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A Hybrid Model for Assessing Student Mathematical Skills

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Abstract. Student assessment is a very important process in education, because it helps the instructor to determine student mistakes and to improve their performance by reforming his/her teaching plans. A hybrid assessment method using qualitative grades for evaluating student mathematical skills is presented in this work. The paper starts with the mathematical background which is necessary for the understanding of its contents. This includes basic information about fuzzy, neutrosophic and soft sets, and about grey numbers. It also includes a description of the use of the center of gravity (COG) defuzzification technique for assessing a student group's quality performance. The COG technique is compared with the classical method of calculating the GPA index. The hybrid assessment method, which is based on all the previous concepts and processes, is developed next and the article closes with the final conclusion and some recommendations for future research.

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Keywords and Phrases: Fuzzy set (FS), Fuzzy logic (FL), Neutrosophic set (NS), Soft set (SS), Grey number (GN), COG defuzzification technique, Rectangular fuzzy assessment model (RFAM), GPA index.

1 Introduction

Student assessment is a very important process in Education, because it helps the instructor to determine student mistakes and to improve their performance by reforming his/her teaching plans. The assessment processes are realized by using either numerical or linguistic (qualitative) grades, like excellent, good, moderate, etc. Traditional assessment methods are used in the former case including the calculation of the *mean value* of the student numerical scores and the calculation of the *Grade Point Average (GPA) Index*, which is a weighted average of the student scores ([19], Chapter 6, p. 125). The first method evaluates the *mean performance* of a student group, whereas the second one evaluates its *quality performance*, where greater coefficients (weights) are assigned to the higher student scores. In many cases, however, the use of numerical scores is either not possible (e.g. in the case of approximate data) or not desirable (e.g. when more elasticity is required for the assessment). In such cases assessment methods are frequently used which are based on principles of fuzzy logic.

The present author developed in earlier works several assessment methods under fuzzy conditions, most of which are reviewed in [20]. More recently, he also introduced a new technique for assessment in a parametric manner using soft sets as tools [22]. It seems, however, that proper combinations of the previous methodologies could give better results; e.g. see [23, 24, 25, 26].

A hybrid assessment method for evaluating student mathematical skills using qualitative grades is presented in this work. The rest of the paper is organized as follows: Section 2 contains the mathematical background which is necessary for the understanding of the papers contents. This includes basic information

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about fuzzy, neutrosophic and soft sets, and about grey numbers. It also includes a description of the use of the *center of gravity (COG) defuzzification technique* for assessing a student group's quality performance. The COG technique is compared with the classical method of calculating the GPA index. The hybrid assessment method is developed in Section 3 and the article closes with the final conclusion and some recommendations for future research contained in its last Section 4.

2 Mathematical Background

2.1 Fuzzy Neutrosophic and Soft Sets

Zadeh [29] introduced in 1965 the concept of *fuzzy set (FS)* as follows:

Definition 2.1. *Let U be the universe, then a FS F in U is of the form*

$$F = \{(x, m(x)) \ : \ x \in U\}. \tag{1}$$

In equation (1), $m : U \to [0, 1]$ is the *membership function* of F and $m(x)$ is called the *membership degree* of x in F. The greater $m(x)$, the more x satisfies the property of F. A crisp subset F of U is a FS in U with a membership function such that $m(x) = 1$ if x belongs to F and 0 otherwise.

Zadeh developed with the help of the concept of FS the infinite-valued in the unit interval $[0, 1]$ *fuzzy logic (FL)* [30] for the purpose of dealing with partial truths. FL, in which truth values are modelled by numbers in the unit interval, is an extension of the classical *bivalent logic (BL)* embodying Lukasiewicz' Principle of Valence. According to this principle propositions are not only either true or false (Aristotle' principle of the Excluded Middle), but they can have intermediate truth-values too.

