Trajectory Optimization of Cable Parallel Manipulators in Point-to-Point Motion

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Abstract

Planning robot trajectory is a complex task that plays a significant role in design and application of robots in task space. The problem is formulated as a trajectory optimization problem which is fundamentally a constrained nonlinear optimization problem. Open-loop optimal control method is proposed as an approach for trajectory optimization of cable parallel manipulator for a given two-end-point task in point-to-point motion. Dynamic equations are organized in a closed form and are formulated in the state space form. A computational technique is developed for obtaining optimal trajectory to maximize dynamic load carrying capacity. By solving the corresponding nonlinear TPBVP, the problem of optimal path and maximum carrying for a 6 DOF spatial cable robot is studied. Finally, dynamic modelling in ADAMS is presented and to validate the optimal control method, optimal trajectory concerned with dynamic motion is compared with the software results.

Keyword: Optimization; Dynamic modeling; Cable robot.

1. Introduction

Parallel link manipulators are in general known for the simplicity of their mechanical design and their high strength and stiffness-to-weight ratios, because their actuators bear no moment loads but act in simple tension or compression. They are also known for their high force and moment capacity since their actuators act all in parallel. Such manipulators with solid adjustable length beams in the place of the cable ropes are fit used for the design of tyre test machines.

Cable-based parallel manipulators are structurally similar to traditional parallel manipulators but the former has some advantages over the latter. Large workspace, high payload-weight ratio, transportability and economical construction are the most important advantages of Cable-based parallel manipulators. These machines may be ideally suited for large scale manufacturing applications. These robots consist of a fixed base and a centrally-located end-effector, attached to a moving payload, which connects to cables whose tension is maintained along the tracked trajectory. One of the early works in Robocrane is

Developed by NIST in order to automate a crane for lifting operations (Albus et al. [1]).

The large payload capacity of Cable-based parallel manipulators allows their application to a large variety of tasks on lift systems. Dynamic Load Carrying Capacity (DLCC) of a manipulator is defined as the maximum payload that the manipulator can repeatedly carry in a defined trajectory.

Several studies have been done about the payload capacity of robots. Wang and Ravani [7] offered a method for determining the maximum load capacity of fixed base robots, and treated the problem as the optimization of trajectory. Korayem and Nikoobin [6] employed an indirect approach based on the open loop optimal control for obtaining the optimal trajectory of robot manipulators to maximize the load carrying capacity for a given point-to-point task. Recently, Korayem and Bamdad [4] determined the dynamic load carrying capacity of a typical cable suspended manipulator regarding tensile capacity of cables and actuators torque capacity for a given trajectory in a specified time, and Korayem et al. [5] have introduced a

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procedure for finding optimal path of maximum load for a cable planar robot with new constraints.

In this paper, dynamic equations are derived using combined Euler–Lagrange formulation and assumed modes method. To solve the optimal control problem, an indirect method via establishing the Hamiltonian function and deriving the optimality condition from Pontryagin's minimum principle is employed. The obtained equations provide a two-point boundary value problem which is solved by numerical techniques.

The optimal solutions with bang-bang controls are found by solving the corresponding nonlinear TPBVP (Two Point Boundary Value Problem). Open-loop optimal control approach is solved by direct and indirect approaches. However, direct method leads to the approximate solution and this approach is time consuming and quite ineffective due to the large number of parameters involved (Chettibi et al. [3]).

Since the problem is very sensitive to the unknown initial costates, a strategy is proposed and used successfully. This strategy leads to a bang-bang control in which the motors operate with the maximum torques changing directions at the switch time. The main advantage of this method is obtaining various optimal trajectories with different characteristics by changing the penalty matrices values which able the designer to choose the best trajectory. Finally, a 6DOF manipulator is simulated to illustrate the performance of the method. The results of open loop optimal control method for maximum payload are applied to ADAMS model and the dynamic response is compared.

2. Modeling of cable parallel robot

The force and/or torque which can be exerted along the various directions of motion under specified conditions of velocity and acceleration should be calculated. The load is a function of mass, moment of inertia, and static and dynamic forces supported by the robot. Below, the kinematics, dynamic modelling and a method for load carrying capacity calculation based on positive cable tensions are presented.

