Study and Design of a Fractional-order Terminal Sliding Mode Fault-Tolerant Control for Spacecraft

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Abstract–In industrial systems, performance of a spacecraft is always affected by the presence of uncertainties and disturbances. In this paper, we study the application of a combination of Fault-Tolerant Control (FTC) designs based on fractional calculus and Terminal Sliding Mode Control (FOTSMC) for a spacecraft in the presence of external disturbances. The proposed controller shows better control performance compared to existing terminal sliding mode control (TSMC).Moreover, in our design, the control law does not need a fault detection and isolation mechanism. The sliding mode control protects controller against disturbances and uncertainties while the fractional calculus provides robust performance. The performance of the proposed fractional-order controller, as compared with other controllers, is provided via numerical simulations. The results clearly demonstrate better performance of the fractional order terminal sliding mode control for an actuator fault in comparison with the terminal sliding mode control. The analytical stability analysis of the closed-loop control system is also provided by the Lyapunov direct method for fractional-order system.

Keywords: Fractional Order, Spacecraft, Sliding Mode Control, Lyapunov Theory

I. Introduction

With the rapid development of advanced technologies and complex industrial systems, it is necessary to provision for the ever increasing requirements for reliability and safety of control systems[1].When controlling real systems, one important aspect is concerned with the occurrence of component faults and their influence on the overall system performance[2].In recent years, fault-tolerant control (FTC) has received considerable attention due to the increasing demands for safety and reliability in modern industrial systems, especially for systems, such as spacecraft, aircraft, and so on[3].The main objective of FTC is to design an appropriate control law, such that the closed-loop system can tolerate some faults in specific control components and maintain the whole system stable with acceptable performance[1]. Among the various design schemes of FTC, the robustness properties of sliding mode control against certain types of disturbances and uncertainties, especially to actuator faults, make it attractive in the field of spacecraft FTC[4].

Researchers have already developed quaternion feedback control algorithms for a variety of applications including: large-angle retargeting maneuvers spacecraft [5],adaptive back-stepping FTC for flexible spacecraft with redundant actuators [4],indirect robust adaptive FTC for attitude tracking of spacecraft [6] and sliding mode control to the spacecraft control [7]. In [1], authors proposed a new nonsingular fast terminal sliding mode FTC for spacecraft control, although it had been already proposed by Zak (1988).Among the suggested algorithms, (Sliding Mode Control) SMC technique has been recognized as efficient one in that it withstands the matched external disturbances and model uncertainties, and thus widely adopted in spacecraft attitude FTC systems [3].

In this paper, we proposed a fractional-order surface for spacecraft system in the presence of disturbances, uncertainties, and actuator failures, in which the system states will converge to zero in finite time.

The rest of the paper is organized as follows. In Section II basic concept of the fractional calculus is presented. In section III, the system model is presented. Section IV is dedicated to the design of the controller and stability analysis. Finally, numerical simulations and conclusions are presented, respectively, in sections V and VI.

II. Fractional Calculus

Definition 1: The Caputo derivative with $\alpha \in R^+$ fractional order of function $x(t)$ is defined as [8]:

$$ {}_0^C D_t^{\alpha} x(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} x^{(m)}(\tau) d\tau \quad (1)$$

Definition 2: For the Caputo derivative, we have [9]

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$$\begin{aligned}&{}_{t_0}^{C}D_t^{1-\alpha}\left({}^{C}D_t^{\alpha}\,\mathrm{f}(t)\right)\\&={}_{t_0}^{C}D_t^{\alpha}\left({}^{C}D_t^{1-\alpha}\,\mathrm{f}(t)\right)\\&={}_{t_0}^{C}D_t^{\alpha}\left({}^{C}D_t^{-\alpha}\dot{f}(t)\right)=\dot{f}(t)\end{aligned} \tag{2}$$

Definition3: If $x(t)\in C^{m}[0,1)$ and $m-1<\alpha<m\in Z^{+}$, then [9],

$${}_{0}^{C}D_t^{\alpha}D_t^{-\alpha}x(t)=x(t) \qquad for\ \ m=1 \tag{3}$$

Lemma 1: Suppose $x(t)\in R$ be a continuous and derivable function, then for any time instant $t\geq t_0$ [10]:

$$\frac{1}{2}\,{}_{0}^{C}D_t^{\alpha}x^{2}(t)\leq x(t)\,{}_{0}^{C}D_t^{\alpha}x(t) \tag{4}$$