It was only in a second moment that FS theory and FL were used to embrace *uncertainty* modeling [5, 28]. This happened when membership functions were reinterpreted as possibility distributions. *Possibility theory* is an uncertainty theory devoted to the handling of incomplete information [6]. Zadeh [28] articulated the relationship between possibility and *probability*, noticing that what is probable must preliminarily be possible. For general facts on FSs and the connected to them uncertainty we refer to the book [8].

Following the introduction of FSs, various generalizations and others related to FSs theories have been proposed enabling, among others, more effective management of all types of existing real world uncertainty. A brief description of the main among those generalizations and theories can be found in [21].

Atanassov [1] added in 1986 to Zadeh's membership degree the *degree of non-membership* and introduced the concept of *intuitionistic fuzzy set (IFS)* as the set of the ordered triples

$$A = \{(x, m(x), n(x)) \ : \ x \in U, \quad 0 \le m(x) + n(x) \le 1\}. \tag{2}$$

Smarandache [14], motivated by the various neutral situations appearing in real life - like ¡friend, neutral, enemy¿, ¡positive, zero, negative¿, ¡small, medium, high¿, ¡male, transgender, female¿, ¡win, draw, defeat¿, etc. introduced in 1995 the degree of *indeterminacy/neutrality* of the elements of the universal set U in a subset of U and defined the concept of *neutrosophic set (NS)*. The term neutrosophic is the result of synthesis of the words neutral and sophia meaning in Greek wisdom. In this work we need only the simplest version of the concept of NS, which is defined as follows:

Definition 2.2. *A single valued NS $(SVNS)$ A in U is of the form*

$$A = \{(x, T(x), I(x), F(x)) \ : \ x \in U, \quad T(x), I(x), F(x) \in [0, 1], \quad 0 \le T(x) + I(x) + F(x) \le 3\}. \tag{3}$$

In (3) $T(x)$, $I(x)$, $F(x)$ are the degrees of truth (or membership), indeterminacy and falsity (or non-membership) of x in A respectively, called the neutrosophic components of x. For simplicity, we write $A < T, I, F >$.

For example, let U be the set of the players of a basketball team and let A be the $SVNS$ of the good players of U. Then each player x of U is characterized by a neutrosophic triplet (t, i, f) with respect to A, with t, i, f in $[0, 1]$. For instance, $x(0.7, 0.1, 0.4) \in A$means that there is a 70% belief that x is a good player, a 10% doubt about it and a 40% belief that x is not a good player. In particular, $x(0, 1, 0) \in A$ means that we do not know absolutely nothing about xs affiliation with A.

In an IFS the indeterminacy coincides by default to $1 - T(x) - F(x)$. Also, in a FS is $I(x) = 0$ and $F(x) = 1 - T(x)$, whereas in a crisp set is $T(x) = 1$ (or 0) and $F(x) = 0$ (or 1). In other words, crisp sets, FSs and IFSs are special cases of $SVNS$s.

When the sum $T(x) + I(x) + F(x)$ of the neutrosophic components of $x \in U$ in a $SVNS$ A on U is < 1, then it leaves room for incomplete information about x, when it is equal to 1 for complete information and when is greater than 1 for paraconsistent (i.e. contradiction tolerant) information about x. An $SVNS$ may contain simultaneously elements leaving room to all the previous types of information. For general facts on $SVNS$s we refer to [27].

The summation of neutrosophic triplets is equivalent to the neutrosophic union of sets. That is why the neutrosophic summation and implicitly its extension to neutrosophic scalar multiplication can be defined in many ways, equivalently to the known in the literature neutrosophic union operators [15]. Here, writing the elements of a $SVNS$ A in the form of neutrosophic triplets we define addition and scalar product in A as follows:

Let (t_1, i_1, f_1), (t_2, i_2, f_2) be in A and let k be an appositive number. Then

- The sum

$$(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1 + t_2, i_1 + i_2, f_1 + f_2). \tag{4}$$