2.1. Kinematic modeling

A 6 DOF model of a cable-suspended with the coordinate system and geometric parameters is shown in Fig. 1 in which the end effector is connected to the base through 6 cables. These cables can extend or retract. It is particularly a cable manipulator based on a modification of the 6 degrees of freedom Gough–Stewart platform where the linear actuators have been replaced by cables (Alp and Agrawal [2]). In the design, the end effector platform can translate and rotate in the inertial frame. 3 transitional movements along the Cartesian coordinate and 3 rotational movements around the coordinated are supported.

The position/orientation vector of the end effector relative to the world coordinate system is denoted by six variables in $\mathbf{x} = [x, y, z, \theta_x, \theta_y, \theta_z]^T$ and the vector of cable lengths can be expressed as $\mathbf{q} = [l_1, l_2, ..., l_m]^T$ and cables' elongation can be obtained from end-effector movement by the aid of inverse kinematics:

$$\dot{L} = J \begin{bmatrix} \dot{X} \\ \omega \end{bmatrix} = J \left(\dot{x}_m, \dot{y}_m, \dot{z}_m, \dot{\psi}, \dot{\theta}, \dot{\phi} \right)^T \tag{1}$$

where $\dot{x}_m, \dot{y}_m, \dot{z}_m$ are the end-effector transitional velocities and $\dot{\psi}, \dot{\theta}, \dot{\phi}$ the angular velocities of the end-effector. Also, J is the Jacobian matrix that can be defined in this way:

$$J = \left[\frac{\partial L_i}{\partial X} \right] = \begin{bmatrix} \dfrac{\partial L_1}{\partial x} & \dfrac{\partial L_1}{\partial y} & \dfrac{\partial L_1}{\partial z} & \dfrac{\partial L_1}{\partial \psi} & \dfrac{\partial L_1}{\partial \theta} & \dfrac{\partial L_1}{\partial \varphi} \\[2mm] \dfrac{\partial L_2}{\partial x} & \dfrac{\partial L_2}{\partial y} & \dfrac{\partial L_2}{\partial z} & \dfrac{\partial L_2}{\partial \psi} & \dfrac{\partial L_2}{\partial \theta} & \dfrac{\partial L_2}{\partial \varphi} \\[2mm] \dfrac{\partial L_3}{\partial x} & \dfrac{\partial L_3}{\partial y} & \dfrac{\partial L_3}{\partial z} & \dfrac{\partial L_3}{\partial \psi} & \dfrac{\partial L_3}{\partial \theta} & \dfrac{\partial L_3}{\partial \varphi} \\[2mm] \dfrac{\partial L_4}{\partial x} & \dfrac{\partial L_4}{\partial y} & \dfrac{\partial L_4}{\partial z} & \dfrac{\partial L_4}{\partial \psi} & \dfrac{\partial L_4}{\partial \theta} & \dfrac{\partial L_4}{\partial \varphi} \\[2mm] \dfrac{\partial L_5}{\partial x} & \dfrac{\partial L_5}{\partial y} & \dfrac{\partial L_5}{\partial z} & \dfrac{\partial L_5}{\partial \psi} & \dfrac{\partial L_5}{\partial \theta} & \dfrac{\partial L_5}{\partial \varphi} \\[2mm] \dfrac{\partial L_6}{\partial x} & \dfrac{\partial L_6}{\partial y} & \dfrac{\partial L_6}{\partial z} & \dfrac{\partial L_6}{\partial \psi} & \dfrac{\partial L_6}{\partial \theta} & \dfrac{\partial L_6}{\partial \varphi} \end{bmatrix}$$