III. System Modeling

Assume a class of second-order nonlinear differential equations as the following:

$$\begin{cases}\dot{X}_1=X_2\\ \dot{X}_2=f(\mathrm{x})+\mathrm{G}(\mathrm{x})\mathrm{u}+\mathrm{d}\end{cases} \tag{5}$$

where $X_1=[x_1,...,x_n]^T\in R^n$, $X_2=[x_{n+1},...,x_{2n}]^T\in R^n$ and $X=[X_1,X_2]^T\in R^{2n}$ denotes system states vector; $f(x)\in R^n, f(0)=0$, $G(x)\in R^{n\times m}$ are smooth and nonlinear functions and $[..]^T$ denotes the transpose of a vector or a matrix, $u=[u_1,...,u_m]^T\in R^m, m\geq n$ the control input, and $d=[d_1,...,d_n]^T\in R^n$, the possible model uncertainties and disturbances where we assumed external disturbance are bounded.

In this paper, we assume that the actuators' faults have been successfully detected and diagnosed by a suitable FDD scheme. The main idea of the FDD mechanism is to decouple the control input through a coordinate transformation, so that any fault associated with a channel can be diagnosed [1]. The actuators are divided into two groups H and F. Thus, System (5) can be rewritten as:

$$\begin{cases}\dot{X}_1=X_2\\ \dot{X}_2=f(\mathrm{x})+\mathrm{G}_H(\mathrm{x})\mathrm{u}_H+\mathrm{G}_F(\mathrm{x})\mathrm{u}_F+\mathrm{d}\end{cases} \tag{6}$$

We assumed the actuators in H are healthy, while those in F experience faults. Where $G(x)=[\mathrm{G}_H(\mathrm{x}):\mathrm{G}_F(\mathrm{x})]$, $\mathrm{u}=[\mathrm{u}_H^T:u_F^T]^T$.

IV. Controller Design : Fractional Order Terminal Sliding Mode Control

The main advantage of fractional order sliding surface is its higher flexibility in improving the robustness and control performance [11]. To design a fractional-order sliding mode controller, the following fractional order terminal sliding surface is proposed:

$$S=X_1+\lambda^{-1}D^{1-\alpha}\left(\left|X_2\right|^{\frac{p}{q}}sign(X_2)\right) \tag{7}$$

Where p and q are positive odd numbers satisfying the relation $1<p/q<2$ and $\alpha\in(0,1)$ represents the order of the fractional operator.

Taking the fractional derivative of the fractional order sliding surface of (7), one can obtain:

$$D^{\alpha}S=D^{\alpha}\left(X_1+\lambda^{-1}D^{1-\alpha}\left(\left|X_2\right|^{\frac{p}{q}}sign(X_2)\right)\right)=0 \tag{8}$$

Applying the Definition 2 results in:

$$\begin{aligned}D^{\alpha}S&=D^{\alpha}X_1+\lambda^{-1}D^{\alpha}D^{1-\alpha}\left(\left|X_2\right|^{\frac{p}{q}}sign(X_2)\right)\\&=D^{\alpha}X_1+\lambda^{-1}\left(\frac{p}{q}\left|X_2\right|^{\frac{p}{q}-1}\dot{X}_2\right)=0\\&=D^{\alpha}X_1+\lambda^{-1}\frac{p}{q}\left|X_2\right|^{\frac{p}{q}-1}\begin{pmatrix}f(\mathrm{x})+\mathrm{G}_H(\mathrm{x})\mathrm{u}_H\\+\mathrm{G}_F(\mathrm{x})\mathrm{u}_F+\mathrm{d}\end{pmatrix}=0\end{aligned} \tag{9}$$

Then, the following equivalent control law can be obtained:

$$\mathrm{u}_{Heq}=-\mathrm{G}_H^{+}(\mathrm{x})\begin{pmatrix}f(\mathrm{x})+\mathrm{G}_F(\mathrm{x})\mathrm{u}_F\\+\lambda\frac{q}{p}\left|X_2\right|^{1-\frac{p}{q}}D^{\alpha}X_1\end{pmatrix} \tag{10}$$

