Optimization of Energy for Tracking of the Magnetic Levitation Ball Using the SDRE Technique

Seyyed Mostafa Ghadami [1*]

Abstract –This paper was dealt with the optimization of energy for tracking the magnetic levitation ball by using the technique of State-Dependent Riccati Equation. The magnetic levitation ball is widely used in various fields. This system includes a steel ball suspended by electromagnetic force. The differential equations of this system are nonlinear. Generally, first, these nonlinear equations are linearized around an equilibrium point. Then, a Linear Quadratic Tracker controller is designed for this system. Note that this system has a non-zero equilibrium point. However, in the technique of State-Dependent Riccati Equation, the nonlinear equations are linearized by using the method of State-Dependent Coefficients. If the system is pointwise stabilizable and detectable, the State-Dependent Riccati Equation has an answer. Then, an optimal control law is extracted so that the energy of tracking is minimized. In the end, it has been shown that four different trajectories are tracked appropriately using the proposed method.

Keywords: Magnetic levitation ball, Optimization energy for tracking, State-Dependent Riccati Equation, State-Dependent Coefficients

1. Introduction

The magnetic levitation ball has special nonlinear features. Using the electromagnetic force, a metal ball is suspended in space and at the desired altitude. Therefore, it is a great challenge for control engineers. The magnetic levitation ball is widely used in various fields. These include a fast electric passenger train, frictionless bearings, the elastic suspension system for wind tunnel to test missile and airborne and so on. Due to the unstable of the magnetic levitation ball, designing a controller for tracking is very important. Generally, first, these nonlinear equations are linearized around an equilibrium point. Note that this system has a non-zero equilibrium point. Then, a Linear Quadratic Regulator(LQR) controller is designed for this system.

In 2004, Scott C. Beeler solved the nonlinear quadratic regulator problem with the State-Dependent Riccati Equation (SDR) method [1]. In [2], Hosseini et al. designed a fuzzy-sliding controller for the magnetic levitation ball. In [3], Mária Hypiusová and Jakub Osuský in 2010 developed the design of the PID controller for the SISO system. This paper, the practical application of the proposed method for designing a PID controller for the magnetic levitation ball system. In 2013, Bharat Bhushan, in [4], investigated and implemented an indirect adaptive control law on nonlinear systems using the Takagi-Sugen fuzzy system. In [5], Bharat Bhushan, in 2013, examined and implemented the direct comparative control stability of nonlinear systems using the Lyapunov function with a fuzzy approach. This paper, the practical application of the proposed method for controlling the robot magnetic suspension. In [6], the variable structure control theory was utilized to derive a discontinuous controller to the magnetic levitation ball. In [7], it was discussed to synthesize an interval type-2 fuzzy logic PID and type-1 fuzzy logic controllers to keep a metal ball suspended in mid-air by changing the field strength of an electromagnet coil. In [8], the mathematical modelling and simulation results of a dynamic non-linear magnetic levitation ball were presented upon which state-space modelling was implemented to achieve linearized and precise position results. In [9], an adaptive proportional-integral-derivative control system

1* **Corresponding Author :** Department of Electrical Engineering, Aliabad katoul Branch, Islamic Azad University, Aliabad Katoul, Iran.
Email: ghadami@aliabadiau.ac.ir
Received: 2021.11.20 ; Accepted: 2022.02.04

was developed to deal with the metallic sphere position control of a magnetic levitation ball, which is an intricate and highly nonlinear system. The proposed control system consisted of an adaptive PID controller and a fuzzy compensation controller. [10] presented an infinite-horizon optimal controller based on a state-dependent Riccati equation approach to solve the tracking for nonlinear systems. In [11], a nonlinear controller was designed for a Magnetic levitation ball. The proposed controller was used input-output feedback linearization using differential geometry in conjugation with a linear state feedback controller in the outer loop to levitate a ferromagnetic ball. In [12], the tracking control problem of the magnetic levitation ball was considered. Three types of nonlinear tracking strategies were proposed: position-velocity-flux feedback, position-velocity feedback, and position feedback. In [13], an electromagnetic levitation system was used with a synchronous motor to navigate the control rod of a small-type research reactor. Also, a controller was designed using state feedback and state feedback integral tracking methods. This paper proposes the SDRE technique for optimal tracking of the magnetic levitation ball. The control law is extracted from solving the Hamilton-Jacobi-Bellman equation for the state-dependent coefficient (SDC) factorized nonlinear system.

