Small Signal Modeling of a Synchronous Isolated Cuk Converter used in Photovoltaic Systems

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Abstract–This paper presents the small-signal modeling of a synchronous Cuk converter using state-space averaging method. In the proposed isolated converter, the conduction and switching losses reduced by replacing the traditional diode with transistor and employing an auxiliary circuit. It causes the conversion efficiency of the photovoltaic system to be increased and better matching of the supplied load. Also this circuit has significant advantage over other topologies since it enables low voltage ripple on both input/output sides of the converter and better dynamic response. The low frequency small-signal model is done using state space averaging technique and two main power-stage transfer functions are derived, which are control-to-output voltage transfer function and input-to-output voltage transfer function. Using the proposed model of the converter we can analyze and simulate the time domain transient behavior of the system accurately and intuitively. Finally, the accuracy and validity of the proposed system model is verified by simulation results.

Keywords: Cuk converter, Small signal model, Averaging Technique, Photovoltaic system

1. Introduction

Through the past years, using of the solar panels was almost limited to the space and military applications. however, nowadays, with raising the importance of the problem of energy provisioning and its related economic value for various applications such as domestic, industrial and commercial in the one hand, and environmental worries on the other, the scope of using photovoltaic system has been expanded and it opened new way into the domestic, commercial and industrial applications. Generally speaking, solar panels can be divided into three categories [1]:

1- Polycrystalline solar panels: the cells in such panels are combined of a large number of high-quality and pure silicon chips. Although these panels price are usually inexpensive, however their efficiency is low, that's mean we need wide surfaces to generate certain electricity power.

2- Mon crystalline solar panels: within these structures, each cell is constructed by a large piece of silicon crystal. Compared to polycrystalline, these technologies offer better performance, however, at higher costs than others.

3- Amorphous solar panels: Typically, these panels are seen as thin layers of silicon with high sensitivity against temperature. Regarding efficiency, they are at the lowest level in comparison with other counterparts; nonetheless, they are inexpensive and occasionally appear more flexible

While performing their functions, i.e., electricity generation, the solar panels can be used as building canopies, too. Among the other advantages of the photovoltaic systems, the followings can be mentioned [2]:

1.They are mature, robust and reliable without moving part
2. They don't need any fuel supply network for operation.
3.Their circuits are relatively easy and quick in systems.
4. Their parts reliability are designed for long time usage.
5. They are resilient against ultraviolet rays and temperature.
6. They are modular, thus the systems can be highly flexible.
7. A stand-alone photovoltaic system is able to operate and
8. An important benefit is their effects on reduction of harmful of greenhouse gas radiation and carbon dioxide.
9. They provided fast-growing market business and benefit.

Cuk converter is a type of DC-DC converter that can be used in a step-up or step-down mode; that is, increase or decrease the output level compared to the input. The name (Cuk) has been adopted off the person' name who first came up with the idea, i.e., Slobodan Cuk [3]. Practically, a Cuk converter is a dual to the buck-boost converter and likewise, the polarity of its output is opposite to that of the input. Within its structure, the capacitor at the beginning of the circuit plays the role to save the energy off the input electricity and subsequently carries it over to the output.

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Although the Cuk converter circuit is consisted of more components compared to the buck-boost, altogether, it is superior to the latter due to some technical features such as its much lower output ripple than a simple buck-boost converter [4]. The following figure illustrates the general scheme wherein a Cuk converter is supposed to function.

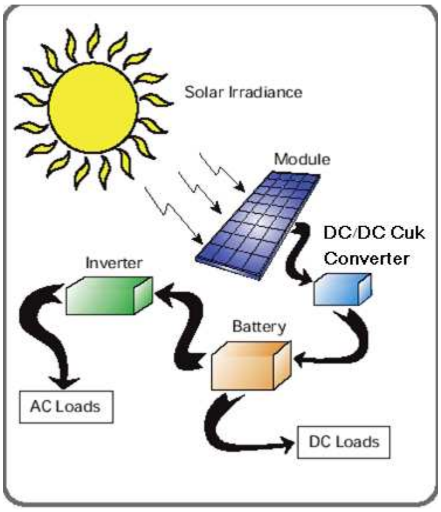

Fig. 1. The General Block diagram of a Photovoltaic System

In practice, the solar cell should not be perceived as an ideal battery, as it comes with its own voltage-current curve (see fig. 2); from the design perspective, the main goal, here, is to take the maximum power out of this device [5].

