

# Disturbance Rejection Nonlinear Control of Flexible Joint Robot Arm

Hadi Hasanpour<sup>1</sup>, Esmail Alibeiki<sup>2\*</sup>, SeyyedMostafa Ghadami<sup>3</sup>

**Abstract-** In this article, we investigate different control methods and control input disturbance removal in the two-link flexible robot. For this purpose, we use the linear feedback control method to control the trajectory tracking of the robot's angle with a nonlinear model by reducing the effects of disturbances in the control signal. The main contribution of this paper is disturbance rejection that effect on performance of robot. In the end, by analyzing and comparing the obtained results, the best method is selected to control and eliminate the control input disturbance in the flexible two-link robot. To analyze the results of this research, MATLAB software and the system simulation are performed and according to the desired trajectory for the robot arms, the optimum control input for the system is obtained. Based on the results, the proposed control structure controls the model property without overshooting and by considering the saturation limit of input control.

**Keywords:** Flexible robot – Nonlinear model-Disturbance rejection

## 1. Introduction

In recent years, fixed-time control algorithms have become increasingly popular in robotics systems due to their fast convergence rates and ability to estimate convergence time without relying on initial states [1-3]. Compared to traditional finite-time control methods, fixed-time control algorithms possess an advantage in convergence time estimation, making them more useful for real-time applications [3]. Various fixed-time control schemes have been developed for systems with unknown uncertainties and multi-agent systems. For example, a distributed fixed-time controller has been designed for an order integrator system with unknown uncertainties caused by inherent dynamics, guaranteeing the convergence of tracking errors in finite time [4]. Nonlinear fixed-time consensus schemes have also been constructed to guarantee the fixed-time consensus problem for a multi-agent system with undirected communication

graphs [4]. Neural network control methods have also become increasingly popular in recent years to deal with uncertainties and nonlinearities in robotics systems [5]. These methods include fault-tolerant controllers, approximation control schemes, cooperative adaptive control schemes, and fuzzy neural network control methods. For example, a neural networks-based fault-tolerant controller has been constructed to estimate actuator failures and uncertain dynamics [5]. Approximation control schemes based on neural networks have been applied to handle model parametric uncertainties and unknown disturbances. The radial basis function neural network control method has been proposed to handle the problems of the unknown dynamic model of coordinated dual arms robot and the saturation nonlinearity of the motor. Cooperative adaptive control schemes have been investigated to deal with nonlinear systems, while fuzzy neural network control methods have been proposed for nonlinear systems [6]. However, the transient performances of these methods are not always satisfying, and the accuracy of the models for robotic systems can be difficult to obtain. Additionally, the methods proposed in the literature often rely on the assumption that all parameters of the robotic systems are known, which may not be practical in real-world applications. As a result, the development of a neural networks-based fixed-time control method for a robot with uncertainties and input

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<sup>1</sup> Department of Electrical Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran. Email: hadihasanpor@gmail.com

<sup>2\*</sup> Corresponding Author : Department of Computer Engineering, Aliabad katoul Branch, Islamic Azad University, Aliabad Katoul, Iran.  
Email: esmail\_alibeiki@aliabadiau.ac.ir

<sup>3</sup> Department of Electrical Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran. Email: ghadami@aliabadiau.ac.ir

deadzone is significant [7]. This approach could provide better transient performance while also addressing uncertainties and input dead zones, which are common challenges in robotics systems. Two-joint flexible robots have many potential applications that can benefit from their unique combination of flexibility and controllability. Here are some examples [8]:

**Medical robotics:** Two-joint flexible robots can be used in medical applications such as minimally invasive surgery and endoscopy. The flexibility of the robot can allow it to navigate through tight spaces and around organs, while the controllability of the system can enable precise manipulation of surgical tools.

**Manufacturing:** Two-joint flexible robots can be used in manufacturing processes that require delicate handling of objects, such as assembling small electronic components or handling fragile materials. The flexibility of the robot can enable it to adapt to different shapes and sizes of objects, while the controllability of the system can ensure precise positioning and manipulation.

**Inspection and maintenance:** Two-joint flexible robots can be used for the inspection and maintenance of structures such as pipelines, bridges, and buildings. The flexibility of the robot can enable it to navigate through complex environments, while the controllability of the system can ensure precise positioning and inspection.