2. The magnetic levitation ball system

In the magnetic levitation ball system, the steel ball is suspended in space with electromagnetic force. The block diagram of the magnetic levitation ball system is shown in figure (1). In addition, the components of this system are given in table (1).

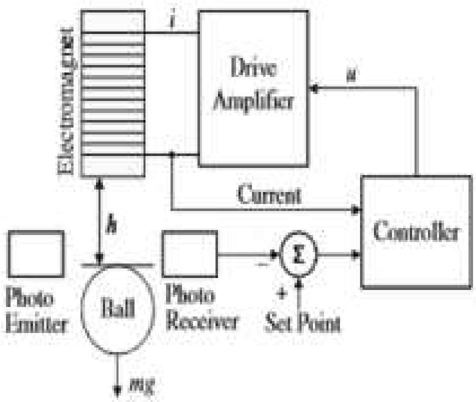

Fig. 1. Block diagram of the magnetic levitation ball system

Table 1. Components of the magnetic levitation ball

Nomenclature	
SetPoint	It regulates the new altitude of the magnetic levitation ball.
Photo Receiver	A sensor detects the altitude of the magnetic levitation ball.
Photo Emitter	A sensor declares the new altitude of the magnetic levitation ball to the regulator.
Controller	According to the altitude of the ball, it controls the input current.
Drive Amplifier	It acts as an input amplifier.
Electromagnet	An electric magnet adjusts the altitude of the magnetic levitation ball.
Ball	A steel ball is in the magnetic field.

In this system, it is assumed:

1. The magnetic flux is uniform in the space between the electric magnet and the levitation ball.
2. The cross-section of the ball and the Magnet core is equal.
3. The leakage of flux is insignificant in the electrical magnet.

Therefore, first, the altitude of the ball is measured by the sensor. Then, the controller adjusts a required current by the coil to the ball can track the desired altitude. An equivalent circuit of the magnetic levitation ball system can be considered as Figure (2). Also, tables (2), (3) show the magnetic levitation ball system parameters.

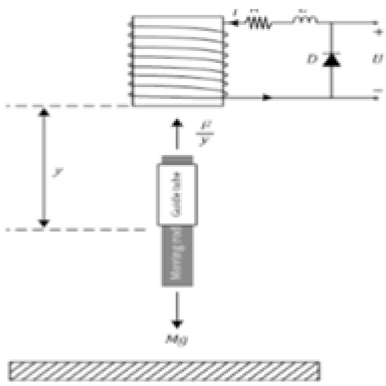

Fig. 2. The equivalent circuit of the magnetic levitation ball system

Table 2 Parameters list of the system

Nomenclature	
m	Mass of the steel ball
y	The distance between the ball and the magnet core
L	The inductance coefficient of the coil
R	The winding resistance
i	The electric current
u	he input voltageT

Table 3 The parameter value of the system[13]

Parameter	value
m	0.05kg
g	$9.8\frac{N}{ms^2}$
R	10Ω
L	1H

The state equations of the system are given as follows[13]:

$$m.\frac{d^2}{dt^2}y = mg - \frac{i^2}{y} \quad (1)$$

$$u(t) = Ri(t) + L\frac{di}{dt} \quad (2)$$

Notice that the magnetic levitation ball system has not a zero point of equilibrium. Therefore, by choosing the following state variables, the state equations are extracted with a zero point of equilibrium.

$$\begin{cases} x_1 = y - y_0 \\ x_2 = \dfrac{dy}{dt} \\ x_3 = i - \sqrt{mgy_0} \\ U = u - R\sqrt{mgy_0} \end{cases} \quad (3)$$

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = \dfrac{gx_1}{x_1 + y_0} - \dfrac{x_3^2 + 2x_3\sqrt{mgy_0}}{m\left(x_{1+y_0}\right)} \\ \dot{x_3} = -\dfrac{R}{L}x_3 + \dfrac{U}{L} \end{cases} \quad (4)$$

3. A proposed method for optimal tracking of the nonlinear system

In this section, we propose a method for the optimal tracking problem. The purpose is to track the desired trajectory to minimize a square performance index. The SDRE technique provides a method for optimal tracking problem solving for a certain class of nonlinear systems. Consider the following nonlinear system:

$$\begin{cases} \dot{x} = f(x) + B(x)u \quad ,x(t_0) = x_0 \\ y = c(x) \end{cases} \quad (5)$$

