Fault Diagnosis Operator in Linear Fractional Order Singular Systems Using Singular Observer and Unknown Input

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Abstract – The singular systems appear in many real occasions of system modeling. Fault occurrence is inevitable in real system; thus to avoid their destructive impacts, new design perspective must be taken. Performance and sensitivity of the fault diagnosis model based methods, however, significantly dependent on the accuracy of the model. In the one hand, it has been shown that many systems naturally follows the fractional order behavior, while on the other, in some scenarios, fractional modeling has improved the accuracy of the model. In this paper, we pay attention to the fault diagnosis in the fractional order singular systems. To this end, a singular observer with an unknown input has been used for diagnosis of the fault in the fractional order singular system, and the proposed observer convergence will be derived in the form of a linear matrix inequality. An advantage of the proposed method is separation of noise from the desired signal, both in inputs and outputs, using only the inputs and outputs signals.

Keywords: fault diagnosis, noise separation, singular system, fractional order system, linear matrix inequality.

I. Introduction

The massive Analysis of the dynamical systems in different fields such as engineering of mechanic, electricity, chemistry, economy and biology, mostly dependent on a mathematical model which describe system behavioral dynamics. The increasing complexity of the systems, in one hand, and the need to more accurate models on the other, has led modeling to be combinatorial, or a combination of subsystems (modular modeling). In this viewpoint of system modeling, usually, interactions between different parts are modeled based on the differential equations describing the behavior of each part and the relationship between subsystems will be described according to algebraic equations. That is, the whole system would be explained by a set of algebraic-differential equations. In control engineering, systems that are modeled in this way are called differential-algebraic systems or singular systems.

The singular models are versatile in practice, however, there might be difficulties when confronting them, which are not likely seen in other usual systems; in singular systems, beside the input derivative on the output, existence of the impulse at the output of the system is likely, too. Analysis of this kind of system has been the topic of many researches of the last years and by which, noticeable advancements have been achieved, providing a mathematical basis for the analysis of the related problems.

Based on the application, these type of systems may be called by different names such as differential-algebraic system, singular, implicit, descriptor, generalized state space, degenerative, semi-state, constrained and reduced order models.

The application of singular systems have been so far seen in engineering system analysis such as electrical circuits, power systems, aerospace engineering, chemical, biological and economical processes, *etc*.[1]. The interested readers are encouraged to refer to [1-9] for more information on the singular systems.

The fault occurrence in inescapable in real systems. Here, fault occurrence means deviation in at least one of the characteristics of the system when it is working at usual and/or standard operating condition. In order to avoid the destructive effects of the fault on the system, there must perspectives to design approaches for diagnosis and control of the fault. The main idea behind the fault diagnosis is the comparison of the system performance under the real conditions with that of usual and faultless one. To do so, there have been plenty of methods which are categorized here into three classes: model based, data based and knowledge based methods. In the model based method, a mathematical model is used to compare the real system with the modeled system using the mathematical analysis

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tools/software and algorithms. At this point, it is necessary to mention that since singular systems are defined under the broader domain of systems modeling, the only method meaningful to identify the right from the wrong is to use the related model. Also, performance and sensitivity of the fault identification mathematical method is significantly dependent on the accuracy of the model; thus, a more accurate model naturally will give rise to better and more reliable results.

The fractional order differential equations have a wide range of applications in modeling of physical systems, especially those in which, there is diffusion or replacement in some physical sense. Also, the fractional derivatives are great tools for description of materials as well as processes' properties such as memory and inheritability; in other words, capabilities of fractional mathematics has made them a convenient tool for system modeling.

In many processes and systems' modeling, it has been shown that either systems follow the fractional laws or fractional mathematics help to more accurate modeling. For example, viscoelastic materials, transmission lines, thermal systems, some biologic materials, electrical capacitors, oil wells erosions and economical systems, all fall into the above mentioned processes and systems [10].