**Entertainment:** Two-joint flexible robots can be used in entertainment applications such as animatronics and robot performances. The flexibility of the robot can enable it to move in a lifelike manner, while the controllability of the system can enable precise and coordinated movement.

**Education and research:** Two-joint flexible robots can be used in educational settings to teach robotics and control theory, as well as in research settings to study the dynamics and control of flexible robotic systems.

Control methods for two-joint flexible robots typically involve a combination of feedforward and feedback control techniques to achieve accurate and robust control of the system [9]. Feedforward control is used to compensate for the dynamic effects of the flexible link, while feedback control is used to track desired trajectories and reject disturbances. Some recent research papers on control methods for two-joint flexible robots have focused on using advanced control techniques such as adaptive control, model predictive control, and sliding mode control [10]. These methods can improve the performance of the

system while also providing robustness to uncertainties and disturbances. Other papers have explored the use of machine learning techniques, such as reinforcement learning and neural networks, to learn control policies for two-joint flexible robots. These methods can be effective in learning complex control strategies that are difficult to design manually. Overall, the field of control methods for two-joint flexible robots is an active area of research, and many new approaches and techniques are being developed and tested. Two-joint flexible robots are a type of robotic system that typically consists of two links connected by a flexible joint. The flexible joint can deform under external loads or forces, which can cause the robot to behave in a non-linear and unpredictable manner. This makes control of the system challenging, as the dynamics of the flexible joint can be difficult to model accurately. To address this challenge, researchers have developed a variety of control methods for two-joint flexible robots. One common approach is to use a combination of feedforward and feedback control techniques. Feedforward control involves estimating the dynamic effects of the flexible joint and compensating for them in the control signal. Feedback control uses sensors to measure the position, velocity, and acceleration of the robot, and adjusts the control signal to track desired trajectories and reject disturbances. Adaptive control is another technique that has been used for two-joint flexible robots [11]. Adaptive control involves continuously updating the control parameters based on feedback from the system, which can improve the accuracy and robustness of the control. Model predictive control is another advanced control technique that has been applied to two-joint flexible robots, which involves predicting the future behavior of the system and optimizing the control signal accordingly [12].

In this paper, the proposed controller aims to reject input control disturbance and to follow the desired path for the links model with two flexible joints. Numerical simulations are done in Matlab software and the proposed controller is designed using linearization input-output feedback with a disturbance rejection block [13]. A simplified-adaptive controller with proper adjustment of the obtained constant values is presented, and the simulation results for the performance of the tracking systems are presented [14-15]. Our research innovations are a multi-controller design-based disturbance rejection structure and the use of a linearization input-output feedback method to achieve the desired path of the links. disturbance rejection from the robot system is very important because its presence can disturb the overall performance of the system and also weaken the

effectiveness of the controller. Therefore, it is necessary to provide a solution for disturbance rejection.

The rest of the paper is organized as follows. In the second section, the model of two flexible joints is presented. In the third section, the proposed controller is designed. In the fourth section, the nonlinear model is simulated in MATLAB software and based on the results, the performance of the method is evaluated. In the fifth section, according to the evaluations performed, the conclusion of the paper is explained.

### 2. Flexible Robot

The model is used to predict the behavior of the robot, including its motion, position, velocity, and acceleration, as well as the internal stresses and strains in the flexible link. The mathematical model typically includes a set of differential equations that describe the dynamics of the system. These equations take into account the forces and torques acting on the robot, as well as the elastic properties of the flexible link. The model can be used to simulate the behavior of the robot under different conditions, such as different input commands or external disturbances. One approach to developing a mathematical model of a two-joint flexible robot is to use a Lagrangian formulation, which describes the system's dynamics in terms of its kinetic and potential energies. This approach can lead to a set of coupled differential equations that describe the motion of the robot and the deformation of the flexible link. Another approach is to use a finite element method, which discretizes the flexible link into a set of smaller elements and solves the behavior of each element separately. This approach can be more computationally intensive, but it can provide more detailed information about the internal stresses and strains in the flexible link. The two-joint flexible robot structure is shown in Figure 1 with one motor attached to each arm. In this robot the arm movement is elastic and its effect is in the form of a spring.