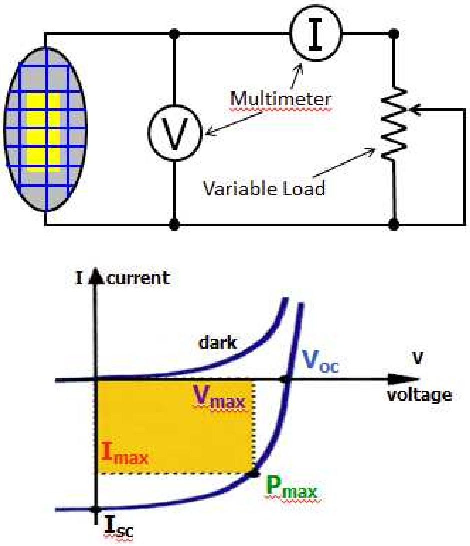

Fig. 2. The Current-Voltage Curve of a Solar Cell

In fig. 2, the value of the load resistance must be tuned in such a way so that the cell always is at the highest power delivery point. The isolated Cuk converter possesses a high potential to realize such a variable resistance, first, because of the existence of the state elements (storage capacitors) and second, due to one degree of freedom as to the transformer winding ratio. The DC output of the converter can both supply DC loads directly and AC loads using an intervening inverter [6].

A mathematical model is the description of a system through circuitry and mathematical language. Such models are highly beneficial and pervasive with wide range of applications, not only in electrical engineering, but also in such natural sciences as physics, biology, geology, meteorology, and engineering sciences such as computer science, artificial intelligence, etc., as well as extensive uses in social sciences like economics, psychology, and sociology. Modeling, specifically through mathematics, helps researchers to methodically analyze a system and enable them to predict its behavior. Among the popular tools applied for modeling, to name a few, dynamic system, statistical model, differential equation, game theory is examples of mathematical modeling to solve engineering problems. The adopted modeling method in this article is a weighted-averaging type applied to the state space. Using this method, we come up with equations describing the transfer functions of the output voltage versus the duty cycle, input voltage and output current. The AC transfer function, Bode diagram, nyquist diagram, geometrical root locus and gain and phase margins, overall and to a large extent, systematically lead to the optimal design of control system, accurate selection of circuit components and accurate output prediction. The resultant data as well as the analyses tools, themselves, are greatly important, especially at higher powers when such energy resources as solar cells or other renewable ones like fuel cells and wind turbines are to be connected to the power grid, so that to provision in the system with more dynamical stable behavior is of critical significance [7].

2. Solar Cell Circuit Model

The layered structure of a solar cell has been shown in fig. 3 [2].

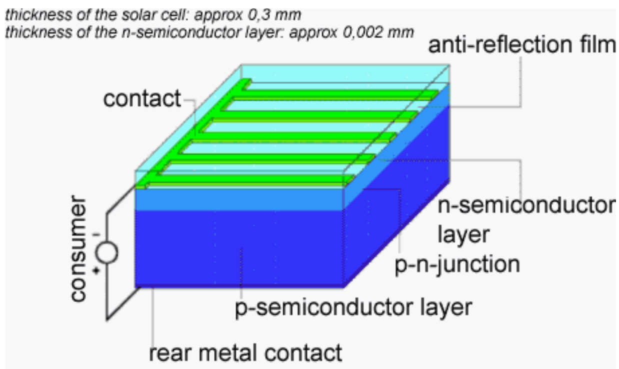

Fig. 3. Structure of Solar Cell

The solar cell can be modeled as a current source parallel with a diode (see fig. 4). In the absent of light, the solar cell generates no power; in this case, it acts only as a diode. As the intensity of the incident light on the cell surface increases, it produces a current proportional to it. The generated current then is divided between the variable resistance of the diode and the load; the amount of current absorbed by the load depends on the ratio of the variable resistance and load impedance in the one hand and on the radiation intensity, on the other. In an ideal cell, the whole current I equals that of produced by the photoelectric cell IL minus the diode current ID.