- The scalar product

$$k(t_1, i_1, f_1) = (kt_1, ki_1, kf_1). \tag{5}$$

A disadvantage connected to the concept of FS is that there is not any exact rule for defining the membership function properly. The methods used for this are usually empirical or statistical and the definition of the membership function is not unique depending on the signals that each observer receives from the environment, which are different from person to person. For example, defining the FS of tall men one may consider as tall all men having heights more than 1.90 meters and another one all those having heights more than 2 meters. As a result, the first observer assigns membership degree 1 to men of heights between 1.90 and 2 meters, in contrast to the second one, who assigns membership degrees < 1. Consequently, analogous differences are logical to appear for all the other heights. The only restriction, therefore, for the definition of the membership function is to be compatible with common sense; otherwise the resulting FS does not give a reliable description of the corresponding real situation. This could happen for instance, if in the FS of tall men, men with heights less than 1.60 meters have membership degrees ≥ 0.5.

The same difficulty appears in all generalizations of FSs in which membership functions are involved (e.g. IFSs, NSs, etc.). For this reason, the concept of interval-valued FS ($IVFS$) [4] was introduced in 1975, in which the membership degrees are replaced by sub-intervals of the unit interval $[0, 1]$. Alternative to FS theories was also proposed, in which the definition of a membership function is either not necessary (grey systems/GNs [2]), or it is overpassed by considering a pair of sets that give the lower and the upper approximation of the original crisp set (rough sets [12]).

Molodstov [10], in order to tackle the uncertainty in a parametric manner, initiated 1999 the concept of soft set (SS) as follows:

Definition 2.3. *Let E be a set of parameters, let A be a subset of E, and let f be a map from A into the power set $P(U)$ of all subsets of the universe U. Then the SS (f, A) in U is defined to be the set of the ordered pairs*

$$(f, A) = \{(e, f(e)) : e \in A\}. \tag{6}$$

The term "soft" is due to the fact that the form of (f, A) depends on the parameters of A. For example, let $U = \{C_1, C_2, C_3\}$ be a set of cars and let $E = \{e_1, e_2, e_3\}$ be the set of the parameters $e_1 = cheap$, $e_2 = hybrid$ (petrol and electric power) and $e_3 = expensive$. Let us further assume that C_1, C_2 are cheap, C_3 is expensive and C_2, C_3 are hybrid cars. Then, a map $f : E \to P(U)$ is defined by $f(e_1) = \{C_1, C_2\}$, $f(e_2) = \{C_2, C_3\}$ and $f(e_3) = \{C_3\}$. Therefore, the$SS(f, E)$ in U is the set of the ordered pairs $(f, E) = \{(e_1, \{C_1, C_2\}), (e_2, \{C_2, C_3\}, (e_3, \{C_3\}\}$.

A FS in U with membership function $m(x)$ is a SS in U of the form $(f, [0, 1])$, where $f(\alpha) = \{x \in U : m(x) \geq \alpha\}$ is the corresponding $\alpha - cut$ of the FS, for each α in $[0, 1]$. For general facts on soft sets we refer to [9].

Obviously, an important advantage of SSs is that, by using the parameters, they pass through the existing difficulty of defining membership functions. We ought to note, however, that, despite the fact that IFSs and SSs have already found many and important applications, there exist reports in the literature disputing the significance of these concepts and considering them as redundant, representing in an unnecessarily complicated way standard fixed-basis set theory [7, 13]. In the Abstract of [13], for example, one reads: In particular, a soft set on X with a set E of parameters actually can be regarded as a $2E - fuzzy$ set or a crisp subset of $E \times X$ [the correct is $ExP(X)$]. This shows that the concept of (fuzzy) soft set is redundant. We completely disagree with this way of thinking. Adopting it, one could claim that, since a FS A in X is a subset of the Cartesian product $Xxm(X)$, where m is the membership function of A, the concept of FS is redundant!