Where $G_H^{+}(x)=G_H^T(x)\left[G_H(x)G_H^T(x)\right]^{-1}$ is a given function. The switching term in control signal in (10) is defined as:

$$u_{Hsw}=-G_H^{+}(x)\left(k_1s+k_2sign(s)\right) \tag{11}$$

Where k_1 and k_2 are positive constants. Hence, the overall control law becomes:

$$\begin{aligned}u_H&=u_{Heq}+u_{Hsw}\\&=-\mathrm{G}_H^{+}(\mathrm{x})\begin{pmatrix}f(\mathrm{x})+\mathrm{G}_F(\mathrm{x})\mathrm{u}_F\\+\lambda\frac{q}{p}\left|X_2\right|^{1-\frac{p}{q}}D^{\alpha}X_1\\+k_1s+k_2sign(s)\end{pmatrix}\end{aligned} \tag{12}$$

To check the stability of the controlled system, the following Lyapunov function is considered:

$$\mathrm{V}=\frac{1}{2}S^2\geq 0 \tag{13}$$

Taking the fractional-order derivative from both sides of (13), using the mentioned property in Lemma 1, and substituting (6), one obtains:

$$\begin{aligned} {}_0^cD_t^\alpha V &= \frac{1}{2}\,{}_0^cD_t^\alpha S^2 \leq S\,{}_0^cD_t^\alpha S \\ {}_0^cD_t^\alpha V &\leq S\left(D^\alpha X_1+\lambda^{-1}\frac{p}{q}\left|X_2\right|^{\frac{p}{q}-1}\dot{X}_2\right) \\ &\leq S\left(D^\alpha X_1+\lambda^{-1}\frac{p}{q}\left|X_2\right|^{\frac{p}{q}-1}\begin{pmatrix} f(\mathrm{x})+\mathrm{G}_H(\mathrm{x})\mathrm{u}_H \\ +\mathrm{G}_F(\mathrm{x})\mathrm{u}_F+\mathrm{d}\end{pmatrix}\right)\end{aligned} \tag{14}$$

Replacing (12) in (14) yields:

$$\begin{aligned} {}_0^cD_t^\alpha V &\leq \lambda^{-1}\frac{p}{q}\left|X_2\right|^{\frac{p}{q}-1}\begin{pmatrix}-k_1S^2-k_2S\ \mathrm{sign(S)} \\ +Sd\end{pmatrix} \\ &\leq \lambda^{-1}\frac{p}{q}\left|X_2\right|^{\frac{p}{q}-1}\left(-k_1S^2+\left(d-k_2\right)\left|S\right|\right)\end{aligned} \tag{15}$$

Where the λ, p and q are positive constants hence one can assume that $\lambda^{-1}\frac{p}{q}\left|X_2\right|^{\frac{p}{q}-1}\geq 0$. We, furthermore, assumed that d_i are bounded; thus, by choosing $d\leq k_2$ and the positive switching gain k, stability of the closed loop system is guaranteed.
Finally, we obtain:

$${}_0^cD_t^\alpha V\leq 0 \tag{16}$$

V. Simulation Results

In this section, results of the simulations implemented in MATLAB/SIMULINK 2013 software are presented. For the purpose of simulation, we have used the overall system dynamics for satellite attitude stabilization control using FOTSMC. The fractional calculus is a useful tool to control and to achieve a significant degree of robustness[12].A spacecraft attitude model as introduced in [1] and [13] is adopted, as described in the same form as (5), with n = 3, in which $X_1=[x_1,x_2,x_3]^T=[\varphi,\theta,\psi]^T$, $X_2=[x_4,x_5,x_6]^T=[\dot{\varphi},\dot{\theta},\dot{\psi}]^T$, $f(x)=[f_1(x),f_2(x),f_3(x)]^T$, $u=[u_1,u_2,u_3,u_4]^T$ and $d=[d_1,d_2,d_3]^T$. Here, φ,θ, and ψ are Euler's angles for the x, y and z axes, respectively. The values considered for parameters are given in Table 1; $f_i(x)$,$i=1,2,3$ and $G(x)$ are described as follow,

