow rates are:

$\dot{m}_a = \dot{m}_1 = \dot{m}_2$ and $\dot{m}_g = \dot{m}_i$ where $i = 3,4,5,11$ and $\dot{m}_s = \dot{m}_6 = \dot{m}_7$

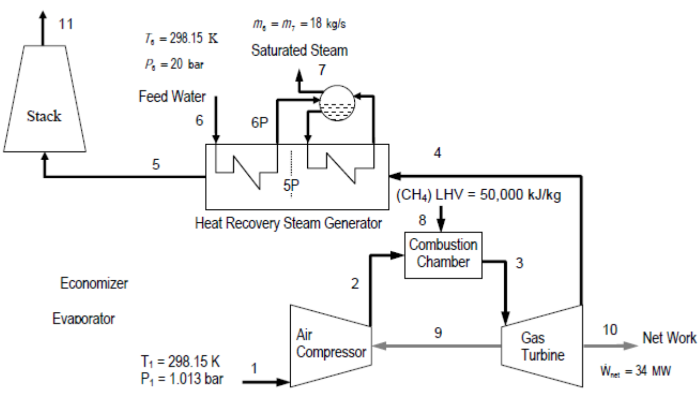

Fig.1: Flow diagram of a cogeneration system

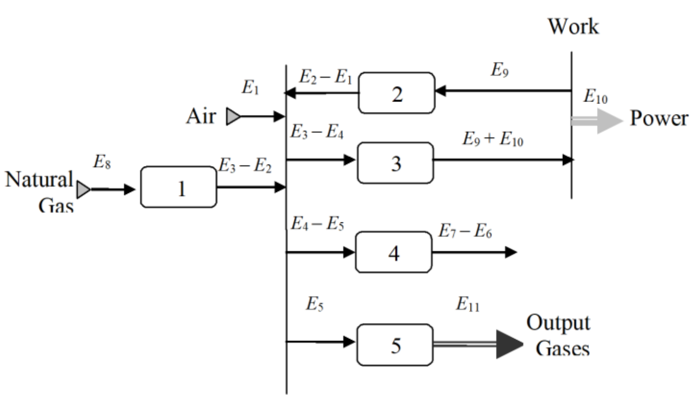

Fig.2: Fuel product diagram of the cogeneration system

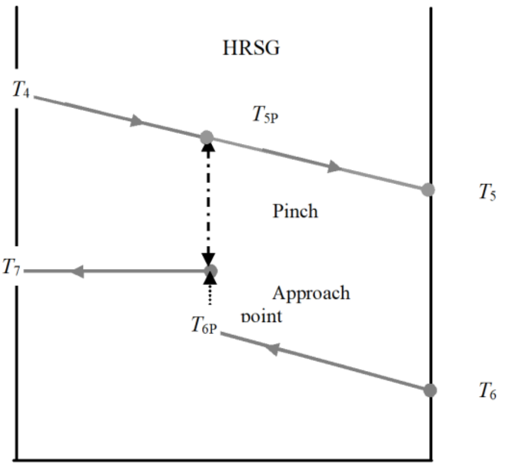

Fig. 3: Temperature profile of the HRSG

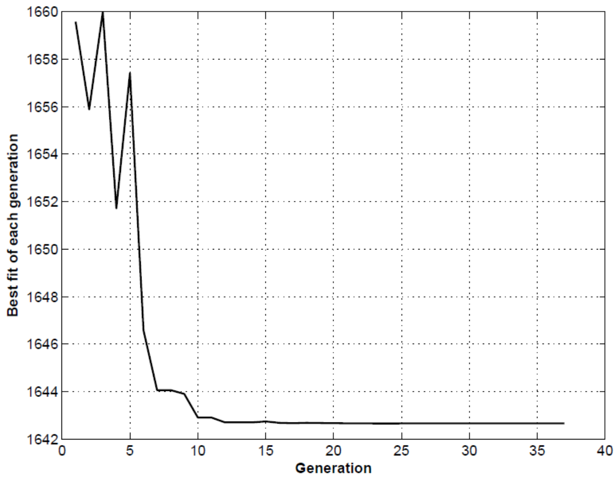

Fig. 4: Best fit of each generation

Nomenclature

c	unit exergoeconomic cost (¢/kWh)	ψ	residue cost distribution ratio
C	exergoeconomic cost ($/h)		Matrices and vectors
CRF	capital recovery factor	**Z**	capital cost vector $(n\times 1)$
c_p	constant-pressure specific heat (kJ/kg.K)	$\mathbf{C}_F$	fuel cost vector $(n\times 1)$
e	specific exergy (kJ/kg)	$\mathbf{C}_P$	product cost vector $(n\times 1)$
E	exergy of a flow (kW)	$\mathbf{C}_R$	residue cost vector $(n\times 1)$
f	fuel/air ratio in reaction of complete combustion in table A.1	$\mathbf{U}_D$	identity matrix $(n\times n)$
F	fuel exergy of a component (kW)	⟨FP⟩	matrix $(n\times n)$ which contains the distribution ratios
h	specific enthalpy (kJ/kg)	⟨RP⟩	matrix $(n\times n)$ which contains the residue ratios
H	enthalpy of a flow (kW)	⟨P*|	cost operator matrix $(n\times n)$
I	irreversibility of a component (kW)		Subscripts
kB	unit exergy consumption	0	index for environment (reference state)
kI	specific exergy destruction	1,2,…,11	(in Fig. 1) refers to thermodynamic states
LHV	lower heating value of fuel (kJ/kg)	a	air
$\dot{m}$	mass flow rate (kg/s)	AC	air compressor
M	molecular weight (kg/kmol)	APH	air preheater
n	number of components	CC	combustion chamber
N	number of the hours of plant operation per year (h/year)	e	system inlet
p	pressure (bar)g gas	F	fuel, related to fuel
P	product exergy of a component (kW)	g	gas
PEC	purchased equipment cost ($)	GT	gas turbine
PR	pressure ratio	HRSG	heat recovery steam generator
Q	heat flow rate (kW)	in	inlet
R	specific gas constant (kJ/kg · K)	i, j	indexes for productive components
RHS	right hand side	L	lower
s	specific entropy (kJ/kg .k)	out	outlet
T	temperature (K)	P	related to product
W	work flow rate (kW)	r	index for dissipative components
x	decision variable	R	related to residue
y	distribution exergy ratios	s	steam
Z	capital cost rate of a component ($/h)	T	total
$\mathcal{P}$	set of productive components	U	upper
$\mathcal{D}$	set of dissipative components	PP	pinch point
		AP	approach point

	Greek letters		
γ	heat capacity ratio	**Superscripts**	
Δ	absolute change in a variable	-1	inverse matrix
ΔP	pressure loss (bar)	e	related to external resources
ε	exergetic efficiency	r	related to residues
η	isentropic efficiency	z	related to capital cost
μ	percent of relative error	$\langle H \rangle$	related to energy, heat and enthalpy
φ	maintenance factor	$\langle S \rangle$	related to entropy

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