2.2 Grey Numbers

Approximate data are frequently used nowadays in many problems of everyday life, science and engineering, because many constantly changing factors are usually involved in large and complex systems. Deng [2] introduced 1982 the grey system (GS) theory as an alternative to the theory of FSs for tackling such kind of data. A GS is understood to be a system that lacks information such as structure message, operation mechanism and/or behaviour document. The GS theory, which has been mainly developed in China, has recently found many important applications [3].

An interesting perspective of the closed intervals of real numbers is their use in the GS theory for handling approximate data. In fact, a numerical interval $I = [x, y]$, with x, y real numbers, $x < y$, can be considered as representing a real number with a known range, whose exact value is unknown. The closer x to y, the better I approximates the corresponding real number. When no other information is given about this number, it looks logical to consider as its representative approximation the real value

$$V(I) = \frac{x + y}{2}. \tag{7}$$

Moore et al. [11] introduced the basic arithmetic operations on real closed intervals. In the present work we shall make use only of the addition and scalar product defined as follows: Let $I_1 = [x_1, y_1]$ and $I_2 = [x_2, y_2]$ be closed intervals, then their sum $I_1 + I_2$ is the closed interval

$$I_1 + I_2 = [x_1 + x_2, y_1 + y_2]. \tag{8}$$

Further, if k is a positive number then the scalar product kI_1 is the closed interval

$$kI_1 = [kx_1, ky_1]. \tag{9}$$

When the real closed intervals are used for handling approximate data, they usually referred to as grey numbers (GNs). A GN $[x, y]$, however, may also be connected to a whitenization function $f : [x, y] \to [0, 1]$, such that, $\forall a \in [x, y]$, the closer$f(a)$ to 1, the better a approximates the unknown number represented by $[x, y]$.

We close this subsection with the following definition, which will be used in the assessment method that will be presented later in this work.

Definition 2.4. *Let $I_1, I_2, \ldots, I_n$ be a finite number of GNs, $n \geq 2$, then the mean value of these GNs is defined to be the GN*

$$I = \frac{1}{n}(I_1 + I_2 + \ldots + I_k). \tag{10}$$

2.3 Comparison of the Rectangular Fuzzy Assessment Model with the GPA Index

Voskoglou [17] developed a fuzzy model for mathematically representing the process of learning a subject matter in the classroom. Later, considering a student classified as a fuzzy system, he calculated the existing in it *total possibilistic uncertainty* for assessing the student mean performance [18].

Subbotin et al. [16], based on Voskoglou's model, properly adapted the *Center of Gravity (COG) defuzzification technique* for use as an assessment method of student learning skills. Since then, Subbotin and Voskoglou have applied jointly or separately the COG technique, termed as *Rectangular Fuzzy Assessment Model ($RFAM$)*, in many other types of assessment problems (e.g. see [19], Chapter 6). Here, we sketch the $RFAM$ and compare its outcomes with the GPA index.

For this, let G be a student group participating in a certain activity (e.g. problem-solving) and let $U = \{A, B, C, D, F\}$ be a set of linguistic labels characterizing the student performance with respect to this activity as follows: $A = excellent$, $B = very\ good$, $C = good$, $D = fair$ and $F = unsatisfactory$. We express G as a FS in U in the form $G = \{(x, \frac{n_x}{n}) \ : \ x \in U\}$, where n is the total number of students of G and n_x is the number of students of G whose performance was characterized by the qualitative grade x in U. In order to be able to sketch the graph of the membership function $y = m(x) = \frac{n_x}{n}$ of the FS G, we correspond to each x in U a real interval as follows: $F \to [0,1)$, $D \to [1,2)$, $C \to [2,3)$, $B \to [3,4)$, $A \to [4,5]$. Consequently, we have that $y_1 = m(x) = m(F) = \frac{n_F}{n}$ for all x in $[0,1)$, $y_2 = m(D) = m(x) = \frac{n_D}{n}$ for all x in $[1,2)$, $y_3 = m(C) = m(x) = \frac{n_C}{n}$ for all x in $[2,3)$, $y_4 = m(x) = m(B) = \frac{n_B}{n}$ for all x in $[3,4)$ and $y_5 = m(x) = m(A) = \frac{n_A}{n}$ for all x in $[4,5]$. Therefore,

$$\sum_{i=1}^{5} y_i = 1. \tag{11}$$

The graph of the membership function $y = m(x)$ of G takes now the form of Figure (1) and the area of the levels section S contained between this graph and the OX axis is equal to the sum of the areas of the rectangles S_i, $i = 1,2,3,4,5$ (wherefrom the term $RFAM$ was generated).