$$\tag{2}$$

2.2. Dynamic modeling

The dynamics of a robot is used to produce motions that extend the payload capability. Since dynamic modeling of cable robot is concerned with relating the motion end-effector to the required active actuator torque, the forces in the cables are derived using the dynamic equations of end-effector and actuators.
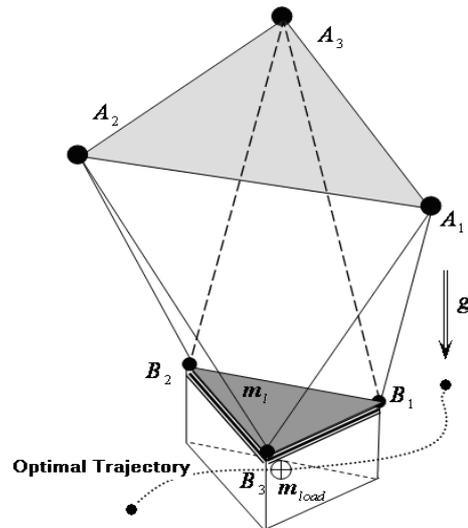

Fig. 1. A caption is positioned left-justified below the figure or scheme.

In the next stage, the actuator dynamics is applied. The combined dynamic effects of the motor, the cable pulley and the end-effector result in a manipulator dynamic modelling:

$$D(\mathbf{x})\ddot{\mathbf{x}} + C(\mathbf{x},\dot{\mathbf{x}}) + g(\mathbf{x}) = \frac{1}{r}\mathbf{J}^T(U_{vel} - U) \qquad (3)$$

$$U_{vel} = J_A\ddot{\beta} + C_A\dot{\beta}$$

where $D(\mathbf{x})$ is the inertia matrix, $C(\mathbf{x},\dot{\mathbf{x}})$ is the vector of velocity terms and $g(\mathbf{x})$ is the gravity vector J_A and C_A are diagonal matrices with rotational inertia and rotational viscous damping coefficients on the diagonal. The vector of pulley angles with pulley radii r is denoted by β.

3. Formulation of the optimal control problem

Let Ω be the set of the admissible control torques. The optimization problem is to find control $U(t) \in \Omega$ and payload m_p, so that the manipulator can carry maximum payload from an initial configuration to a final motion target in final time t_f. Therefore, the objective function that must be minimized is defined as

$$\underset{U(t),m_p}{\text{Minimize}} J_0 = -\frac{1}{t_f}\int_0^{t_f} m_p dt \qquad (4)$$

The optimal control problem is controlling all active joints so as to achieve the best dynamic coordination of joint motions while minimizing the actuating inputs together bound the velocities. The control forces are bounded as

$$U_i^- \leq U_i \leq U_i^+ \qquad (5)$$

The indirect methods are difficult to converge but easy to determine the optimality condition. Thus, necessary conditions for optimality are mentioned to find the optimal path for a specified payload and then maximum payload is obtained via an iterative algorithm in terms of the state variables. The performance index now looks like

$$\underset{U(t)}{\text{Minimize}} J_0 = \int_{t_0}^{t_f} L(X,U)dt \qquad (6)$$

where

$$L(X,U) = \frac{1}{2}\|X_1\|_{W_1}^2 + \frac{1}{2}\|X_2\|_{W_2}^2 + \frac{1}{2}\|U\|_R^2 \qquad (7)$$

Integrand L(.) is a smooth, differentiable function in the arguments, $\|X\|_K^2 = X^T K X$ is the generalized squared norm, W_1 and W_2 are symmetric, positive semi-definite (m×m) weighting matrices and R is symmetric, positive definite (m×m) matrices.

Pontryagin's minimum principle calls for a Hamiltonian state equation; a dynamic model is thoroughly derived using canonical variables and an optimal path is designed in order to achieve the predefined objective. By implementing Pontryagin's minimum principle for solving

optimization problems, the necessary conditions for optimality are obtained on the basis of variational calculus. The Hamiltonian function of the problem is determined and then PMP derives the optimality conditions and the Hamiltonian function is defined as:

$$H(X,U,Y,m_p,t) = Y^T f(X,U,t) + L(X,U,m_p,t). \qquad (8)$$

In addition, costate time vector-function Y(t) that verifies the costate vector-equation (or adjoint system) is obtained as:

$$\dot{Y}^T = -\partial H/\partial X. \qquad (9)$$

Moreover, the minimality condition for the Hamiltonian formulation is obtained by differentiating the Hamiltonian function with respect to control and costates as follows:

$$\begin{cases} \partial H/\partial U = 0 \\ \dot{X} = \partial H/\partial Y \end{cases} \qquad (10)$$