Where $x \in R^n, u \in R^m, y \in R^k, f(x) \in R^n$, $B(x)$: $\Re^n \to \Re^{n\times m}, c(x) \in R^n$ and $x_0 \in R^n$ is the initial condition. Also, x = 0 is the hyperbolic equilibrium point.In the SDRE technique, nonlinear systems are presented as state-dependent coefficients. Therefore, assuming that $f\,(.) \in C^1(\Re^n), c\,(.) \in C^1(\Re^n)$, $f(0_{n\times1}) = 0_{n\times1}$, $c(0_{n\times1}) = 0_{k\times1}$, there is always a quasi-linear representation $f(x) = A(x)x$ and $c(x) = C(x)x$ .Where, $A(x): \Re^n \to \Re^{n\times n}$ and $C(x): \Re^n \to \Re^{k\times n}$are unique for n> 1.An appropriate choice for the matrix $A(x)$ and $C(x)$ is $A(x) = \int_0^1 \frac{\partial f}{\partial x_{|x=\alpha x}}\, d\alpha$, $C(x) = \int_0^1 \frac{\partial c}{\partial x_{|x=\alpha x}}\, d\alpha$where α is an additional variable, and it is usually used only for integration[1].

Theory: For the nonlinear system (6), the control law for optimal tracking with the performance index (7) is (8).

$$\begin{cases} \dot{x} = f(x) + B(x)u \quad ,x(t_0) = x_0 \\ y = c(x) \end{cases} \quad (6)$$

$$J = \left(\left(y(t_F)\right) - r(t_F)\right)^T q_F \left(\left(y(t_F)\right) - r(t_F)\right) + \int_{t_0}^{t_F} ((y - r(t))^T Q(x)(y - r(t)) + u^T R(x)u)dt \quad (7)$$

$$u \quad (8)$$

Where $q_F, Q(x): \Re^n \to \Re^{m\times m}$ and $R(x): \Re^n \to \Re^{m\times m}$are the semi-positive definite and positive definite matrices,respectively. *If the* pair {A(x), $B(x)$} and {A(x), C(x)} be point wise *stabilizable* and *detectable* for ∀x. In addition, if the closed loop matrix $A(x) - B(x)R^{-1}(x)B^T(x)P(x)$ be Hurwitz for $\forall x \in \Omega$, then SDRE has an answer. Ω are points where the lyapunov function $V(x) = x^T\left(\int_0^1 \alpha P(\alpha x)d\alpha\right)x$ is a locally lipschitz function [1].

$$\dot{P}(x) + P(x)A(x) + A^T(x)P(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + C^T(x)Q(x)C(x) = 0_{n\times n}, \quad P\left(x(t_F)\right) = C^T\left(x(t_F)\right)q_F C\left(x(t_F)\right) \quad (9)$$

$$\dot{v}(x) + \left(A(x) - B(x)R^{-1}(x)B^T(x)P(x)\right)^T v(x) + C^T(x)Q(x)r(t) = 0_{n\times1}, \quad v\left(x(t_F)\right) = C^T\left(x(t_F)\right)q_F r(t_F) \quad (10)$$

Proof:

In the tracking problem, the control law u is extracted to minimize the performance index J. Also, y is converged to the desired trajectory of $r(t)$ .In the first step, a Hamiltonian function $\mathcal{H}$ for the auxiliary vector $\lambda \in R^n$ is defined as follows:

$$\mathcal{H}(x,u,t) = \frac{1}{2}(y - r(t))^T Q(x)(y - r(t)) + \frac{1}{2}u^T R(x)u + \lambda^T (f(x) + B(x)u) \tag{11}$$

According to the optimal control theory, the conditions for optimization are as follows.