There is a commonly used in fuzzy logic approach to represent the fuzzy data with the coordinates (x_c, y_c) of the COG, say F_c, of the levels section S, which can be calculated by using the following well-known from Mechanics formulas:

$$x_c = \frac{\iint\limits_S x \,\mathrm{d}x\,\mathrm{d}y}{\iint\limits_S \mathrm{d}x\,\mathrm{d}y}, \quad y_c = \frac{\iint\limits_S y \,\mathrm{d}x\,\mathrm{d}y}{\iint\limits_S \mathrm{d}x\,\mathrm{d}y}. \tag{12}$$

In equations (12), $\iint\limits_S \mathrm{d}x\,\mathrm{d}y$ is the area of S which is equal to $\sum_{i=1}^5 y_i = 1$.

Also $\iint\limits_S x\,\mathrm{d}x\,\mathrm{d}y = \sum_{i=1}^5 \iint\limits_{S_i} x\,\mathrm{d}x\,\mathrm{d}y = \sum_{i=1}^5 \int_0^{y_i} \mathrm{d}y \int_{i-1}^{i} x\,\mathrm{d}x = \sum_{i=1}^5 y_i \int_{i-1}^{i} x\,\mathrm{d}x = \frac{1}{2}\sum_{i=1}^5 (2i-1)y_i$, and $\iint\limits_S y\,\mathrm{d}x\,\mathrm{d}y = \sum_{i=1}^5 \iint\limits_{S_i} y\,\mathrm{d}x\,\mathrm{d}y = \sum_{i=1}^5 \int_0^{y_i} y\,\mathrm{d}y \int_{i-1}^{i} \mathrm{d}x = \sum_{i=1}^5 \int_0^{y_i} y\,\mathrm{d}y = \frac{1}{2}\sum_{i=1}^5 y_i^2$. Therefore, equations

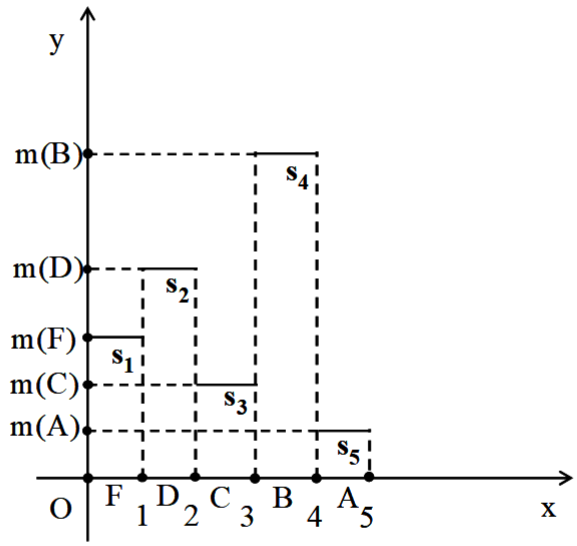

Figure 1: Graph of the membership function of the fuzzy set G

(12) give:

$$x_c = \frac{1}{2}(y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5), \quad y_c = \frac{1}{2}(y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2). \tag{13}$$

But $y_i^2 + y_j^2 \geq 2y_iy_j$, for all $i, j = 1, 2, 3, 4, 5$, with the equality holding if, and only if, $y_i = y_j$. Adding by members all the previous inequalities one finds that

$$6\sum_{i=1}^{5} y_i^2 \geq (\sum_{i=1}^{5} y_i)^2 + \sum_{i=1}^{5} y_i^2 \iff 5\sum_{i=1}^{5} y_i^2 \geq 1 \iff \sum_{i=1}^{5} y_i^2 \geq \frac{1}{5}, \tag{14}$$

with the equality holding if, and only if$y_1 = y_2 = y_3 = y_4 = y_5 = \frac{1}{5}$.