The fractional order singular systems are singular systems that their dynamic behavior are described using fractional order equations; thus, they have the algebraic limitations of singular systems in one hand, and benefit from the advantages of fractional order modeling, on the other. In other words, with increasing of the use of fractional order calculus in system modeling and their applications in singular systems, acquaintance and analysis with the fractional order singular systems deemed necessary [11]. In this respect, electrochemical processes, large size fractional order systems, electrical circuits comprising fractional circuits and power systems encompassing hydropowers, are just samples of fractional order singular systems.

Because of singularity and fractionality of derivatives, fractional order singular systems do not obey many rules governing systems of integer derivatives or at least, do need some changes in the rules; this lead to computational complexities in derivation of solutions and their interpretations.

The fractional order singular systems are right choices of modeling for such complicated systems and systems with memory as economical, biological, electrical circuits which includes fractionally behavioring elements, mechanical, chemical systems and processes, *etc,* due to the inclusion of singularity in their models, in one hand, and fractional order behavior, on the other.

Although the research history of singular as well as fractional order systems goes back to the latest years of the 20th century and before, studies specifically focusing on the fractional order singular systems in limited to the recent decade. In the recent years, there have been inquiries in a varieties of contexts, including, but not limited to, stability [12], stabilization [11], dynamical analysis [13], and observer designs for these system. In the field of fault diagnosis in fractional order singular systems, number of researches are very limited. This is the main motive of the authors for doing of this study.

The suggested method of the operator's fault diagnosis in the fractional order singular systems are based on using an observer with an unknown input; in this point of view, the error in the output estimation has been used to produce the residual signal, which later would be utilized for the fault diagnosis; the observer differentiate between the disturbance and the desired signals in order to do a robust diagnosis. It must be noted that in the proposed method, in order to estimate the fault of the operator, an observer that uses only the matrix coefficients of the main system, has been applied.

II. Theoretical Introduction and Problem Statement

In this section, first, the necessary definitions and lemmas are presented and then the problem would be described. There a few definitions of fractional integral and derivatives in which the most important ones are those proposed by Riemann-Liouville, Gronvald-Letnikov, and Caputo [17].

Definition 1: the Caputo definition of the fractional order derivation is as,

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad n-1 \le \alpha < n \tag{1}$$

In which, n is an integer and α is a real number.

Lemma 1: the matrix relationship

$$X\Theta = \Omega \tag{2}$$

Has a solution if and only if the following relation is satisfied,

$$\text{rank}\begin{bmatrix}\Theta \\ \Omega\end{bmatrix} = \text{rank}\Theta \tag{3}$$

in this case, the solution would be of the form,

$$X = \Omega\Theta^{+} + \Upsilon\left(I - \Theta\Theta^{+}\right) \tag{4}$$

where Θ^{+} is the generalized inverse matrix satisfying $\Theta\Theta^{+}\Theta = \Theta$, and Υ is an arbitrary matrix with desired dimensions.

Lemma 2: the pair (E, A) is regular, commensurate and stable if and only if there is a matrix X that satisfies,

$$E^T X = X^T E \geq 0, \quad A^T X + X^T A < 0 \tag{5}$$

An LTI commensurate with linear fractional order singular system, in the presence of disturbance in both input and output and fault in the operator would be modeled as:

$$\begin{cases} ED^{\alpha}x(t) = Ax(t) + Bu(t) + B_d d(t) + B_f f(t) \\ y(t) = Cx(t) + D_d d(t) \end{cases} \tag{6}$$

where, α is the order of fractional derivative, $x \in R^n$, the semi-state vector, $u \in R^{k_u}$ and $y \in R^m$, respectively control inputs and outputs, $d \in R^{k_d}$, the unknown limited input vector, $f \in R^{k_f}$, the fault of the system's operator. $E, A \in R^{n \times n}$ can be all singular. B, B_d, B_f, C, D_d, are real matrices with appropriate dimensions. Without any loss of generality, we might assume that the matrix D_d is full rank in its columns.