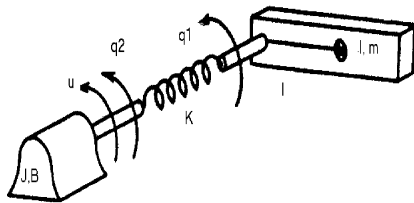


Fig 1. Structure of FJR robot with a single link

The dynamic equation of the FJR robot for the n-joint state is in the form of the following relationships, which include the joint rotation relationships and the motor rotation angle of the joints:

$$M(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + G(q_1) = K(q_1 - q_2) \tag{1}$$

$$J\ddot{q}_2 + B\dot{q}_2 + K(q_2 - q_1) = t \tag{2}$$

In the above relation,  $q_1$  and  $q_2$  are the vectors of the joint angle and the motor rotation angle, respectively. The matrix  $M, G, C,$  and  $B$  are the general components of inertia, the Coriolis vector, the gravity vector, and the actuator damping component, respectively.  $K$  indicates the coefficient of elastic force. Also in the motor equation, the motor inertia matrix ( $J$ ), the drive attenuation matrix, and the matrix coefficient of articulation are considered. In single robot mode, the above equations will be in scalar values.

### 3. Disturbance rejection method

Based on the results in [14], a motion control design based on a continuous single-output (SISO) control system using a progressive control system is proposed. In this system, only one perturbation is applied to the primary plant input. In some cases, such as flexible systems controlled by full closed-loop position control systems, a single disturbance model is not sufficient if physical factors that are not present in the plant model affect the output of small and large loops. To explain this point, consider a flexible joint that is modeled as a two-mass system, depending on Figure 2, where  $J_m$  and  $D_m$  are investigated.

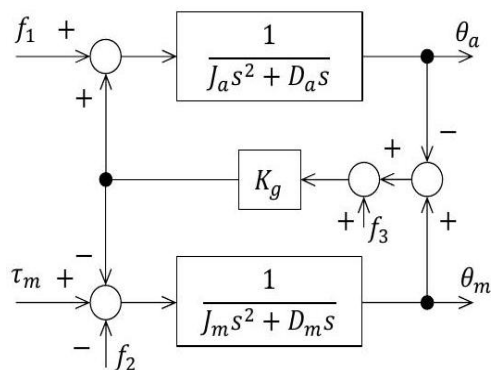


Fig. 2. Two-mass system with physical disturbances.

moment of inertia and viscous damping factor of the motor side;  $J_a$  and  $D_a$  are the moment of inertia and viscous damping factor of the arm side;  $K_g$  is the spring factor of the elastic component in the gear;  $f_1$  is the physical disturbance torque applied at the arm side including the interference force generated from the other axis movement and the external force;  $f_2$  is the physical disturbance torque applied at the motor side such as nonlinear friction;  $f_3$  is the physical disturbance applied within the elastic component such as the gear's angular transmission error. The input/output relationship of this system is expressed as follows:

$$\begin{aligned} \theta_m &= G_1(s)(\tau_m - G_{f_1,m}(s)f_1 - G_{f_2,m}(s)f_2 - \\ &G_{f_3,m}(s)f_3), \theta_a = G_2(s)(\theta_m - G_{f_1,a}(s)f_1 - \\ &G_{f_2,a}(s)f_2 - G_{f_3,a}(s)f_3) \end{aligned} \quad (3)$$

where the transfer functions are computed as:

$$G_1(s) = \frac{P_{n,m}(s)}{P_d(s)}, \quad G_2(s) = \frac{P_{n,a}(s)}{P_{n,m}(s)} \quad (4)$$

$$G_{f_1,m}(s) = -\frac{K_g}{J_a s^2 + D_a s + K_g}, \quad G_{f_2,m}(s) = 1 \quad (5)$$

$$G_{f_3,m}(s) = \frac{K_g(J_a s^2 + D_a s)}{J_a s^2 + D_a s + K_g} \quad (6)$$

$$G_{f_1,a}(s) = -\frac{1}{K_g}, \quad G_{f_2,a}(s) = 0, \quad G_{f_3,a}(s) = -1 \quad (7)$$

$$P_{n,a}(s) = K_g \quad (8)$$

$$P_{n,m}(s) = J_a s^2 + D_a s + K_g, \quad (9)$$

$$P_d(s) = J_m J_a s^4 + (J_m D_a + J_a D_m) s^3 + (D_m D_a + K_g J_m + K_g J_a) s^2 + K_g (D_m + D_a) s \quad (10)$$