When considering more precise models for a solar cell, the equivalent circuit must be constituted of two diodes instead of one. Given the above explanations and using the mathematical relationships of a solar cell, the following formula describes, in general, the behavior of the output current as a function of the involving variables [4]:

$$I = I_L - I_o\left(E^{\frac{q(V+I.R_s)}{nkT}} - 1\right) - \frac{V + I.R_S}{R_{SH}} \qquad (1)$$

In this relationship, n is the ideal coefficient of the diode (usually between 1 and 2) and R_S and R_{SH} indicate the serial and parallel resistors, respectively; the equivalent circuit of the solar cell based upon the above descriptions has been shown in fig. 4. It must be noted that throughout a realistic solar cell operation, the corresponding efficiency will be decreased due to [8]:

1) Power losses caused by the internal resistances including those related to the device's connections and
2) Through the leakage currents on the either side of device.

As seen in the fig. 4, these parasitic resistances have been modeled as parallel and serial resistors R_{SH} and R_S.

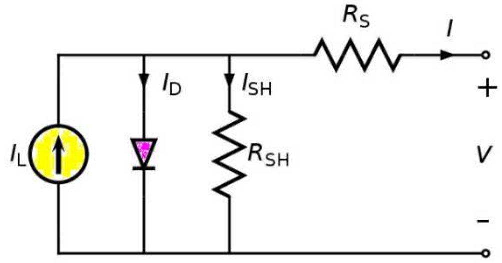

Fig. 4. Simplified circuit of a solar cell

In an ideal cell, value of the parallel resistor is infinite and thus there exists no alternative path for the current leakage. Likewise, an ideal serial resistor must be zero, which means there would be no voltage drop before the electrical energy reaches the load. Contrariwise, as the parallel resistance decreases, the value of V_{oc} decreases, too; similarly, an increase in RS would cause a decrease in ISC. As a result of these imperfections, i.e., increase and decrease in the parallel and serial resistances, respectively, the fill factor (FF) and the maximum delivered power of the cell will be reduced, in total. The values of the parallel and serial resistances, either for the experimental or practical purposes, can be extracted from current-voltage curve [4].

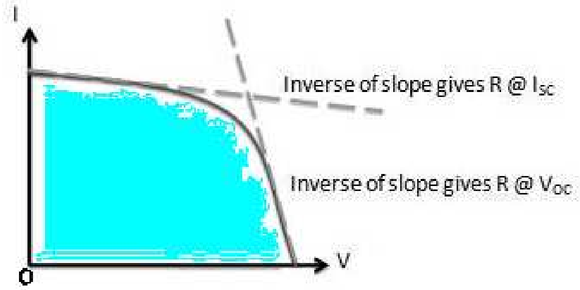

Fig. 5. Serial and parallel resistances of a PV

In the frequency domain, when the solar cell is analyzed from the frequency perspective, the parasitic capacitors and inductors also come into play; however, as their values are negligible compared to the converters' capacitor and inductor, their related pole and zero within the transfer function can be harmlessly ignored. It is worthy to mention that the existence of the serial resistance is rooted in the resistance of the cell material against the current flow, especially between the upper surface and the connections, as well as the resistance of connections per se; this might lead to a real issue specifically at high current densities, e.g., under an intensive focused light incident. On the other hand, the parallel resistance as the other innate property of the solar cell, is caused by the current leakage through the cell around the edges of constituent segments of the device and also between the connections of different poles; as a result, the effect of the parallel resistance is, to a large extent, related to the underlying semiconductor design.

3. The CUK Converter Modeling

Fig. 6 illustrates a simple Cuk converter [3].

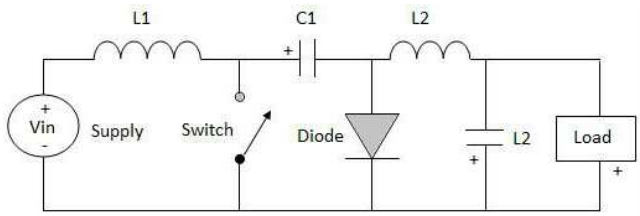

Fig. 6. The traditional CUK converter circuit

From the figure, when the switch is closed, the current of the inductor increases and the diode turns off by the capacitor C_1. The main task of $C_1$is to transfer energy of the inductor current to the output when the switch opens. During this stage, the inductor energy is carried over to the capacitor C_1 through the diode.