$$\dot{x} = \left(\frac{\partial \mathcal{H}}{\partial p}\right)^{\mathrm{T}} = f(x) + B(x)u \, , x(t_0) = x_0 \tag{12}$$

$$\dot{\lambda} = -\left(\frac{\partial \mathcal{H}}{\partial x}\right)^{\mathrm{T}} = -c_x(x)^T . Q(x)\big(y - r(t)\big) - \frac{1}{2}\big(c(x) - r(t)\big)^T \frac{\partial Q(x)}{\partial x}\big(c(x) - r(t)\big) - \frac{1}{2}u^T \frac{\partial \mathrm{R}}{\partial x} u - f_x(x)^T \lambda - \lambda^T \frac{\partial \mathrm{B}}{\partial x} u \, , \tag{13}$$

$$\lambda(t_F) = \frac{1}{2}\left(c_x{}^T q_F \left(c\big(x(t_F)\big) - r(t_F)\right)\right)^T$$

$$0 = \frac{\partial \mathcal{H}}{\partial u} = R(x)u + B^T(x)\lambda \tag{14}$$

From Equation (14), since B(x) and R (x) are non-zero, the optimal control law will be proportionate to the vector λ.

$$u = -R^{-1}(x)B^T(x)\lambda \tag{15}$$

By choosing the SDC representation of $\lambda(x) = P(x)x - v(x)$ and by derivation of $\lambda(x)$ can write:

$$\dot{\lambda} = P(x)\dot{x} + \dot{P}(x)x - \dot{v}(x) \tag{16}$$

Where $P(x)$ is a symmetric, unique and positive definite matrix. By placing (15) and (16) in (13), we have:

$$\big[\dot{P}(x) + P(x)A(x) + A^T(x)P(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + C^T(x)Q(x)C(x)\big]x - \Big[\dot{v}(x) + \big(A(x) - B(x)R^{-1}(x)B^T(x)P(x)\big)^T v(x) + C^T(x)Q(x)r(t)\Big]$$

$$\Bigg[\sum_{i=1}^{n} x_i \left(\frac{\partial A_i}{\partial x}(x)\right)^T \big(P(x)x - v(x)\big) + \sum_{i=1}^{n} x_i \left(\frac{\partial C_i}{\partial x}(x)\right)^T Q(x)(c(x) - r(t)) + \frac{1}{2}\sum_{i=1}^{n}((c(x) - r(t))^T)_i \frac{\partial Q_i}{\partial x}(x)(c(x) - r(t)) + \frac{1}{2}\sum_{i=1}^{m}\left(\big((P(x)x - v(x))^T B(x)R^{-1}(x)\big)_i \frac{\partial R_i}{\partial x}(x)\right) R^{-1}(x)B^T(x)\big(P(x)x - v(x)\big) - \sum_{i=1}^{m}\left(\big((P(x)x - v(x))^T B(x)R^{-1}(x)\big)_i \left(\frac{\partial B_i}{\partial x}(x)\right)^T\right)\big(P(x)x - v(x)\big)\Bigg] = 0_{n\times n}, \tag{17}$$

$$P\big(x(t_F)\big) = C^T\big(x(t_F)\big)q_F C\big(x(t_F)\big),$$
$$v\big(x(t_F)\big) = C^T\big(x(t_F)\big)q_F r(t_F)$$

Where the partial derivative for A(x), B(x),C(x),Q(x) and R(x) are defined as follows:

$$\frac{\partial M_i}{\partial x} = \begin{pmatrix} \partial M_{1i}/\partial x_1 & \cdots & \partial M_{1i}/\partial x_n \\ \vdots & \ddots & \vdots \\ \partial M_{ni}/\partial x_1 & \cdots & \partial M_{ni}/\partial x_n \end{pmatrix} \tag{18}$$

The first part is called a state-dependent reticulate equation (SDRE). The second part is the generalized tracking of LTI systems for nonlinear affine systems. The third part is usually neglected from the interior of [1] because of its small magnitude.

4. Simulation results

By comparing the relations (4) and (5) for the magnetic levitation ball system, $A(x), f(x), B(x)$ and $c(x)$ are determ ined as $A(x) = \int_0^1 \frac{\partial f}{\partial x}_{|x=\alpha x} d\alpha$, $f(x) = \left(x_2, \quad \frac{g x_1}{x_1 + y_0} - \frac{x_3^2 + 2x_3\sqrt{mgy_0}}{m\,(x_1 + y_0)}, \quad -\frac{R}{L}x_3\right)^T$, $B(x) = \left(0, \quad 0, \quad \frac{1}{L}\right)^T$ and $y = c(x) = x_1$, respectively. In this section, for four different trajectorie s(constant, sinusoidal, square, complex), the simulation res ults of the optimal tracking problem are examined for the m agnetic levitation ball system.