$$f_1(x)=\omega_0x_6cx_3cx_2-\omega_0x_5sx_3sx_2 + \frac{I_y-I_x}{I_x}\begin{pmatrix} x_5x_6+\omega_0x_5cx_1sx_3sx_2 \\ +\omega_0x_5cx_3sx_1+\omega_0x_6cx_3cx_1 \\ +\frac{1}{2}\omega_0^2s(2x_3)c^2x_1sx_2+\frac{1}{2}\omega_0^2c^2x_3s(2x_1) \\ -\omega_0x_6sx_3sx_2sx_1-\frac{1}{2}\omega_0^2s^2x_2s^2x_3s(2x_1) \\ -\frac{1}{2}\omega_0^2s(2x_3)sx_2s^2x_1-\frac{3}{2}\omega_0^2c^2x_2s(2x_1)\end{pmatrix} \tag{17}$$

$$\begin{aligned} f_2(x)&=\omega_0x_6sx_3cx_1+\omega_0x_4cx_3sx_1+ \\ &\omega_0x_6cx_3sx_2sx_1+\omega_0x_5sx_3cx_2sx_1+\omega_0x_4sx_3sx_2cx_1 \\ &+\frac{I_z-I_x}{I_y}\begin{pmatrix} x_4x_6+\omega_0x_4cx_1sx_3sx_2 \\ +\omega_0x_4cx_3sx_1-\omega_0x_6sx_3cx_2 \\ -\frac{1}{2}\omega_0^2s(2x_2)s^2x_3cx_1 \\ -\frac{1}{2}\omega_0^2cx_2sx_1s(2x_3) \\ +\frac{3}{2}\omega_0^2s(2x_2)cx_1\end{pmatrix}\end{aligned} \tag{18}$$

$$\begin{aligned} f_3(x)&=\omega_0x_4sx_1sx_3sx_2-\omega_0x_6cx_1cx_3sx_2 \\ &-\omega_0x_5cx_1sx_3cx_2+\omega_0x_6sx_3sx_1-\omega_0x_4cx_3cx_1 \\ &+\frac{I_x-I_y}{I_z}\begin{pmatrix} x_4x_5+\omega_0x_4cx_3cx_1-\omega_0x_4sx_3sx_2sx_1 \\ -\omega_0x_5sx_3cx_2-\frac{1}{2}\omega_0^2s(2x_3)cx_2cx_1 \\ +\frac{1}{2}\omega_0^2s^2x_3sx_1s(2x_2)-\frac{3}{2}\omega_0^2s(2x_2)sx_1\end{pmatrix}\end{aligned} \tag{19}$$

$$G(x)=\begin{pmatrix} 0.67 & 0.67 & 0.67 & 0.67 \\ 0.69 & -0.69 & -0.69 & 0.69 \\ 0.28 & 0.28 & -0.28 & -0.28\end{pmatrix} \tag{20}$$

Where c and s denote the cosine sine functions. Initial condition and disturbance are set as $x(0)=(0.7,0.07,-1.2,0.15,-1.3,0.3)$, $d=0.05\left(\sin(t),\cos(2t),\sin(3t)\right)^T$, respectively. In order to alleviate the chattering phenomenon of the presented FTC, we use the tangent hyperbolic function with boundary layer to be $\varepsilon=0.02$ .Other design parameters values are: $p=9,q=9,\alpha=0.1,k_1=k_2=0.5$. We have assumed that the actuator u_2 fails to work at time $t=4s$.

It was shown in [13] using the observer can provide FDD for any single actuator fault which could then be used to estimate the output value.

The simulation results for the proposed controller are depicted in Figs1-5.In Figure 1 and 2, it can be seen that the states converge to zero in a finite time. The results also show that FOTSMC have comparable performance (faster tracking) with respect to its integer counterpart suggested TSMC in [1].The sliding surfaces are shown in Fig.3, from which convergence of sliding surface to zero can be seen. Fig.4- 5 show control laws, one can see that the chattering-free control input converges to zero.

VI. Conclusions

In this paper, a fault tolerant (FT) control based on fractional order terminal sliding mode control is proposed for a spacecraft. The diagnosis of the fault actuator via a mixture FT control, fractional order calculus and terminal sliding mode control has been considered. The stability analysis and simulations were studied for the proposed controller. It was shown that the closed-loop system can achieve the stability, even if some of the actuators fail to operate. According to simulation results, the proposed controller have a good performance in the sense of disturbance rejection and detection of actuator fault.