The output-input voltage relationship can be written as,

$$V_{out} = \left(\frac{D}{1-D}\right)V_{in} \tag{2}$$

The corresponding relationship for the current is,

$$I_{out} = \left(\frac{1-D}{D}\right)I_{in} \tag{3}$$

When using Cuk converter in solar cells, from the design point of view, the main goal must be to reduce the power losses to the most so that efficiency of the system reaches its highest possible level. Practically, all the elements of the circuit, more or less, causes some power losses; in such a context, elements with highest shares out of converter' power losses must be dealt with specifically and designed properly so that their behaviors and as a result, their corresponding power losses be adjusted to the desired level where is depicted in Fig. 7 according to losses [6].

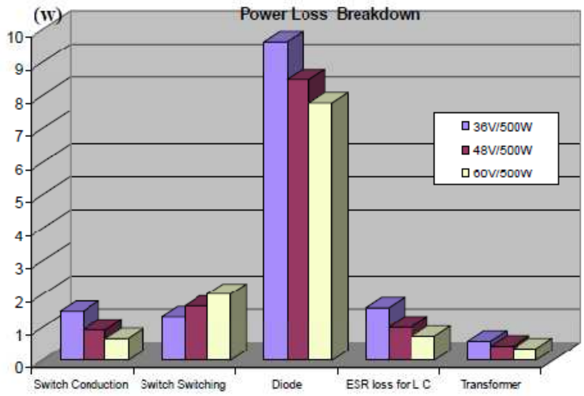

Fig. 7. Comparison of the Converter Elements Power Losses

As can be seen from the figure, the highest power loss is due to the diode; for this reason, the diode would be replaced by a transistor as the power loss of the latter is much less than the first. As an illustration, when a 10Acurrent passes through a diode with a voltage drop of 0.7 volts, it causes 7 watts power losses; if this diode, however, be replaced with a MOSFET of 0.01 ohm internal resistance, the power loss will only be 1 watt. The structure of a synchronous isolated Cuk converter (where in the diode is replaced by a transistor) can be seen in Fig. 8.

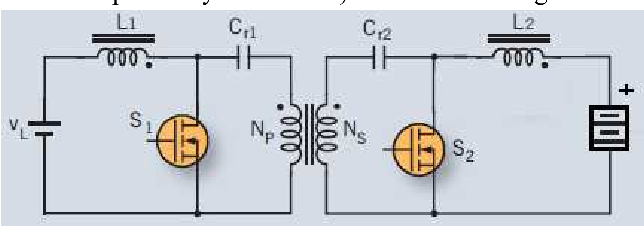

Fig. 8. The Synchronous Isolated Cuk converter

For the given Cuk converter, it is enough to multiply and divide (2) and (3) by the transformer turn winding ratio ($N=N_S/N_P$) in order to achieve the desired relationships. From this, the resulting state space equations thus becomes,

$$\begin{aligned} \dot{x} &= Ax + Bv_g + B_d d \\ v_o &= Cx \\ x &= [v_2 \;\; v_1 \;\; i_2 \;\; i_1]' \end{aligned} \tag{4}$$

The corresponding block diagram of the state space equations has been shown in fig. 9 [9].

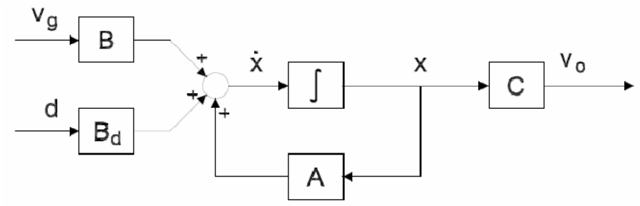

Fig. 9. The State Space Block Diagram

In order to drive the complete state space equations of the whole converter, we begin by writing, firstly, the transformer equations; here, however, for the purpose of simplification, we have integrated the L_1 and L_2 with the primary and secondary inductances of the transformer. The results would be,

$$v_{L1} = v_g - i_1R_1 - v_1 \tag{5}$$

$$v_{L2} = v_2 - i_2R_2$$

$$\sigma^2 = L_1L_2 - M^2 \tag{6}$$

$$\frac{di_1}{dt} = \frac{M}{\sigma^2}v_2 + \frac{-L_2}{\sigma^2}v_1 + \frac{MR_2}{\sigma^2}i_2 + \frac{-L_2R_1}{\sigma^2}i_1 + \frac{L_2}{\sigma^2}v_g \tag{7}$$

$$\frac{di_2}{dt} = \frac{-L_1}{\sigma^2}v_2 + \frac{M}{\sigma^2}v_1 + \frac{-L_1R_2}{\sigma^2}i_2 + \frac{MR_1}{\sigma^2}i_1 + \frac{-M}{\sigma^2}v_g \tag{8}$$

To avoid confusing details, here, only the final results of calculations and applied equations, enough for the purpose of analysis and simulation of the Cuk converter, have been presented. Within the state space model, first the state matrices (A,B) are calculated in the switching on and off conditions in a sequence and then weight-average according to their duty cycles of the off/on states (D) [10].