4.1 First case: The fixed trajectories ($\boldsymbol{r(t) = constant}$)

In this case, the purpose is that the magnetic levitation ball tracks the fixed trajectories. Figure (3) shows the fixed trajectories at 6, 8 and 10 cm.

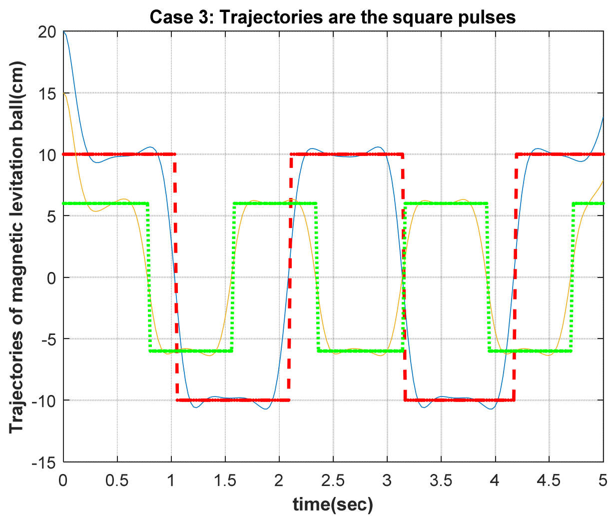

Fig. 3. The trajectories of the magnetic levitation ball for $r(t) = constant$

4.2 Second Case: The sinusoidal trajectories($r(t) = sinusoidal$)

In this case, the purpose is that the magnetic levitation ball track sinusoidal trajectories. Figure (4) shows these trajectories.

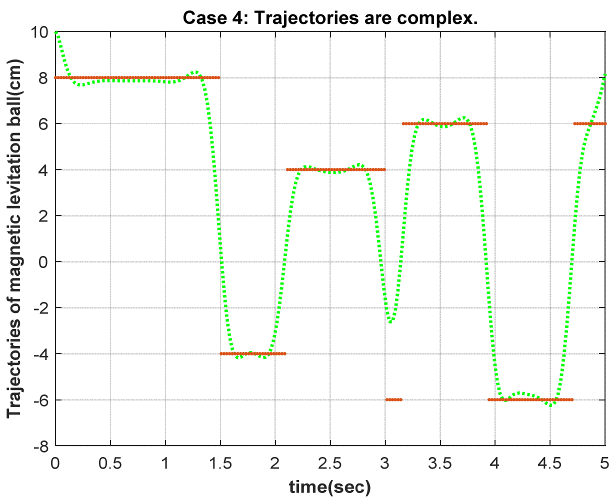

Fig. 4. The trajectories of the magnetic levitation ball for $r(t) = sinusoidal$

4.3 Third case: The square pulse trajectories, ($r(t) = square\ pulse$)

In this case, the purpose is that the magnetic levitation ball tracks square pulse trajectories. Figure (5) shows these trajectories.

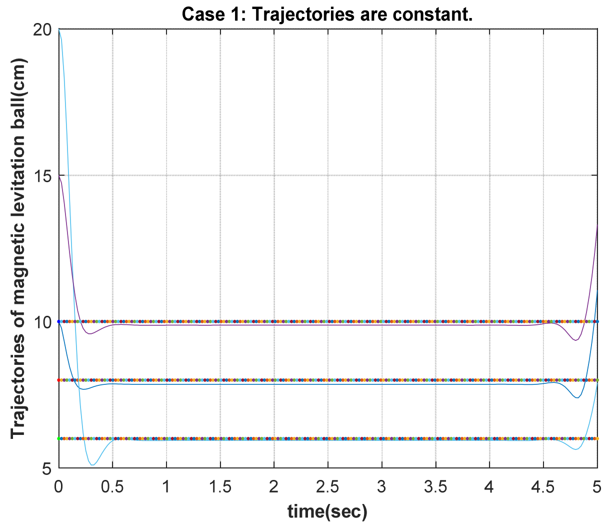

Fig. 5 The trajectories of the magnetic levitation ball for $r(t) = square\ pulse$

4.4 Fourth case: The complex trajectories

In this case, the purpose is that the magnetic levitation ball track the complicated trajectories. Figure 6 shows these trajectories.