The Eqs. (3) and (4) can be described by the block diagram shown in Fig. 3, where:

$$d_1 = G_{f_1,m}(s)f_1 + G_{f_2,m}(s)f_2 + G_{f_3,m}(s)f_3 \quad (11)$$

$$d_2 = G_{f_1,a}(s)f_1 + G_{f_2,a}(s)f_2 + G_{f_3,a}(s)f_3 \quad (12)$$

To capture the impact of physical disturbances on both the inner and outer loop outputs of a fully closed control system, the double-disturbance model shown in Figure 3 must be examined. This is particularly crucial for multi-joint flexible robots, as the joints'

simultaneous motion generates significant interference force [10], causing  $f_1$  to become substantial and  $d_2$  to become non-negligible. In this model,  $d_1$  and  $d_2$  represent the overall effects of physical disturbances on the minor and major loops, respectively, of a cascade positioning control system.

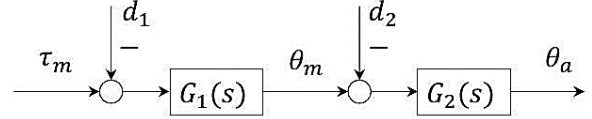


Fig. 3. Double-disturbance model

The controlled output  $\theta_a$  can be then expressed as follows:

$$\begin{aligned} \theta_a &= \frac{C_p(s)C_v(s)G_1(s)G_2(s)}{1+C_v(s)G_1(s)s+C_p(s)C_v(s)G_1(s)G_2(s)} r \\ &+ \frac{C_v(s)G_1(s)G_2(s)}{1+C_v(s)G_1(s)s+C_p(s)C_v(s)G_1(s)G_2(s)} c_\omega \\ &+ \frac{G_1(s)G_2(s)}{1+C_v(s)G_1(s)s+C_p(s)C_v(s)G_1(s)G_2(s)} c_\tau \quad (13) \\ &- \frac{G_1(s)G_2(s)}{1+C_v(s)G_1(s)s+C_p(s)C_v(s)G_1(s)G_2(s)} d_1 \\ &- \frac{G_2(s)+C_v(s)G_1(s)sG_2(s)}{1+C_v(s)G_1(s)s+C_p(s)C_v(s)G_1(s)G_2(s)} d_2 \end{aligned}$$

From the above transfer functions, the effect of disturbances on the controlled output can be eliminated by using either one of two following sets of compensation inputs:

- Set 1 :

$$c_\omega = \frac{1+C_v(s)G_1(s)s}{C_v(s)G_1(s)} d_2, \quad c_\tau = d_1 \quad (14)$$

- Set 2 :

$$c_\omega = s d_2, \quad c_\tau = d_1 + G_1^{-1}(s) d_2. \quad (15)$$

Although the first set of equations in (14) is a more intuitive solution, as it uses  $c_\tau$  to compensate for  $d_1$  in the minor loop and  $c_\omega$  to compensate for  $d_2$  in the major loop, the second set of equations in (15) should be chosen due to its independence from feedback controllers.

(19)

#### 4. Simulation

The purpose of this paper is to track the desired sinusoidal path as well as disturbance rejection at both

angles of the robot joint. The minimum disturbance effect on the input control signal and the elimination of this effect in the output signal is the aim of this paper. For this purpose, we analyze the behavior of the system in a linear model and the designed controller is used to achieve the desired path track based on [13]. In this simulation, the robot parameter is given in Table 1.

Table 1: The Parameters of the FJR Two-Joint Robot Model

Unit	Value	Parameter
Nm/rad	100	$K_1$
Nm/rad	100	$K_2$
Nms/rad	0.9	$B_2$
Nms/rad	0.9	$B_2$
Kg	0.5	$m_1$
Kg	0.5	$m_2$
M	1	$L_1$
M	1	$L_2$

In this simulation, the optimal path for the first and second joint angles is  $0.25\cos(t)$ . In this simulation, the sampling rate is 0.01 s and the initial values of the angles of the robot are set to zero. This simulation is performed in 9 seconds. The simulation results with the conditions expressed in the form of two outputs  $q_1$  and  $q_2$  are flexible robots and the control inputs in the form of  $\tau_1$  and  $\tau_2$  are shown in Figures 2 and 3. The sinusoidal disturbance is added at 4.9s with attitude 0.5 N.m.