In case of equality the first of equations (12) gives that $x_c = \frac{5}{2}$. Further, combining the inequality (14) with the second of equations (13), one finds that $y_c \geq \frac{1}{10}$.

Therefore the unique minimum for y_c corresponds to the COG $F_m(\frac{5}{2}, \frac{1}{10})$.

The ideal case is when $y_1 = y_2 = y_3 = y_4 = 0$ and $y_5 = 1$. Then from equations (13) one gets that $x_c = \frac{9}{2}$ and $y_c = \frac{1}{2}$. Therefore the COG in this case is the point $F_I(\frac{9}{2}, \frac{1}{2})$. On the other hand, in the worst case $y_1 = 1$ and $y_2 = y_3 = y_4 = y_5 = 0$. Then, from equations (13) one gets that the COG is the point $F_x(\frac{1}{2}, \frac{1}{2})$. Therefore the area where the COG F_c lies is the triangle $F_xF_mF_I$ of Figure (2).

Figure (2) shows that the greater the value of x_c the better the groups performance. Also, for two groups with the same value of x_c, if $x_c \geq \frac{5}{2}$, then the group having the COG which is situated closer to F_I is the group with the greater value of y_c; on the contrary, if $x_c < \frac{5}{2}$, then the group having the COG which is situated farther to F_x is the group with the lower value of y_c. Based on the previous considerations, one obtains the following criterion for comparing the group performance:

- Between two groups, the group with the greater value of x_c performs better.

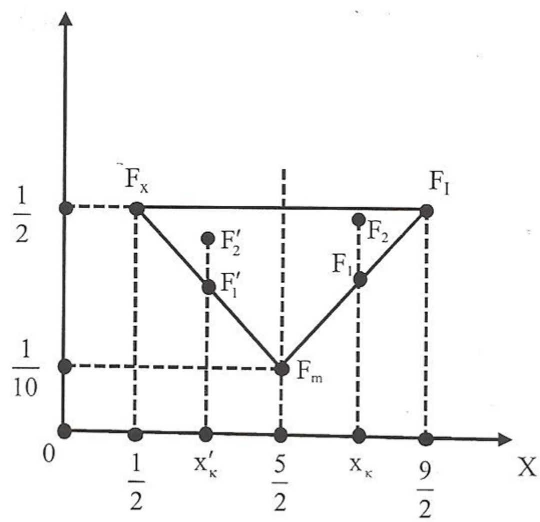

Figure 2: Graphical representation of the area where the COG lies

- Between two groups having the same value of x_c, if $x_c \geq \frac{5}{2}$, then the group with the higher value of y_c performs better, and if $x_c < \frac{5}{2}$, then the group with the lower value of y_c performs better.

As it becomes evident by the above criterion and the first of equations (13), the $RFAM$ model assigns greater coefficients to the higher student grades, i.e. it focuses on a student group *quality performance*.

Another classical method, very popular in USA and other western countries, for evaluating a student group quality performance is the calculation of the GPA index. Keeping the same notation, the corresponding formula is ([19], Chapter 6, p. 125)

$$GPA = \frac{0n_F + n_D + 2n_C + 3n_B + 4n_A}{n} = y_2 + 2y_3 + 3y_4 + 4y_5. \tag{15}$$

Combining equations (13) and (15) one finds that $x_c = \frac{1}{2}(2GPA + 1)$ or

$$x_c = GPA + \frac{1}{2}. \tag{16}$$

Thus, with the help of the first case of the previous criterion, one concludes that, if the GPA value of two student groups is different, then the $RFAM$ model and the GPA index give the same outcomes concerning the assessment of the qualitative performance of the two groups. If the GPA index is the same for the two groups, however, one $MUST$ apply the $RFAM$ model for comparing the qualitative performance of the two groups.