The relationships have been derived in such a way that different transfer functions can be extracted with appropriate while simple manipulations. For example, in order to determine the transfer function of the output voltage versus the duty cycle, it suffices to set the input AC voltage source to zero, or to derive the transfer function of the output/input voltage, we need only to eliminate the AC source off the equations throughout the duty cycle interval. The state matrices of both on / off conditions of the switch are given in the following tables,

Table 1. The characteristics state space matrix when the switch 1 is on

S1=ON ; S2=OFF
$A_1 = \begin{bmatrix} -\frac{1}{RC_2} & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & -\frac{1}{C_1} & 0 \\ -\frac{L_1}{\sigma^2} & \frac{L_1}{\sigma^2} & -\frac{L_1 R_2}{\sigma^2} & \frac{MR_2}{\sigma^2} \\ \frac{M}{\sigma^2} & -\frac{M}{\sigma^2} & \frac{MR_2}{\sigma^2} & -\frac{L_2 R_1}{\sigma^2} \end{bmatrix}$
$B_1 = \begin{bmatrix} 0 \\ 0 \\ -\frac{M}{\sigma^2} \\ \frac{L_2}{\sigma^2} \end{bmatrix}$
$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$
$D_1 = [0]$

Table 2. The characteristics state space matrix when the switch 1 is off

S1=OFF ; S2=ON
$A_2 = \begin{bmatrix} -\frac{1}{RC_2} & 0 & \frac{1}{C_2} & 0 \\ 0 & 0 & 0 & -\frac{1}{C_1} \\ -\frac{L_1}{\sigma^2} & \frac{M}{\sigma^2} & -\frac{L_1 R_2}{\sigma^2} & \frac{MR_1}{\sigma^2} \\ \frac{M}{\sigma^2} & -\frac{L_2}{\sigma^2} & \frac{MR_2}{\sigma^2} & -\frac{L_2 R_1}{\sigma^2} \end{bmatrix}$
$B_2 = \begin{bmatrix} 0 \\ 0 \\ -\frac{M}{\sigma^2} \\ \frac{L_2}{\sigma^2} \end{bmatrix}$
$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$
$D_2 = [0]$

Now, the state matrices should be integrated and weight-averaged according to the duty cycle of the switch. This process has been shown in the following equations.

$$A = D_S A_1 + D'_S A_2$$
$$B = D_S B_1 + D'_S B_2 \qquad (9)$$
$$C = D_S C_1 + D'_S C_2$$
$$D = D_S D_1 + D'_S D_2$$

$$x = X + \tilde{x}$$
$$v_g = V_g + \tilde{v}_g \qquad (10)$$
$$d = D_S + \tilde{d}$$
$$v_o = V_o + \tilde{v}$$

$$X = -A^{-1} B V_g$$
$$B_d = (A_1 - A_2)X + (B_1 - B_2)V_g \qquad (11)$$
$$D_d = (C_1 - C_2)X + (D_1 - D_2)V_g$$

Through applying the above relationships, the ultimate state matrices would be as,

$$A = \begin{bmatrix} -\frac{1}{RC_2} & 0 & \frac{1}{C_2} & 0 \\ 0 & 0 & -\frac{D_S}{C_1} & \frac{1-D_S}{C_1} \\ -\frac{L_1}{L_1L_2 - M^2} & \frac{D_S L_1 + M - D_S M}{L_1L_2 - M^2} & 0 & 0 \\ \frac{M}{L_1L_2 - M^2} & \frac{-D_S M - L + L_2 D_S}{L_1L_2 - M^2} & 0 & 0 \end{bmatrix} \qquad (12)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -\frac{M}{L_1L_2 - M^2} \\ \frac{L_2}{L_1L_2 - M^2} \end{bmatrix} \qquad (13)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$
$$D = [0]$$

$$B_d = \begin{bmatrix} 0 \\ -\frac{D_S V_g}{R(1-D_S)^2 C_1} \\ \frac{V_g (L_1 - M)}{(1-D_S)(L_1L_2 - M^2)} \\ \frac{V_g(-M + L_2)}{(1-D_S)(L_1L_2 - M^2)} \end{bmatrix} \qquad (14)$$

$$D_d = [0]$$

4. The Simulation Results

Fig. 10 depicts the I-V curve of a typical solar cell.