By defining U as a set of admissible control torque over the time interval, the imposed bound of torque for each motor can be expressed. The bounds on the control input, U^- and U^+ in the optimization problem limit the motor torques. The objective function specified by Eq. (18) and Eq. (19) is minimized over the entire duration of the motion. Finally, the optimization problem is completed by applying the boundary conditions:

$$X_1(0) = X_{10}, \; X_2(0) = X_{20} \qquad (11)$$

must reach the terminal conditions

$$X_1(t_f) = X_{1f}, \; X_2(t_f) = X_{2f} \qquad (12)$$

which represent the characteristics of states at initial and final times. The aforementioned equations lead to transforming the problem of optimal control into a nonlinear two-point boundary value problem. There are numerical techniques for solving such problems; e.g. available commands in some softwares such as MATLAB are good candidates for this issue.

4. The optimal trajectory constraint

In general, robotic manipulators are used at their limited capacities for obvious reasons of productivity. The tensionability is a necessary and sufficient condition for designing spatial cable-suspected robots . The dynamics of manipulator can be used to extend its payload capability while taking into account joint torque as realistic constraints. This leads, however, to quite significant joint torque and velocity magnitudes which can be harmful to the system. If the torque speed characteristic curves of the actuator are available, the load carrying capacity for any given end-effector motion trajectory can be determined.

The permanent magnet D.C. motors are commonly used for the actuators. The torque speed characteristic of such

D.C. motors may be represented by a linear equation and the bounds on the control input, so U^- and U^+ in (5) are

$$U^+ = K_1 - K_2 X_2 \qquad (13)$$

$$U^- = -K_1 - K_2 X_2$$

Where $K_1 = [\tau_{s1} \ \tau_{s2} \cdots \tau_{sm}]^T$, $K_2 = \text{dig}[\tau_{s1}/\omega_1 \cdots \tau_{sm}/\omega_m]$, τ_s is the stall torque and ω is the maximum no load speed of the motor (Korayem & Bamdad [4]).

5. The Algorithm for maximum payload calculation

The trajectory optimization includes dynamical constraints (controls, states), boundary constraints (conditions that the initial and final states must satisfy) and path constraints (conditions which must be satisfied at all points of the trajectory). The algorithm iterates on the initial values of the costate until the final boundary conditions are satisfied in the desired degree of accuracy in TPBVP solving:

$$\frac{1}{2}\|X_1(t_f) - X_{1f}\|_{W_p}^2 + \frac{1}{2}\|X_2(t_f) - X_{2f}\|_{W_v}^2 \leq \varepsilon \qquad (14)$$

The component of W_p and W_v can be changed to achieve the relative importance of position and velocity errors of end-effector during the trajectory. The final error obtained of (14) must be less than the desired accuracy ε.

The payload value is known and the solution of optimal control problem is obtained. The solution method is based on increasing the minimum value of payload until its maximum value is found. For a known payload, the obtained equations are in the standard form of TPBVP which bvp4c command in MATLAB is used to solve it. Desired accuracy ε in TPBVP solution for $m_p \leq m_{p\max}$ is achievable thus 22 is satisfied and payload increases in each step until the payload value becomes lager than its maximum value ($m_p > m_{p\max}$). At this condition (14) will not be satisfied, because for carrying the payload more than $m_{p\max}$, the torque more than their limits is required and on the other hand, the torque constraints are satisfied in TPBVP solution.

6. Simulation

In this section, a simulation study is presented to investigate the application and efficiency of the proposed algorithm. A typical cable suspended robot is considered for the simulation study. Its suspended movable platform and the overhead support are typically two equilateral triangles. The side lengths of base and movable platforms are 0.47 *m* and 0.14 *m* respectively. The problem is to find optimal path for moving between the point (-0.15, -0.25, 1.6) and point (0.15, 0.25, 1.9) in XYZ coordinate.All the linear and rotational velocities and rotation angles are supposed to be zero in these two boundary points. The overall time is t_f=1s for this trajectory.