Table1: System parameters

Parameters	*Values*
I_x	$2000\ N.m.s^2$
I_y	$2000\ N.m.s^2$
I_z	$400\ N.m.s^2$
ω_0	$1.0312\times10^{-3}\ rad/s$

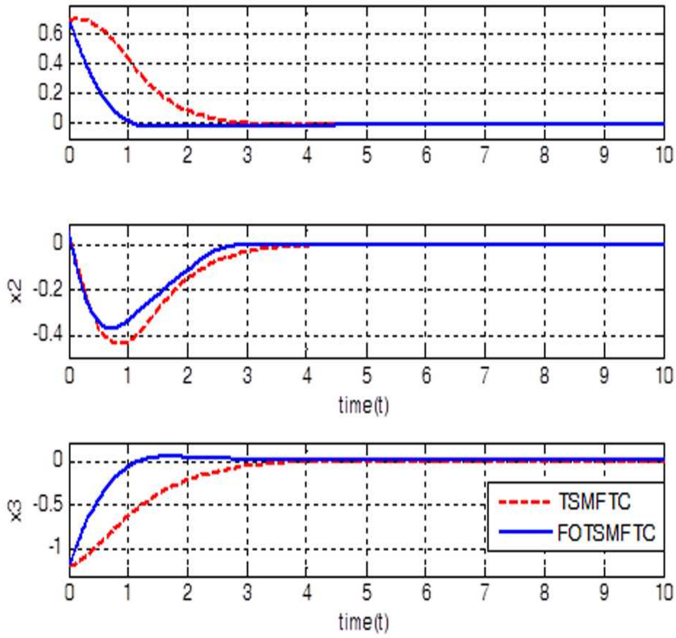

Fig.1: Time history of system states x1, x2, x3

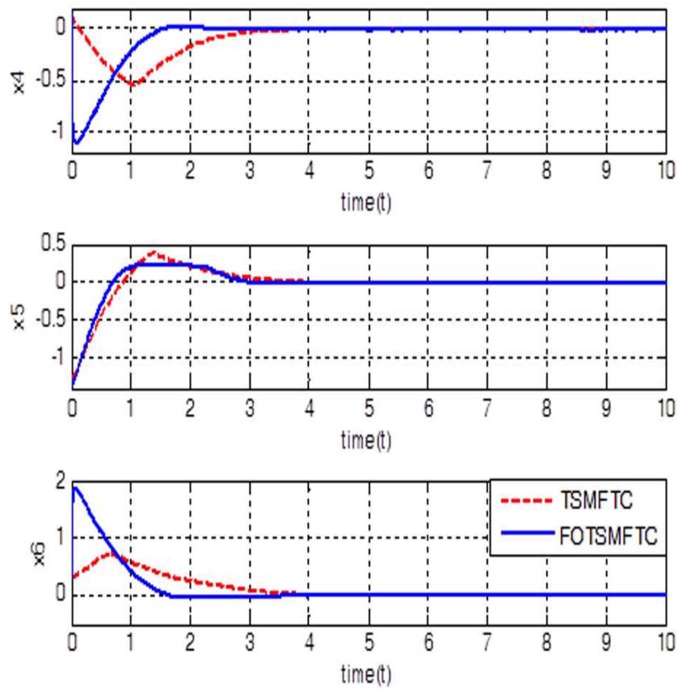

Fig.2: Time history of system states x4, x5, x6