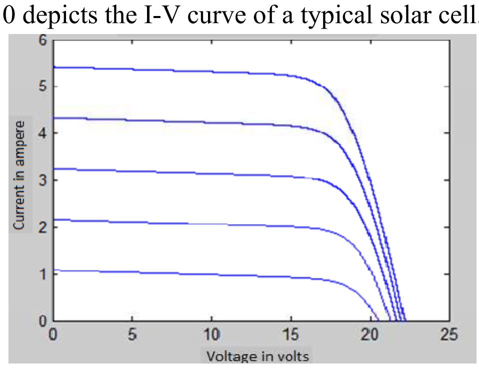

Fig. 10. The Voltage-Current Curve of a Typical Cell

The power versus voltage of the typical cell of fig. 10 has been illustrated in fig. 11.

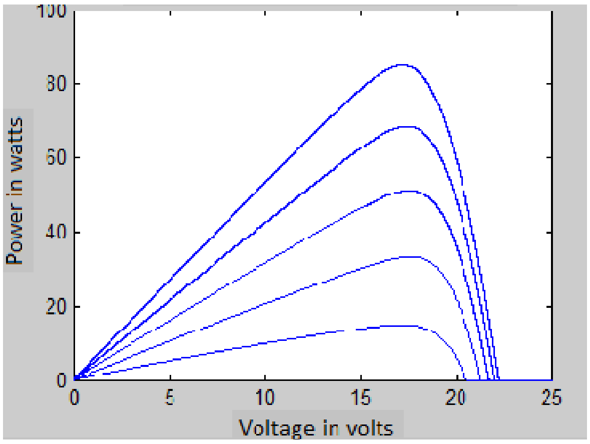

Fig. 11. The Power-Voltage Curve of the Cell

The AC transfer function, derived from the state equations, for the system under study, has been shown in the following:

$$\begin{bmatrix} G_{vg}(s) \\ G_{vd}(s) \end{bmatrix} = \left[C(sI-A)^{-1}B+D\right]^T \quad (15)$$

$$G_{vd}(s)=\frac{C_1L_1RV_gs^2/D' + L_1V_gD^2(D'-D)/D'^2 s+RV_g}{L_1C_1L_2RC_2s^4+L_1C_1L_2s^3+(D'^2L_2RC_2+L_1C_1R+L_1D^2RC_2)s^2+(D'^2L_2+L_1D^2)s+D'^2R} \quad (16)$$

$$G_{vg}(s)=\frac{-D'R}{L_1C_1L_2RC_2s^4+L_1C_1L_2s^3+(-D'^2L_2RC_2-L_1C_1DR-L_1D\ RC_2)s^2+(D'^2L_2-L_1D^2)s+D'^2DR} \quad (17)$$

Table 3 summarizes the parts value of the converter adopted through the simulation process for the typical converter.

Table 3. The Characteristics of the Typical Converter Parts Value

Part	Value	Unit
V_g	12	V
L_1	0.5	mH
L_2	7.5	mH
M	1.5	mH
C_1	2	uF
C_2	20	uF
R_1	0.01	ohm
R_2	0.01	ohm
Duty	0.666	---
Freq.	100	KHz

Fig. 12 illustrates the Bode diagram of the transfer function, derived within MATLAB environment.

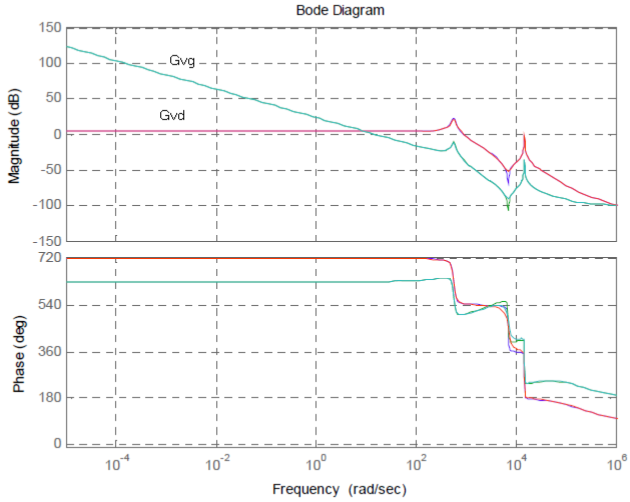

Fig. 12. Bode Diagram of the Converter Transfer Function

Fig. 13 depicts the root locus of the transfer function.