III. Designing a singular observer with unknown input for operator's fault diagnosis in the fractional order singular systems

The Intrusion In order to estimate the state and diagnose the fault in system (6), consider the observer with unknown input PI^{α} in the following,

$$ED^{\alpha}\hat{x}(t) = G\hat{x}(t) + Hu(t) + Ly(t) \tag{7}$$

in which, $\hat{x} \in R^n$ is the estimated values of the system states. G, H and L are real matrices with suitable dimensions, which must be chosen in such a way that guarantee convergence of the estimation errors to zero.

The estimation error vector has been defined as,

$$e = \hat{x} - x \tag{8}$$

The dynamics of the state's estimation error would be derived by,

$$\begin{aligned} ED^{\alpha}e = {} & Ge + (LC + G - A)x \\ & + (LD_d - B_d)d + (H - B)u \\ & - B_f f. \end{aligned} \tag{9}$$

Now, if the following conditions are satisfied,

$$LC + G - A = 0 \tag{10}$$

$$H = B \tag{11}$$

$$LD_d = B_d \tag{12}$$

The dynamic of the state estimation error would be,

$$ED^{\alpha}e = Ge - B_f f. \tag{13}$$

Therefore, both disturbance and input will not have any impacts on the dynamics of the state estimation error.

Taking into account the lemma 1, in order to exist a response for the (12), the following condition must be guaranteed,

$$\operatorname{rank}\begin{bmatrix} D_d \\ B_d \end{bmatrix} = \operatorname{rank} D_d \tag{14}$$

Since matrix D_d is full rand in column by assumption, the condition is already satisfied. Then, answer to (12) can be derived by,

$$L = B_d D_d^{+} - \Upsilon\left(I - D_d D_d^{+}\right) \tag{15}$$

replacing (15) into (10), now we have,

$$G = A - B_d D_d^{+} C + \Upsilon\left(I - D_d D_d^{+}\right)C \tag{16}$$

consequently, dynamics of the state estimation error would be,

$$ED^{\alpha}e = (A_1 + \Upsilon C_1)e - B_f f. \tag{17}$$

where, we have $A_1 = A - B_d D_d^{+} C$ and $C_1 = (I - D_d D_d^{+})C$.

In this stage, the observer coefficients must be determined such that to guarantee the convergence of the state estimation error. To this end, matrix Υ should be determined so that dynamics of the state estimation error (17) becomes regular, commensurate and stable. In order to have such a matrix, the following relationship must be kept fulfilled,

$$\operatorname{rank}\begin{bmatrix} sE - A + B_d D_d^{+} C \\ (I - D_d D_d^{+})C \end{bmatrix} = n, \quad \forall s \in R^{+} \tag{18}$$

$$\operatorname{rank}\begin{bmatrix} E & 0 \\ A + B_d D_d^{+} C & E \\ (I - D_d D_d^{+})C & 0 \end{bmatrix} = n + \operatorname{rank}(E) \tag{19}$$

Condition of the stability of the state estimation error can be derived using the next theorem.

Theorem 1: assuming that (18) and (19) are fulfilled already, if there are matrices X and $Y = \Upsilon^T X$ such that the following linear matrix inequalities have solutions,

$$\begin{aligned} &E^T X = X^T E \geq 0, \\ &A_1^T X + X^T A_1 + C_1^T Y + Y^T C_1 < 0 \end{aligned} \tag{20}$$

Then, the dynamics of the state estimation error (17) would be regular, commensurate and stable.

Proof. Using lemma 2 and replacing $G = A_1 + \Upsilon C_1$ the condition (20) would be true for the dynamical state estimation error (17).

IV. Conclusion

In this paper, fault diagnosis in fractional order singular system has been studied. To this purpose, a singular observer with unknown input was suggested to diagnose fault of the operator of the fractional order singular systems and moreover, convergence conditions of the proposed observer have been derived in the form of linear matrix inequalities. An advantage of this method is to separate the disturbance from the desired signals of both input and output, merely based on the obtained input and output signals.

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