All the other used parameters relative to the robot actuators consisting of motors and pulleys are presented in Table 1.

The dynamic model is completed for end-effector motion analysis by Eqs. (1-3). By applying the derived equations in the previous sections, $m_{p\max}$ is obtained. Desired accuracy in TPBVP solution is considered as ε=0.01, and the matrices are considered to be Wp=Wv=diag(1).

Table 1
Actuator parameters in simulation

Parameter	Value	Unit
Max. no load speed	330	RPM
Stall torque	2.84	N.m
Pulley radius	0.05	m
Pulley-motor rotational inertia	8×10^{-4}	Kg.m^2
Motor shaft viscous damping coefficient	0.02	N.m.s

6.1. Simulation Software

The software package used to develop, simulate, and analyze the dynamic models is ADAMS by MSC Software. It has a 3D environment for modeling mechanical problems and it uses its own solver to formulate and solve problems. Because of high capability of ADAMS/view software, it is used to simulate the six- cable robot in order to have a second opinion on the results from Matlab codes.

A model is developed by defining all parts of a system including masses and inertial properties, defining forces acting on or between these parts, and constraining the motions of the parts to each other (or totally). ADAMS develops the equations of motion of the system from the model and then solves the equations numerically in the time frame. The output of the program is the solution of the equations, from which any force acting through the system or motion of any part in the system can be obtained.

6.2. Verification of Optimal Trajectory for Cable robot

As mentioned before, using the indirect method, the optimization problem is converted to a two-point boundary value problem as by solving that, we can have a precise solution of problem. This method could be used for any kind of systems that state space form of equations is achievable. It is used as a successful tool for analyzing nonlinear systems and path planning of different types of systems. For capacity prediction, optimality conditions are obtained using the Pontryagin's minimum principle (PMP) which leads to the bang-bang control in a two-point boundary value problem (TPBVP) solution.

By increasing the payload from mp to m_{pmax}, the required torque increases and torque curves lay on their own limits, until the payload reaches to its maximum value, i.e. m_{pmax}=25.9 kg that is the maximum payload for the considered penalty matrices, while by choosing the other penalty matrices, the other optimal trajectories with different specifications can be obtained.

The tracking point is initially at the center of the triangular end-effector. The displacement, velocity, and acceleration between the two points can be used to compare their relative motions. Fig. 2 shows the optimal trajectory between the two given points including the time variation of six states. The relative motion is examined by comparing two points in space.

As always there must be a positive torque in the system of cable driven robots, the lower bound must be larger than zero.

Fig. 3 shows the required motor torques over a segment of the 2-second simulation to carry the maximum payload.

The upper and lower limits of motor torques are presented with dashed lines. As the figure indicates, the motor torques stay in a relatively narrow upper band.
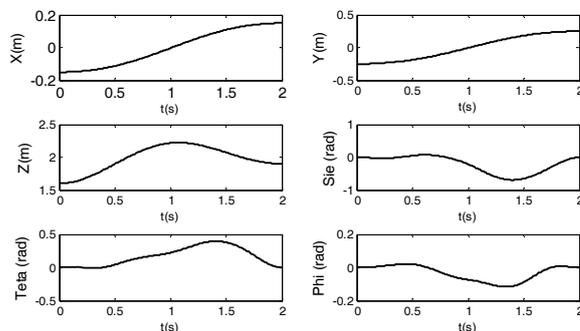