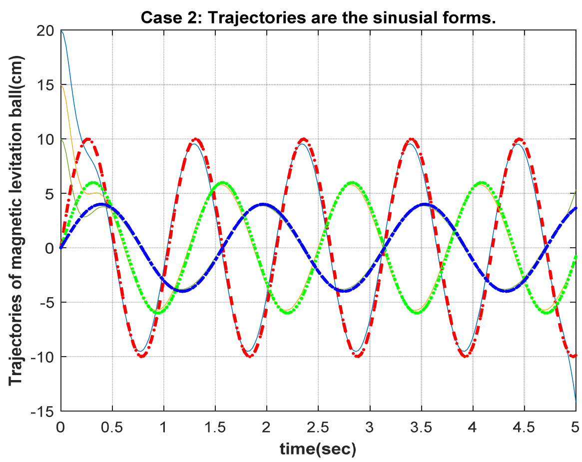

Fig. 6 The trajectory of the magnetic levitation ball

5. Conclusion

In this paper, a nonlinear optimal tracker is proposed using the SDRE technique to track the magnetic levitation ball. In the SDRE technique, the nonlinear equations were linearized using the SDC method. Then, an optimal control law was extracted to minimize the tracking energy. In the end, for four different trajectories, the simulation results were examined for the magnetic levitation ball system. It has been shown that the magnetic levitation ball tracks the desired trajectories appropriately.

References

[1] Scott C. Beeler,State-Dependent Riccati Equation Regulation of Systems with State and Control Nonlinearities: National Institute of Aerospace, Hampton, Virginia, Report No. 2004-08.

[2] Saeed Mohammad-Hosseini, Ali-Reza Alfie, Mohammad Farrokhi, Mohammad-Reza Jahed-Motlagh, “Fuzzy sliding state feedback control of nonlinear ball suspension system”, IFAC Proceedings Volumes,Vol.35, No.1, 2002, pp 229-234.

[3] Mária Hypiusová, Jakub Osuský, “PID controller design for magnetic levitation model”,in Proceedings of International Conference cybernetics and informatics,Feb. 2010.

[4] Bharat Bhushan, Sudarshan K.Valluru, Madhusudan Singh, “Takagi-Sugeno Fuzzy System based Stable Direct Adaptive Control of Nonlinear Systems”, International Journal of Computer Applications, Vol.68, No.15, Apr. 2013.

[5] Bharat Bhushan, “Direct Adaptive Control of Robot Manipulator and Magnetic Ball Suspension System”, International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, Vol.2, No.11, Nov. 2013.

[6] Shibly Ahmed Al-Samarraie, “Variable Structure Control Design for a Magnetic Levitation System”, Journal of Engineering, Mechanical and Energy Engineering, Vol.24, No.12,Dec. 2018.

[7] Ahmed A. Oglah, Samara H. Abed Al-Hady, “Design and implementation of an interval Type-2 Fuzzy Logic and type-1 fuzzy logic controls for Magnetic Levitation System”, American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS), Vol.46, No1,Aug. 2018.

[8] Muhammad Junaid Khan, Muhammad Junaid,, Sahibzada Bilal, Sadia Jabeen Siddiqi,Haseeb Ahmad Khan, “Modelling, Simulation & Control of NonLinear Magnetic Levitation System”, in Proceedings of IEEE, 21st International Multi-Topic Conference (INMIC),Nov. 2018,

[9] Chih-Min Lin,Ming-Hung Lin, Chun-Wen Chen, “SoPC-Based Adaptive PID Control System Design for Magnetic Levitation System”, IEEE Systems Journal, Vol.5, No.2, Jun. 2011.

[10] Fernando Ornelas-Tellez, J. Jesus Rico-Melgoza, Edgar N. Sanchez, “Optimal tracking for a class of nonlinear systems based on the state-dependent Riccati equation”, in Proceedings of 10th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE),Mexico city, Mexico,Sep. 2013.

[11] Raghuwansh Sing, “Nonlinear control of a magnetic levitation system using feedback linearization”, Journal of Technology Research & Innovation, Vol.4, No.2, July 2016.

[12] Guimei Qi,Zhaojing Wu, “Filter-based tracking of magnetic levitation ball system”,in Proceedings of Chinese Control And Decision Conference (CCDC), June 2018.

[13] Mohammad Divandari, Mehdi Hashemi-Tilehnoee, Masoud Khaleghi, Mohammadreza Hosseinkhah, “A novel control-rod drive mechanism via electromagnetic levitation in MNSR”, NUKLEONIKA, Vol.59, No.2:73−79, June2014.