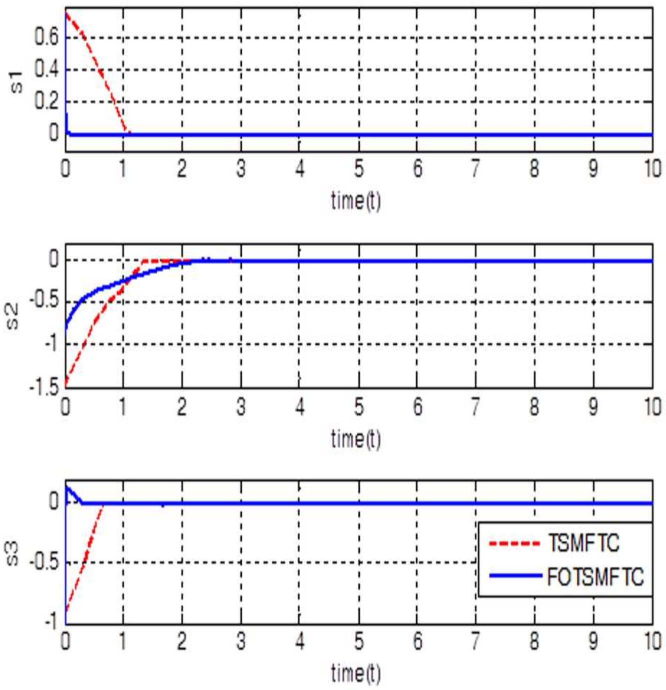

Fig.3: Comparison of sliding surface

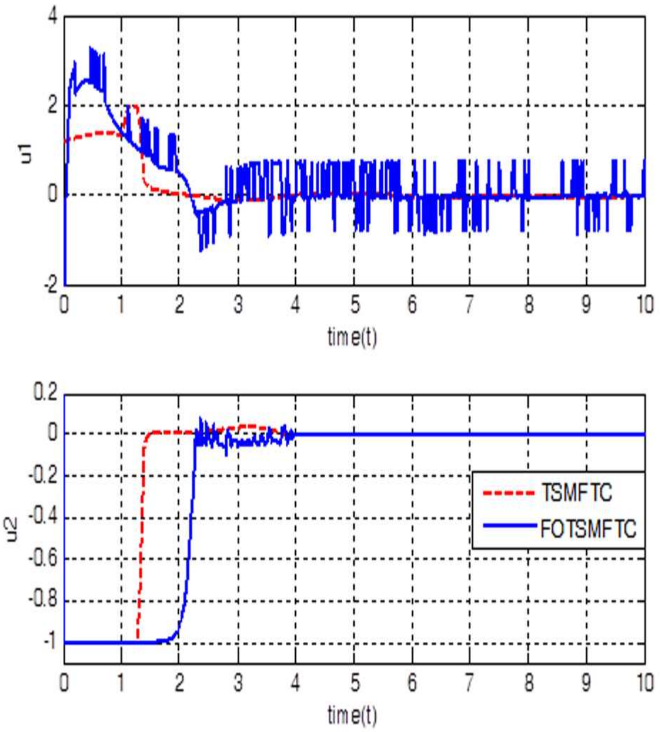

Fig.4: Comparison of control signals of the controllers u1, u2

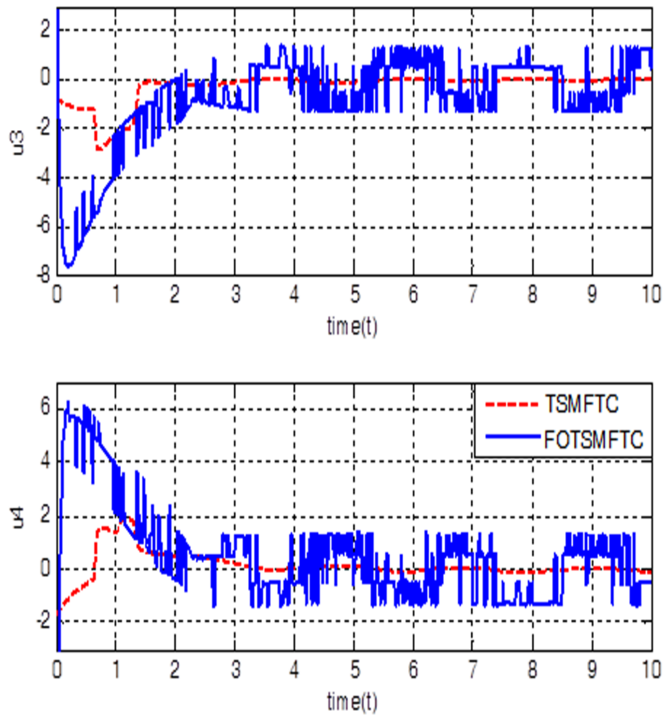

Fig.5: Comparison of control signals of the controllers u3, u4

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