3 The Hybrid Assessment Method

Here we develop a hybrid method for assessing student mathematical skills with qualitative grades. More explicitly, we use SSs for a parametric evaluation of a student group performance, the GPA index and the $RFAM$ model for assessing the groups quality performance and GNs for estimating its mean performance. When the teacher is not sure about the grades assigned to some (or all) students of the group, however, then the use of NSs is more appropriate for estimating the student group's overall performance. We start with the following example:

Example 3.1. The students of two classes obtained the following grades in Mathematics: Class I: $A = 5$ students, $B = 3$, $C = 7$, $D = 0$, $F = 5$, Class II: $A = 4$, $B = 4$, $C = 7$, $D = 1$, $F = 4$.
i) Evaluate parametrically the two classes performance
ii) Which class demonstrated the better quality performance and which one the better mean performance?

Solution: i) Let $U = \{S_1, S_2, \ldots, S_{19}, S_{20}\}$ be the set of students of each class put in order according to their grades, starting from the higher grades. Consider $E = \{A, B, C, D, F\}$ as the set of parameters an let $f_i : E \to P(U)$, $i = I, II$ be the maps assigning to each parameter (grade) of E the subset of U consisting of all students of the corresponding class whose performance was assessed by this grade. Then the $SSs(f_I, E) = \{(A, \{S_1, S_2, \ldots, S_5\}), (B, \{S_6, S_7, S_8\}), (C, \{S_9, S_{10}, \ldots, S_{15}\}), (D, \varnothing), (F, \{S_{16}, S_{17}, \ldots, S_{20}\})\}$ and $(f_{II}, E) = \{(A, \{S_1, S_2, S_3, S_4\}), (B, \{S_5, S_6, S_7, S_8\}), (C, \{S_9, S_{10}, \ldots, S_{15}\}), (D, \{S_{16}\}), (F, \{S_{17}, S_{18}, S_{19}, S_{20}\})\}$ represent the parametric assessment of the performance of the classes I and II respectively.
ii) Equation (15) gives that $GPA_I = GPA_2 = \frac{43}{20}$. One, therefore, must use the $RFAM$ model for comparing the two classes quality performance. Thus, by the first of equations (13) one gets that $x_{c_I} = x_{c_{II}} = \frac{53}{20} > \frac{5}{2}$. But the second of equations (13) gives that $y_{c_I} = 54$ and $y_{c_{II}} = 49$, therefore, by the second case of the $RFAM$ assessment criterion, one concludes that Class I demonstrated better quality performance.

Further, observe that equation (15) gives in the ideal case ($n = n_A$) that $GPA = 4$, whereas in the worst case ($n = n_F$) gives that $GPA = 0$. Consequently, we have that $0 \leq GPA \leq 4$. Therefore, since $GPA_I = GPA_2 = \frac{43}{20} > 2$, both groups demonstrated satisfactory quality performance.

Since the student individual assessment was realized with qualitative grades, the two classs mean performance cannot be assessed by calculating the mean value of the student scores. To overcome this difficulty, using the numerical climax $1 - 100$ we assign to each of the student qualitative grades a closed real interval (GN), denoted for simplicity with the same letter, as follows: $A = [85, 100]$, $B = [75, 84]$, $C = [60, 74]$, $D = [50, 59]$ and $F = [0, 49]$. Then, under the light of equation (10), it is logical to accept that the $GN_s M_I = \frac{1}{20}(5A + 3B + 7C + 0D + 5F)$ and $M_{II} = \frac{1}{20}(4A + 4B + 7C + 1D + 4F)$ respectively can be used for estimating the two classes mean performance. Straightforward calculations with the help of equations (8) and (9) give that $M_I = \frac{1}{20}[1070, 1515] = [53.5, 75.75]$ and $M_{II} = \frac{1}{20}[1110, 1509] = [55.5, 75.45]$. Equation (7) gives, therefore, that $V(M_I) = 64.625$ and $V(M_{II}) = 64.75$. Thus, both classes demonstrated good (C) mean performance, with the mean performance of Class II being better.