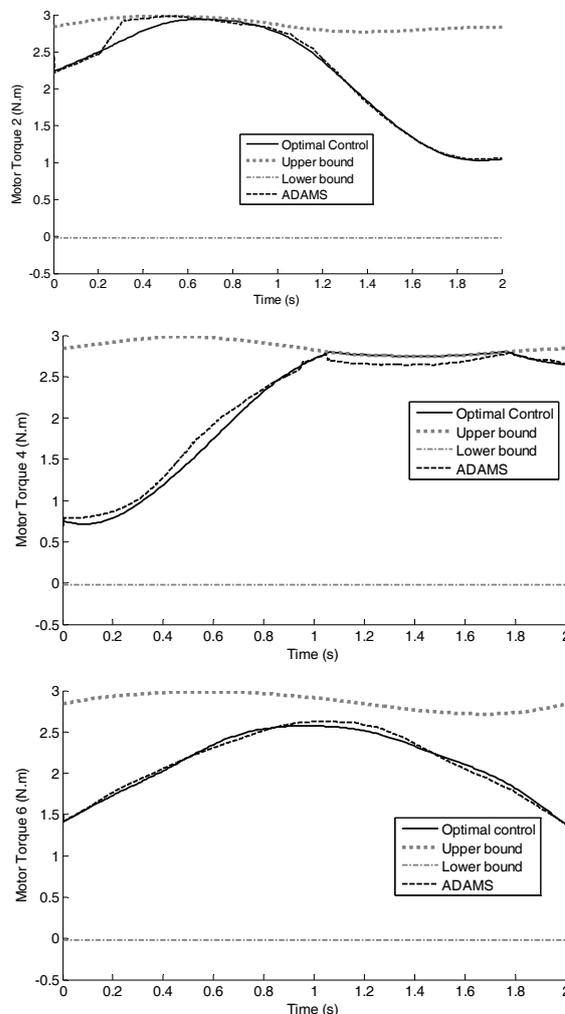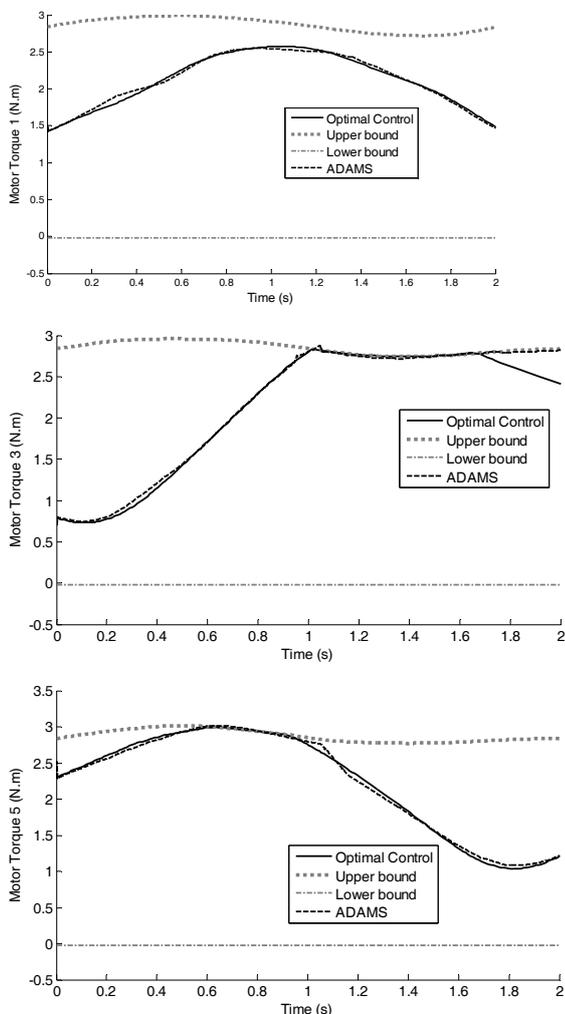Fig. 2. The optimal trajectory with payload 25.9 Kg.

Fig. 3. The motor torques to carry the maximum payload.

7. Conclusion

In the present paper, open loop optimal control method is used and the optimization problem is converted to TBVP as by solving that, a precise solution for maximum payload with considering the Kinematics and Dynamics of cable robot is obtained. The efficient optimal control scheme has the capacity to incorporate multiple criteria in the formulation, and the designer is allowed to add some terms to the objective function. The solving strategy makes it possible to get any possible objective functions for the optimality solution such as energy consumption, actuating torques, travelling time or bounding the velocity magnitude or maximization of payload. The procedure is capable to determining the states, costates, and the switching functions with a high numerical accuracy. This method could be used for any kind of systems that state space form of equations is achievable. It is used as a successful tool for analyzing nonlinear systems and path planning of different types of systems.

Acknowledgments

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