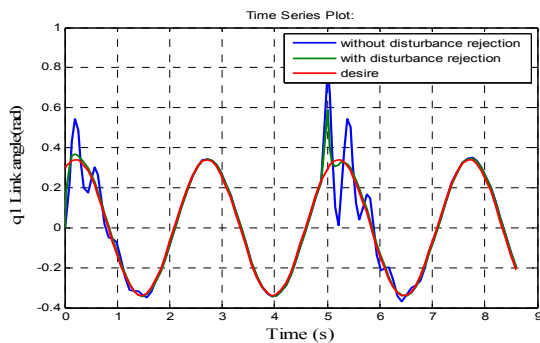


Fig.4 Angle Related to Dynamics  $q_{11}$  and  $q_{12}$

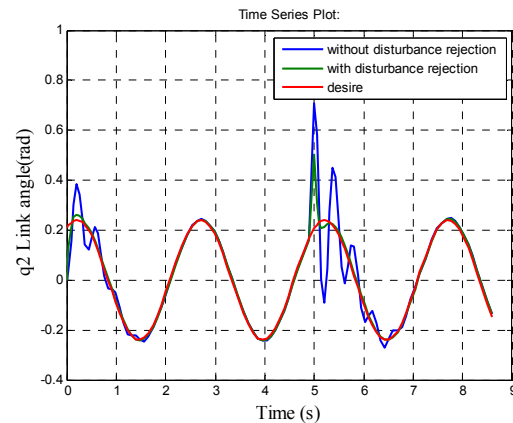


Fig.5 The related angles  $q_{21}$  and  $q_{22}$

In this section, the adaptive backstepping controller design is discussed in [15]. The simulation results of input control for the controller are shown in Figures 4 and 5.

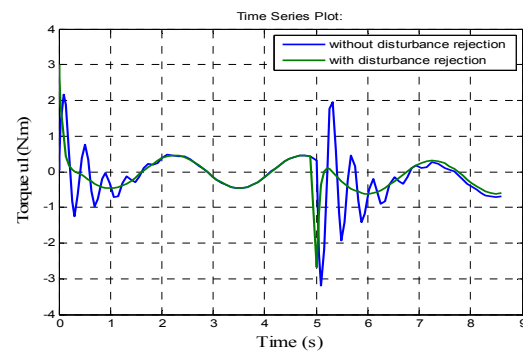


Fig. 6. Input controller of the first joint

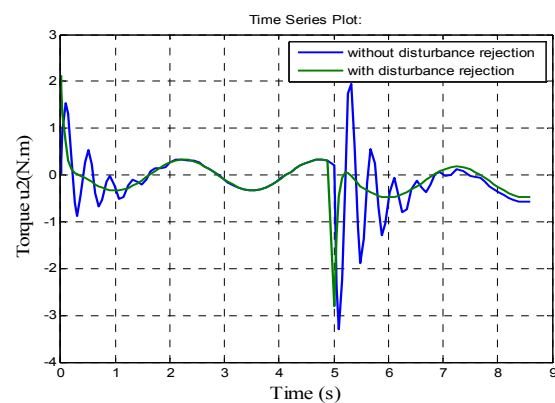


Fig.7. Input controller of second joint

According to the results, the first and second joint angular followed the desired path, and the sinusoidal disturbance by considering the proposed method is rejected in minimum time (0.1s). Based on these results, with input disturbance in this model, the controller acts properly.

## 5. Conclusion

In this paper, a two-degree-of-freedom flexible robot system is modeled and the control law is designed based on the disturbance rejection. The nonlinear model is considered a linear model with feedback linearization and based on multiple inputs of input controller and disturbance effects, a controller designed to reject disturbance. By using the backstepping control method in [15], the control input is designed. The proposed control law is simulated and analyzed in a flexible two-joint robot in MATLAB software. The simulation results show the ability of the proposed control law to reject the disturbance effect quickly and track the desired path in a minimum time. According to the method proposed in this article for flexible joint robots, this method can be used for systems with high coupling that face disturbances. Therefore, for future works, this method can be used for similar systems despite the disturbance. Another suggestion for researchers is to use a disturbance observer for the model, which can be used to observe the disturbance and reduce or eliminate its effect accordingly.

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