It is of worth noting that, although the GNs assigned to the qualitative grades satisfy generally accepted assessment standards, the previous assignment is not unique, depending on the teachers personal goals. For a more strict assessment, for example, the teacher could choose $A = [90, 100]$, $B = [80, 89]$, $C = [70, 79]$, $D = [60, 69]$, $F = [0, 59]$, etc.

Frequently, however, the teacher has doubts about the grades assigned to some (or all) students. In such cases the use of NSs is more appropriate for estimating the student group overall performance. This process is illustrated in the following example:

Example 3.2. A teacher was not sure for the grades earned in a mathematics examination by some of the 35 in total students of a class, because their answers were not so clear. He considered, therefore, the NS of the good (at least) students of the class and characterized the individual performance of each student by a neutrosophic triplet as follows: Students $S_1 - S_{10}$ by $(1, 0, 0)$, $S_{11} - S_{12}$ by $(0.8, 0.2, 0.1)$, $S_{13} - S_{14}$ by $(0.7, 0.3, 0.2)$, $S_{15} - S_{17}$ by $(0.7, 0.2, 0.3)$, $S_{18} - S_{19}$ by $(0.5, 0.3, 0.6)$, $S_{20} - S_{25}$ by $(0.3, 0.5, 0.5)$, $S_{26} - S_{27}$ by $(0.2, 0.4, 0.6)$, $S_{28} - S_{30}$ by $(0.1, 0.5, 0.7)$, S_{31} by $(0.1, 0.3, 0.8)$ and $S_{32} - S_{35}$ by $(0, 0, 1)$. How he can estimate the overall performance of the class in this examination?

Solution: It is logical to accept that the groups overall performance can be estimated by the neutrosophic triplet $\frac{1}{35}[10(1,0,0)+2(0.8,0.2,0.1)+2(0.7,0.3,0.2)+3(0.7,0.2,0.3)+2(0.5,0.3,0.6)+6(0.3,0.5,0.5)+2(0.2,0.4,0.6)+3(0.1,0.5,0.7)+(0.1,0.3,0.8)+4(0,0,1)]$, which by equations (4) and (5) is equal to $\frac{1}{35}(18.7,7.8,13.8)=(0.534,0.223,0.394)$. This means that a random student of the group has a 53.4% probability to be at least a good student, but there exists also a 22.3% doubt about it and a 39.4% probability to be not a good student. The teacher could work in the same way by considering the NSs of excellent, very good, mediocre and weak students and obtaining analogous results.

4 Discussion and Conclusions

A hybrid assessment method was applied in this work for assessing student mathematical skills under fuzzy conditions (with qualitative grades). The discussion performed leads to the following conclusions:

• SSs can be used for realizing a parametric assessment of the student groups overall performance.

• The quality performance of a student group (where greater coefficients are assigned to the higher grades) can be measured either by the classical method of calculating the GPA index, or by applying the $RFAM$ model, which is based on the COG defuzzification technique. When two groups have the same GPA index, then the $RFAM$ model must be applied to find which group demonstrates better performance.

• In the case of using qualitative grades for assessing student performance, the assessment of a student groups mean performance cannot be realized by the classical way of calculating the mean value of the student's individual scores. The student mean performance in this case can be estimated by using GNs (real closed intervals).

• When the teacher has doubts about the grades assigned to some (or all) students, NSs can be used for assessing the overall performance of a student group.

Our experience from the present and earlier works implies that hybrid methods, like the previous one, usually give better and more complete results, not only in the assessment processes, but also in decision-making, in tackling the existing real world uncertainty and possibly in various other human or machine activities. This is, therefore, an interesting subject for further research.

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