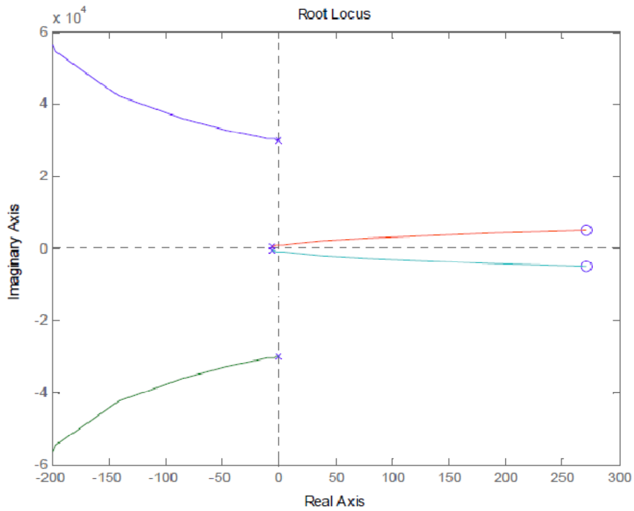

Fig. 13. The diagram of the root locus

In fig. 14 , the output voltage of the converter has been drawn versus the time.

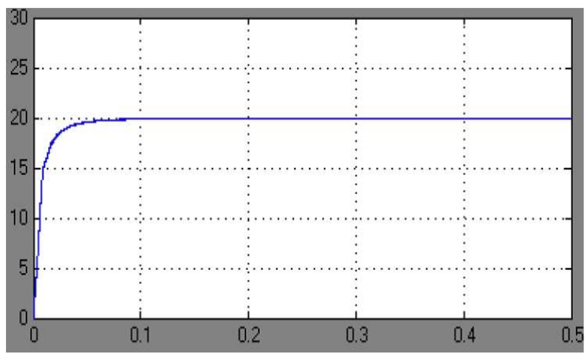

Fig. 14. The Output Voltage of the Proposed Cuk Converter

Finally, fig. 15 states the efficiency versus the output power of converter for different levels of input voltages.

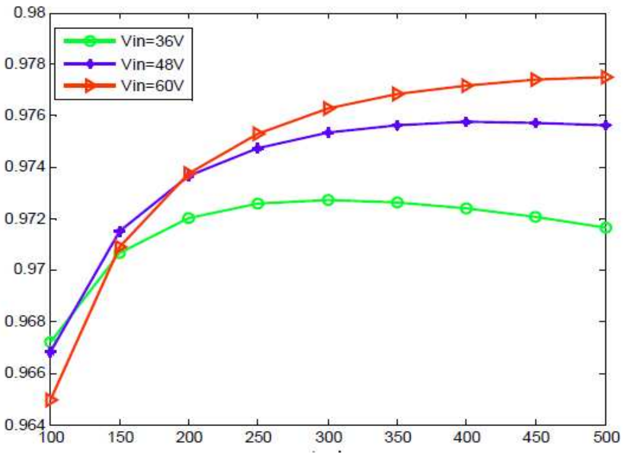

Fig. 15. The Efficiency of the converter versus the its output power

5. Conclusion

This article dealt with the modeling and analysis of a photovoltaic converter based on Cuk configuration. Modeling was performed through weighted-averaging of the state space equation. The proposed Cuk converter has been chosen and designed as synchronous so that it reduces the power losses of its individual components and isolated to cover wide ranges of voltage levels. These characteristics provide high potential for this converter for solar cell system applications. The transfer functions of the output/input voltages to duty cycle ratio have been calculated and simulated by Matlab. As expected, the transfer functions were of the degree four, even though the output of the control transfer function has a zero on the right half plane (RHPZ); this caused a slight delay in the system dynamics, which can easily be ignored in non-critical applications. The final results of this converter modeling and analysis provide the designers to choose the best parameters values of the circuit elements. Finally, the simulation and modeling results show the validity of the proposed model.

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